Advances in Three-Dimensional Reconstruction Methods for Coronal Mass Ejections: Postprint

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Abstract

Coronal Mass Ejection (CME) is a large-scale magnetized plasma cloud ejected from the Sun, carrying substantial magnetic flux and plasma into interplanetary space. CMEs that enter interplanetary space are also referred to as Interplanetary CMEs (ICMEs). When propagating toward Earth and arriving in its vicinity, they interact with the magnetosphere, generating geomagnetic storms and other space weather phenomena. The two-dimensional data provided by existing observational results cannot fully characterize the true magnetic field structure and plasma distribution of CMEs. To better predict the arrival time of ICMEs at Earth and their potential effects on Earth and its surrounding environment, understanding the three-dimensional structure of CMEs and the three components of their velocity is essential. This study introduces methods for the three-dimensional reconstruction of CMEs based on existing imaging observations, encompassing two categories of reconstruction approaches utilizing coronagraph data and heliospheric imaging data, as well as three-dimensional reconstruction methods for CME-driven shocks that are closely associated with CME imaging reconstruction. Each method is applicable to CME events with distinct characteristics and possesses its own limitations and required constraint conditions. By comparing results obtained from several different reconstruction methods, we find that the estimated propagation speeds and directions of CMEs from these methods are relatively consistent, demonstrating their high reliability. Finally, current hot topics and future development trends in CME three-dimensional reconstruction are discussed.

Full Text

Preamble

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Abstract

Coronal mass ejections (CMEs) are large-scale magnetized plasma structures ejected from the Sun, carrying substantial magnetic flux and plasma into interplanetary space. When a CME propagates Earthward and reaches near-Earth space, it interacts with the magnetosphere, producing geomagnetic storms and other space weather phenomena. Existing observational data provide only two-dimensional information, which cannot fully describe the true magnetic structure and plasma distribution of CMEs. To better predict the arrival time of interplanetary CMEs (ICMEs) at Earth and their potential impacts on Earth and its surrounding environment, it is essential to understand the three-dimensional structure of CMEs and all three components of their velocity. This paper introduces methods for three-dimensional reconstruction of CMEs based on existing imaging observations, including two categories: reconstruction methods based on coronagraph data and heliospheric imager data, as well as CME-driven shock wave reconstruction methods closely related to CME imaging reconstruction. Each method is suitable for CME events with different characteristics, and each has its own limitations and required constraints. By comparing results obtained from several different reconstruction methods, we find that the estimated propagation speeds and directions are relatively consistent across methods, demonstrating their high reliability. Finally, we discuss current hot topics and future development trends in CME three-dimensional reconstruction.

Keywords: corona; coronal mass ejection; three-dimensional reconstruction Classification: P182.6

1 Introduction

Coronal mass ejections (CMEs) result from the disruption of large-scale magnetic field structures in the solar corona, during which magnetized plasma (10^{16} g) carrying enormous magnetic flux (10^{21}–10^{22} Mx) is rapidly ejected outward, releasing substantial energy (10^{25} J). CME speeds typically range from 200 to 3,000 km/s, with an average of approximately 500 km/s [1]. CMEs exhibit various shapes, including cloud-like, fan-like, and loop-like structures. The most typical CMEs possess a three-part structure consisting of a bright front, a dark cavity, and a bright core [2]. Figure 1 [Figure 1: see original paper] shows a CME with this typical three-part structure observed by the Large Angle Spectrometric Coronagraph (LASCO) [4] onboard the Solar and Heliospheric Observatory (SOHO) [3]. Although this structure is frequently observed, current visible-light observations represent only a two-dimensional projection of the three-dimensional structure onto the sky plane, with different projections obtained from different viewing angles. Consequently, the true three-dimensional structure of the three-part CME remains unclear.

Since CMEs are large-scale magnetized plasma structures from the Sun, they produce intense disturbances in the surrounding environment while propagating through interplanetary space. When these plasma-laden magnetic structures reach Earth, they interact with the magnetosphere, generating auroras and triggering geomagnetic storms (currently considered the most severe geomagnetic disturbances). They also disrupt the space environment (space weather), damaging communication systems, power grids, high-altitude aircraft, and even oil pipelines, while also harming spacecraft and threatening astronaut safety. Therefore, CMEs represent a critically important research topic in both solar physics and space science.

Ground-based observations are limited by time, weather, and wavelength constraints, so space-based instruments are typically used for CME observations. The SOHO satellite [3], located near the first Lagrangian point (L1), has observed more CMEs than any other space instrument to date, providing 24-hour continuous solar observation. SOHO's LASCO/C2 and C3 coronagraphs [4] can observe the region from 2.0 R to 30 R (where R represents solar radius), but these instruments observe CMEs from only a single perspective. In 2006, the Solar Terrestrial Relations Observatory (STEREO) [5] was successfully launched, consisting of two identical spacecraft (STEREO-A and STEREO-B) orbiting the Sun, one ahead of Earth and one behind, at distances of 0.9 AU and 1.1 AU respectively (these distances are variable). The separation angle between STEREO-A and STEREO-B increases by approximately 45° per year. The SECCHI [6] instrument suite onboard STEREO includes two white-light coronagraphs (COR1, COR2), an extreme ultraviolet imager (EUVI), and two heliospheric imagers (HI), with the primary mission of obtaining CME images from different perspectives. The heliospheric imagers can perform imaging observations of ICMEs in the inner heliosphere [7]. However, these coronagraphs and heliospheric imagers can only obtain two-dimensional projections of CME three-dimensional structures onto the sky plane, which cannot fully reflect the true three-dimensional structure. To obtain complete geometric information about CMEs, we must reconstruct their three-dimensional structures from these images and related information, combined with reasonable assumptions. This process is called three-dimensional reconstruction of CMEs.

CME imaging reconstruction methods can be broadly divided into two categories based on the observational data used: reconstruction based on coronagraph observations and reconstruction based on heliospheric imager observations. These methods employ different observational data for three-dimensional reconstruction and yield different three-dimensional parameters. Observations from coronagraphs and heliospheric imagers can provide information about CME three-dimensional morphology and plasma distribution. Section 2 introduces various CME imaging reconstruction methods in detail, Section 3 briefly describes two CME-driven shock reconstruction methods, Section 4 compares and discusses the characteristics and applicability of some CME imaging reconstruction methods, and Section 5 summarizes this work and provides an outlook for future research.

2 CME Imaging Reconstruction Methods

CMEs possess very complex dynamic structures and are in a highly dynamic state of motion, with varying morphology and size, including expanding loop-like structures, three-part structures, and fan-like structures. During propagation through interplanetary space, most CMEs/ICMEs appear as large magnetic flux ropes. Understanding and ultimately predicting solar eruptions requires clear knowledge of the internal structure of magnetic flux ropes before CME eruption. Here, a magnetic flux rope refers to a magnetic field structure formed by helically wound magnetic field lines. Such structures may exist prior to eruption or may form during the eruption process. Generally, magnetic flux ropes existing before eruption are thought to form in the convection zone and rise to the photosphere and chromosphere due to Parker instability, eventually entering the corona; alternatively, they may result from magnetic reconnection driven by shearing and converging motions of coronal magnetic structures at photospheric footpoints, transforming originally simple magnetic arches into complex magnetic configurations with helical structures, such as filaments (or prominences) and S-shaped magnetic structures frequently observed before eruptions [8]. Magnetic flux ropes formed during eruption result from magnetic reconnection occurring in current sheets formed behind sheared magnetic arches as they expand outward, converting sheared magnetic arches into helically wound magnetic structures. In actual coronal magnetic structures, sheared magnetic arches and magnetic flux ropes may coexist [9].

In either case, the magnetic structure of a CME is essentially a large-scale magnetic flux rope with both ends connected to the Sun and its middle portion extending far into interplanetary space [10] (as shown in Figure 2 [Figure 2: see original paper]). As CMEs/ICMEs propagate through interplanetary space, they expand with increasing distance from the Sun [11, 12]; simultaneously, they may also be deflected and deformed by the solar wind or other structures. Isavnin [13] simulated the complete propagation process of CME magnetic ropes and their deformation in the solar wind, demonstrating that the internal magnetic structure of CMEs changes during propagation due to deformation. The impact of CMEs/ICMEs on the space environment is directly related to their magnetic structure at specific locations, which in turn depends on the overall three-dimensional geometry and morphology of the CME. Therefore, correct three-dimensional reconstruction of CMEs based on observational data is crucial for establishing reliable space weather forecasting.

2.1.1 Cone Model Method

Zhao et al. [14] proposed a cone model to estimate the geometric structure and kinematic properties of three-dimensional halo CMEs. In the cone model, a CME is described as a hydrodynamic plasma characterized by a spherically self-similarly expanding structure with constant angular width, propagation direction, and speed. Although the lack of magnetic field structure within the ejected plasma prevents the cone model from reliably predicting the magnetic field of CMEs at Earth or other locations, this model has been successfully used to predict whether a CME will reach Earth and when it will arrive [15].

The cone model is based on three assumptions: (1) the CME propagates through the corona with nearly constant angular width in the radial direction; (2) the source region of halo CMEs is near the corresponding active region and at the solar center; (3) the overall CME velocity is radial and the expansion is isotropic. The cone model selects a heliocentric coordinate system (x , y , z), where z points toward Earth, y points north, and the x -y plane defines the sky plane. A right circular cone is then established with its apex at the center, along with a corresponding coordinate system (x_c, y_c, z_c), where x_c aligns with the cone's symmetry axis and the y_c-z_c plane is parallel to the cone's base. In the heliocentric coordinate system, (, λ) defines the cone's direction, representing the longitude and latitude angles of the cone's axis relative to the ecliptic plane, while the cone's angular width is defined as 2ω (as shown in Figure 3 [Figure 3: see original paper]).

Since this method requires substantial computational resources and fitting the model to LASCO images introduces uncertainties, Xie et al. [16] improved upon Zhao et al.'s [14] cone model to reduce computational load and determine the actual speed, angular width, and propagation direction of halo CMEs.

