Post-print of Gravitational Wave Detection of Black Hole Mergers

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Abstract

Globular clusters and galactic nuclei host extremely dense populations of stellar-mass black holes. When two black holes approach each other, they form highly eccentric orbits and emit gravitational waves. The detection of such signals will contribute to further research in black hole physics and enhance our understanding of black hole populations. However, such gravitational waves have not yet been detected and have not been widely considered in current research. Therefore, studying their detectability is important. Based on numerical relativity simulations, recent studies have provided accurate waveforms for parabolic black hole encounters. Using the gravitational-wave search tool Minke, we constructed a large-scale dataset in the two-dimensional parameter space of total mass and distance, and quantitatively calculated the detection range. The results indicate that gravitational waves from black hole encounters are likely potential detection targets for current ground-based detectors, while next-generation gravitational-wave detectors significantly expand the detection range. Finally, we discuss the hot topic of how to identify merger signals as black hole capture. By utilizing the posterior distributions of signals under two samplers, we propose a discrimination method. Through comparing different posteriors, we find that binary black hole mergers and black hole capture signals exhibit significant differences in the merger time t0, which will help distinguish between these two types of sources in merger samples.

Full Text

The Detection of Gravitational Waves from Black Hole Encounters

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Abstract

Globular clusters and galactic nuclei harbor extremely dense populations of stellar-mass black holes. When two black holes approach each other, they can form highly eccentric orbits and emit gravitational waves. Detection of such signals would advance research in black hole physics and enhance our understanding of black hole populations. However, these gravitational waves have not yet been observed and have not been widely considered in current studies. Therefore, investigating their detectability is essential. Based on numerical relativity simulations, recent research has provided accurate waveforms for parabolic black hole encounters. Using the gravitational-wave search tool Minke, we have generated a large-scale dataset across the two-dimensional parameter space of total mass and distance, quantitatively calculating detection ranges. Our results indicate that gravitational waves from black hole encounters are likely detectable by current ground-based detectors, while next-generation detectors will significantly expand the detection horizon. Finally, we address the critical question of how to distinguish black hole capture signals from binary black hole mergers. By utilizing posterior distributions of signals under two different samplers, we propose a discrimination method. Comparing these posteriors reveals significant differences in merger time t_0 between binary black hole merger and black hole capture signals, which will aid in distinguishing these two sources among detected merger events.

Keywords: gravitational waves; stellar-mass black holes; binary black hole systems

1 Introduction

In 1915, Einstein published the theory of general relativity, predicting the existence of gravitational waves [1] and initiating a century-long quest for their detection. This endeavor achieved a breakthrough in September 2015 when the ground-based laser interferometer LIGO [2] observed GW150914 [3], a gravitational-wave signal from a binary black hole merger, providing a new observational window for astrophysics and cosmology. This marked humanity' s first direct detection of gravitational waves, yet the mission of LIGO and other detectors remains far from complete. They will continue to detect and interpret signals, ushering in the era of gravitational-wave astronomy, which brings the challenge of extracting countless signals from noise and gaining deep insights into the astrophysical mechanisms that produce them.

Over the past few years, the number of detected compact binary coalescence events has increased dramatically. The LIGO-Virgo [4] detector network' s Gravitational-Wave Transient Catalogs 1 [5] and 2 [6] contain a total of 55 gravitational-wave events. In the recently published GWTC-3 [7], the collaboration identified 35 additional events, 32 of which are most likely binary black hole mergers [8], bringing the total number of detected events since 2015 to 90.

Binary black hole systems have thus become the most important gravitational-wave sources, sparking intense interest and debate about their formation channels. The prevailing view holds that there are two primary formation pathways for stellar-mass binary black hole mergers: isolated evolution of massive binary stars [9-12] and formation through dynamical interactions between stellar-origin black holes in dense environments [13]. The latter can be subdivided into several scenarios [14-17], one of which—black hole encounters [18]—has not been widely considered in current searches.

