Advances in Dark Matter Halo Boundary Research (Postprint)

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Abstract

This section addresses the issue of boundary definition for dark matter halos. The conventional spherical overdensity algorithm, which defines a dark matter halo as a spherical high-density region exceeding a certain density threshold, suffers from the problem of "pseudo-evolution". Early halo-finding algorithms also include the widely utilized "friends-of-friends" algorithm that connects neighboring particles. Additionally, various alternative approaches have been explored, such as the halo static radius defined according to the farthest position in the static interval, the splashback radius proposed in recent years based on the characteristic steep decline of the density profile in the outer regions, and the characteristic depletion radius and inner depletion radius introduced based on the dynamical features of halos—these definitions are capable of reflecting the dynamic nature of dark matter halos. The discussion also introduces two computational programs for determining the splashback radius along with observational support, as well as the detection of the inner depletion radius for the Milky Way. Furthermore, the merger tree and growth process of dark matter halos are described. Finally, a summary and future prospects for these various boundary definitions are presented.

Full Text

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The Recent Progress of the Boundaries of Dark Matter Haloes

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Abstract

This paper reviews the longstanding problem of defining boundaries for dark matter haloes. The traditional spherical overdensity algorithm defines a halo as a spherical region exceeding a certain density threshold, but this approach suffers from the "pseudo-evolution" problem. Early halo-finding methods also include the widely used Friends-of-Friends algorithm that links neighboring particles. Various alternative approaches have been explored, such as the static radius defined by the outermost extent of the static region, the splashback radius proposed in recent years based on the steep decline of the density profile in outer regions, and the characteristic and inner depletion radii defined according to the dynamical properties of haloes. These definitions better reflect the dynamic nature of dark matter haloes. We also introduce two algorithms for calculating the splashback radius and discuss observational support for it, as well as measurements of the inner depletion radius for the Milky Way. The merger tree and growth process of dark matter haloes are also discussed. Finally, we summarize the various boundary definitions and provide an outlook for future research.

Keywords: dark matter halo; boundary; overdensity; splashback radius; depletion radius
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1 Introduction

According to our current understanding of large-scale structure formation, the universe consists of approximately 27% dark matter, 5% baryonic matter, and the remaining 68% dark energy [**?**]. Cold dark matter undergoes hierarchical collapse and cooling to form dark matter haloes (hereafter "haloes"), which serve as the fundamental building blocks of the universe and host galaxies at the centers of their gravitational potential wells [**?**, **?**].

Our understanding of the basic properties of dark matter haloes remains incomplete, and our knowledge of their size—a crucial aspect—is still in its infancy. Initially, inspired by the spherical collapse model [**?**], researchers proposed the spherical overdensity (SO) algorithm [**?**, **?**], which defines a halo as a spherical region whose density exceeds a certain threshold. However, this density-based definition fails to capture the true physical nature of haloes and leads to what is now known as "pseudo-evolution" [?, ?, ?, ?, ?], where a halo' s mass and radius grow substantially according to the definition while no significant physical processes occur internally. In addition to SO, traditional halo definitions include the famous Friends-of-Friends (FoF) algorithm [?, ?], and both represent early methods for halo identification.

Subsequently, numerous explorations have moved beyond simple density considerations to adopt more dynamical perspectives. Cuesta et al. (2008) [?] introduced the concept of static radius, which reflects the dynamical characteristics of haloes and offers certain advantages. In recent years, the concept of the splashback radius ($R_{\rm sp}$) has been proposed [?, ?, ?, ?, ?], initially discovered by Diemer and Kravtsov [?] who found that the slope of the halo density profile drops sharply in the outer regions. This radius corresponds to the apocenter of dark matter particles' first orbit after falling into the halo, where recently accreted material exhibits splashback behavior—ejected and then returning like a splash guard. This radius provides a clear distinction between the inner and outer regions of a halo and is considered a very natural and physical definition that has attracted considerable attention. Observational support has also emerged: More et al. [?] were among the first to detect splashback features, identifying a sharp halo edge in the projected number density profile of photometric galaxies around galaxy clusters in the Sloan Digital Sky Survey (SDSS) [?], corresponding to the first apocenter of satellite galaxies after infall. Additionally, Chang et al. (2018) [?] used data from the Dark Energy Survey (DES) [?] to measure steep, splashback-like features in galaxy density profiles. Bianconi et al. (2021) [?] utilized data from the Local Cluster Substructure Survey (LoCuSS) [?] to directly probe splashback features in galaxy clusters and study how their dynamical state affects these features.