To better describe the cone's direction and propagation process from the observer's perspective (i.e., in the heliocentric coordinate system), it is necessary to transform the cone coordinate system to the heliocentric coordinate system. For this purpose, they introduced a transitional coordinate system (x'_c) (as shown in Figure 3).

The relationships among these three coordinate systems are as follows: First, rotate axes x and y counterclockwise around axis z by angle α to align axis x with axis x'_c and axis y with axis y'_c (z and x'_c coincide, x'_c and z'_c coincide); then rotate axis z'_c clockwise around axis y_c (y'_c, y'_c, z'_c) by angle to align axis z'_c with x_c. This achieves the transformation from the heliocentric coordinate system to the cone coordinate system, and vice versa [16].

The relevant parameters of the cone model are introduced below. In the cone coordinate system (x_c, y_c, z_c), x_c coincides with the cone's axis, r is the cone's generatrix, and the cross-section of the cone at distance r (shown as a circle in Figure 3) can be expressed as: x_c = r cos ω y_c = r sin ω cos δ z_c = r sin ω sin δ where $\delta = \tan^{-1}$(z_c/y_c) is the azimuthal angle in the cone's cross-section.

Transforming Equation (1) to the projection equation of the cone model's circular cross-section onto plane c (the sky plane): = 1, h = r cos ω cos a = r sin ω sin b = r sin ω

Equation (3) can be rewritten as: sin = tan ω = , = $\sin^{-1}$ cos , $\omega = \tan^{-1}$ cos

Equation (2) represents the elliptical equation of the cross-section projected onto the sky plane (see Figure 4 [Figure 4: see original paper]), where parameters a, b, and h are the semi-minor axis, semi-major axis, and displacement of its center from the origin in the heliocentric system, respectively, which can be determined from SOHO-LASCO-C2 and C3 images. Since the image size of 512 pixels corresponds to 30 R at both the poles and equator, Xie et al. [16] limited the C3 field of view to within 30 R , and a, b, h, and α can be calculated based on the geometric structure shown in Figure 4.

The traditional longitude and latitude λ are calculated using Equation (5): $\lambda = \tan^{-1} = \tan^{-1}$ cos sin α $\cos^2$ $\cos^2 \alpha + \sin^2$ cos cos α sin

Thus, Equations (4) and (5) determine the cone's direction and angular width. Figure 5 [Figure 5: see original paper] shows the reconstruction of a halo CME using the cone model.

If a = b and h = 0, then cos = 0. In this case, the solution is degenerate because the angular width ω is not uniquely determined. This occurs when the direction of a cone-shaped CME is parallel to the line of sight, which is the same situation encountered by Zhao et al. [14]. Since V_{rad} cannot be obtained due to geometric reasons, Xie et al. [16] used the empirical formula V_{rad} = 0.88 V_{exp} [17] to compensate for this limitation.

Using Equations (2) and (5), the relationship between the CME's actual radial velocity V_r and the projected velocities V_x' and V_y' is obtained: (cos ω cos $-$ sin ω sin sin δ) sin ω cos δ

Similarly, the relationship between the CME's actual radial velocity V_r and the projected velocities V_{xh}, V_{yh} is: cos ω cos $-$ sin ω sin sin δ sin ω cos δ

Here V_{xh}, V_{yh}, V_x' are the projected velocity components along axes x_h, y_h, x'_c in the sky plane. Both Equations (7) and (8) can be used to calculate the actual radial velocity, and by comparing the results from (7) or (8) at different δ (P_A), the validity of the cone model can be tested. The position angle δ is defined in the cone coordinate system as $\delta = \tan^{-1}$(z_c/y_c), while the position angle (P_A) in LASCO's sky plane is defined as $P\_A = -\tan^{-1}(x\_h/y\_h)$. There is a complex and unique transformation relationship between P_A and δ for each event, which depends on the three free parameters of the cone: ω, α, and .

Xie et al. [16] used the CME's actual radial velocity to estimate the propagation time for both fast and slow halo CMEs, finding that the model-based results agreed well with observations.

The cone model method improves modeling accuracy and efficiency. It was the first to quantitatively determine the actual speed, angular width, and source location of halo CMEs using coronagraph data—parameters that are crucial for space weather simulations. However, the cone model still has some uncertainties. For instance, the prediction accuracy of CME propagation time depends on measurements of the CME's actual speed and acceleration, where speed is derived from the observed CME positions at different times, and acceleration is further derived from the derived speeds. Additionally, these observations and derivations can only be performed within 30 R from the Sun's center (the LASCO/C3 field of view); beyond this distance, measurements are impossible. Second, the cone model requires substantial computational resources, and fitting the model to LASCO images introduces uncertainties. Xie et al. [16] improved this method, reducing computational load and determining the CME's actual speed, angular velocity, and propagation direction.

2.1.2 Polarization Ratio Method

The polarization ratio (PR) method was first proposed by Moran and Davila [18] to convert LASCO-C2 polarization observation data into three-dimensional distances from the sky plane (POS). Later, Dere et al. [19] validated the PR method using high temporal resolution (1 hour) LASCO polarization measurements, obtaining clear CME structures. Mierla et al. [20] and Moran et al. [21] applied this method to STEREO coronagraph polarization observations, demonstrating that polarization effects can be used to reconstruct three-dimensional CMEs. Dai et al. [22] used the PR method to calculate CME three-dimensional mass, improving traditional CME mass calculation methods and obtaining more accurate CME masses. Lu et al. [23] combined the PR method with the GCS method, proposing a new GCS-PR method to obtain three-dimensional parameters of CMEs observed from a single viewpoint at Earth.

Coronal white-light radiation originates from Thomson scattering of photospheric radiation by free electrons; therefore, coronal light signals are polarized. We can estimate the effective scattering angle from the intensity ratio of radiation with different polarization states and then use this scattering angle to calculate the distance from the sky plane. The physical principle of the PR method is Thomson scattering, whose cross-section depends on the angle between the scattering direction and the electric field vector. Light from the photosphere is unpolarized and can be decomposed into two equal, perpendicular components: one perpendicular to the scattering plane with intensity I_{tan}, and one parallel to the scattering plane with intensity I_{rad}. According to Thomson scattering theory, I_{tan} is independent of the scattering angle , while I_{rad} is proportional to $\sin^2$. Polarized brightness I_{pol}, total brightness I_{tot}, and polarization degree P are defined as [23]:

$I_{pol} = I_{tan} - I_{rad}$, $I_{tot} = I_{tan} + I_{rad} = 2I_{tan} - I_{pol}$ $I_{tan} = I_{pol} =$ dz N_e(, z)[(1 − u)C + uD] dz N_e(, z)[(1 − u)A + uB] $\sin^2$

where I_0 is the intensity at the solar disk center, σ_e is the Thomson scattering cross-section, u is the limb darkening coefficient (Dai et al. [22] take u = 0.63), N_e is electron density, is the projected distance along the sky plane (see Figure 6 [Figure 6: see original paper]), z is the distance along the line of sight from the sky plane, ω is the half-angle subtended by the scattering point on the solar disk, and A, B, C, D are the van de Hulst coefficients [24]:

$B(r) = -1/1 - 3 \sin^2\Omega(r) - \cos^2\Omega(r) \sin \Omega(r) \times [1 + 3 \sin^2\Omega(r)] \times \ln(1 + \sin \Omega(r)/\cos \Omega(r))$ $A(r) = \cos \Omega(r) \sin^2\Omega(r)$, $C(r) = -\cos \Omega(r) - \cos^3\Omega(r)$ $[5 + \sin^2\Omega(r) - \cos^2\Omega(r) \sin \Omega(r)$ $[5 - \sin^2\Omega(r)] \times \ln(1 + \sin \Omega(r)/\cos \Omega(r))]$ $D(r) = -1$

where the angle Ω is given by $\sin \Omega(r) = 1/r$ (r is the heliocentric distance in units of solar radius R , with $r^2 = {}^2 + z^2$).

For each line of sight, the PR method assumes that all electrons contributing to I_{tan} and I_{pol} are located at the same position ($_0$, z_0) (shown as P_1 in Figure 6), called the effective scattering center. The electron density at the effective scattering center is N_e($_0$, z_0), and the polarization degree P can be expressed as the ratio of polarized brightness to total brightness:

$P = [(1 + u)A + uB] \sin^2$ / $[2[(1 + u)C + uD] - [(1 - u)A + uB] \sin^2]$

where A, B, C, D, and are all functions of $_0$ and z_0. P and $_0$ can be obtained from observations, and solving Equation (18) yields the distance z_0 of the scattering point from the sky plane along the line of sight. A disadvantage of this method is that the result is only z^2, and the sign of the distance between the effective scattering center and the sky plane is uncertain—that is, the effective scattering center could be on either the front or back side of the Sun. We can use observations from another viewpoint [23] or EUV images of the CME source region to determine the sign of z_0. PR reconstruction cannot obtain the distribution of CME along the line of sight; the information provided is only the weighted average of CME plasma density along the line of sight [20].

Mierla et al. [20] used the PR method to reconstruct the CME of August 31, 2007, as shown in Figure 7 [Figure 7: see original paper]: panels a) and b) show reconstruction results using STEREO-A/COR1 data, while c) and d) show results using STEREO-B/COR1 data; a) and c) are side views, b) and d) are front views. Figure 7 shows that the reconstructed CME has only a rough outer surface, providing no information about the CME's depth structure. However, the advantage of this method is that it requires only single-viewpoint observation data for reconstruction.

