Noise Suppression Techniques in Gravitational Wave Detection: A Review (Postprint)

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Abstract

In recent years, significant progress has been made in weak gravitational wave detection. Addressing the noise suppression challenges in gravitational wave measurement, this review covers several typical detection methods including resonant bar detectors, ground-based laser interferometers, and space-based laser interferometer arrays, explains the principles of gravitational wave detection, introduces noise sources such as thermal noise, shot noise, and residual gas noise in various detection approaches, elaborates on noise suppression methods, and presents novel noise suppression technologies including quadruple suspension, signal recycling, and squeezed light fields. For space-based gravitational wave detection, particular emphasis is placed on noise suppression techniques distinct from ground-based approaches, such as time-delay interferometry and arm-locking techniques.

Full Text

Preamble

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Review of Noise Suppression Technologies in Gravitational Wave Detection

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Abstract

In recent years, significant progress has been made in the detection of weak gravitational waves. This review addresses the critical challenge of noise suppression in gravitational wave measurements by examining several representative detection methods, including resonant bar detectors, ground-based laser interferometers, and space-based laser interferometer arrays. We explain the fundamental principles of gravitational wave detection, introduce major noise sources such as thermal noise, shot noise, and stray gas noise in various detection schemes, and elaborate on noise suppression techniques. Novel technologies including quadruple suspension, signal recycling, and squeezed light fields are discussed. For space-based gravitational wave detection, we focus on specialized noise suppression techniques that differ from ground-based approaches, particularly time-delay interferometry and arm-locking.

Keywords: gravitational waves; laser interferometry; squeezed light; drag-free control

1 Introduction

Gravitational waves represent a crucial prediction of Einstein' s general theory of relativity, manifesting as propagating disturbances in the gravitational field. Generated by coherent bulk motion of matter, gravitational waves carry information about both the wave source and its macroscopic dynamics. General relativity establishes that gravitational waves are transverse waves with two polarization states propagating at the speed of light [1]. From momentum conservation and the fact that mass can only appear as a positive quantity, gravitational waves can only be produced by time-varying quadrupole moments [2]. High-energy astrophysical events such as supernova explosions typically generate gravitational waves. Due to their extremely weak interaction with matter, gravitational waves can penetrate substances opaque to electromagnetic radiation, enabling us to study the interiors of high-energy celestial objects. While neutrinos also pass through matter with minimal attenuation, they may undergo multiple scattering during propagation, preventing them from preserving pristine information about the source interior. Research objects associated with gravitational waves include supernovae, newborn neutron stars, black hole formation, collisions of compact objects, and the internal structure of common envelopes in binary systems [1].

Interest in gravitational wave detection has grown substantially in recent years. Since the Laser Interferometer Gravitational-wave Observatory (LIGO) first detected gravitational wave signals, multiple ground-based detectors have been constructed worldwide. Additionally, the Laser Interferometer Space Antenna (LISA), led by the European Space Agency (ESA), represents a new generation of space-based gravitational wave detectors currently under development. China is also developing space-based gravitational wave detectors, primarily through two proposed missions: the TaiJi program and the TianQin Gravitational Wave

Research Project [3;4]. Japan has proposed the Deci-hertz Interferometer Gravitational Wave Observatory (DECIGO) as its space-based detection program [5].

The magnitude of gravitational waves is typically measured by a dimensionless quantity h, where $h = \Delta \mathrm{l}/\mathrm{l}$ represents the relative strain between two points in space caused by the passing wave. Typical gravitational wave amplitudes are generally smaller than 10^{-21} [6], imposing extremely stringent requirements on detector sensitivity. Consequently, regardless of the detection method employed, noise suppression is essential to achieve the sensitivity necessary for gravitational wave detection.

2 Early Gravitational Wave Detection

Early gravitational wave detection primarily relied on resonant bar detectors [1]. Limited by the technological capabilities of the time, researchers developed a series of ingenious noise suppression methods that dramatically improved detector sensitivity. Although the final sensitivity remained insufficient for gravitational wave detection, these methods provide important insights and continue to influence modern detector design.

2.1 Resonant Bar Detectors

The resonant bar detector, also known as the Weber bar, was the first gravitational wave detector, originally designed and built by Joseph Weber, hence the name "Weber bar." In 1969, Weber [7] claimed to have detected gravitational wave signals using this method, attracting widespread attention. Subsequently, several similar detectors were constructed and put into operation. To date, resonant mass detectors have undergone three generations of development, with the most advanced detectors operating at temperatures as low as 0.1 K [2], yet none have produced conclusive results.

The structure of a Weber bar (using ALLEGRO as an example) is shown in [Figure 1: see original paper], while [Figure 2: see original paper] illustrates the overall schematic principle. The Weber bar consists of a solid aluminum cylinder that can oscillate along its axial direction, with piezoelectric sensors attached near the center of the cylinder' s side surface. The entire assembly is suspended in a vacuum chamber equipped with vibration isolation systems to shield against external disturbances such as seismic vibrations [8]. The innermost 2 K shell shown in [Figure 1: see original paper] represents the vacuum chamber. When a gravitational wave passes through, the cylinder periodically expands and contracts along its axis, generating electrical signals via the piezoelectric crystals. The earliest detectors had a natural frequency of 1,657 Hz [9]. When the gravitational wave frequency matches the Weber bar' s natural frequency, axial resonance occurs, producing an output vibration signal that enables gravitational wave detection.