More recently, Fong and Han [?] introduced the concept of the depletion radius, comprising the inner depletion radius ($R_{\rm id}$) and characteristic depletion radius ($R_{\rm cd}$), defined respectively as the location of maximum mass inflow rate and the location where the bias profile reaches its minimum. The inner depletion radius separates growing haloes from their surroundings, which are being "depleted" by accretion, and the region within it has roughly constant density. Subsequently, Li and Han (2021) [?] used radial motions of nearby dwarf galaxies to measure, for the first time, the inner depletion radius and turnaround radius ($R_{\rm ta}$) [?, ?] of the Milky Way's dark matter halo, revealing the Galaxy's unique evolutionary history and dynamical state.

Section 2 introduces several methods for defining halo boundaries, focusing on the traditional spherical overdensity algorithm and its problems, with particular emphasis on the discovery of the splashback radius, algorithms for calculating it, and the introduction of depletion radii. Section 3 discusses observational applications and measurements of halo boundaries. Section 4 describes halo merger trees and their growth processes. Section 5 provides a summary and discussion of various boundary definitions.

2 Several Definitions of Dark Matter Halo Boundaries

This chapter introduces the main definitions of halo boundaries: the traditional spherical overdensity (SO) and FoF algorithms, and later proposals including the static radius, splashback radius, and depletion radii. Density-based algorithms suffer from the "pseudo-evolution" problem and fail to reflect the true physical situation of haloes. The famous FoF algorithm connects particles within a certain distance in numerical simulations to define haloes, and many halo tracking and merger tree construction programs first run the FoF algorithm. Additionally, researchers have sought new ways to define halo boundaries. The static radius, based on radial velocity, and the corresponding static mass concept offer certain advantages. This chapter focuses on the splashback radius, discovered through the observation that the logarithmic slope of the density profile in halo outskirts drops dramatically. We also introduce two programs for calculating the splashback radius. Recently, the characteristic and inner depletion radii have been defined based on halo bias profiles and the dynamic process of radial mass flow, offering what is argued to be a more natural characterization of halo boundaries with clearer physical meaning than the splashback radius. It is important to emphasize that the pseudo-evolution problem is unique to density-based definitions; other definitions reflecting dynamical features do not suffer from this issue.

2.1 Density-Based Definitions

Inspired by the spherical collapse model [**?**], researchers developed the spherical overdensity (SO) algorithm [**?**, **?**], which treats a spherically symmetric region exceeding a certain density as a halo.

The SO algorithm proceeds by first ranking all particles in a numerical simulation by their local density. Starting from the highest density location, it draws a spherical region whose radius grows until its average density drops below a specified threshold. The halo center is then recalculated as the mass center of the selected region, and the process iterates until both the halo center and its membership converge. All particles assigned to the halo are then removed from further calculation. This method is repeated iteratively to identify all haloes. Eventually, smaller haloes or those overlapping with larger ones are merged into larger structures.

Haloes found through this method satisfy the relation:

$$M = \frac{4}{3}\pi\Delta\rho_{\mathrm{ref}}R^3$$

where ρ_{ref} is the reference density and Δ is the overdensity factor. The reference density can be either the mean matter density or the critical density, both of which evolve with redshift. Common choices include $200\rho_{\mathrm{mean}}$ and $180\rho_{\mathrm{crit}}$, with corresponding radii R_{200m} and R_{180c}, and masses M_{200m} and M_{180c}. The widely used virial density proposed by Bryan and Norman [?] in 1998 (hereafter BN98) is:

$$\rho_{\rm vir} = (18\pi^2 - 82q - 39q^2)\rho_{\rm crit}$$

where $q = \Omega_{M,0}a^{-3}/(\Omega_{M,0}a^{-3} + \Omega_{\Lambda,0})$, with a being the scale factor, $\rho_{\rm crit}$ the critical density, and $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$ the present-day matter density parameter and cosmological constant parameter, respectively. The corresponding radius and mass are $R_{\rm vir}$ and $M_{\rm vir}$, commonly known as the virial radius and virial mass.

However, the SO algorithm' s attempt to define halo boundaries through density regions has significant problems and cannot reflect the true physical processes of haloes, leading to the so-called "pseudo-evolution" phenomenon where a halo' s mass and radius grow substantially without corresponding changes in internal physics. We now discuss pseudo-evolution in detail.