2.1.3 Graduated Cylindrical Shell Method

Thernisien et al. [25] proposed a magnetic flux rope forward modeling technique called the graduated cylindrical shell (GCS) method, also known as the croissant method, which was developed based on the ice cream cone method [26]. While the ice cream cone method is mainly applied to three-dimensional reconstruction of halo CMEs, the GCS method assumes self-similar CME expansion and essentially constant source region neutral lines and CME angular width to perform fitting inversion of CME morphology. Cremades and Bothmer (CB04) [27] systematically studied 124 CME magnetic flux ropes using the cone model, analyzing the relationship between source region characteristics and CME morphology observed by LASCO, and derived CME projection effects. Thernisien et al. [25] reconstructed the three-dimensional morphology of CMEs based on CB04 results, obtaining CME propagation direction and three-dimensional propagation characteristics.

The CME morphology in the GCS model resembles a croissant tube, consisting of two parts: the flux rope itself and two conical legs supporting it, with the leg ends connected to the solar surface. Figure 8 [Figure 8: see original paper] shows the magnetic flux rope structure and its spatial position used in the GCS model [28], which has nine parameters, six of which are free parameters: (1) CME tilt angle γ, the angle between the corresponding filament or magnetic neutral line on the solar disk and the adjacent latitude line; (2) CME half-angular width α, related to the length of the filament or magnetic neutral line (when $\alpha = 0$, the GCS model is identical to the "ice cream cone" model); (3) leg height h of the flux tube; (4) parameter characterizing the flux tube thickness, defined as the ratio of the perpendicular distance from a point on the flux tube surface to the axis and its distance to the Sun's center, i.e., $= \sin \delta$; (5) Carrington longitude of the CME source region; (6) Carrington latitude of the CME source region. The first four parameters define the geometric structure of the magnetic flux rope, while the last two determine the source region location. Detailed descriptions of the model geometry can be found in reference [29].

By initializing these parameters, the CME's projection onto the sky plane is first obtained, yielding the CME's three-dimensional structure. After obtaining the morphological fit, a Gaussian fitting function [25] is selected to assume the electron density distribution at the CME edge:

N_e(d) = N_e e^{-(d−a/σ_s)^2} for trailing (d < a) and leading (d > a)

where d is the distance from any point inside or outside the shell to the skeleton (the dashed line in Figure 8a). This function introduces three additional parameters for fitting: electron density N_e, Gaussian characteristic lengths σ{*trailing*} *and* σ{leading}. The function can represent an asymmetric Gaussian distribution, with electron density primarily located in the shell. This asymmetry makes the simulation results more consistent with actual conditions. Thernisien et al. [28] used this method to reconstruct 26 CMEs and compared the results with CME directions determined by Colaninno and Vourlidas [30]. Except for one halo CME, the other results were quite similar, with differences in propagation direction within 10°.

Figure 9 [Figure 9: see original paper] shows an event on April 26, 2008. In STEREO-A's field of view (right panel), the CME propagated eastward, while in STEREO-B's field of view (left panel) it appeared as a halo CME. Due to different observation angles of the two STEREO coronagraphs, the CME front appeared differently, so they manually traced the front to fit the GCS model (second row in Figure 9), achieving a high goodness-of-fit of 83% for this example. However, for another case, such as the three CMEs erupting on December 31, 2007 shown in Figure 10 [Figure 10: see original paper], although Thernisien et al. [28] had already divided the front into two parts for fitting, the effect was still poor due to strong distortion of the CME front, with the lowest goodness-of-fit reaching only 34%.

The GCS model requires images from two different viewpoints, with the most suitable observation equipment being the STEREO spacecraft. However, since the STEREO-B satellite ceased operation in 2014, current applications of the GCS model generally use data from both STEREO-A and SOHO/LASCO for fitting. The greatest advantage of this model is that it can provide the CME's three-dimensional initial velocity and initial direction. Based on this, Zhao et al. [31] adopted the simple analytical model proposed by Liu et al. [32], extending CME studies beyond 1 AU and obtaining the relationship between distance and time from near the Sun to 5.34 AU.

Of course, GCS also has limitations: it assumes all CMEs have a croissant shape. Clearly, in reality, many CMEs are not croissant-shaped, as illustrated by the fitting results for the December 31, 2007 event. Additionally, GCS can only simulate individual CMEs and struggles to handle multiple CMEs erupting in rapid succession, yielding unsatisfactory fitting results that cannot distinguish different CMEs well [33].

2.1.4 Three-Dimensional Coronal Rope Ejection Method

Möstl et al. [34] widely applied the GS method to reconstruct magnetic clouds in the solar wind and proposed another forward modeling semi-empirical model for flux rope reconstruction called the Three-Dimensional Coronal Rope Ejection (3DCORE) model [35]. This is the first model to include interplanetary propagation as well as CME deceleration and expansion. In addition to reconstructing CMEs, it can also derive geomagnetic indices (Dst). Theoretical studies [36] have shown that the axial magnetic field in CMEs is approximately conserved during propagation and can provide sufficient southward magnetic field components. After magnetic reconnection with the northward magnetic field component at Earth's magnetopause, the magnetic flux and energy brought by the

CME are delivered to the magnetotail, triggering intense geomagnetic storms. Therefore, based on the reconstructed CME axial field strength, the southward magnetic field strength after the CME reaches near-Earth space can be derived, allowing estimation of geomagnetic storm intensity and the corresponding Dst index.

The geometric shape of CMEs in the 3DCORE model is shown in Figure 11 [Figure 11: see original paper]. It features a magnetic flux rope structure from the 2.5-D Gold-Hoyle [37, 38] model with a circular cross-section and uniformly twisted magnetic field configuration. The flux rope ends are connected to the Sun, with both the cross-section and overall shape being circular, but the cross-section radius varies, gradually approaching zero at the sections connected to the solar surface. When reconstructing actual events, the CME's initial velocity and propagation direction are obtained from STEREO HI (heliospheric imager) or COR (white-light coronagraph) observations, while the flux rope chirality and axial field direction are derived from magnetograms and extreme ultraviolet monochromatic images of the CME source region. The front's motion follows the drag-based model (DBM) [39, 40], other parts expand self-similarly, and the magnetic field along the flux rope axis decreases with increasing distance.

The 3DCORE model is primarily used to study CME propagation in interplanetary space and to predict CME velocity and magnetic field information at 1 AU. Its initial parameters, such as the flux rope's initial position and CME initial velocity, can be obtained from STEREO/HI observations or from GCS modeling results. The most important parameter is the drag parameter in the DBM, which ranges from 0.05 to 2 [40, 42] and is crucial for predicting CME arrival time. Results from the 3DCORE method can infer the southward component of the magnetic field in the flux rope, which is essential for predicting geomagnetic storms [43]. However, the 3DCORE method employs the DBM model for CME front motion, and the drag parameter is set empirically, often leading to deviations between model results and actual CME conditions.

2.1.5 Triangulation Method

Deriving CME velocity from coronagraph observations is affected by projection effects, with the magnitude depending on the CME source region's location on the solar surface. The triangulation method can determine the CME source region location, eliminating projection effects to obtain the true velocity. Inhester [44] described in detail the triangulation analysis method for determining CME three-dimensional spatial positions using two-point observations. Temmer et al. [45] used observations from STEREO and LASCO to reconstruct CME spatial positions by tracking the CME front.

The basic idea of the triangulation method is shown in Figure 12 [Figure 12: see original paper]. Relevant parameters include the angle ϕA *between the sky planes seen by the two spacecraft, the projected distances* d{0A} and d_{0L} of the CME front from the Sun in the sky planes of STEREO-A and LASCO respectively, the true distance d between the CME front and the Sun, and the elongation angles λ and α of the CME in images from LASCO and STEREO. The elongation angle is defined as the angle between the line connecting the spacecraft and the CME front and its projection onto the ecliptic plane, and it can be converted to distance—for example, for LASCO observations d_{0L} = 216λ [46–48]. Based on this relationship, we can first obtain d_{0L}, then calculate the elongation angle α between STEREO-A and the CME front.

From Figure 12, Ac is the projection of a point on the CME front onto STEREO-A's sky plane, and c is the projection of Ac onto the ecliptic plane; the same point on the CME front projected onto LASCO's sky plane is Lc, and c is the projection of Lc onto the ecliptic plane; point D is the intersection of line LLc and line AAc, with C being D's projection onto the ecliptic plane; the heliocenter is at point O. Clearly, point C is also the intersection of line LL'c and line AAc. Angle AcAA'c = λ;   is the angle between LA'c and LL'c; π is the angle between OC and LC. Since LOC forms a planar triangle,   + π + ϕ = 180°. The CME source region's longitude and latitude on the solar surface are denoted by ϕ and δ, respectively. Based on this setup, the following results can be obtained [45]:

$$= \arctan \pi = 180° - \; - \phi, \sin \pi \; \delta = \arctan[\tan(P\_A - A) \sin(\phi \pm \phi\_A)] \sin \;, p^2 = d^2 + r^2 - 2 \sin \delta \cos \delta \cos(\phi + \phi\_A), \alpha = \arccos(AU + p^2 - d^2)$$

By varying the source region's longitude ϕ and latitude δ and applying an iterative algorithm, the minimum deviation between the model and measured projected distances can be found, yielding the best estimate of the CME source region location. Using the measured projected distance (d_{0A}) and inserting the derived longitude into Equations (21)–(23), the true distance d can be estimated. It should be noted that the triangulation method has two assumptions: first, that CME velocity is not affected by CME expansion, and second, that the CME propagates radially. Clearly, these assumptions are not reasonable in some cases, leading to significant deviations in the results. Additionally, since CME images result from brightness integration along different lines of sight, identifying corresponding points in two CME images is difficult, causing deviations from actual conditions and preventing true three-dimensional reconstruction of CMEs—only the three-dimensional propagation direction can be obtained. However, the triangulation method can quantify and correct projection effects in most measurements without prior knowledge of the CME's three-dimensional shape [49]. In Temmer et al.'s [45] study of 11 events, most reconstruction results were good. If there are no problems with CME distance measurements and the same CME structure can be clearly identified in all three instruments, the triangulation method is relatively effective. Currently, triangulation has achieved good results in rapidly estimating source regions, directions, and correcting projection effects [45].