Dense stellar environments in globular clusters and galactic centers provide active arenas for black hole interactions. Various gravitational processes occur in these regions: some black hole binaries form bound orbits, emitting periodic gravitational waves with three distinct phases—inspiral, merger, and ringdown. During inspiral, gravitational radiation carries away energy, circularizing the orbit and evolving into a binary system with circular or low-eccentricity elliptical orbits that eventually merge. These systems are called binary black hole mergers. Other black hole binaries form unbound orbits, leading to possible scattering or merger events. The condition for unbound orbits is $E_{\mathrm{orb}} > 0$, where E_{orb} is the orbital energy of the binary system. We refer to black hole binaries in unbound orbits as black hole encounters, which emit strong burst-type gravitational waves. Compared to binary black hole mergers, their waveforms lack an inspiral phase and exhibit very significant eccentricity when entering the sensitive band of LIGO detectors (90% with eccentricity $e > 0.9$), allowing them to be distinguished physically from the former. For small trajectory deflection angles, this process can be considered analogous to bremsstrahlung in electromagnetic radiation [19,20], but gravitational radiation becomes more complex when considering larger deflection angles and spins. Approximate models have been proposed for black hole encounter waveforms in the bremsstrahlung regime, applicable to low-velocity cases with arbitrary orbital deflection angles [21] and head-on collisions of black holes [22]. Based on orbital energy conditions, numerical relativity modeling can classify black hole encounter waveforms into hyperbolic ($E_{\mathrm{orb}} > 0$) and parabolic ($E_{\mathrm{orb}} = 0$) types. Recent advances in numerical simulations have enabled the generation of 3.5 PN waveforms for hyperbolic encounters of non-spinning black hole binaries [23] and accurate waveforms for parabolic encounters [24]. Furthermore, when two black holes approach each other, if the energy carried away by gravitational wave radiation exceeds the orbital energy, the system will still form a bound orbit and produce a merger event. This subset of black hole encounters is called black hole capture. Conversely, the binary will be scattered, approaching each other through gravitational interaction before separating permanently.

Regarding such sources, beyond the waveform simulations mentioned above, existing research has focused on calculating their energy spectra [25,26] and estimating detection rates. The detection rate of black hole encounter events can constrain the initial mass function of stars and the mass of central black holes in galactic nuclei [18]. Several studies [27-29] have estimated the per-unit-distance detection rates for such events in Advanced LIGO and LISA, all yielding results on the order of ~ 1 Gpc^{-3} yr^{-1}. However, these estimates lack calculations of the detection range for gravitational-wave signals.

This paper employs parabolic black hole encounter waveforms to generate simulated datasets for both merger and non-merger scenarios and investigates detection ranges across multiple detectors. The primary motivation is to study the detectability of black hole encounter gravitational waves by the latest ground-based and space-based detectors. Due to the lack of suitable templates and search algorithms for black hole encounters, no such events have been detected and confirmed during the first three observation runs of the Advanced LIGO, Advanced Virgo, and KAGRA detector network. Future detectors such as LISA, DECIGO, and the Einstein Telescope (ET) will join the network, contributing observations with higher signal-to-noise ratios and extending the detection band to lower frequencies and broader ranges, making the study of detectability for such events highly significant.

Another important question regarding the merger waveforms from this source class is how to correctly distinguish them from detected high-mass binary black hole merger signals [30]. As previously mentioned, typical binary black hole merger gravitational waves have three phases—inspiral, merger, and ringdown—with waveform shapes very different from burst-like black hole capture gravitational waves. However, in high total mass scenarios, the inspiral phase of the former lies below the cutoff frequency of LIGO detectors and is not reflected in the recorded data, making these two event types have similar waveforms that may be misclassified in practice. A typical example is GW190521 [31,32], emitted by a binary black hole system with component masses of 85 $M_\odot$ and 66 $M_\odot$, merging to produce an intermediate-mass black hole of 150 $M_\odot$. Multiple studies [33] suggest that its orbit may have high eccentricity, sparking interest in its formation mechanism. Recent research has demonstrated that analyzing black hole capture with the mainstream circular-orbit binary black hole merger template IMRPhenomPv2 [34,35] can still yield Bayesian inference results similar to those for high-mass binary black hole mergers [30], posing a challenge for correct identification and classification of gravitational-wave events.