Diemand et al. [?] presented the mass accretion history of a Milky Way-mass dark matter halo from the Via Lactea simulation [?], shown in [Figure 1: see original paper]. The figure reveals that physical mass accretion essentially ceased after redshift $z \approx 1.7$ (scale factor $a = 0.37$), with over 80% of the material within 400 kpc already accreted by $z = 1$. Yet during the same period, the virial mass $M_{\rm vir}$ and M_{200} increased substantially.

Cuesta et al. [?] showed the evolution of density profiles for galaxy-sized haloes [Figure 2: see original paper]. The density profiles have remained nearly unchanged since $z = 1$, while the virial radius at $z = 0$ has doubled compared to its value at $z = 1$.

In response, Diemer et al. [?] found that considerable pseudo-evolution occurs in many haloes after $z = 1$, with most low-mass haloes ($< 10^{12} M_\odot/h$) experiencing pseudo-evolution.

2.2 Friends-of-Friends (FoF) Algorithm

The FoF algorithm [?, ?] connects particles separated by less than $bn^{-1/3}$, where n is the mean particle density and $b = 0.2$ is typically used (20% of the mean interparticle distance). This creates chains of particles, and those with sufficient mass are identified as haloes. A characteristic feature of FoF haloes is their irregular shapes, though occasionally bridging particles connect two distinct haloes. While this method defines haloes themselves, it does not provide a strict "boundary."

The FoF algorithm serves as the foundation for many halo tracking and merger tree construction algorithms, including SUBFIND (subhalo finder) [?, ?], HBT (Hierarchical Bound-Tracing) [?, ?], ROCKSTAR (Robust Overdensity Calculation using K-Space Topologically Adaptive Refinement) [?], and SUBLINK (subhalo linking) [?].

2.3 Static Radius

Cuesta et al. [?] presented the average radial velocity profiles for haloes in three mass bins, each averaged over hundreds of individual haloes [Figure 3: see original paper]. The region between the dashed lines represents their defined static region (within 5% of the virial velocity). Cluster-mass haloes exhibit strong infall (negative radial velocities), while regions beyond this strong infall, even with near-zero average radial velocity, are not considered part of the static region. Low-mass and galaxy-mass haloes show slight outflow before the Hubble flow. They defined the static radius as the outermost location of the static region, with the enclosed mass termed static mass, arguing this is superior to virial mass and radius based on the SO algorithm.

2.4 Splashback Radius

The splashback radius (R_{sp}), proposed in recent years [?, ?, ?, ?, ?], is considered a more natural halo boundary. Diemer and Kravtsov [?] showed the median density profile slope for massive haloes ($\nu > 3.5$, $M_{\text{vir}} > 10^{15} M_{\odot}/h$) at $z = 0$ [Figure 4: see original paper]. The profile exhibits a dramatic decline in the outskirts, deviating significantly from traditional NFW [?] and Einasto [?] profiles.

Adhikari et al. [?] explained that the locally steep slope arises from caustics formed by shell crossing, related to the splashback phenomenon of recently accreted material (ejected and then returning like a splash guard). Particle orbits pile up at similar locations, particularly near apocenters, strengthening the caustic. This prominent caustic exists in both spherical [?] and non-spherical [?] configurations. Based on this, the splashback radius was defined.

More et al. [?] presented a slice projection of density for a halo with $M_{\text{vir}} = 1.1 \times 10^{14} M_{\odot}/h$ and mass accretion rate $\Gamma = 0.8$ at $z = 0$ [Figure 5: see original paper]. The density field clearly drops sharply at R_{sp}, effectively separating the halo' s multi-stream region from the infall region. Consequently, R_{sp} is considered a very natural definition of the halo boundary. The mass within R_{sp} is correspondingly defined as the splashback mass, $M_{\text{sp}} \equiv M(< R_{\text{sp}})$.

More et al. [?] defined the splashback radius as the location where the density profile slope is steepest. They provided a fitting formula:

$$\frac{R_{\text{sp}}}{R_{200m}} = 0.54\,[1 + 0.53\Omega_m(z)]\left(1 + 1.36e^{-\Gamma/3.04}\right)$$

where Γ is the mass accretion rate:

$$\Gamma \equiv \frac{\Delta \log M_{\text{vir}}}{\Delta \log a}$$

with R_{200m} defined by Equation (1) using 200 times the mean matter density, a the scale factor, M_{vir} the virial mass, and $\Omega_m(z)$ the matter density parameter.