2.1.6 Tie-Point Method

Another commonly used method in CME reconstruction is the tie-point (TP) method. The TP method requires observational data from the STEREO twin spacecraft for three-dimensional reconstruction. Two STEREO spacecraft and a point in the corona define an epipolar plane [44]. When a target appears simultaneously in images from STEREO-A and STEREO-B, triangulation can be used to obtain its three-dimensional heliocentric coordinates, as shown in Figure 13 [Figure 13: see original paper]. The two lines of sight intersect at the target's location. Finding corresponding points can be done manually [50] (most commonly tracking prominent features of the front such as density changes, shape structures, or the CME core) or automatically using local correlation tracking (LCT) methods. Once the correspondence between pixels is established, they are projected back toward the Sun along their respective lines of sight, and the three-dimensional coordinates of corresponding points are calculated to achieve three-dimensional reconstruction. Since the lines of sight must lie in the same epipolar plane, their intersection in this plane is unambiguous and independent of the selected target. All epipolar planes intersect along the line connecting the two spacecraft. This method is commonly called "tie-point" or TP reconstruction [44].

Srivastava et al. [51] performed TP reconstruction of a partial halo CME that erupted on May 20, 2007. They obtained total brightness images from STEREO/COR1 polarization images, then combined these with COR2 white-light images for reconstruction. Before reconstruction, the images needed to be corrected so that STEREO-A and STEREO-B images had the same resolution and solar center coordinates. The Solar Software (SSW) SECCHI package could then be used to reconstruct three-dimensional coordinates of points on the front and determine the true height and velocity of selected points on the front in three-dimensional space. The reconstructed true height was 1.80 times higher than the projected height.

The TP method assumes that affine geometry is valid in coronagraphs. This assumption depends on the ratio between the Sun-target distance and the Sun-observer distance; the smaller this ratio, the more reasonable the assumption. Typically, the distance to the object being reconstructed is greater than 200 R , which is much larger than both the object's size and its distance from the Sun. Affine geometry assumes the observer is at infinity, so all lines of sight can be considered parallel, independent of the distance h between projections of the target on the sky plane at different observation angles. This assumption introduces errors because h/200 R is not zero, leading to overestimation of target size in reconstruction results. Additionally, the TP reconstruction method itself has certain errors that depend on the separation angle between the two spacecraft; when the separation angle is appropriate, the error is relatively small.

The TP reconstruction has spawned many derivative methods, including the three-dimensional height-time reconstruction technique (3D-HT [52]) that can accurately calculate CME three-dimensional velocity and propagation direction, the LCT-TP technique combining local correlation tracking with triangulation [20], and methods using combinations of two or three observational instruments (STEREO or STEREO + SOHO) for triangulation [45, 53]. The 3D-HT technique derived from TP reconstruction is also based on the principle of affine geometry. According to geometric calculations (see Figure 14 [Figure 14: see original paper]), the reconstructed point's coordinates in spherical coordinates (R_{3D}, , λ) are obtained, where and λ are latitude and longitude, respectively. This method can infer the CME's position and propagation direction at each moment, and its actual velocity can be calculated by estimating the time derivative of R_{3D}.

Mierla et al. [52] used the 3D-HT technique to obtain height-time plots of a well-identifiable feature in CMEs from two STEREO images, yielding two independent projected velocity vectors that could be further combined to construct a three-dimensional velocity vector. The HT technique has been widely applied for some time, but mainly to determine CME projected velocities in the sky plane—that is, selecting a specific target in coronagraph images and obtaining the CME propagation direction by tracking its position changes over time. With the advent of STEREO twin spacecraft data, the HT technique is no longer limited to solving for CME two-dimensional velocity. Sheeley et al. [54] developed algorithms that can automatically detect the faintest moving features, enabling the 3D-HT technique to not only display CME motion and velocity in real time but also calculate three-dimensional velocities of ambiguous targets with high sensitivity.

2.1.7 Mask Fitting Method

In 2012, Feng et al. [55] proposed a new method for reconstructing CMEs based on back-projection of CME peripheries from multiple coronagraphs, called the mask fitting (MF) method, aimed at reconstructing CME position, shape, and shape evolution in three dimensions. The MF method uses coronagraph data from three viewpoints, yielding more accurate results than two-viewpoint reconstructions, and unlike forward modeling, the MF method does not require assumptions about CME geometry.

Traditional triangulation or tie-point methods select the same feature point in a pair of coronagraph images, then project along different lines of sight connecting the spacecraft to the Sun to obtain their three-dimensional coordinates. In contrast, the MF method selects a point in 3D space containing the Sun and projects it onto three coronagraph observation images. If the projections along the line of sight all lie within the defined CME region, that point is considered to be inside the CME cloud. By sequentially finding all points on the CME edge, the three-dimensional boundary of the CME can be obtained. To better describe the three-dimensional structure of the CME cloud, the reconstructed three-dimensional CME region must be sufficiently close to the true CME boundary.

The specific steps of the MF method [56] are as follows: First, create a CME region on each of the three coronagraph images from STEREO-A, LASCO, and STEREO-B, setting pixel values inside the CME edge curve to 1 and those outside to 0. Then, discretize a three-dimensional cube centered on the Sun, projecting each point onto the image planes of the three images obtained by STEREO-A, STEREO-B, and SOHO. Only points projected onto the CME regions in all three images are considered to belong to the CME. Finally, Bézier curves are used to smooth the boundary, obtaining the three-dimensional boundary shape of the CME as shown in Figure 15 [Figure 15: see original paper].

The MF method does not require assumptions about CME shape, can reconstruct CME surfaces well, obtain fine surface structures [57], and is adaptable to various irregular CME shapes, making it convenient and flexible to use [56]. The three-dimensional CME shape obtained by the MF method allows analysis of its geometric center and principal axis scale. The method's shortcomings are that the results do not include CME internal structure, and it requires suitable separation angles among the three observational instruments.

2.2.1 Geometric Triangulation Method

The triangulation method and tie-point method described earlier can both determine CME propagation direction and radial distance, but these two methods and other similar methods [58–60] require identification and tracking of the same features in image pairs from two spacecraft, which is impossible at large distances where CME signals become very weak and diffuse. Liu et al. [61] proposed a geometric triangulation method based on time-elongation maps from imaging observations. The advantage of this method is that it can apply geometric triangulation to faint features in HI-1 and HI-2 for the first time based on time-elongation maps, enabling continuous tracking of CMEs in the heliosphere and prediction of their impact on Earth. Building on this, Liu et al. [61] and Lugaz et al. [63] introduced harmonic mean geometry into the geometric triangulation framework, while Davies et al. [64] introduced self-similar expansion geometry into the framework. These methods have all been successful, proving that the geometric triangulation concept and framework are effective. Subsequent work has further confirmed this [65–67]. More importantly, comparing this geometric triangulation concept and implementation method with other methods, such as in-situ solar wind measurements and interplanetary type II radio burst frequency drift, yields very similar results, further revealing the propagation characteristics of typical fast and slow CMEs throughout the Sun-Earth space [68, 69].

In principle, geometric triangulation is based on Figure 16 [Figure 16: see original paper], where white-light features can be seen by moving along the direction between two spacecraft. The feature's elongation angle (the angle between the feature and the Sun-spacecraft line) is represented as α_A and α_B in STEREO-A and STEREO-B, respectively, obtained from time-elongation maps generated by stacking dynamic difference images along the ecliptic plane. Based on the simple geometric relationship in Figure 16, we can obtain:

$r \sin(\alpha_A + \beta_A) / \sin \alpha_A = d_A$, $r \sin(\alpha_B + \beta_B) / \sin \alpha_B = d_B$, $\beta_A + \beta_B = \gamma$,

where r is the radial distance of the feature from the Sun, β_A and β_B are the propagation angles of the feature relative to the Sun-spacecraft line, d_A and d_B are the distances of STEREO-A and STEREO-B from the Sun, and γ is the longitudinal separation angle between the two spacecraft. Once the elongation angles (α_A and α_B) are measured from imaging observations, the above equations can be solved for r, β_A, and β_B, with a unique solution (compared to model fitting). If the STEREO spacecraft distances are similar (d_A d_B), we obtain:

$\tan \beta_A = [\sin \alpha_A \sin(\alpha_B + \gamma) - \sin \alpha_A \sin \alpha_B] / [\sin \alpha_A \cos(\alpha_B + \gamma) + \cos \alpha_A \sin \alpha_B]$

However, Liu et al. [62] pointed out that the CME propagation direction can be quickly obtained even when $d_A \neq d_B$.

The geometric triangulation method is relatively reliable and has no free parameters. The only assumption is that the same feature can be tracked in time-elongation maps from both spacecraft, allowing even faint features to be tracked to near 1 AU. This method has three advantages [61]: First, based on time-elongation maps, geometric triangulation is applied to faint features in HI-1 and HI-2 for the first time; second, compared to single-spacecraft fitting techniques, this method relies on fewer assumptions, yielding more accurate solutions; third, it can determine the propagation direction and true distance of CME features (or other white-light features) from the Sun all the way to 1 AU. The disadvantage of this method is that it cannot obtain CME three-dimensional geometry and magnetic field information.

2.2.2 Self-Similar Expansion Method

The self-similar expansion (SSE) method proposed by Davies et al. [70] uses HI data for CME reconstruction. The SSE method obtains CME time-elongation profiles from single-viewpoint imaging to investigate CME propagation direction and speed. As early as 1999, Sheeley et al. [54] proposed a fixed-ϕ fitting (FPF) method to obtain CME time-elongation profiles. Another similar method is harmonic mean fitting (HMF) [72, 73]. FPF assumes the CME is a radially propagating point source with constant propagation direction (hollow black point in Figure 17 [Figure 17: see original paper]a); HMF assumes the CME front is circular, with the expanding circle fixed at the Sun's center and the CME propagating along a fixed radial trajectory (solid black point and large gray circle in Figure 17a). Both methods are extreme cases of the SSE method.