This paper introduces black hole encounters as a potential gravitational-wave source expected to have considerable detection rates in next-generation ground-based and space-based detectors. Using numerical relativity simulation waveforms, we create large-scale gravitational-wave datasets for this source class across multiple parameter spaces, then reveal the relationship between detection range, total mass, and luminosity distance through signal-to-noise ratio calculations, discussing their detectability. We also compare the detection performance of the next-generation Einstein Telescope with Advanced LIGO. Finally, to address the potential confusion between black hole encounters and high-mass binary black hole mergers in practical data analysis, we propose a method to distinguish them. For the two posterior probability distributions of black hole encounter signals, we calculate their differences on several main parameters and measure the magnitude using JS divergence, then compare them with baseline values from binary black hole mergers.

The structure of this paper is as follows. Section 2 describes the method for generating large-scale black hole encounter gravitational-wave waveforms, using the latest ten numerical relativity waveforms (five merger waveforms and five non-merger waveforms) to create simulated datasets and calculate their signal-to-noise ratios and detectability. Section 3 proposes a method to distinguish binary black hole mergers from black hole encounters by comparing posterior distributions from two samplers, analyzing four parabolic black hole capture gravitational-wave waveforms. Section 4 presents our conclusions and outlook.

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2 Detectability Estimation

Black hole encounters represent an as-yet undetected class of gravitational-wave sources. In this section, we use parabolic black hole encounter waveforms [24] to generate simulated data and study their detectability in current and future detectors.

2.1 Simulated Data Injection

Gravitational-wave detector strain $h(t)$ has two components, h_+ and $h_\times$:

$$h(t) = F_+(t)h_+(t) + F_\times(t)h_\times(t),$$

where F_+ and $F_\times$ are the antenna response functions corresponding to the two gravitational-wave polarization components, δ is the source declination relative to the detector plane, α is the azimuth angle relative to one arm, and ψ is the polarization angle of the gravitational wave. The sky location of the source thus affects both the waveform shape and amplitude, while the antenna response also reduces detector sensitivity to certain sky regions. To calculate the optimal signal-to-noise ratio, we select appropriate sky positions in our simulations to ensure the detector operates at peak sensitivity.

We use the Minke software package to generate simulated gravitational-wave signals. Minke is a Python-based gravitational-wave search tool that can produce large-scale datasets based on specified numerical relativity waveforms and injection parameter distributions. The process of generating simulated data with Minke proceeds as follows: (1) Specify numerical relativity waveforms. This study uses parabolic black hole encounter waveforms, including both merger and non-merger types, each further divided into five mass ratios (1, 2, 4, 8, 16), totaling ten waveforms. (2) Set distributions for injection parameters, including total mass of the black hole binary, source luminosity distance, sky location, source polarization, and other relevant parameters. (3) Call LALSimulation to generate simulated waveforms for each set of injection parameters. (4) Specify detectors and calculate the antenna response for each simulated waveform based on the detector’ s antenna pattern, obtaining the simulated signal set. Here we use Advanced LIGO, Einstein Telescope, and DECIGO. (5) Finally, inject detector noise into each gravitational-wave signal to obtain the simulated dataset.

We set the injected binary total mass to a uniform distribution with 50 points ranging from 5 $M_\odot$ to 150 $M_\odot$. Black holes can be categorized as stellar-mass, intermediate-mass, and supermassive, with known intermediate-mass black holes being extremely rare, creating a gap in the black hole mass distribution between stellar-mass and supermassive black holes. Due to detector frequency limitations, all observed merger events to date have originated from binary systems of two stellar-mass black holes. Therefore, we adopt an upper limit of 150 $M_\odot$ for the total mass, derived from GW190521, the most massive merger event observed to date. Within this mass range, individual black holes are stellar-mass black holes that are abundant in the universe, and the sources have indirect optical observations for cross-validation. We set the minimum injected luminosity distance to 1 Mpc and determine different maximum values for each waveform, also forming a uniform distribution of 50 injection values. The signal start GPS time is set to $t_s = 1126259642.0$ s.