Additionally, Diemer et al. [?, ?] developed the SPARTA (Subhalo and PARticle Trajectory Analysis) algorithm. This algorithm analyzes all particle orbits within host haloes in cosmological simulations and accurately extracts the first apocenter after infall—the splashback location. They define the halo' s splashback radius as the smoothed average of particle apocenter radii. [Figure 6: see original paper] shows a 30 Mpc/h wide and deep, 15 Mpc/h thick slice from a numerical simulation, comparing traditional virial radii with splashback radii from SPARTA. The displayed haloes have $N_{200m} > 1000$ particles, corresponding to masses $\geq 1.4 \times 10^{11} M_\odot/h$, with the central massive halo having $M_{200m} = 1.2 \times 10^{14} M_\odot/h$.

Mansfield et al. [?] developed the SHELLFISH (SHELL Finding in Spheroidal Halos) algorithm, which identifies halo "splashback shells" from a single snapshot of the density field in numerical simulations without requiring particle dynamical information. [Figure 7: see original paper] illustrates the four steps of the SHELLFISH algorithm: (1) From the center, tens of thousands of random rays (red lines) trace the halo' s density field. (2) The density profile along each ray is measured (black line) and smoothed using a Savitzky-Golay filter [?] (red line); arrows indicate the steepest point on the smoothed profile. (3) All steepest points are marked and classified; the white curve is a filtering spline generated during point selection, with nearby points marked white and distant ones red. (4) Based on the distribution of splashback points across 100 random planar orientations, a Penna-Dines surface [?] is fitted. SHELLFISH thus obtains the splashback shell, and the equivalent volume radius is defined as:

$$R_{\rm sp} \equiv \left(\frac{3V_{\rm sp}}{4\pi}\right)^{1/3}$$

where $V_{\rm sp}$ is the volume enclosed by the shell.

2.5 Depletion Radius

Recently, Fong and Han [?] introduced the novel concepts of inner depletion radius ($R_{\rm id}$) and characteristic depletion radius ($R_{\rm cd}$), arguing they provide a more natural characterization of halo boundaries. [Figure 8: see original paper] shows the bias factor b:

$$b(r) = \frac{\xi_{\rm hm}(r)}{\xi_{\rm mm}(r)} = \frac{\langle \delta(r) \rangle}{\xi_{\rm mm}(r)}$$

where $\xi_{\rm hm}$ and $\xi_{\rm mm}$ are the halo-matter and matter-matter correlation functions, and $\delta(r)$ is the density contrast around haloes. A clear trough is visible in the middle, leading to the definition of the characteristic depletion radius ($R_{\rm cd}$) as the location where the bias factor b is minimized.

[Figure 9: see original paper] shows profiles of average bias factor, radial velocity, and mass flow rate (MFR) for different halo mass bins. The total radial velocity decomposes as $v_r = v_p + v_h$, where the right-hand terms are peculiar and Hubble velocities (negative values indicate infall). The MFR represents mass crossing a spherical surface per unit time: $\mathrm{MFR} = \rho(r) v_r(r) 4\pi r^2$, normalized by $M_{\mathrm{vir}} \cdot v_{\mathrm{vir}} \cdot r_{\mathrm{vir}}^{-1}$, where $v_{\mathrm{vir}} = \sqrt{GM_{\mathrm{vir}}/r_{\mathrm{vir}}}$ is the virial velocity and G is the gravitational constant. The prominent troughs reveal that massive haloes actively accrete surrounding material, while low-mass haloes do not.

Since MFR determines density profile evolution, they identified the location of maximum inflow shown in [Figure 10: see original paper] as the maximum inflow radius. Within this radius, material pours into the halo; beyond it, material appears to be gradually depleted. The maximum inflow location marks the transition from halo construction to material consumption by accretion, signifying the inner boundary of the active depletion zone around haloes. As shown in [Figure 9: see original paper], R_{cd} lies 10-20% beyond the maximum inflow radius. Based on this dynamical interpretation, they termed this maximum inflow radius the "inner depletion radius" (R_{id}). R_{id} heralds the beginning of continuous depletion, while R_{cd} at the minimum bias location represents the characteristic or "deepest depletion" radius, together characterizing the scale of the depletion trough.

2.6 Relationship between Depletion Radius and Splashback Radius

Fong and Han [?] presented radial velocity distributions for haloes in different mass bins [Figure 11: see original paper], with virial masses M_{vir} in units of $M_\odot \cdot h^{-1}$ and all haloes at $z = 0$ in their study. The depletion radius lies where the phase-space distribution of the density profile is narrowest.