In the SSE model, the CME cross-section is also circular. In the plane corresponding to the position angle of interest to the observer, the radius increases as the CME propagates outward. This circle is no longer fixed at the Sun (as shown in Figure 17b) but moves outward with the CME, making the angle subtended by the CME at the Sun's center constant. Therefore, in the SSE model, the radial distance R_{SSEa}, which is the distance between the CME apex a (the point farthest from the Sun, solid black point in Figure 17b) and the Sun, can be expressed as [70]:

R_{SSEa}(t) = d_0 sin _a(t) / [sin(_a(t) +) + sin λ]

where d_0 is the observer-Sun distance, _a is the angle between the Sun-observer line and the tangent from the observer to the CME's circular front, λ is the CME half-width angle (0 $\leq$ *λ{SSE} $\leq$ 90°), and is the angle between the Sun-observer line and the CME propagation direction, which is assumed constant during CME propagation. Similarly, the distance R*{SSEb} between the CME tail point b (the point closest to the Sun, hollow black point in Figure 17b) and the Sun can also be derived [70]:

R_{SSEb}(t) = d_0 sin _b(t) / [sin(_b(t) +) − sin λ]

where _b is the angle between the Sun-observer line and the tangent to the rear of the CME. For CMEs, the rear differs from the front and is less likely to be distorted by interaction with the solar wind ahead [74]. Derivations of these expressions can be found in reference [75]. In the above formulas, when λ = 0°, FP reconstruction is obtained, i.e., R_{PF}(t) = d_0 sin[(t)] / sin[(t) +]; when λ = 90°, HM reconstruction is obtained, i.e., R_{HM}(t) = 2d_0 sin[(t)] / [sin((t) +) + 1]. We note that both the formula expressions and model geometric constructions show that the FPF method does not distinguish between the CME front and rear, while in the HMF method the CME rear is always located at the Sun's center. Therefore, both methods are extreme cases of the SSE method.

Implementation of the SSE method requires further calculation of angles _a(t) and _b(t) [70], i.e.:

(t) = $\cos^{-1}$[−bc + a / √(a^2 + b^2 − c^2)] a^2 + b^2 = d_0(1 + c) / V_{SSE} t − cos(_{SSE}), b = sin(_{SSE}), c = ± sin(λ_{SSE})

where the sign of c is determined as follows: use "+" when calculating _a(t) and "−" for _b(t). The SSE method fits the time-elongation profile of a CME transient observed from a single viewpoint to this equation to obtain the best-fit radial velocity and propagation direction. From these parameters, other parameters of interest can be derived, such as predicted arrival time at a specified location. Davies et al. [70] used Monte Carlo simulations to verify the applicable conditions of the SSE model, finding that elongation angles should be between 19° and 74°; angles that are too large or too small lead to unreliable results. The limitation that elongation angles cannot be too large restricts the applicability of the SSE model for space weather forecasting. Since the SSE model assumes all CMEs have circular transient cross-sections, which clearly differs from reality, the reconstruction results will certainly differ from the true CME configuration. Researchers are currently working to optimize this technique for CME reconstruction with elliptical transient cross-sections (such as the elliptical evolution model introduced in Section 2.2.3 [76]).

2.2.3 Ellipse Evolution Model

On January 7, 2014, a fast CME (with a sky-plane projected speed of about 2,400 km/s) erupted from an Earth-facing active region. Since fast CMEs erupting from source regions near the solar center typically affect Earth [77, 78], many observers predicted this CME would have a significant impact on Earth. However, no geomagnetic storm subsequently occurred, indicating that this event did not propagate radially and that this non-radial propagation direction was determined very close to the Sun rather than being caused by deflection during interplanetary propagation [79]. To investigate the specific reasons for this CME's propagation, Möstl et al. [76] proposed a new Ellipse Evolution (ELEvo) model specifically for the non-radial propagation characteristics of such fast CMEs.

The ELEvo model describes the CME-driven shock as an ellipse in the ecliptic plane (see Figure 18 [Figure 18: see original paper]). Using analytical expressions in the ELEvo model, the shock shape can be projected onto the sky plane of a given planet or local observer, and the velocity and arrival time at the observer can be calculated for any point along the ellipse (shock). Initial conditions such as CME initial velocity, direction, and width obtained from coronagraph observations can be used as inputs.

The ELEvo model employs the following assumptions: (1) The CME boundary's heliospheric longitude angular width remains constant; (2) One principal axis of the ellipse (shock) is along the propagation direction; (3) The ratio of the ellipse's major to minor axes is constant; (4) The direction of the ellipse's symmetry axis remains unchanged. For CMEs propagating in interplanetary space, the DBM model [40] is used. Therefore, the ELEvo model including DBM involves four free parameters: (1) The reciprocal of the major-to-minor axis ratio f; (2) The ellipse's half-angular width λ; (3) The drag parameter γ (approximately (0.1–2) $\times$ 10^7 km^{-1}); (4) The background solar wind speed w. Using the DBM model with a small drag parameter can describe shock propagation within 1 AU from the Sun, allowing calculation of the CME shock arrival time and velocity at Earth.

We now derive the formulas used in the ELEvo model and the velocity at each point along the elliptical front [76]. The ellipse vertex's R(t) (the point on the ellipse farthest from the Sun along the direction of the ellipse's center) is given by DBM [40]. With the reciprocal of the major-to-minor axis ratio f = b/a and a_r = a/b, we obtain:

f = b/a $\beta = \lambda \quad = \tan^{-1}(b^2/a^2 \tan \beta)$

where angle β is the angle between the semi-major axis a and the normal tangent at point T (see Figure 18a). T's position is the tangent point on the ellipse of a line starting from the Sun. From geometric relationships, $\beta = \lambda$, and is the polar angle of the ellipse. The geometric relationship between β and is given by Equation (37), and r is the distance between the ellipse's center and point T. Combining Equation (37) yields:

$= \tan^{-1}(f^2 \tan \beta)$

From the sine law on the large orange triangle in Figure 18a, we derive: $\sin \lambda$ / $\sin \alpha = R(t) - b$

where angle α is obtained from the triangle's geometric relationship: $\alpha = 90° + - \lambda$. The distance r in polar coordinates is defined as: r = b / $\sqrt{}[(f^2 - 1) \cos^2 + 1]$

Substituting the equations for α and r into Equation (39) and letting $\omega = \sqrt{}[(f^2 - 1) \cos^2 + 1]$, we obtain the ellipse parameters a, b, c:

a = b/f $R(t)\omega \sin \lambda / \cos(\lambda -) + \omega \sin \lambda$ c = R(t) − b

Equation (41) provides the final description of the ELEvo ellipse model parameters. From Equations (38) and (40), we see that the ellipse's minor axis b depends on all known variables [R(t), f, λ], while the major axis a simply follows the definition of f in Equation (37). After obtaining the model's geometric shape, the velocity at any point on the ellipse can be calculated. For convenience, Möstl et al. [76] introduced a Cartesian coordinate system in the ellipse (Figure 18b), where the X-axis is perpendicular to the CME propagation direction, Y is orthogonal to X, c is the vector from the Sun's center to the ellipse's center, d is a straight line along the Sun-Earth connection ending at the ellipse boundary (from Figure 18b we can see d intersects the ellipse at two points), r connects the ellipse center to the endpoint of d on the ellipse, and Δ is the angle between the CME center direction and the Sun-Earth line (Earth could also be another planet or spacecraft). Based on the above formulas, the two intersection points of d with the ellipse can be calculated:

$d_{1,2} = c \cos \Delta \pm \sqrt{}[(b^2 - c^2)f^2 \sin^2\Delta + b^2 \cos^2\Delta] / [f^2 \sin^2\Delta + \cos^2\Delta]$

where the solution with the positive sign before the square root is the "front" solution (d_1), and the negative sign gives the "rear" solution (d_2). The velocity at a position defined by angle on the ellipse is derived from the ellipse's self-similar expansion, meaning the half-width remains constant. The self-similar expansion assumption implies that the CME shape does not change with time, so the ratio of velocity to distance for all points on the ellipse must be constant [75]:

$V_\Delta(t) = V(t) / d_1(t)$

Furthermore, when $d_1(t)$ equals the planet's heliocentric distance, the local observer can provide the time when the ellipse reaches a certain local position and the corresponding velocity $V_\Delta(t)$. Since the major-to-minor axis ratio in the ELEvo model is a free parameter, control over the ellipse shape is more flexible than with self-similar expansion circles, making it more suitable for self-consistent modeling of multi-point local observations. Möstl et al. [76] used multi-point local observation data from Earth and Mars to constrain the ELEvo model shape (major-to-minor axis ratio of 1.4 ± 0.4), finding that the ellipse's major axis is perpendicular to the CME's motion direction [80]. These results demonstrate that this CME event did not cause a geomagnetic storm not because of deflection during interplanetary propagation, but due to non-radial propagation determined near the CME source region caused by strong magnetic fields there. Since observed CME images result from line-of-sight integration, studying CME non-radial motion in heliospheric longitude is difficult, but the ELEvo model provides an effective method for investigating CME non-radial propagation.

2.2.4 Correlation-Aided Reconstruction Method

Among reconstruction methods based on heliospheric imager data, there is also the CORrelation-Aided Reconstruction (CORAR) method [81], which can automatically identify and locate non-uniform structures in the solar wind, particularly small-scale transient structures. Later, Li et al. [82] improved this method, enabling its application to three-dimensional CME reconstruction and automatic identification of CMEs propagating along the Sun-Earth line from STEREO/HI dual images. Compared with other methods, the CORAR method is much simpler and makes no special morphological assumptions about the target of interest.