2.2 Noise Suppression Methods for Bar Detectors

The primary noise sources confronting Weber bars include thermal noise, seismic noise, air disturbances, and electromagnetic interference [10]. Thermal noise produces particularly significant disturbances; for instance, Weber observed thermal noise amplitudes of 10^{-16} [9], far exceeding typical gravitational wave amplitudes. Weber' s mitigation approach employed high-quality-factor aluminum bars, which enhanced the peak amplitude of vibrations induced by gravitational waves. By applying filtering techniques, signals with relatively short periods characteristic of gravitational waves could be extracted [2], making detection feasible. Weber deployed two detectors separated by 2 km; if thermal noise caused a false signal in one detector, the probability of simultaneous false signals in the other was extremely small, enabling rejection of thermal noise effects. Additionally, a control group was established approximately 1,000 km away [7]; if signals originated from seismic activity, time delays would appear between the two locations. Weber set a time threshold of 0.44 s, considering only signals that overlapped within this threshold across all detectors as potential gravitational wave candidates [7]. This method was later adopted by LIGO and other gravitational wave detectors. To avoid air and electromagnetic interference, the apparatus was placed in a vacuum chamber [2] and equipped with electromagnetic shielding [7].

In second-generation detectors, the apparatus was cooled to 4.2 K to further suppress thermal noise, and ultra-low-noise amplifiers were employed [2]. Subsequent research revealed that the detector' s quality factor (including both the antenna and circuit) also increased at cryogenic temperatures, further enhancing sensitivity. Since piezoelectric transducers exhibit relatively low quality factors and poor electromechanical coupling performance at cryogenic temperatures, capacitive or inductive transducers are typically used in cryogenic detectors. These cryogenic transducers utilize superconducting circuits resonant at the same frequency as the detector, enabling preliminary signal amplification with lower noise. The transducer output signals are processed by ultra-low-noise amplifiers, converted by analog-to-digital converters, and recorded on magnetic tape.

Currently, advanced resonant bar detectors exist worldwide, such as the NAUTILUS detector, which operates at temperatures below 0.1 K and achieves a peak sensitivity of 10^{-21} $\mathrm{Hz}^{-1}/^{2}$ (see [Figure 3: see original paper]). For millisecond-duration gravitational wave bursts, its sensitivity across a broad bandwidth (30 Hz) reaches approximately $h = 3 \times 10^{-19}$ [11]. With the advent of laser interferometer gravitational wave detectors, Weber bars have gradually faded from the detection landscape due to their inferior bandwidth and sensitivity compared to interferometric instruments. Although Weber did not detect gravitational waves, his efforts stimulated academic interest in gravitational wave detection and laid the foundation for the field. Resonant bar detectors provided a stable means of gravitational wave detection for a considerable period, with operational duty cycles exceeding 95%, and their noise suppression methods have informed subsequent detection technologies.

3 Ground-based Laser Interferometer Gravitational Wave Detectors

Laser interferometers currently represent the primary instruments for gravitational wave detection. In addition to LIGO, which first detected gravitational waves, the Virgo Interferometer in Italy, GEO600 in Germany, and the Kamioka Gravitational Wave Detector (KAGRA) in Japan have all been constructed and integrated into a global collaborative gravitational wave observation network. Most existing laser interferometer gravitational wave detectors (except GEO600 [12]) employ Michelson interferometers with Fabry-Pérot cavities (FP cavities) and utilize power recycling techniques to enhance sensitivity [6]. Laser interferometer gravitational wave detectors surpass resonant detectors like Weber bars in both precision and bandwidth, enabling response to gravitational waves from various astrophysical processes.

Since LIGO' s first detection of gravitational wave signals in 2015, global laser interferometer detectors have observed multiple gravitational wave events [13-15] and identified electromagnetic counterparts to some signals [16]. Gravitational wave observations have revealed details of black hole mergers and binary neutron star coalescences, advancing gravitational wave astronomy and opening a new chapter in observational cosmology.

3.1 Principles of Laser Interferometric Gravitational Wave Detection

The core of laser interferometer gravitational wave detection is the Michelson interferometer. Using LIGO [17] (upgraded to advanced LIGO, aLIGO [18]) as an example, the interferometer arms suspend test masses (TMs) with reflective mirrors that detect gravitational wave effects (see [Figure 4: see original paper]). When a gravitational wave passes, the test masses undergo displacement due to the wave' s influence. The interferometer arms are of equal length, and in the absence of gravitational waves, the output port (also called the antisymmetric port) remains at a dark fringe. As shown in [Figure 5: see original paper], when a gravitational wave passes with orientation matching the interferometer plane, it induces relative length changes in the two arms. Due to the transverse nature of gravitational waves, a wave arriving perpendicular to the interferometer plane periodically elongates one arm while shortening the other, producing laser phase modulation. Without gravitational waves, the interferometer output exhibits destructive interference; only light modulated by gravitational waves produces constructive interference at the antisymmetric port, with output intensity proportional to the gravitational wave amplitude [17;19;20]. In this operating state, only the differential mode signal generated by gravitational waves reaches the output port, while the common-mode laser light returns to the laser source. To facilitate control of the interferometer' s operating state, radio-frequency signals are modulated onto the input laser.

