ChinaRxiv

AI translation · View original & related papers at
chinarxiv.org/items/chinaxiv-202306.00379

Postprint of the Polytropic Gas Hydrostatic Model of Prestellar Cores

Authors: Li Dalei

Date: 2023-06-07T00:00:00+00:00

Abstract

Prestellar cores are important samples for studying the initial conditions and earliest stages of star formation. Density and temperature constitute the two most fundamental physical parameters for characterizing prestellar cores, and are also essential for studying molecular excitation conditions, calculating molecular column densities using non-local thermodynamic equilibrium radiative transfer methods, and conducting astrochemical simulations. Barnard 68 (B68) is a highly typical prestellar core possessing a near-spherical morphology, rendering it exceptionally suitable for testing spherically symmetric theoretical models. In a spherically symmetric coordinate framework, a polytropic hydrostatic model has been employed to fit the dust extinction data of B68, yielding a mass of 0.8 M for the prestellar core B68. Compared with previous isothermal hydrostatic models, this model not only successfully reproduces the dust extinction data, but its results also align with observational findings of the non-isothermal gas distribution in B68, and furthermore show excellent agreement with observational pressure data.

Full Text

The Polytropic Static Gas Model of Prestellar Core

LI Da-lei
Xinjiang Astronomical Observatory, Chinese Academy of Sciences, Urumqi 830011, China

Abstract

Prestellar cores are crucial samples for investigating the initial conditions and earliest stages of star formation. Density and temperature are the two most fundamental physical parameters for characterizing prestellar cores, and they are also essential inputs for studying molecular excitation conditions, calculating molecular column densities, and conducting astrochemical simulations using non-local thermodynamic equilibrium radiative transfer calculations. Barnard 68 (B68) is a prototypical prestellar core with a nearly spherical structure, making it an ideal target for testing spherically symmetric theoretical models. In a spherical coordinate system, we fit the dust extinction data of B68 using a polytropic static gas model, which yields a mass of 0.8 M for the prestellar core. Compared with previous isothermal static gas models, this model not only successfully reproduces the dust extinction data but also agrees with observational results of non-isothermal gas temperature distribution in B68 and matches the observed pressure data well.

Keywords: molecular cloud core; star formation; prestellar core; hydrodynamics

1 Introduction

Dense cores in molecular clouds are the sites of star formation. Prestellar cores are self-gravitationally bound dense molecular cloud cores that can collapse under their own gravity to form protostars and eventually stars [1]. Representing the most initial state and earliest stage of the entire star formation process, prestellar cores provide strong constraints for setting initial conditions in numerical simulations and theoretical studies of star formation [1-3]. Therefore, prestellar cores are important samples for studying the initial conditions and earliest stages of star formation.

The physical conditions of prestellar cores, such as density and temperature, can be measured through observations of dust extinction, dust emission, and molecular spectral lines. For example, observations of dust extinction can yield the column density distribution of prestellar cores, which combined with model simulations can be used to obtain their density structure [4], while multi-band observations in the far-infrared and submillimeter wavelengths can simultaneously provide both the density and dust temperature structures [5, 6]. The gas temperature of prestellar cores must be obtained through observations of molecular spectral lines, such as NH_3 [7-9].

The density and temperature structures estimated from dust and molecular spectral line observations are the most basic physical parameters for describing prestellar cores. In non-local thermodynamic equilibrium radiative transfer calculations, density and temperature are necessary input parameters for studying molecular line excitation conditions, simulating molecular line profiles, estimating molecular abundances, and investigating molecular depletion and ice formation on dust grains [10-14]. Additionally, density and temperature are the most fundamental physical quantities in theoretical studies of star formation [2, 3], numerical simulations [1], and astrochemical modeling [15].

Barnard 68 (B68) is a typical prestellar core with a nearly circular structure (see [Figure 1: see original paper]), making it suitable for testing star formation models [4]. Using optical and near-infrared observations, Alves et al. [4] precisely measured the extinction of background starlight by dust in B68 and constructed the radial distribution of dust extinction as a function of core radius (see Figure 1b). The dust extinction can be converted to hydrogen molecular column density using N(H+2H_2)/A_V = 2$×10^{21}$ cm^{-2} mag^{-1} [16], which allows investigation of B68' s density structure. Further studies found that B68' s density structure can be well described by an isothermal static model (i.e., the isothermal Lane-Emden equation) [4]. However, subsequent observations, particularly molecular spectral line data, revealed that B68' s gas temperature is not isothermal but rather higher in the center (~10 K) and lower at the boundary (~6-7 K) [17]. Therefore, it is necessary to reinvestigate the density and temperature structures of this prototypical prestellar core B68 using more accurate models. Section 2 introduces the polytropic static gas model and the basic steps for model fitting, Section 3 presents the simulation results and discussion, and Section 4 provides a summary and outlook.