The CORAR method first requires processing HI images to eliminate F-corona and background star field effects. With known spacecraft positions, fields of view, and baselines (Sun-Earth line), HI-1 images are projected onto the meridional plane containing the baseline based on the temporal and spatial positions of the two STEREO spacecraft in the Heliocentric Earth Ecliptic (HEE) coordinate system. The images are then radially projected using latitude and distance from the Sun's center as polar coordinates, marking the CME's spatial position on this meridional plane. A radial-latitude sampling box is used to obtain two-dimensional local brightness variations, and correlation coefficients cc are calculated along the "baseline." If the target is on the baseline, the two segments overlap well (see Figure 19 [Figure 19: see original paper]a), and they should be highly correlated; if the target is far from the baseline, the two line segments overlap little or not at all (see Figure 19b), resulting in low correlation.

The following describes how to project HI images onto the meridional plane. First, assume P is the HEE coordinate of a pixel's projection point on the meridional plane with longitude :

P(x, y, z) = O + kn

where n is the direction of the pixel in the HI-1 image relative to the observer O, obtainable from STEREO/HI-1 header files, and k is an unknown positive number. The latitude λ and distance D from the Sun's center of the projection point are two key unknown parameters in the projection process, with the relationship:

$x = D \cos\lambda \cos \quad y = D \cos\lambda \sin \quad z = D \sin\lambda$

where is the longitude of the projection point. Solving these two equations yields x, y, z, k, λ, and D. Based on the derived λ and D, the two HI projection images are plotted on the meridional plane with longitude . The correlation coefficient cc of the overlapping region of the two projection images on the same meridional plane is calculated. A dynamic 3D (radial-latitude-time) region of appropriate size (latitude $\pm$5°, radially about 8 R) is delineated and sampled for cc calculation. Brightness variations in the sampling region mainly depend on CME density spatial variations. To highlight density spatial variation patterns, the average value is subtracted from the brightness data in the sampling region. The correlation coefficient cc is calculated using the linear Pearson correlation coefficient:

$$cc = \Sigma_{i=1}^{n} (p_i - \bar{p})(q_i - \bar{q}) / \sqrt{[\Sigma_{i=1}^{n} (p_i - \bar{p})^2 \, \Sigma_{i=1}^{n} (q_i - \bar{q})^2]}$$

where p and q are two datasets (corresponding to data in the same sampling box), sample size is n, and $\bar{p}$ and $\bar{q}$ are the mean values of p and q, respectively. In our selected sampling region, $n = 11 \times 41 \times t$, where t equals the number of time steps, which can be 1 or 5. The cc value ranges from −1 to 1: larger cc values indicate better correlation; negative cc values are considered uncorrelated.

Equation (46) shows that as long as the signal-to-noise ratio is sufficiently large, the cc value is not affected by feature density but rather by density variation patterns. CME position, signal-to-noise ratio, and other factors may affect the calculated cc value, requiring further research and correction [82]. Figure 20 [Figure 20: see original paper] shows the CORAR method's reconstruction result for a CME. To verify this method's reliability, Li et al. [82] applied the GCS method to the same CME event, finding that the two methods yielded basically consistent CME angular width and propagation direction. The difference lies in CME geometric structure: Figure 20 shows that the CME structure obtained by the CORAR method is no longer the traditional magnetic flux rope shape but rather a highly distorted structure.

The CORAR method's advantage is that it requires no assumptions about CME geometry and can automatically reconstruct CMEs. Moreover, through improvements by Li et al. [83], this method can also obtain the radial velocity distribution of CMEs in three-dimensional space. However, this method also has some limitations. For example, the coplanarity effect means that if a CME is located in the central part of the plane formed by the line connecting the two spacecraft (see Figure 19a)—that is, the two spacecraft have the same line of sight—then even if the CME is far from the baseline (see Figure 19b), we will obtain similar projection images, resulting in high cc values. This leads to very low longitudinal positioning accuracy for CMEs near the "coplanarity" region, and reconstruction of large-scale CMEs will be biased toward the central plane between the two spacecraft. The CORAR method can only track CMEs near the preset baseline, not CMEs in the entire HI field of view. Additionally, like the triangulation method, the CORAR method uses simple optical projection, which may incorrectly treat two different parts of a large-scale CME as the same part [84]. Therefore, the CORAR method is more accurate for small-scale transient localization.

2.3 Summary

This section describes ten different CME three-dimensional reconstruction methods. We summarize their data requirements, viewing perspectives, obtained parameters, and method characteristics in Table 1 .

3 CME-Driven Shock Reconstruction Methods

Closely related to CME imaging reconstruction is the three-dimensional reconstruction of CME-driven shocks, which represents an important new development trend in recent years. After a CME eruption, energy can be released not only in the form of accelerated particles but also as waves; in the latter case, fast magnetosonic waves are observed as Moreton waves and EUV waves [85–93], white-light waves [93, 94], and type II radio bursts [95, 96]. When a CME's speed relative to the surrounding solar wind exceeds the local fast magnetosonic speed or Alfvén speed, a shock wave is generated ahead of the CME. In coronagraph images, shocks typically appear as bow shock morphologies or double-front structures [97]. The importance of shock reconstruction lies in its ability to provide the three-dimensional structure and kinematic characteristics of shocks, which can be compared with CME properties [98, 99], energetic particle characteristics [100–102], and in-situ solar wind observations [99]. This is of great significance for hazardous space weather forecasting and research on high-energy particle acceleration. This chapter briefly introduces three-dimensional reconstruction methods for CME-driven shocks.

3.1 Spherical Shock Model

Spherical shock fitting was first proposed by Vourlidas and Ontiveros [97] and later applied by Hess and Zhang [103] to study shock evolution. On July 23, 2017, an extremely high-speed CME erupted (front speed about 3,000 km/s), which exhibited a spherical structure in observations from three viewpoints (STEREO-A, STEREO-B, and SOHO) with extremely high speed. To study the structure, propagation, and expansion of the shock associated with this halo CME, Liu et al. [99] assumed a spherical structure to simulate the shock (as shown in Figure 21 [Figure 21: see original paper]).

Assuming this moving spherical shock has constant propagation direction, the model uses four free parameters to describe the shock: longitude and latitude of propagation direction, sphere radius (r), and distance between sphere center and Sun (d). Liu et al. [99] proposed a distance calculation method for shock propagation and expansion applicable to both coronagraph and heliospheric imager observations, similar to the triangulation method proposed by Liu et al. [61, 62]. As shown in Figure 21, the distance relationships determined from three different perspective observations by STEREO-A, STEREO-B, and SOHO are:

$$\sin \alpha_A + d = d_A,\ d^2 + r^2 - 2dr \sin(\gamma - \alpha_B) = d^2_B \sin^2\alpha_B + d^2 \cos^2(\gamma - \alpha_B),$$

where d_A and d_B are the distances of STEREO-A and STEREO-B from the Sun, α_A and α_B are the shock elongation angles measured tangentially from STEREO-A and STEREO-B, and γ is the longitudinal separation angle between STEREO-A and STEREO-B. The distance between the shock front and Sun is defined as r + d, and propagation velocity is calculated from distance-time plots. The expansion velocity can be separated from the shock sphere's propagation velocity. The distances and velocities from forward modeling are basically consistent with those from triangulation using SOHO and STEREO-B, but slightly lower than the latter [104]. Figure 22 [Figure 22: see original paper] shows the reconstruction results of the CME shock using the spherical shock model at three viewpoints. The results indicate that both coronagraph imaging reconstruction results and type II radio bursts and in-situ observations can be reasonably simulated with a spherical structure.

3.2 Ellipsoidal Shock Model

To determine the three-dimensional structure and kinematic properties of multiple CME fronts, Kwon et al. [98] developed a composite model. In this composite model, the bubble-like shock front is described by an ellipsoidal model, while the internal CME front is fitted with the GCS model. To construct this composite model, two coordinate systems are defined (see Figure 23 [Figure 23: see original paper]): the reference coordinate system (x_{ref}, y_{ref}, z_{ref}) and the local coordinate system (x, y, z). The origin O_{ref} of the reference coordinate system is at the Sun's center, with the z_{ref} axis being the Sun's rotation axis. The x_{ref} axis intersects the solar central meridian as seen from Earth. A sphere with radius h is defined in the reference coordinate system, while the origin O of the local coordinate system is located on this sphere's surface. The x-axis is tangent to the longitudinal great circle passing through O and the z_{ref} axis, the z-axis points in the sphere's radial direction, and the local coordinate system moves synchronously with the CME's radial motion.

Figure 24 [Figure 24: see original paper] shows the creation of the ellipsoidal model and GCS model in the defined coordinate systems. The ellipsoidal model requires seven geometric parameters: three parameters for the origin $O_E(x'_{ref,E}, y'_{ref,E}, z'_{ref,E})$ of the local coordinate system in the reference coordinate system, or $O_E(h_E, {}_E, {}_E)$, where h_E, ${}_E$, and ${}_E$ are the height, latitude, and longitude of origin O_E in the reference coordinate system; three other parameters are the lengths of the ellipsoidal model's three semi-axes a, b, and c; and the final parameter γ_E is the angle between the major axis a and the x-axis. As shown in Figure 24a, in the local coordinate system, the ellipsoidal model is determined by:

x'_E = a cos cos , y'_E = b cos sin , z'_E = c sin ,

where latitude 0 ≤ ≤ 180° and longitude 0 ≤ ≤ 360°. When a = b = c, the ellipsoid becomes a sphere. The GCS model differs from Thernisien et al. [28] in that its origin is not at the Sun's center and its skeleton lies in the x-z plane.

Composite model construction is an iterative process. First, initial guesses for free parameters are made to establish the ellipsoidal and GCS models in the reference coordinate system. Next, the structure's view in the two-dimensional plane is calculated and compared with actual spacecraft observations. The calculated observations are then compared with observed fronts. Finally, the geometric structure constructed in the reference coordinate system is transformed to the observation coordinate system (X_{sc}, Y_{sc}, Z_{sc}), whose origin O_{sc} is at the Sun's center, axis X_{sc} points toward the observer, and axes Y_{sc} and Z_{sc} point west and north, respectively, on the image plane.