We first inject signals into Advanced LIGO. Based on the aforementioned mass and distance distributions, each waveform yields 2,500 signal samples. Figure 1 [Figure 1: see original paper] shows the comparison between the characteristic strain spectrum of one example signal and detector sensitivity. This signal, denoted “black hole encounter,” originates from a black hole capture waveform with mass ratio $q = 1$, total mass of 150 $M_\odot$, and luminosity distance of 5,000 Mpc. Figure 2 [Figure 2: see original paper] displays one example signal for each of the ten black hole encounter waveforms, differing in binary mass ratio q and whether the binary ultimately merges. All ten waveforms have a total mass of 150 $M_\odot$ and a luminosity distance of 50 Mpc. Compared to non-merging black hole encounters, black hole capture exhibits unique waveform shapes with merger and ringdown phases.

2.2 Signal-to-Noise Ratio Limits Detection Range

The optimal signal-to-noise ratio ρ_{opt} of a gravitational-wave signal can be expressed as:

$$\rho_{\text{opt}} = 2\sqrt{\int_{f_{\min}}^{f_{\max}} \frac{|h(f)|^2}{S_h(f)} df},$$

where $h(f)$ is the frequency-domain representation of the gravitational-wave signal, $S_h(f)$ is the one-sided noise power spectral density in units of Hz^{-1}, and $f_{\min} \le f \le f_{\max}$ corresponds to the instrument’ s operational band (e.g., Advanced LIGO has $(f_{\min}, f_{\max}) \approx (10, 10^4)$ Hz). The optimal signal-to-noise ratio accounts for the noise power spectral density, meaning frequency bands with higher noise contribute less to the signal-to-noise ratio than quieter bands.

For each waveform, we calculate the signal-to-noise ratio for every signal using equation (5) and plot the results as shown in Figures 3 and 4. Figure 3 [Figure 3: see original paper] presents the signal-to-noise ratio contours for black hole capture waveforms as a function of luminosity distance and total mass. The standard signal-to-noise ratio threshold is 8, above which a signal is considered detectable and is plotted with a thicker line. Figure 3 clearly shows that gravitational waves from such a system can be detected out to 6,500 Mpc (for black hole binaries with mass ratio 1) under the assumption of a maximum total mass of 150 $M_\odot$, a value that may be higher when considering the sensitivity enhancement from joint observations. Meanwhile, binaries with mass ratios of 2, 4, 8, and 16 have maximum detection distances of 5,500, 1,800, 1,250, and 580 Mpc, respectively, with systems having mass ratios closer to 1 having larger detection ranges.

Figure 4 [Figure 4: see original paper] shows the signal-to-noise ratio contours for non-merging black hole encounter waveforms. Maximum detection distances are achieved within a total mass range of 20-60 $M_\odot$, with values of 57, 118, 115, 275, and 160 Mpc for black hole binaries with mass ratios of 1, 2, 4, 8, and 16, respectively. Therefore, black hole capture has a greater detection range in Advanced LIGO than non-merging black hole encounters and is more likely to be discovered.

When considering the detection capabilities of next-generation detectors for black hole encounter gravitational waves, we performed the same signal injection and signal-to-noise ratio analysis using the Einstein Telescope and DECIGO detectors as with Advanced LIGO. The results indicate that for black hole capture, the Einstein Telescope achieves a maximum detection distance of 5×10^4 Mpc for binary mass ratio $q = 1$. For non-merging black hole encounters, detection is possible out to 2,500 Mpc, corresponding to the $q = 8$ waveform. On the other hand, DECIGO has moderate detection efficiency for black hole capture, reaching a maximum of 3,000 Mpc, which is lower than the detection capability of currently operating Advanced LIGO. However, DECIGO has the largest detection range for non-merging black hole encounters, reaching up to 5,000 Mpc for waveforms with mass ratio $q = 1$ (as shown in Figure 5 [Figure 5: see original paper]).

