Determination of Projection Coefficients for Spatial Signal Ranging Errors in LEO Constellations (Post-Print)

Authors: Tian Qiuning[1], Cao Yueling[2], Yang Jianhua[2,3], Liu Jinhuo[2,3], Xiang Yu , Yang Zuo , Lü Hao , Peng Hanbing

Date: 2023-06-07T00:00:00+00:00

Abstract

With the continuous advancement of low Earth orbit (LEO) constellation deployment, the calculation of Signal In Space Range Error (SISRE) is no longer targeted exclusively at ground users of Global Navigation Satellite System (GNSS), but also extends to LEO spaceborne users of GNSS and ground users of LEO navigation systems. To better support SISRE calculations for LEO constellations, this study investigates the characteristics of SISRE error projection coefficients for LEO spaceborne users and LEO satellite ground users based on the principles of SISRE computation. The results demonstrate that the error projection coefficients of GNSS satellites for ground users are not applicable to LEO spaceborne users or ground users of LEO navigation constellations. When LEO satellite altitude decreases from 2,000 km to 300 km, the orbit radial error projection coefficient of GNSS satellites for LEO spaceborne receivers increases from 0.96 to 0.98, while the orbit tangential/normal plane error projection coefficient decreases from 0.20 to 0.15. For LEO satellites, the orbit radial error projection coefficient for ground users decreases from 0.72 to 0.37, while the orbit tangential/normal plane error projection coefficient increases from 0.49 to 0.66. These findings provide important references for future LEO satellite-related space signal ranging error analysis and LEO integrity research.

Full Text

Determination of Projection Coefficients for SISRE of LEO Constellations

Tian Qiuning[1], Cao Yueling[2], Yang Jianhua[2,3], Liu Jinhua[2,3], Xiang Yu[4], Yang Zuo[4], Lü Hao[4], Peng Hanbing[4]

[1]Beijing Research Institute of Telemetry, Beijing 100076, China
[2]Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China
[3]University of Chinese Academy of Sciences, Beijing 100049, China
[4]Beijing Satellite Navigation Center, Beijing 100094, China

Abstract

With the continuous advancement of Low Earth Orbit (LEO) constellation construction, the calculation of Signal-In-Space Range Error (SISRE) is no longer limited to ground users of Global Navigation Satellite Systems (GNSS), but now also includes LEO spaceborne users of GNSS and ground users of LEO navigation systems. To better support SISRE calculations for LEO constellations, this study investigates the characteristics of SISRE error projection coefficients for both LEO spaceborne users and LEO satellite ground users based on the fundamental principles of SISRE computation. The results demonstrate that the error projection coefficients derived for GNSS ground users are not applicable to LEO spaceborne users or LEO navigation constellation ground users. As the orbital altitude of LEO satellites decreases from 2,000 km to 300 km, the radial orbit error projection coefficient for GNSS satellites relative to LEO spaceborne receivers increases from 0.96 to 0.98, while the tangential/normal plane error projection coefficient decreases from 0.20 to 0.15. For LEO satellites relative to ground users, the radial orbit error projection coefficient decreases from 0.72 to 0.37, while the tangential/normal plane error projection coefficient increases from 0.49 to 0.66. These findings provide important references for future SISRE analysis related to LEO satellites and LEO integrity research.

Keywords: GNSS; LEO constellation; Signal-In-Space Range Error; error projection coefficient

1 Introduction

GNSS service accuracy is a critical metric for evaluating satellite navigation system performance and represents a core competency of GNSS systems. Service accuracy can be calculated through User Equivalent Range Error (UERE) and Dilution of Precision (DOP), where DOP is determined by satellite constellation geometry. UERE comprises satellite navigation message errors, atmospheric delay model correction errors, and receiver measurement errors projected onto the ranging direction. Receiver measurement errors primarily include ranging noise and multipath errors, reflecting GNSS receiver equipment performance. Atmospheric delay model correction errors consist mainly of ionospheric and tropospheric delay model correction errors. Satellite navigation message errors include broadcast ephemeris errors and broadcast clock parameter errors, which are reflected through SISRE.