To reconstruct CMEs in three-dimensional space, the actual light path of each pixel must be considered. The coordinates of the constructed three-dimensional structure need to be converted to angular distances (u, v) westward and northward from the image center point on the image plane. The angular distances of point (X', Y', Z') in the observation coordinate system are defined as:

$u = \tan^{-1}(Y'' / d)$, $v = \tan^{-1}(Z'' / d)$

where Y' ' = Y' − ΔY, Z' ' = Z' − ΔZ, and d = D − X'. Parameters ΔY and ΔZ are the offsets of the image center relative to the Sun's center on the image plane. Additionally, D is the observer's distance to the YZ plane, i.e., D = √(d_ ^{2} − ($\Delta Y^2 + \Delta Z^2$)), where d_ is the observer's distance to the Sun's center. These processes are repeated iteratively until the reconstructed geometric structure can well reproduce observations from all instruments.

The composite geometric model (including ellipsoidal and GCS models) using three-viewpoint observation data can well reproduce the three-dimensional structure of CMEs and their driven shocks. For the three-part structure of CMEs, the faint outermost shock boundary can be well reproduced by the ellipsoidal model, while the bright CME front is reproduced by the GCS model. Simultaneous fitting of the two fronts with two different geometries shows that they are not projections of a single three-dimensional structure onto the sky plane but have fundamentally different three-dimensional morphologies [98]. Liu et al. [105] used the ellipsoidal shock model to reconstruct the three-dimensional geometry of CMEs and shocks, comparing ENLIL MHD simulations with in-situ measurements, and found that shock expansion plays an important role in the extent of heliospheric shocks. Based on Liu et al.'s [105] shock geometric fitting, Zhu et al. [100] combined steady-state MHD data of the background solar wind to study energetic particle characteristics, demonstrating effective particle acceleration at the shock flanks.

Since the ellipsoidal model imposes restrictions on shock shape, inversion results based on it are difficult to provide the shock's true shape and structure. Feng et al. [57] combined epipolar geometry methods [44] to improve the mask fitting method introduced in Section 2.1.7, reconstructing the three-dimensional surface of CME shocks when analyzing shock structures in specific events, and obtaining the evolution of this surface shape over time. They reconstructed the three-dimensional shock surface using the spherical shock model, ellipsoidal model, and MF method. All three methods gave consistent shock propagation directions, and the MF method could also capture the concave structure of the shock front.

Feng et al. [57] further pointed out that accurately reconstructing the three-dimensional structure of shock surfaces and obtaining corresponding shock parameters is crucial for understanding the relationship between shock structure and solar energetic particles accelerated by the shock. Additionally, based on data from different observational instruments, the improved MF method can be used for multi-faceted studies of events of interest.

4 Comparison of Results from Different Reconstruction Methods

In the previous sections, we introduced the techniques and methods commonly used by international colleagues for three-dimensional reconstruction of CME spatial structures and discussed their respective advantages and areas for improvement. We found that each reconstruction method can reproduce certain features of a particular type of CME to some extent, but all differ from actual conditions in other aspects. Due to limitations in observational conditions and positions, such differences will continue to exist for some time. In this chapter, we compare the application results of these reconstruction methods for specific events and provide a comprehensive evaluation of their advantages and shortcomings.

4.1 May 15, 2007 Event

The first event we discuss is a CME that occurred on May 15, 2007, with its source region AR10956 located at N02°E47°. The projected speed of the CME front given by LASCO was 491 km/s. At this time, the separation angle between the two STEREO spacecraft was about 8.63°.

Mierla et al. [52] used the 3D-HT method to obtain a CME speed of about 169 km/s, longitude of about E70°, and estimated latitude around N14°. Temmer et al. [45] used the TP method, analyzing STEREO-A and STEREO-B data to obtain longitudes of E46° and E50°, and latitudes of N02° and N01°, respectively, with a CME front speed of about 445 km/s. The results from these two methods differ significantly, mainly because Mierla et al. [52] reconstructed the CME interior, while Temmer et al. [45] reconstructed the CME edge. Comparing these two results shows that reconstructing the CME edge better reflects the CME's comprehensive information and large-scale propagation characteristics.

Mierla et al. [20] analyzed this event again using three other methods, deriving the CME propagation direction and feature positions (as shown in Table 2). The first method was the graduated cylindrical shell (GCS) method. The second method first used local correlation tracking (LCT) to identify the same features in STEREO-A and STEREO-B coronagraph images, then used the TP method to reconstruct the CME—this is called the LCT-TP method. The third method used TP reconstruction to find the centroid's three-dimensional coordinates (CM-TP method), first assuming Thomson scattering is isotropic, so pixel brightness is proportional to the integral of electron mass density along respective lines of sight, and the centroid of integrated brightness should equal the projection of the center of gravity of electron surface mass distribution on the epipolar plane. Then, based on TP reconstruction, the distribution of CME centroids on each intersecting epipolar plane was obtained, and the propagation direction of CME centroids was studied.

Table 2 shows that the latitudes and longitudes of the CME obtained by Mierla et al. [20] using the three methods are basically consistent, with deviations not exceeding 10°. However, the CME propagation directions obtained by different methods (−51°, 7°) differ significantly from the source region location, possibly due to radial deflection during CME propagation [106].

4.2 May 20, 2007 Event

The second event is a filament eruption-related partial halo CME that occurred on May 20, 2007. The separation angle between the two STEREO spacecraft was about 9°. The projected speed of the CME front observed by SOHO-LASCO was 275 km/s, and this CME propagated toward Earth. Srivastava et al. [107] applied the TP method to reconstruct the front in the COR1 and COR2 fields of view, with the CME source region at E2°S28°, front speed of 510 km/s, and reconstructed sky-plane projected speed of about 272 km/s. Mierla et al. [52] applied the 3D-HT technique to the same features, obtaining a CME source region at E2°S27°, three-dimensional front speed of 548 km/s, and sky-plane projected speed of about 250 km/s.

Srivastava et al. [107] further calculated the CME's three-dimensional propagation speed and arrival time at Earth based on the reconstruction, which was very close to the actual arrival time measured near Earth. The differences may be caused by measurement errors and incompleteness of the reconstruction method. This also indicates that using projected speed alone to estimate propagation time would produce large errors. Therefore, three-dimensional reconstruction of CMEs is of great significance for hazardous space weather forecasting.

4.3 November 16, 2007 Event

The third event is a southwestward CME observed on November 16, 2007, when the separation angle between the two STEREO spacecraft was 40°. This CME had a typical three-part structure, including a bright front, dark cavity, and bright core. The projected speed of the CME front obtained by SOHO-LASCO was 326 km/s.

Howard and Tappin [59] used the TP method to reconstruct the middle part of the CME front in this event, obtaining a projected speed of 274.11 km/s, three-dimensional speed of 322.71 km/s, and CME source region at S14°W73°.

Temmer et al. [45] used triangulation reconstruction on the front using STEREO-A+SOHO and STEREO-B+SOHO coronagraph data. The average source region location derived from the two datasets was W120°S10°, with an average three-dimensional speed of 403 km/s. Liu et al. [62] studied the CME kinematics (propagation direction and radial distance) from the Sun to 1 AU using their geometric triangulation method, comparing CME imaging reconstruction with MC in-situ reconstruction results, and calculated the CME source region location as S7°W123° and CME speed as 388 km/s.

Thernisien et al. [28] used the GCS method to reconstruct the magnetic flux rope of this event, obtaining a CME source region at S14°W123° and three-dimensional speed of 345 km/s, with a projected speed of 289 km/s.

These results show that speeds obtained by several reconstruction methods are all in the 350–400 km/s range, with three-dimensional speeds slightly higher than projected speeds, but each method yields slightly different speeds. These differences mainly arise because the three methods reconstruct different positions of the CME.

4.4 March 25, 2008 Event

The fourth event is a filament eruption-related CME on March 25, 2008, when the separation angle between the two STEREO spacecraft was 47°. The projected speed of the CME front given by LASCO was 1,103 km/s, with the source region at S10°E86°.

Liewer et al. [108] used the TP method to reconstruct this event, obtaining a three-dimensional front speed of 1,087 km/s and front position S9°E86°. Mierla et al. [20] applied LCT-TP, M-TP, and PR methods to this event. For COR1 data, the source region locations obtained by LCT-TP and CM-TP were S15°E89° and S14°E88°, respectively, while for COR2 data the results were S2°E88° and S7°E92°. The source region location obtained by the PR method differed from other methods by up to 40° in longitude, possibly because the polarization degree of radiation from the CME's high-density plasma center (filament material) is too low, naturally affecting the reliability of PR reconstruction based on the target's polarization state. Mierla et al. [20] mainly aimed to study CME propagation angles, so they did not provide three-dimensional speeds from the reconstruction. Thernisien et al. [28] used the GCS method to reconstruct the CME's magnetic flux rope, obtaining a CME speed of 1,130 km/s and source region location S12°E84°.

The reconstruction results from different methods for this event show basically consistent source region locations, with deviations from actual observations not exceeding 10°, within the error range. Although the methods used by Liewer et al. [108] and Thernisien et al. [28] differ significantly, the CME front speeds they provide are similar, both about 1,100 km/s. The CME propagation directions obtained by these reconstruction methods differ by less than 10°, while the propagation directions obtained by Mierla et al. [20] using COR1 and COR2 data differ by about 12°, possibly because the CME was deflected near the source region.

4.5 August 7, 2010 Event

The fifth event is a halo CME on August 7, 2010, when the separation angle between the two STEREO spacecraft was about 150°, and the projected speed of the CME front given by LASCO was 871 km/s.

Feng et al. [55] applied several different reconstruction methods to this event, including the MF method, GCS method, and local correlation tracking plus triangulation (LCT-TR) method. Based on existing results and accumulated experience, results obtained by the GCS method are more accurate, so this paper uses the GCS method as the standard to verify the reliability of the MF method, while calculating the three-dimensional CME centroid and its longitudinal and latitudinal extents using different reconstruction methods (as shown in Table 3).