3.2 Noise Sources and Suppression Methods in Laser Interferometers

Laser interferometer noise can be broadly categorized into displacement noise and detection noise. Displacement noise causes motion of test masses and mirror surfaces, interfering with arm length measurements, while detection noise limits the precision of test mass monitoring [17].

Displacement noise primarily includes thermal noise, seismic noise, and control system noise. In the low-frequency band, seismic noise—ground motion caused by wind, ocean waves, human activity, and minor earthquakes—represents a major source of displacement noise, typically filtered by suspension systems and seismic isolation platforms.

Thermal noise constitutes another primary source of displacement noise, arising from random molecular motion. It can be subdivided into pendulum thermal noise, mirror thermal noise, and suspension thermal noise, affecting all frequency bands. Pendulum thermal noise originates from energy exchange between the mirror as a whole and surrounding gas, which applies fluctuating forces and generates thermal motion [21]; it also arises from dissipative forces in pendulum motion, such as residual gas, structural damping, and thermoelastic loss. Consequently, vacuum chambers are necessary to minimize thermal exchange between mirrors and the external environment [22]; vacuum chambers also provide acoustic isolation for mirrors, preventing acoustic disturbances and reducing thermal fluctuations in the optical path. Mirror thermal noise results from thermoelastic waves in the mirror substrate and coating, while suspension thermal noise causes vibrations in the suspension system that alter arm length [17]. Most thermal noise concentrates at resonant frequencies, which are designed to lie far from the detection band [17]. Within the detection band, thermal noise is associated with mechanical dissipation far from resonance. Therefore, designing low-mechanical-loss suspension systems reduces thermal noise impact. Nevertheless, thermal noise remains a significant contributor, requiring advanced techniques such as cryogenic mirror technology similar to that employed by KAGRA [23].

Additionally, local variations in Earth’ s mass distribution (from atmospheric or soil changes) and motion of nearby objects (such as vehicles) cause local gravitational field fluctuations, introducing gravity gradient noise [24]. Gravitational effects cannot be eliminated by seismic isolation systems and must be addressed by locating detectors away from noise sources, such as building interferometers underground or launching them into space. Monitoring mass density fluctuations near test masses for real-time correction is also necessary [22].

Detection noise primarily includes shot noise (also called quantum noise), dark current, laser frequency stability, and laser power stability. Although laser frequency and power noise are common-mode at the output port, differences in resonant reflectivity between the two arms cause carrier light to “leak” into the antisymmetric port and interfere with modulated RF sidebands, generating noise [17;25].

Above 100 Hz, shot noise dominates, arising from the Poisson statistics of photon detection [11]. The ideal shot noise density is given by [26]:

$$\tilde{h}(f) = \frac{\pi\hbar}{\eta P_{BS} c}\sqrt{\frac{1+(4\pi f\tau_s)^2}{4\pi\tau_s}}$$

where π is pi,   is the reduced Planck constant, c is the speed of light, τs *is the optical storage time in the arm cavity,* f *is the gravitational wave frequency,* P{BS} is the optical power at the beam splitter, and   is the detector' s quantum efficiency. Imperfect interference at the beam splitter and interference from modulated RF sidebands at the antisymmetric port also increase shot noise [17;26]. One approach to suppressing shot noise is increasing laser power.

In addition to shot noise, mirrors experience radiation pressure noise from laser light. Both shot noise and radiation pressure noise originate from the quantum nature of light and are collectively termed quantum noise. Unlike shot noise, radiation pressure noise primarily affects low frequencies, with amplitude proportional to [18]. Consequently, increasing laser power to reduce shot noise inevitably increases radiation pressure noise. When these two effects reach equilibrium, further power increases no longer reduce quantum noise; this quantum noise level is called the standard quantum limit (SQL). [Figure 6: see original paper] shows aLIGO' s pre-upgrade sensitivity estimates relative to the SQL. The figure depicts sensitivity curves without signal recycling mirrors, a general configuration with relatively good performance across all bands, and a configuration optimized specifically for neutron star mergers. In aLIGO, this limit is $1.8 \times 10^{-22}/\sqrt{\text{Hz}}$ [27;28]. Within the detection band, the SQL is approximately 10^{-24} [27;28]. The SQL fundamentally reflects the Heisenberg uncertainty principle and can only be surpassed through quantum optics methods [29].

Other noise sources in laser interferometers include stray photon noise, where laser light scattered from the main beam and reflected back from non-isolated surfaces couples with the main beam, introducing additional noise. This can be mitigated by installing optical baffles [30]. Residual gas noise also exists, and thermal noise can be further subdivided. Due to space limitations, detailed discussions of these mechanisms and treatments are omitted here; interested readers may consult reference [21].

To extract signals from noise, interferometer output signals require preliminary filtering. Different source types necessitate different filtering approaches. For instance, setting "data quality flags" can exclude erroneous data from instrument malfunctions, such as actuator saturation states. These flag algorithms are tested using theoretically calculated gravitational waveforms, enabling interferometers to effectively identify target waveforms [31].