SISRE has been a focal point in GNSS research, describing the average projection of satellite broadcast ephemeris errors and clock parameter errors onto the line-of-sight direction for users within the service area. SISRE can be calculated by determining satellite broadcast ephemeris errors and clock parameter errors using broadcast navigation messages and precise products, then applying projection coefficients. Current SISRE values for major systems are 0.14 m for Galileo, 2.59 m for GLONASS, and 0.35 m for BDS-3. For GPS, SISRE values are 0.35 m and 1.27 m depending on the onboard atomic clock type. Comprehensive assessments of radial and tangential/normal plane broadcast ephemeris errors, clock parameter errors, and integrated SISRE for various GNSS systems are presented in Table 1 .

Space signal ranging error projection coefficients, including radial orbit error projection coefficients and tangential/normal plane error projection coefficients, are essential parameters for calculating SISRE. Montenbruck et al. provided SISRE projection coefficients for GPS, GLONASS, Galileo, and BDS systems. These coefficient calculations relate to satellite orbital altitude, the beam angle of the satellite's downlink L-band antenna, and the average altitude of users. GNSS system interface control documents provide projection coefficients for calculating GNSS SISRE, assuming users on the geoid surface.

With advancing LEO satellite technologies, LEO navigation augmentation systems have become a crucial direction for next-generation satellite navigation development. Europe's Kepler program demonstrates that next-generation navigation constellations will consist of mixed medium and low Earth orbit satellites. China also plans to enhance GNSS service capabilities using LEO satellites, developing constellations such as "Hongyan" and "Weili-1." SpaceX's Starlink program aims to launch tens of thousands of LEO satellites to provide communication and navigation services.

Traditional SISRE projection coefficients for ground users cannot meet the requirements of LEO-related applications. Therefore, there is an urgent need to develop a set of space signal ranging error projection coefficients adapted for LEO satellite navigation users and for LEO spaceborne receivers calculating GNSS SISRE. Currently, most LEO navigation constellations remain in the demonstration phase, and research on SISRE error projection coefficient calculation for LEO satellites themselves has not been fully conducted. This paper investigates SISRE error projection coefficients for GNSS satellites relative to LEO spaceborne users and for LEO satellites of different orbital altitudes relative to ground users, providing important support for LEO satellite development.

This paper first derives the calculation method for space signal ranging error and error projection coefficients and validates the algorithm's effectiveness. Based on this calculation method, we investigate GNSS satellite SISRE projection coefficients for LEO users and LEO satellite SISRE projection coefficients for ground users.

2 Ground Users' GNSS Satellite Space Signal Error Projection Coefficients

The concept of Signal-In-Space Range Error dates back to early GPS programs, based on the geographic averaging of satellite-side errors. Assuming users are uniformly distributed over a surface area S, the instantaneous average projection of satellite-side errors onto the user line-of-sight direction within that region can be expressed as:

SISRE (cid:0) Errds

where Err represents the projection of broadcast ephemeris error and clock parameter error in the satellite-to-user direction. Correspondingly, the root mean square error of SISRE, RM S(SISRE), can be expressed as:

RM S(SISRE) (cid:0) (cid:217) (cid:219)(cid:219)(cid:219)(cid:219)(cid:219) Err2ds

Satellite-side errors consist of satellite clock parameter errors and satellite orbit errors. Defining the satellite orbit error vector as E, clock parameter error as T, and the unit vector from satellite to user as l, and assuming Earth is approximately spherical with users uniformly distributed on Earth's surface S, the projection relationship of satellite-side errors in the orbital reference frame is shown in Figure 1 [Figure 1: see original paper]. RM S(SISRE) can then be further expressed as:

RM S(SISRE) (cid:0) (cid:0) (cid:217) (cid:219)(cid:219)(cid:219)(cid:219)(cid:219)(cid:219)(cid:219) (cid:219)(cid:219)(cid:219)(cid:219)(cid:219) (cid:217) 2(cid:25) (cid:25) (cid:14)(cid:18)max E sin (cid:11)d(cid:11)d(cid:12) (l (cid:0) E (cid:14) cT)2r2 (cid:14)(cid:18)max (cid:25) 2(cid:25) (cid:14)(cid:18)max (l (cid:0) E (cid:14) cT)2 sin (cid:11)d(cid:11)d(cid:12) E sin (cid:11)d(cid:11)d(cid:12) 2(cid:25) (cid:25) 2(cid:25)(1 (cid:14) sin (cid:18)max)

In Figure 1, O represents Earth's center, axis X points opposite to the satellite's along-track direction, axis Y points to the satellite's normal direction, and axis Z points opposite to the satellite's radial direction. The angle (cid:11) is the Earth-centered angle between satellite and user, (cid:12) is the angle between the user's projection onto the XOY plane and the X axis, RS is the satellite's geocentric distance, (cid:18) is the angle with the satellite as vertex between user and Earth's center, and (cid:18)max is determined when the satellite line-of-sight is tangent to Earth.

In the orbital reference frame, E can be decomposed into radial orbit error, along-track orbit error, and normal orbit error:

E (cid:0) ((cid:14)eC; eA; (cid:14)eR)

where eR, eA, eC are the three basis vectors in radial, along-track, and normal directions in the orbital reference frame. Based on the geometric relationship in Figure 1, l in the orbital reference frame can be calculated as:

l (cid:0) SD (cid:0) (rE sin (cid:11) cos (cid:12); rE sin (cid:11) sin (cid:12); rE cos (cid:11) (cid:14) RS) (cid:17) R2 (cid:14) 2rERS cos (cid:11)

Substituting equations (4) and (5) into equation (3) yields:

RM S(SISRE) (cid:0) (cid:14) 2!Rc2T eR (cid:17) c2T 2 (cid:17) !2

where !R and !AC are the radial orbit error projection coefficient and tangential/normal plane error projection coefficient, respectively, calculated as:

!R (cid:0) (cid:14)(cid:18)max rE cos (cid:11)(cid:14)RS (cid:17)R2 1 (cid:14) sin (cid:18)max (cid:14)2rERS cos (cid:11) sin (cid:11)d(cid:11)

!AC (cid:0) rE sin (cid:11) (cid:14)2rERS cos (cid:11) 2(1 (cid:14) sin (cid:18)max)

In traditional GNSS space signal ranging error projection coefficient calculations, users are typically assumed to be at rE (cid:0) 6 371 km. Knowing the GNSS satellite altitude, the space signal ranging error projection coefficients for different altitudes can be calculated using the above formulas, with results shown in Table 2 . Here, !R1 and !AC1 are the radial and tangential/normal plane error projection coefficients obtained using equations (7) and (8), while !R2 and !AC2 are the coefficients proposed by Montenbruck et al.

The GNSS space signal ranging error projection parameters calculated through this method are essentially consistent with those of Montenbruck et al., thereby validating the algorithm's feasibility. Table 2 shows that GNSS satellite radial orbit error projection coefficients are close to 1, while tangential/normal plane direction error projection coefficients are less than 0.2, indicating that GNSS radial orbit errors have a significantly greater impact on ground user space signal ranging errors than tangential/normal plane errors.

---

3 Ground Users' LEO Satellite Space Signal Error Projection Coefficients

To facilitate ground user analysis of space signal ranging error characteristics for different LEO navigation constellations, it is necessary to calculate the space signal ranging error projection coefficients for LEO satellites of different orbital altitudes relative to ground users. Substituting different LEO satellite orbital design altitudes into equations (7) and (8) yields the radial and tangential/normal plane ranging error projection coefficients for LEO satellites relative to ground users.