Regarding the splashback radius, R_{id} can be roughly considered the outermost boundary of the splashback population. In [Figure 11: see original paper], the splashback radius lies not far inside the narrowest velocity distribution boundary. To clarify, [Figure 12: see original paper] shows the radial velocity phase-space distribution for a single halo with $M_{\mathrm{vir}} = 1.35 \times 10^{15} M_\odot \cdot h^{-1}$, where the location of maximum infall clearly marks the outer boundary of splashback. Thus, the inner depletion boundary can be understood as encompassing the complete splashback population.

3 Observational Studies and Applications of Halo Boundaries

This chapter introduces observationally used boundaries such as R_{500} and observational support for the splashback radius (R_{sp}) and inner depletion radius (R_{id}). The steep decline in density profiles at the splashback radius has observational counterparts in galaxy number density and mass profiles around clusters. Shortly after the depletion radius concept was proposed, the Milky Way' s inner depletion radius was measured.

3.1 Observational Application of Density-Defined R_{500}

Observationally, M_{500} and R_{500} are frequently used to estimate galaxy cluster masses and sizes through the Sunyaev-Zel' dovich (SZ) effect [?, ?]. For example, Ricci et al. (2020) [?] used XXL survey [?] data to study the XLSSC 102 cluster [?], employing the relation in Equation (1) with $500\rho_{\rm crit}$—500 times the critical density at the cluster' s redshift—as shown in [Figure 13: see original paper].

3.2 Observational Evidence for the Splashback Radius

In 2016, More et al. [?] first measured the splashback radius using the projected surface density profile of galaxies ($\Sigma_g(R)$) around clusters from SDSS DR8 [?], finding a steep decline analogous to that seen by Diemer and Kravtsov [?] [Figure 14: see original paper]. Discrepancies between observed and simulated splashback radii may arise from parameter measurement biases or potentially from unknown new physics in dark matter self-interactions that could reduce the splashback radius.

Furthermore, Chang et al. (2018) [?] used DES data to study galaxy number density and weak lensing mass profiles around clusters, also finding steep, splashback-like features [Figure 15: see original paper]. Importantly, their weak lensing measurements provided the first detection of this phenomenon in cluster mass profiles. The steepest slope locations in both galaxy number density and weak lensing mass profiles agree, with both being steeper than NFW profiles, consistent with splashback radius predictions.

3.3 Measurement of the Milky Way' s Inner Depletion Radius

Shortly after the depletion radius concept was introduced, Li and Han [?] measured the Milky Way' s inner depletion radius using catalogs of galaxies within 3 Mpc [?, ?] and proper motions of available nearby dwarf galaxies [?, ?] to convert heliocentric radial velocities to galactocentric ones. They obtained $R_{\rm id} = (559 \pm 107)$ kpc and $R_{\rm ta} = (839 \pm 121)$ kpc [Figure 16: see original paper].

4 Evolution of Dark Matter Haloes

This chapter describes merger trees that reflect halo merging processes, which are essential for understanding halo structure. We also present examples of halo growth defined by virial radius, FoF haloes, static radius, and splashback radius. The virial radius case clearly shows two growth phases: early rapid growth and late slow growth. Other definitions have their own characteristics, all presented here for reference.

4.1 Dark Matter Halo Merger Trees

Numerical simulations show that when two haloes merge, the lower-mass halo does not completely disappear but continues to exist as a self-bound subhalo orbiting within the more massive halo's potential. [Figure 17: see original paper] illustrates a sequence of merging events, with purple spheres representing the main progenitor' s merger history (zeroth-order progenitor). The zeroth-order progenitor accretes first-order progenitors (orange spheres), which become first-order subhaloes at redshift $z = z_0$. Similarly, first-order progenitors accrete second-order progenitors (blue spheres), which become second-order subhaloes or sub-subhaloes at $z = z_0$. This process continues, with the branch tracking the main progenitor called the main branch [?]. The algorithms mentioned in Section 2.2 first identify dark matter clumps via FoF and then construct merger trees like [Figure 17: see original paper] according to their respective criteria.

4.2 Halo Growth Histories

4.2.1 Virial Radius Description [Figure 18: see original paper] shows halo radius evolution from Zhao et al. [?]. The three different density-defined radii are similar. In the commonly used NFW profile [?], the inner characteristic radius r_s is defined where the logarithmic density slope equals -2, with concentration $c = r_{\rm vir}/r_s$ and $r_{\rm vir}$ the virial radius.