LIGO' s data analysis team has also developed instrument-based filtering methods to exclude anthropogenic noise. LIGO' s primary source categories include: transient known sources such as compact binary mergers; transient unknown sources like gravitational wave bursts; continuous narrowband sources such as gravitational waves from rotating asymmetric neutron stars; and continuous broadband sources like stochastic cosmic background radiation [17].

For well-modeled astrophysical processes like compact binary mergers, precise waveform templates can be calculated from existing models, enabling signal extraction through matched filtering [17;32]. The matched filtering algorithm requires templates corresponding to detection targets; LIGO employs 7,000 templates covering binary masses from 7 to 35 solar masses. Optimization algorithms reduce template count while maintaining signal-to-noise ratio degradation below 3%. When filter output exceeds a threshold, the algorithm generates a trigger signal. When triggers from two LIGO detectors share similar template parameters and fall within the same time window, the signal becomes a gravitational wave candidate. To distinguish genuine signals from noise-induced false alarms, time-shift analysis is performed on each coincident pair by shifting one signal in time before comparison. If signals originate from noise, they remain similar after time shifting. Only signals passing this time-shift test are considered gravitational wave candidates [17;33;34].

For gravitational wave burst events ($t \leq 1$ s), precise modeling is difficult due to complex generation mechanisms, requiring alternative filtering algorithms. Three main approaches exist [17]: (1) Fourier spectral analysis or wavelet analysis to identify components with power significantly above the detector' s baseline noise level in the time-frequency domain, followed by further analysis using the methods described above; (2) direct identification of common signals across different detectors, typically by calculating cross-correlation coefficients between signals (though differing detector response characteristics may reduce signal similarity); and (3) combined analysis using data from multiple detectors, which enhances result reliability.

These burst search methods perform well, achieving sensitivities typically half that of matched filtering for known sources while offering higher computational efficiency [17].

Continuous gravitational wave signals can also be searched using cross-correlation methods, as discussed in reference [17]. Additionally, machine learning techniques are being investigated for noise rejection [35].

3.3.1 Quadruple Suspension System

To suppress thermal and seismic noise, current interferometers worldwide employ combinations of seismic isolation platforms and suspension systems [36]. The suspension system acts as a mechanical filter, attenuating horizontal disturbances above its resonant frequency. This approach reduces seismic noise to 10^{-10}, enabling detection sensitivity around $10^{-22}/\sqrt{\text{Hz}}$ [36]. Similar to Weber bars, high-quality-factor materials can suppress thermal noise. As previously discussed, suspension thermal noise directly correlates with mechanical losses, so designing low-mechanical-loss systems reduces thermal noise. Suspension systems inherently exhibit extremely low mechanical loss, providing a “dilution” effect that makes the overall system loss far lower than that of the suspension material itself [36;37].

However, even with suspension systems, thermal noise remains a major interferometer noise source [17]. Further reduction of suspension thermal noise is necessary. Current interferometers widely adopt the quadruple suspension system originally designed for GEO600 [38].

[Figure 7: see original paper] shows a simplified diagram of GEO600’ s quadruple suspension system. Early suspension systems typically used piano wire [39], but subsequent research demonstrated that fused silica fibers exhibit superior mechanical loss performance—approximately 1/100th that of metal wires [40]. Using hydroxide catalysis bonding (HCB) technology to weld fused silica fibers to mirror sides achieves quality factors of 2×10^7 [41], improved to $(1.1 \pm 1.6) \times 10^9$ in aLIGO [18]. To suppress thermoelastic noise, fused silica fibers are designed with a dumbbell shape [42;43], where the middle section diameter is precisely chosen so that Young’ s modulus-induced deformation cancels thermoelastic effects [36].

Cryogenic cooling of mirrors represents an important future upgrade for thermal noise suppression [23;36]. However, at low temperatures, fused silica fibers exhibit significantly increased mechanical loss and reduced thermal conductivity, making them unsuitable for cryogenic mirrors. Japan’s KAGRA has successfully implemented cryogenic mirrors using sapphire for both mirrors and suspension fibers [23;44]. Sapphire achieves quality factors near 10^8 at cryogenic temperatures [23;45], making it more suitable than fused silica for cryogenic applications. KAGRA also employs a multi-stage suspension system that thermally isolates mirrors and intermediate masses from upper stages to prevent cryogenic effects on mirrors [45].

3.3.2 Dual Recycling

Laser interferometers employ two types of recycling: power recycling and signal recycling [46]. Power recycling enhances laser power within the interferometer without increasing laser source power or sacrificing bandwidth, thereby suppressing shot noise. Signal recycling increases the storage time of gravitational-wave-modulated signal sidebands within the interferometer, enabling enhancement of specific frequency bands [47].

When the interferometer’ s antisymmetric port is at a dark fringe, the symmetric port (where laser enters) is at a bright fringe, causing most laser energy to return to the source. Placing a mirror at the symmetric port redirects this energy back into the interferometer. A Michelson interferometer operating at a dark fringe can be equivalently modeled as a compound mirror; the power recycling mirror (PRM) combines with this equivalent mirror to form another resonant cavity. Combined with FP cavities, this technique increases stored laser power in the arms by approximately 8,000 times compared to a simple Michelson interferometer [17]. From equation (1), when laser power increases by ΔP, shot noise decreases by $\sqrt{\Delta P}$, achieving shot noise suppression. However, this improvement is limited by mirror absorption, reflection losses, and wavefront distortion [46], resulting in slightly reduced practical gains. In the initial LIGO design, actual shot noise exceeded theoretical values by about $3 \times 10^{-24}/\sqrt{\text{Hz}}$ [17].