Using LEO satellite orbital altitude as a variable, the corresponding SISRE projection coefficients for different orbital altitudes are shown in Figure 2 [Figure 2: see original paper]. The blue and orange lines represent the radial and tangential/normal plane error projection coefficients for LEO satellites of different altitudes relative to ground users. The figure clearly shows that SISRE error projection coefficients vary significantly with LEO satellite orbital altitude. As LEO satellite orbital altitude increases gradually from 300 km to 2,000 km, the radial error projection coefficient for ground users increases from 0.374 to 0.7164, while the tangential/normal plane error projection coefficient decreases from 0.656 to 0.493. This indicates that as LEO satellite orbital altitude increases, radial orbit errors have an increasingly greater impact on SISRE, while tangential/normal plane errors have a decreasing impact.

Comparison with the SISRE error projection coefficients for GNSS satellites relative to ground users in Section 2 reveals that GNSS satellite coefficients are not applicable to LEO navigation satellites relative to ground users, as this would introduce risks in ground user navigation performance assessment. For ground users, tangential/normal direction errors of LEO satellites have a greater impact on positioning errors compared to traditional GNSS satellites.

We selected four common LEO constellation orbital altitudes to calculate SISRE error projection coefficients for LEO satellites relative to ground users, including maximum satellite zenith angle (cid:18)max, radial orbit error projection coefficient !R, and tangential/normal plane error projection coefficient !AC. Table 3 presents these space signal ranging error projection coefficients for ground users relative to LEO satellites at specific altitudes.

Figure 2 and Table 3 show that when LEO satellite orbital altitude is 970 km, !R and !AC are essentially equal. When satellite altitude exceeds 970 km, !R is greater than !AC, meaning radial orbit errors contribute more to space signal ranging errors. When satellite altitude is below 970 km, !R is less than !AC, indicating that tangential/normal plane orbit errors contribute more to space signal ranging errors.

---

4 LEO Spaceborne Receiver's GNSS Satellite Space Signal Error Projection Coefficients

In addition to serving ground users, GNSS satellites provide navigation services to numerous LEO satellites. Therefore, to assess GNSS navigation and positioning performance for LEO satellites, it is necessary to calculate GNSS satellite space signal error projection coefficients for LEO spaceborne receivers.

Assuming LEO spaceborne receiver users are distributed in near-circular orbits of 300–2,000 km altitude, the calculations of various physical quantities are no longer based on rE (cid:0) 6 371 km, but on the average LEO satellite altitude rref. In equations (7) and (8), rE is replaced by the LEO satellite orbital altitude rref, and (cid:18)max also varies with LEO satellite altitude. Figure 3 [Figure 3: see original paper] shows the GNSS satellite space signal ranging error projection coefficients for LEO spaceborne receiver users at different orbital altitudes, with Figure 3a presenting radial error projection coefficients and Figure 3b showing tangential/normal plane error projection coefficients. The blue, red, and yellow lines represent error projection coefficients for GPS, Galileo, and GLONASS satellites relative to LEO spaceborne receivers at different altitudes, the purple line represents coefficients for BDS MEO satellites, and the green line represents coefficients for BDS GEO/IGSO satellites.

Figure 3 indicates that as LEO satellite orbital altitude increases, the satellite orbit radial projection error coefficient decreases while the tangential/normal plane error projection coefficient increases. When LEO satellite orbital altitude increases from 300 km to 2,000 km, the tangential/normal plane error projection coefficient !AC for GNSS MEO satellites can increase from 0.15 to 0.20. Compared to ground users, tangential/normal plane direction errors of GNSS MEO satellites have a more significant impact on LEO spaceborne receiver users.

Using "Weili-1" and "Hongyan" LEO spaceborne receivers as examples, we calculated GNSS satellite space signal ranging error projection coefficients for LEO spaceborne receivers at orbital altitudes of 970 km and 1,100 km, as shown in Table 4 and Table 5 .