Table 3 shows that the CME centroid longitude and latitude obtained by the MF method are similar to those from the GCS method, with similar latitudinal extents but significantly different longitudinal extents. The LCT-TR method yields large deviations in all results because the LCT method is based on the correlation of plasma fluctuations in two STEREO images, and this correlation's reliability is low when the separation angle between STEREO twin spacecraft is too large. At the time of the August 7, 2010 event, the separation angle between STEREO-A and STEREO-B was about 150°, causing large deviations in the LCT-TR reconstruction results.

5 Summary and Outlook

CMEs are the main source of disturbances in the interplanetary environment and space weather. Real-time prediction of CME or ICME propagation direction and arrival time at Earth is needed for hazardous space weather forecasting. Although the arrival time of ICMEs at Earth is an important content of hazardous space weather forecasting, more important are the speed and southward magnetic field component (Bz) of ICMEs when they reach Earth. Additionally, information about ICME three-dimensional structure and three-dimensional velocity also has important reference value. This paper provides a detailed introduction to several commonly used CME imaging reconstruction methods and two CME-driven shock reconstruction methods. Each of these methods emphasizes and reconstructs certain important features of CMEs from different perspectives, but their descriptions in other aspects differ significantly from actual conditions. This indicates that the information obtained from current observational methods cannot reflect the full picture of CMEs, nor can it extract complete information about CMEs. New detection methods and observational techniques are worth further exploration and development.

Currently used CME imaging reconstruction methods are divided into two major categories: reconstruction based on small and medium field-of-view coronagraph data and reconstruction based on large field-of-view or ultra-large field-of-view coronagraph (or heliospheric imager) data. Depending on different mathematical methods and targeted physical characteristics, each major category contains different subcategories. The CME reconstruction methods based on coronagraph data introduced in this paper include the cone model method, polarization ratio method, GCS model method, 3DCORE model method, triangulation method, TP method, and MF method. Except for the polarization ratio method, all other methods can obtain CME three-dimensional geometric structure. Among them, the cone model is an early model describing CME geometric structure. It does not assume CME front features and is more suitable for describing halo CMEs. However, the cone model has no magnetic field structure, making it difficult to reliably predict the magnetic field of CMEs near Earth or other locations. Nevertheless, this model has been successful in predicting whether CME-driven shocks will reach Earth and when they will arrive. The GCS method works well for reconstructing three-dimensional velocity and propagation direction of individual CMEs, but its fitting accuracy is very low for consecutive CME eruptions. When using the 3DCORE model for reconstruction, GCS fitting results are often used for setting initial parameters. The reconstruction results mainly provide magnetic field information at 1 AU, which is particularly important for geomagnetic storm prediction. The polarization ratio method converts single-viewpoint polarization observations into 3D distances from the sky plane, but can only obtain the distance-weighted average of CME plasma density along each line of sight, providing no depth information, velocity, or position information. Triangulation and TP methods require no assumptions about CME geometry. As long as CME features can be clearly distinguished in images, good results can be obtained, but neither method can reconstruct electron density in CMEs. Moreover, one assumption of the TP method is that the basic framework of affine geometry holds, so there is a magnification factor, and the results will have some deviation from actual conditions. The HT technique developed from the TP method has a unique advantage: it can display motion processes and velocities in real time with high sensitivity, extracting the faintest features and detecting their motion. The MF method can reconstruct CME surfaces well without assuming CME shape, is more flexible to use, and can analyze geometric centers and principal axis scales after obtaining three-dimensional CME shape. The MF method is based on three-viewpoint reconstruction, yielding more accurate results than two-viewpoint reconstruction. Its disadvantage is that the results do not include CME internal structure and plasma distribution.

Reconstruction methods based on heliospheric imager data include geometric triangulation, SSE method, ellipse evolution model method, and correlation-aided reconstruction method. Both the SSE method and ellipse evolution model require assumptions about CME geometry. The SSE method yields unreliable results when CME elongation angles are too large or too small, and the limitation that elongation angles cannot be too large restricts the SSE model's application to space weather forecasting. Additionally, the SSE model assumes CME fronts have circular cross-sections. The ellipse evolution model assumes elliptical cross-sections, making the model more flexible and more suitable for self-consistent modeling of multi-point observations. The unique advantage of geometric triangulation over the previous two methods is that it has no free parameters, makes fewer assumptions, and can track CMEs continuously from the Sun to 1 AU. The correlation-aided reconstruction method was proposed for small-scale solar wind transients. For large-scale CME reconstruction, it decomposes them into numerous small CMEs. This method's advantages are that it requires no assumptions about CME geometry, can be automated, is simple to operate, and can obtain the radial velocity distribution of CMEs in three-dimensional space. Its disadvantages are that it can only be applied to CMEs near the Sun-Earth line, and the coplanarity effect leads to very low longitudinal positioning accuracy for CMEs near the coplanarity region.

Three-dimensional reconstruction of CME-driven shocks is also very important because CMEs are closely related to their shocks, and shock reconstruction can provide three-dimensional structure and kinematics of shocks. The ellipsoidal shock model superimposed on the GCS model, applied to three-viewpoint observation data, can well reproduce the three-dimensional structure of CMEs and their driven shocks. The spherical shock model can be applied to shocks driven by extreme CMEs, which are very fast and drive shocks with very wide angular widths. Currently, comparing shock reconstruction results with CME properties, energetic particle characteristics, and in-situ solar wind observations is of great significance for studying space weather variations and high-energy particle acceleration, and is also a hot topic of concern in space science and solar physics research.

As mentioned above, any fitting contains multiple free parameters that are degenerate (especially in non-linear fitting), so the solution is not unique and is unstable. The more free parameters, the more uncertain the solution. Therefore, introducing more free parameters, while bringing certain degrees of freedom, also makes the method's disadvantages more obvious. This is also the superiority of the geometric triangulation method, which has no free parameters. These reconstruction methods have played important roles in studying CME kinematics and dynamics, but they all have some shortcomings. First, these reconstruction methods make many assumptions and add corresponding constraints. Second, according to the principles of three-dimensional structure, constructing a three-dimensional structure of a target from two-dimensional images requires at least three two-dimensional images of the target, and the lines connecting the observation points to the target cannot lie in the same plane. Moreover, these three lines are preferably perpendicular to each other. However, current imaging observations of the Sun and its activities are basically carried out in the ecliptic plane, so when reconstructing our research targets based on such observational data, we are obviously still missing one dimension of information.

CME internal magnetic field structure, density extrema, and magnetic field Bz component (i.e., the southward magnetic field component) are all important parameters for studying the physical nature of CMEs, with the southward magnetic field component being a very critical quantity for predicting geomagnetic storms. CME imaging reconstruction generally cannot provide magnetic field information inside CMEs, so it is necessary to first use some magnetic cloud reconstruction methods based on local observation data to calculate magnetic field-related information. For example, the earliest cylindrically symmetric force-free field magnetic flux rope model [109, 110] and the recent elliptical-cylindrical analytical magnetic flux rope model [111] can both provide various components of the magnetic field in magnetic clouds and other related information. There is also the Grad-Shafranov (GS) method that does not need to assume the magnetic cloud cross-section. Initially used to study Earth's magnetosphere magnetic field structure [112], Hu et al. [113, 114] and Liu et al. [115, 116] later found that this method could be used for magnetic field analysis in magnetic clouds. GS reconstruction results can be verified by observation data from multiple spacecraft with separation angles, which requires the satellites to be separated by relatively large distances (but not too large, otherwise the MC would be distorted by the solar wind) to provide observational consistency. Therefore, Liu et al. [116] used observation results from STEREO and Wind/ACE satellites for verification. Previously, verification only used Wind and ACE, but since these two satellites are too close together, they essentially observe the same structure and cannot provide effective verification. There are also some early representative works related to magnetic cloud reconstruction, see references [12, 117–119]. In the future, comparative studies between CME imaging reconstruction and magnetic cloud (or ICME) local reconstruction will receive increasing attention because they can provide important information about magnetic flux rope structure and magnetic field orientation, such as the orientation of the magnetic flux rope axis and whether rotation occurs during propagation. Currently, some scholars have already carried out work in this area, such as Liu et al. [62] using GCS imaging reconstruction to obtain the overall CME structure, then using the GS local reconstruction method to obtain information about local magnetic flux ropes near 1 AU; Chen et al. [120] have similar work. Hazardous space weather forecasting requires not only prediction of CME arrival time at Earth but also prediction of the southward magnetic field component (Bz) and speed. This highlights the importance of comparing CME imaging reconstruction results with magnetic cloud (or ICME) local reconstruction results.

Obtaining correct values for these parameters depends on complete and correct reconstruction of CME three-dimensional structure. "Complete" means that no assumptions need to be added during reconstruction, and no models need to be relied upon—only observation data from three mutually perpendicular viewing angles are required. Of course, current observational data cannot meet this requirement, so the reconstruction methods used all contain corresponding assumptions or rely on certain models. The only way to overcome these difficulties is to carry out observations outside the ecliptic plane. The Solar Orbiter [121] launched by Europe in 2020 has achieved observations outside the ecliptic plane. The Solar Orbiter carries a coronagraph and heliospheric imager at a distance of about 0.28 AU from the Sun. Although not as close to the Sun as the Parker Solar Probe [122], its inclination relative to the ecliptic plane is larger, enabling observations of the Sun from higher latitudes. Therefore, using Solar Orbiter observation data to reconstruct three-dimensional CMEs will reveal more three-dimensional CME information, helping us study CME internal magnetic field structure and plasma distribution more deeply and comprehensively. If we can also conduct close-up observations (probes) of the Sun [123], our reconstruction of CMEs/ICMEs will be more reliable, and the corresponding forecasts will be more accurate.

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