Next-generation interferometers led by aLIGO have adopted a new recycling scheme: signal recycling [18]. A signal recycling mirror (SRM) placed at the interferometer’ s antisymmetric port reflects gravitational-wave-modulated signal sidebands back into the interferometer. This approach reduces requirements for mirror perfection and improves detection efficiency [18]. The SRM also reduces carrier light loss at the antisymmetric port [47]. When carrier frequency mismatches the signal recycling cavity frequency, signal recycling creates correlation between shot noise and radiation pressure noise, enabling the interferometer to surpass SQL limits and achieve sensitivities half the SQL in specific frequency bands [48].

3.3.3 Mode Cleaners and DC Readout

Mode cleaners are specially designed optical resonant cavities that filter out higher-order mode components unnecessary for gravitational wave detection while preserving the TEM_{00} mode [49]. In first-generation interferometers, an input mode cleaner (IMC) at the interferometer input filtered higher-order modes and high-frequency noise from the laser source [17]. In second-generation interferometers led by aLIGO, an output mode cleaner (OMC) was added at the antisymmetric port to implement DC readout. The OMC eliminates non-TEM_{00} components generated by resonant arm cavities and RF sideband control signals modulated onto carrier light for interferometer control [17;50], reducing unwanted signals reaching photodetectors and dramatically improving signal-to-noise ratio [18]. In practice, OMC parameters including finesse, length, and Gouy phase shift must be tuned to approach ideal performance, limiting shot noise contribution to no more than 5% [50].

Gravitational-wave-modulated optical sidebands have frequencies around 10^{14} Hz [51], making direct photodetector readout impossible without an additional local oscillator for demodulation. First-generation interferometers employed heterodyne detection by adding an RF sideband to the laser and introducing a few-centimeter arm length offset (Schnupp asymmetry) [52] to allow RF sidebands to reach the output port as a local oscillator. This method enables measurement of all quadrature components and optimal sensitivity across the full band [53], but couples various error sources, significantly reducing interferometer sensitivity. Second-generation interferometers universally adopt DC readout [18;44;54] by slightly offsetting the interferometer from the dark fringe, allowing a small fraction of carrier light to reach the antisymmetric port for direct photodetector readout [51]. This readout method is severely affected by laser power stability, which prevented its adoption in first-generation interferometers. DC readout improves shot-noise-limited signal-to-noise ratio [51;53], reduces coupling of other noise sources [51;55], and simplifies photodetector design [51]. Additionally, this approach creates conditions for subsequent introduction of squeezed light fields.

3.3.4 Squeezed Light States

In next-generation interferometers, most noise sources are effectively suppressed, with sensitivity primarily limited by the quantum nature of light, reaching or approaching the SQL [29]. Increasing interferometer laser power suppresses shot noise, but excessive power affects optical components and increases radiation pressure noise [56]. Effective shot noise suppression requires introducing squeezed light fields.

In quantum mechanics, optical fields can be described by quadrature phase and amplitude operators, which are linear combinations of creation and annihilation operators forming a set of orthogonal components. The uncertainties of these two components obey the Heisenberg uncertainty principle. In vacuum and coherent states, both components exhibit identical fluctuations; in squeezed states, fluctuations in one component are suppressed while the other increases, maintaining the uncertainty product [57]. The quadrature diagram [58] provides clearer understanding of squeezed states. In [Figure 8: see original paper], the ellipse formed by dashed lines represents the "uncertainty cloud" from optical fluctuations, with E_1 and E_2 denoting quadrature amplitude and phase fluctuations, respectively. Mirror displacement caused by gravitational waves manifests as phase changes in light, aligning with the quadrature phase direction (E_{GW}). Radiation pressure couples quadrature amplitude fluctuations into phase changes through mirror motion, creating additional fluctuations in this direction (E_{RP}) [21;58]. Squeezed light fields suppress one quadrature component, such as quadrature phase, reducing its fluctuations and improving signal-to-noise ratio. However, according to the uncertainty principle, quadrature phase squeezing inevitably increases quadrature amplitude fluctuations. Due to radiation pressure noise, quadrature amplitude coupling increases noise in the quadrature phase, resulting in degraded low-frequency sensitivity. Appropriate squeezing angle adjustment is necessary: quadrature amplitude squeezing suppresses radiation pressure noise at low frequencies, while quadrature phase squeezing addresses shot noise at high frequencies. By implementing frequency-dependent squeezing, squeezed light fields can improve interferometer sensitivity across a wide frequency band [59]. Detailed descriptions of squeezed light fields can be found in references [21, 58].