Tables 4 and 5 reveal that compared to MEO satellites, GEO and IGSO satellites have greater radial orbit error impact and smaller tangential/normal plane error impact on LEO spaceborne receiver users' SISRE. For MEO satellites of the four major GNSS systems, the differences between radial and tangential/normal plane error projection coefficients are relatively small. Among them, Galileo satellites have relatively smaller tangential/normal plane error impact and relatively larger radial orbit error impact on SISRE.

---

5 Conclusions

Based on the principles of satellite space signal ranging error, this paper calculated GNSS satellite space signal error projection coefficients for ground and LEO users, as well as LEO satellite space signal error projection coefficients for ground users at different orbital altitudes. The results are summarized as follows.

The radial orbit error projection coefficient of LEO satellites for ground users is smaller than that of traditional GNSS satellites for ground users, while the tangential/normal plane error projection coefficient of LEO satellites for ground users is larger than that of traditional GNSS satellites. This indicates that ground users have higher accuracy requirements for LEO satellite tangential/normal plane orbit precision.

As LEO satellite orbital altitude increases from 300 km to 2,000 km, the radial orbit error projection coefficient for GNSS satellites relative to LEO spaceborne receiver users decreases from 0.98 to 0.96, while the tangential/normal plane error projection coefficient increases from 0.15 to 0.20. The variation in tangential/normal plane error projection coefficient is greater than that of the radial projection coefficient. Combined with Table 1, Galileo satellites have slightly larger radial orbit errors than BDS-3 satellites but significantly smaller tangential/normal plane orbit errors than other GNSS satellites. This suggests that Galileo may have advantages in future LEO applications and that higher tangential/normal plane orbit accuracy will be required as integrated LEO-GNSS applications develop.

This research provides support for LEO satellite-related space signal ranging error analysis and LEO integrity studies.

References

[1] Department O. GPS and its Augmentation Systems, 2008, 35(2): 197
[2] Montenbruck O, Steigenberger P, Hauschild A. Advances in Space Research, 2018, 61(12): 3020
[3] Warren D L, Raquet J F. GPS Solutions, 2003, 7(3): 151
[4] Montenbruck O, Steigenberger P, Hauschild A. Gps Solutions, 2015, 19(2): 321
[5] Yang J H, Song Y Z, Chang Z Q, et al. SCIENTIA SINICA Physica, Mechanica & Astronomica, 2021, 51(1): 9508
[6] Montenbruck O, Steigenberger P, Hauschild A. 2020 IEEE/ION PLANS, 2020: 407
[7] 贾小林, 陶清瑞, 王利军, 等. 测绘科学, 2021, 46(1): 62
[8] Chen J P, Hu X G, Tang C P, et al. Science China: Physics, Mechanics and Astronomy, 2020, 63(6): 105
[9] 陶清瑞, 贾小林, 王利军, 等. 测绘工程, 2021, 30(1): 24
[10] Heng L, Gao G X, Walter T, Enge P. Proceedings of ION ITM, San Diego: ION, 2011: 312
[11] Chen L, Jiao W, Huang X, et al. CSNC, 2013, 243: 423
[12] Wang K, El-Mowafy A, Rizos C. GPS Solutions, 2021, 26(1): 1
[13] Mistrapau F, Bija B, Cueto-Felgueroso G, et al. Portland, Oregon: ION, 2016: 1793
[14] Hauschild A, Montenbruck O. Navigation, 2021, 68(2): 419
[15] Zhao X L, Zhou S S, Ying C, et al. GPS Solutions, 2020, 24(4): 102
[16] 杨文翰, 花国良, 冯岩, 等. 天地一体化信息网络, 2021, 2(1): 9
[17] Glaser S, Michalak G, Mnnel B, et al. Journal of Geodesy, 2020, 94(12): 1

Note: Figure translations are in progress. See original paper for figures.