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3 Distinguishing Black Hole Capture from Binary Black Hole Merger Using JS Divergence

Current parameter estimation work primarily employs the binary black hole merger template IMRPhenomPv2 for Bayesian inference. According to Bayesian theory, the posterior probability distribution can be expressed as:

$$p(x|y,H) \propto p(y|x,H)p(x,H),$$

where x represents model parameters, y is the observational data, H is the model, $p(x|y,H)$ is the posterior, $p(y|x,H)$ is the likelihood function, and $p(x,H)$ is the prior on these parameters. We omit a proportionality factor on the right side—the inverse of the Bayesian evidence $p(y)$—because $p(y)$ is constant and we are only interested in the shape of the posterior distribution. Such analyses exhibit degeneracy when inferring binary black hole mergers versus black hole captures [30]. We obtain posterior probability distributions for the main parameters of black hole capture and binary black hole merger events from a recent study [30], which performed parameter estimation using two samplers—dynesty [36] and VItamin [37]—under the zero-spin IMRPhenomPv2 approximation for binary black hole mergers. Dynesty employs nested sampling methods, while VItamin provides posterior estimates for binary black hole mergers based on deep learning techniques.

We propose a discrimination method based on JS divergence [38] to determine in actual detections whether a binary-black-hole-merger-like signal is a black hole capture. The fundamental idea is that a model yields better inference results for data that conform to its physical mechanism. Under the binary black hole merger model, such signals themselves can be inferred very accurately, while black hole encounter signals yield biased inferences. In the latter case, the inference results are merely mathematical and lack physical meaning when mapped to each parameter. Therefore, when comparing the biased inferences from two samplers, the posterior distributions for each parameter may differ significantly. For signals that conform to the physical mechanism, inferences from different samplers will converge. We quantify this difference using JS divergence. For an unknown signal with binary-black-hole-merger-like posteriors, the magnitude of JS divergence can indicate whether the signal originates from the physical mechanism of that model. This method provides a criterion for identifying unmodeled or poorly modeled transient signals like black hole capture among binary black hole merger events.

JS divergence is a measure of the difference between two distributions $p(x)$ and $q(x)$, defined as:

$$D_{\mathrm{JS}}(p\|q) = \frac{1}{2}[D_{\mathrm{KL}}(p\|s) + D_{\mathrm{KL}}(q\|s)], \quad D_{\mathrm{KL}}(p\|q) = \int p(x)\log_2\frac{p(x)}{q(x)}dx,$$

where $s = \frac{1}{2}(p+q)$ and D_{KL} is the KL divergence. JS divergence takes values in $[0,1]$, with larger values indicating greater differences between distributions.

Next, we consider the parameters selected for this analysis. The non-spinning circular-orbit binary black hole merger model has nine parameters: the two black hole masses (m_1, m_2), luminosity distance (d_L), sky position (α, δ), binary orbital inclination (Θ_{jn}), gravitational-wave polarization angle (ψ), merger time (t_m), and merger phase (ϕ_0). Selecting a reference time $t_{\text{ref}} = 1126259642.5$ s, the relative merger time is $t_0 = t_m - t_{\text{ref}}$. We perform divergence analysis on m_1, m_2, d_L, and t_0. For the posterior distributions output by dynesty and VItamin, we first randomly sample the posterior with more samples to match the number of samples in the smaller posterior. We then calculate and compare JS divergence for the distributions of these four parameters. We selected four parabolic black hole capture waveforms with mass ratios $q = 1, 4, 8, 16$ and one zero-spin binary black hole merger signal for this analysis. Due to space limitations, we only show the posterior distribution for the black hole capture waveform with mass ratio 1 in Figure 6 [Figure 6: see original paper], along with the binary black hole merger waveform as a comparison standard in Figure 7 [Figure 7: see original paper]. All JS divergence values are recorded in Table 1 .

Figures 6 and 7 show that VItamin produces sharper peaks for m_1 and m_2 sampling for both waveforms compared to dynesty. Additionally, dynesty and VItamin have very similar estimates for the merger time t_0 of binary black hole mergers, but not for black hole capture, where VItamin' s t_0 posterior peak is delayed by approximately 0.03 s relative to the true value and dynesty's posterior peak. This is more clearly reflected in Table 1 , with JS divergences of 0.05 and 0.22, respectively. Except for the waveform with mass ratio 16, other black hole captures show similar results: the VItamin sampler always tends to estimate t_0 as a bimodal posterior distribution with the main peak far from the true value, resulting in high JS divergence for t_0. The waveform with mass ratio 16 has smaller divergence in t_0 because its two posterior main peaks almost coincide. For other parameters, black hole capture does not show obvious distinction from binary black hole merger in terms of JS divergence.