[Figure 19: see original paper] presents halo mass growth histories along the main branch from Zhao et al. [?]. They provide the following formula for average growth:

$$\frac{M_{\rm vir}(z)}{M_{\rm vir,tp}} = \left(1 - a + ax^{-1.8a}\right)^{-1}$$

where $x = \rho_{\rm vir,tp}/\rho_{\rm vir}(z)$ represents "time,"the subscript "tp"denotes the turning point, and $\rho_{\rm vir}$ is the virial density. The parameter a takes values of 0.75 and 0.42 for the fast and slow growth phases, respectively.

Halo growth divides into early rapid growth and late slow growth phases. During early violent relaxation, material becomes well mixed, allowing both virial radius ($r_{\rm vir}$) and characteristic radius (r_s) to grow synchronously, keeping the concentration nearly constant. In the late phase, material can no longer reach the halo core and only accumulates in the outskirts, causing r_s to remain constant while $r_{\rm vir}$ increases, leading to dramatic concentration growth.

4.2.2 FoF Haloes Poole et al. [?] presented the evolution of FoF halo and substructure masses with redshift [Figure 20: see original paper]. They selected FoF systems with masses $> 10^{13} M_\odot \cdot h^{-1}$ at $z = 0.5, 1, 2$ from numerical simulations. FoF halo masses increase with time, while substructure masses first increase to a peak and then decline.

4.2.3 Static Radius Cuesta et al. [?] showed the evolution of the ratio of static radius to virial radius with time (scale factor) [Figure 21: see original paper]. In virial radius terms, the static region of low-mass haloes first increases to a maximum then decreases, galaxy-mass haloes are approaching this maximum, and cluster-mass haloes' static radii continue to grow.

4.2.4 Splashback Radius O' Neil et al. [?] presented splashback radius evolution for haloes with masses $(10^{13} - 10^{15})M_\odot$ [Figure 22: see original paper], comparing N-body simulations from the IllustrisTNG project [?] (TNG300-1-DM, panel a) with hydrodynamical simulations (TNG300-1, panels b and c). The latter includes dark matter only (panel b) and all matter (panel c), while the former includes only dark matter. Overall, N-body and hydrodynamical simulations yield consistent $R_{\rm sp}$ results. They also compared with analytic models from More et al. [?] (dashed lines) and Diemer [?] (dotted lines), which apply only to dark matter but are shown for reference in all panels.

5 Summary and Outlook

This paper has summarized several definitions of dark matter halo boundaries. The spherical overdensity algorithm, inspired by the spherical collapse model, defines haloes as spherical high-density regions exceeding a certain density threshold, commonly using virial radius and mass. This definition fails to reflect the true physical situation and suffers from "pseudo-evolution," where halo mass and radius grow according to the definition without corresponding physical changes. Besides SO, the famous FoF algorithm connects nearby particles in numerical simulations to identify haloes.

Subsequent explorations have proposed alternative methods. The static radius, defined by the outermost extent of the static region, offers certain advantages. The splashback radius was introduced after discovering that the density profile slope declines dramatically in halo outskirts, steeper than traditional NFW and Einasto profiles. We introduced two algorithms for calculating splashback radius: SPARTA, which tracks particle orbital apocenters, and SHELLFISH, which identifies the steepest density slope points in various directions to fit a surface. The inner and characteristic depletion radii better reflect halo dynamical properties, together characterizing the scale of the depletion trough. The inner depletion radius, in particular, effectively captures material flow at halo boundaries and has clearer physical meaning than the splashback radius.

We also discussed observational aspects of halo boundaries. Density-related boundaries are relatively easy to observe; M_{500} and R_{500} are commonly used when estimating cluster masses via the SZ effect. The splashback and depletion radii are more challenging to observe, but measurements of galaxy number density profiles around clusters and weak lensing mass profiles have revealed steep, splashback-like features. The Milky Way' s inner depletion radius has also been measured.

The merging process of haloes is described by merger trees. Growth histories based on virial mass and radius show two phases: early rapid growth and late slow growth. Pseudo-evolution occurs during the late slow-growth phase, as shown in [Figure 19: see original paper].

Previous halo research focused primarily on inner regions relevant to galaxy formation, but outer regions have gained attention in recent years, partly due to the splashback radius proposal. Discrepancies between observed and simulated splashback radii may relate to dark matter self-interactions, offering potential to constrain dark matter properties. Research on the newly proposed depletion radii has not yet extensively explored halo growth processes, and we anticipate future work in this area. Finally, with the widespread application of machine learning, especially deep learning, we expect these techniques to help identify halo boundaries more efficiently and accurately.

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