If the coherent amplitude of photons in a squeezed state is zero, it is called a squeezed vacuum state [60]. By injecting squeezed vacuum at the antisymmetric port, the squeezed state overlaps with the gravitational wave signal field and becomes entangled with it [29], creating correlations between light in the two interferometer arms that cancel quantum noise between them. This method reduces quantum noise impact; higher squeezing factors produce more pronounced effects [61]. A 3 dB squeezing factor doubles shot-noise-limited signal-to-noise ratio, while a 10 dB factor produces effects equivalent to a tenfold increase in laser power. However, high squeezing factors are constrained by optical losses in the system. In GEO600, 10 dB of squeezed light yielded only 3.5 dB of effective squeezing after passing through the optical system [57], while LIGO achieved only 2.2 dB from 10.3 dB of injected squeezing [62]. Improving noise reduction requires both increasing squeezing factors and reducing system optical losses.

Currently, squeezed states are generated through nonlinear optical processes. [Figure 9: see original paper] shows a typical squeezed state generation device, where red light represents the original laser, green light is the second harmonic, and dashed lines indicate the generated squeezed state. A laser beam at the interferometer frequency undergoes spatial mode filtering, is converted to half its original wavelength by a second harmonic generator (SHG), and then produces squeezed states via an optical parametric amplifier (OPA). Both SHG and OPA are made from MgO:LiNbO$_3$ crystals with dielectric back-surface coatings that form resonant cavities with additional mirrors.

The squeezed states described above are frequency-independent. Frequency-independent squeezing only improves sensitivity in specific frequency bands and may degrade sensitivity elsewhere [58] because squeezing-induced shot noise suppression increases radiation pressure noise. To achieve sensitivity improvement across broader frequency bands, frequency-dependent squeezing is required—where the squeezed quadrature component rotates with frequency [64] (see [Figure 10: see original paper]), matching the squeezing angle to frequency for full-band effectiveness. Current nonlinear optical squeezing generation produces frequency-independent states; obtaining frequency-dependent squeezing requires passing squeezed light through detuned FP cavities [63], known as "filter cavities" [28]. Squeezing rotation can be performed either before entering the interferometer (input filtering) or after leaving it (output filtering or variational readout). Studies show no significant difference in optical loss between these methods [63], though output filtering is more sensitive to optical losses [65].

Frequency-dependent squeezing remains experimental and has not yet been implemented in interferometers, though several teams have included it in future upgrade plans [18;66]. Introducing squeezed light fields will substantially improve interferometer sensitivity, enhancing the ability to detect weak gravitational wave signals.

4 Space-based Gravitational Wave Detection

Space-based gravitational wave detection also employs laser interferometry. Due to large separations between satellites in space, space-based detection enables long baselines: Japanese DECIGO plans 1,000 km arms [5], China' s TianQin plans 10^5 km arms [3], while other missions including LISA and TaiJi plan 10^6 km arms [31;67]. Since optimal interferometer frequency is inversely proportional to arm length [31], space-based detection operates at lower frequencies than ground-based interferometers. Compared to ground-based detectors like

LIGO, space-based gravitational wave detection can observe a wider variety of astrophysical targets [5;67], holding significant scientific importance. Multiple space-based interferometer arrays are expected to be operational around 2030, enriching our understanding of the universe and opening new windows for gravitational wave astronomy.

4.1 Principles of Space-based Gravitational Wave Detection

Space-based gravitational wave detection also uses laser interferometry to measure gravitational waves. Current space missions employ a three-satellite equilateral triangle formation (see [Figure 11: see original paper]), with laser links established between each pair of satellites. Each link terminal features heterodyne interferometry equipment transmitting several watts of 1,064 nm laser light (515 nm for DECIGO) [3;5;31;67]. Received laser power is only a few hundred picowatts, making direct reflection impractical. Instead, phase-locking technology is used: the receiving satellite actively transmits a laser phase-locked to the received beam with a slight frequency offset, enabling the transmitting satellite to recover phase difference information and measure relative distance changes between the two satellites. Laser links are bidirectional, with this process occurring in both directions for each satellite pair.

The light received by satellites does not directly interfere with light from the other arm. Instead, displacement information between satellites is recorded and later processed to synthesize a "virtual" Michelson interferometer by adding phase information from both arms [68]. This method combines four laser links into one Michelson interferometer: two transmitted to the other satellites and two received from them. In LISA, laser interferometry between three satellites is synthesized into two virtual Michelson interferometers and one Sagnac interferometer [67]. The two Michelson interferometers operate independently, enabling simultaneous measurement of two gravitational wave polarization components. Since the Sagnac interferometer is insensitive to gravitational wave signals, it characterizes the interferometer' s background noise level.

During orbital motion, the triangular formation experiences continuously changing inter-satellite distances, but this variation occurs over months and thus has negligible impact in the target mHz band. Furthermore, as the three-satellite formation orbits the Sun with a non-fixed orbital plane, the interferometer can localize gravitational wave sources, facilitating electromagnetic counterpart searches [67].

4.2 Drag-free Control in Space-based Detection

Besides gravitational forces, various other forces act on satellites in space, including solar wind and magnetic fields, which disturb satellite motion and prevent free-fall conditions, interfering with observations [69]. Typically, displacements caused by external disturbance forces far exceed those from gravitational waves, making detection nearly impossible [5]. Deng et al. [69] proposed drag-free control methods where satellites track test mass motion. Since satellites protect test masses and isolate other forces, test masses experience only gravitational forces and remain in free-fall states [69], enabling gravitational wave detection through relative displacement measurement.