Divergence analysis provides a criterion for distinguishing binary black hole mergers from black hole capture: when an unknown binary-black-hole-merger-like signal is detected, use dynesty and VItamin to obtain posterior distributions for merger time t_0 and calculate their JS divergence. If the value is much larger than that for binary black hole mergers (0.05), it likely indicates a black hole capture.

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4 Summary and Outlook

This paper discusses the detection of gravitational waves from parabolic black hole encounters. Our study has two main objectives: (1) to generate large batches of gravitational-wave signal samples across the parameter spaces of total mass and distance to evaluate the detection capabilities of Advanced LIGO and next-generation gravitational-wave detectors for multiple waveforms; and (2) to propose a discrimination method that addresses the posterior degeneracy between black hole capture and high-mass binary black hole mergers in practical data analysis by comparing two posterior distributions of signals.

The results of the first study show that, assuming a maximum binary total mass of 150 $M_\odot$, current Advanced LIGO can achieve a detectable signal-to-noise ratio for black hole encounters out to 6,500 Mpc. Notably, even the smallest detection range among different waveforms reaches 57 Mpc, exceeding the diameter of the Local Supercluster, making such events very promising gravitational-wave source candidates. Advanced LIGO achieves maximum detection range under different conditions for merger versus non-merger waveforms. For black hole capture, larger total mass yields greater detection distance, while non-merging black hole encounters achieve maximum detection distance at total masses of 20-60 $M_\odot$. We also injected waveforms into the Einstein Telescope and DECIGO to compare the detection capability improvements of next-generation detectors. The Einstein Telescope will greatly increase the maximum detection distance for black hole capture, reaching up to 5×10^4 Mpc, while DECIGO will extend the detection range for non-merging black hole encounters to 5,000 Mpc. In Section 3, we proposed a JS divergence-based discrimination method to address the misclassification problem of black hole encounters in parameter estimation work. We compared posterior distributions from two samplers, VItamin and dynesty, using data from a previous study [30] including four black hole capture waveforms and one binary black hole merger waveform. The analysis revealed that black hole capture has much higher JS divergence in merger time t_0 than circular-orbit binary black hole mergers, which will help distinguish these two gravitational-wave sources among detected merger samples.

According to a recent estimate, the event rate for black hole encounters is approximately 0.9 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$ [29]. The detection ranges calculated in this paper suggest that the event rate for black hole encounters may be quite substantial, making their search an effective utilization of existing observational resources that will continue with next-generation detectors. To extract accurate signals from gravitational-wave data, we require more precise relativistic simulation waveforms, which may facilitate these searches. For the second study, we will conduct more tests and validations based on this work. First, due to computational resource limitations, the number of events used is currently small; future work will consider more simulated waveforms for merger time t_0 analysis, such as extending binary black hole merger waveforms to different mass ratios and injecting black hole capture waveforms with different total masses. Second, the posterior distributions used in this paper come from the zero-spin binary black hole merger model IMRPhenomPv2, which can be improved in three aspects: spin, orbital eccentricity, and precession. Black hole binaries formed through dynamical capture likely have isotropic spin orientations [39], a feature not present in binaries formed through isolated massive star evolution; thus, spin analysis is important for distinguishing such events. Additionally, this model uses single-spin post-Newtonian precession to describe precession effects, while newer models allow for double-spin precession. The model is also limited to circular-orbit binary black hole systems, so we could consider introducing the eccentric-orbit SEOBNRE model in the next step. Although its modeling can-not reach the ultra-high orbital eccentricities of these sources, it is expected to provide better fitting than circular-orbit models. Third, mainstream parameter estimation work commonly uses multiple samplers, including CPnest [40] and emcee [41] in addition to the two mentioned here. Testing their posterior sampling performance for the same waveforms will help improve and refine the models. We will conduct in-depth research on these aspects in future work.

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