The core principle of drag-free control involves measuring relative displacement between quasi-free-falling test masses and satellites, then controlling micro-Newton thrusters to counteract external disturbances. Drag-free control operates in two primary modes: displacement mode and accelerometer mode [69;70]. Since laser ranging equipment is mounted on satellites, drag-free control precision directly determines laser ranging precision and thus gravitational wave detection sensitivity. To eliminate other force effects, satellite internal crosstalk and other noise sources must be suppressed [71]. Cosmic rays may charge test masses, causing interference from interstellar magnetic fields, necessitating UV illumination systems to neutralize test mass charges [72].

Drag-free technology was first proposed by Lange in 1964 and has been implemented in multiple scientific satellites. Currently, the satellite with the highest drag-free control precision and performance is ESA' s LISA Pathfinder (LPF), achieving control precision of 5.0×10^{-15} $\mathrm{m \cdot s^{-2}/\sqrt{Hz}}$ in the detection direction [71], meeting gravitational wave detection requirements. [Figure 12: see original paper] illustrates LPF' s drag-free control implementation.

LPF' s drag-free control system differs from traditional drag-free satellites. LPF carries two test masses corresponding to two laser links. Each test mass is a gold-platinum alloy cube placed in an electrode housing that provides electrostatic shielding and control [72]. Since the two test masses cannot simultaneously follow satellite motion via drag-free control, one test mass is controlled drag-free while the other is electrostatically controlled to follow the satellite [71].

4.3 Noise in Drag-free Control

Drag-free control noise primarily originates from displacement measurement noise, control force noise, and environmental noise [73]. Displacement measurement noise mainly arises from thermal effects in circuits, while circuit instabilities may amplify displacement signals from electrodes, causing measurement inaccuracies. Control force noise depends on control current stability, affecting the stability of electrostatic forces applied to test masses. Environmental noise includes test mass charging and thermal gradient effects. Thermal gradient fluctuations between electrodes disturb test masses through thermal radiation and outgassing effects. LPF uses low thermal conductivity materials to successfully suppress thermal gradient noise within requirements.

Drag-free control uses micro-thrusters as actuators; thrust noise in the measurement band must also meet requirements [69]. Besides cold gas thrusters, LPF carries eight colloid thrusters provided by NASA, achieving noise levels of 0.1 $\mathrm{N/\sqrt{Hz}}$ in the target band [74]. In low Earth orbit, larger disturbances require higher-thrust thrusters (such as cold gas thrusters) to meet control demands

[69].

In LPF, noise below 1 mHz is dominated by star trackers [71]. LPF' s star trackers primarily provide attitude control signals. In LISA, attitude sensing is accomplished through differential wavefront sensing of laser beams [67], reducing attitude control noise by four orders of magnitude to 10^{-8} rad/$\sqrt{}$Hz [71]. Above 1 mHz, noise is dominated by test mass displacement measurement noise and control force noise [71].

4.4 Noise Suppression in Space-based Laser Interferometry

Similar to ground-based laser interferometers, space-based gravitational wave detection is also limited by quantum noise. As previously mentioned, inter-satellite laser power is low, making shot noise the dominant noise source [75]. Due to satellite platform limitations, increasing mirror power or laser power raises mission complexity and reduces satellite lifetime and stability. Additionally, implementing inter-satellite FP cavities is impractical given the large distances; only DECIGO plans to employ FP cavities [5]. As arm length increases, the balance frequency between shot noise and radiation pressure noise shifts lower, moving overall noise toward low frequencies [5]. LISA' s designed shot noise is approximately 4.7 pm/$\sqrt{}$Hz [67].

Another significant noise source in space-based detection is laser frequency noise or phase noise. Laser frequency noise levels are typically 12-15 orders of magnitude higher than LISA' s shot noise [76], representing a major noise contributor. LISA employs PDH methods [77;78] similar to ground-based interferometers for frequency pre-stabilization. Since LISA' s arm length changes occur at frequencies far below the measurement band and are negligible within it, arm-locking technology can lock laser frequency to interferometer arm length for further stabilization (see [Figure 13: see original paper]). After pre-stabilization and arm-locking, laser frequency noise can be suppressed to 10^{-4} Hz/$\sqrt{}$Hz [76], meeting subsequent data processing requirements [77]. Finally, time-delayed interferometry (TDI) [79] processing techniques suppress residual frequency noise [67].

[Figure 13: see original paper] illustrates the arm-locking principle. Similar to PDH techniques that lock laser frequency to FP cavity resonance, arm-locking exploits LISA' s relatively stable arm lengths to lock laser frequency to the interferometer arm length. Considering Doppler shifts, comparing current laser phase with phase delayed by τ in the interferometer arm yields frequency noise signals that are fed back to the laser controller for stable phase [76]. To address long transients and frequency-limited control gains in arm-locking, Sutton and Shaddock [80] proposed dual arm-locking, utilizing unequal time constants τ between LISA' s adjacent arms to suppress initial transients, reducing time constants from 10^6 s to 10 s and concentrating transient energy at high frequencies while eliminating zero-frequency points within LISA' s detection band.

Constrained by clock noise, satellite displacement noise, and shot noise in arm-locking control, practical applications can suppress laser frequency stability to only 10^{-2} [77], still far exceeding the 10^{-6} Hz/√Hz required for gravitational wave detection [81]. TDI techniques are therefore necessary for signal processing to reduce frequency noise effects.

Building upon first- and second-generation TDI, Vallisneri et al. [82] proposed third-generation TDI, addressing limitations in earlier versions regarding arm length variations. TDI’ s core concept involves time-shifting and combining measurements so that laser frequency noise between arms cancels [83]. Using first-generation TDI as an example (see [Figure 14: see original paper]), the phase measurement expression for a satellite is:

$$y_i(t) = h_i(t) + C(t - 2L_i) - C(t) + n_i(t)$$

where $i = 1$, 2 indexes adjacent interferometer arms, $y\_i(t)$ is the laser phase measurement at time t, $h\_i(t)$ is the gravitational wave contribution, $C(t$ - $2L\_i)$ represents frequency noise at laser emission, $2L\_i$ is the round-trip light travel time in the corresponding arm, and $n\_i(t)$ is phase readout noise. The middle two terms represent the difference between laser frequency noise at emission time t - $2L\_i$ and current time t.

Subtracting measurements from both arms yields a new quantity $\Delta_1(t)$:

$$\Delta_1(t) \equiv y_1(t) - y_2(t) = h_1(t) - h_2(t) + C(t - 2L_1) - C(t - 2L_2) + n_1(t) - n_2(t)$$

Delaying $y_1(t)$ by arm 2’ s round-trip time and $y_2(t)$ by arm 1’ s round-trip time yields combined quantity $\Delta_2(t)$:

$$\Delta_2(t) \equiv y_1(t-2L_2)-y_2(t-2L_1) = h_1(t-2L_2)-h_2(t-2L_1)+C(t-2L_1)-C(t-2L_2)+n_1(t-2L_2)-n_2(t-2L_1)$$

Noting that $\Delta_1(t)$ and $\Delta_2(t)$ contain frequency noise in identical form, subtracting them yields $\Sigma(t)$:

$$\Sigma(t) \equiv \Delta_2(t)-\Delta_1(t) = h_1(t-2L_2)-h_1(t)-h_2(t-2L_1)+h_2(t)+n_1(t-2L_2)-n_1(t)-n_2(t-2L_1)+n_2(t)$$

Equation (5) shows complete frequency noise cancellation. This derivation assumes constant inter-satellite distances, limiting first-generation TDI practicality [82]. Second-generation TDI accounts for varying arm lengths but only applies to linear or slowly varying distance changes [82;84]; nevertheless, it sufficiently suppresses frequency noise below LISA requirements [82;85;86].

TDI implementation also requires absolute inter-satellite distance measurement and clock synchronization for subsequent data processing. LISA’ s satellite clocks require precision better than 3 ns, with inter-satellite synchronization better than 1 ns [67]. Absolute distance measurement uses pseudo-random codes [87;88]: modulating a pseudo-random code onto the laser allows the receiving satellite to determine laser emission time and, using onboard clocks, calculate light propagation time between satellites—the absolute inter-satellite distance. Modulation signals typically operate in the MHz band to avoid interfering with gravitational wave detection [87;89]. This method achieves root-mean-square distance measurement errors of 0.19 m [87], meeting LISA design specifications.

5 Summary

Gravitational wave detection has achieved remarkable results, not only confirming the existence of gravitational waves but also discovering previously unobserved compact object merger events [13]. Over decades of development, gravitational wave detection technology has evolved from early resonant mass bars to today' s precision laser measurements, achieving dramatic sensitivity improvements from 10^{-16} to 10^{-23} [7;18]. Noise suppression technology is an indispensable component of gravitational wave detection; aLIGO detected gravitational waves shortly after implementing new suspension systems, signal recycling, and DC readout [90]. With Japan' s KAGRA interferometer beginning operations in 2020 [91], a global network of laser interferometer gravitational wave detectors has been established. Current detectors have ongoing upgrade plans [92], such as implementing frequency-dependent squeezed light fields and cryogenic mirrors similar to KAGRA. Next-generation interferometer proposals including Europe' s Einstein Telescope [93] and America' s Cosmic Explorer [94] aim to further improve sensitivity to 10^{-24}. These future interferometers will feature longer arms, higher laser power, and lower thermal noise to achieve extreme sensitivity [6]. Undoubtedly, existing noise suppression technologies must be further optimized and enhanced to enable such precise measurements.

Space-based gravitational wave detection represents another major future direction. Space-based detection enables observation of low-frequency gravitational wave signals from important astrophysical sources [67]. The complex space environment and satellite payload constraints require specialized noise suppression techniques different from ground-based interferometers to achieve comparable sensitivity. Technologies including arm-locking [76], TDI [83], and drag-free control [95] have matured over decades and successfully demonstrated space-based detection feasibility on LPF [71;74]. Upon completion of space-based gravitational wave detectors in 2030, they will undoubtedly open new windows for cosmic observation.

Gravitational wave detection not only advances fundamental sciences like astronomy but also enhances national engineering capabilities and comprehensive strength. The extremely low noise requirements for gravitational wave detection will drive domestic technological advancement, playing a crucial role in breaking foreign technology monopolies and developing independent technological industries.

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