Note: (a) Optical image from the Digitized Sky Survey, with the red '+' marking the position of maximum dust extinction; (b) Radial distribution of dust extinction.
[Figure 1: see original paper] Optical image of prestellar core B68 and radial distribution of dust extinction

2.1 Dust Extinction Data

The data for prestellar core B68 used here are from reference [4]. Using optical and near-infrared observations of starlight in the B68 region, Alves et al. [4] precisely measured the extinction of background stars by dust in B68 and obtained the radial distribution of dust extinction.

2.2 Polytropic Static Gas Model

Prestellar core B68 can be treated as a spherically symmetric ideal gas sphere in hydrostatic equilibrium under the combined action of gravity and thermal pressure. Since B68' s gas temperature distribution is higher in the interior and lower at the boundary [17], we adopt a spherically symmetric polytropic static gas model [18] to more accurately describe its density and temperature structures. The mass density is given by:

$$\rho = \rho_0 \theta^n = \mu_g m_{H_2} n_{H_2} \theta^n$$

where ρ_0 and n_{H_2} are the central mass density and number density of the prestellar core, m_{H_2} is the mass of H_2, μ_g is the mean molecular weight, and n is the polytropic index. The dimensionless variable θ is determined by the following differential equation:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right)+\theta^n=0$$

where ξ is a dimensionless radius related to the physical radius by:

$$\xi=\frac{r}{\sqrt{(n+1)K\rho_0^{1/n-1}/(4\pi G)}}$$

with K being a constant and G the gravitational constant. The central density should be finite, and the boundary conditions for Equation (2) are $\theta(0)=1$ and $\theta'(0)=0$.

The gas pressure of the prestellar core is:

$$P=K\rho^{1+1/n}$$

From the ideal gas equation of state and using Equations (1) and (5), the gas temperature structure is:

$$T=\frac{\mu m_H K\rho^{1/n}}{k_B}$$

where m_H is the mass of a hydrogen atom, k_B is the Boltzmann constant, and μ is the mean molecular weight. Setting the central temperature of the prestellar core as $T_0=10$ K [8, 9], from Equation (6) we obtain:

$$T_0=\frac{\mu m_H K\rho_0^{1/n}}{k_B}=\frac{\mu m_H K(\mu_g m_{H_2} n_{H_2})^{1/n}}{k_B}$$

The mass of the prestellar core is:

$$M=-4\pi\left[\frac{(n+1)T_0 k_B}{\mu m_H}\right]^{3/2}\frac{1}{(4\pi G)^{3/2}}\int_0^{\xi_{\max}}\xi^2\frac{d\theta}{d\xi}d\xi$$

where $\xi_{\max}$ is the maximum dimensionless radius. Through this polytropic static gas model, we can investigate the density structure, temperature structure, and pressure structure of prestellar core B68 and calculate its mass.

2.3 Model Fitting

The basic steps for fitting the dust extinction data of B68 using the polytropic static gas model are as follows. First, for a given polytropic index n, a numerical solution to Equation (2) can be obtained using the fourth-order Runge-Kutta method. Second, given a central number density n_{H_2}, the number density distribution of the prestellar core is obtained as $n_{H_2}\theta^n(\xi)$. Third, with the central gas temperature $T_0 = 10$ K [8, 9], the values of n, n_{H_2}, and T_0 are substituted into Equation (7) and combined with Equation (3) to derive the relationship between the physical radius r and the dimensionless radius ξ. Fourth, for a given value of ξ, the radius of the prestellar core is obtained. Additionally, since the observed dust extinction data are a function of angular scale, the distance D to the prestellar core must be specified to convert the model’ s linear scale to angular scale. Finally, the obtained number density distribution as a function of radius is integrated along the line of sight to obtain the column density, which is converted to dust extinction using N(H+2H$_2$)/A_V = 2$×10^{21}$ cm^{-2} mag^{-1} [16] for comparison with the observed dust extinction data. In summary, the model includes four free parameters: n, n_{H_2}, ξ, and D. To find the parameter values that best match the observational data, we employ a Monte Carlo optimization algorithm implemented through the MAGIX software package [19].

3 Results and Discussion

Using the Monte Carlo optimization algorithm, we obtain a theoretical model that matches the observational data (see [Figure 2: see original paper]). Figure 2 compares the fits to the observed data from both isothermal and polytropic static gas models. The polytropic model yields a larger radius compared to the isothermal model. [Figure 3: see original paper] shows the posterior distributions of the four free parameters in the model. These one-dimensional posterior functions are nearly Gaussian. The maximum likelihood values of the distributions give the best-fit parameters, with errors taken from the 16% and 84% percentiles. The best-fit parameter values are: polytropic index $n = 8.02^{+1.17}_{-0.92}$, distance to the prestellar core $D = 74.58^{+0.51}_{-0.47}$ pc, dimensionless radius $\xi = 2.46^{+0.12}_{-0.14}$, and central H_2 number density $n_{H_2} = 3.16^{+0.02}_{-0.02} \times 10^5$ cm^{-3}.

Note: The dots with error bars represent the measured dust extinction data, the black solid line shows the isothermal static gas model, and the blue solid line shows the polytropic static gas model adopted in this work.
[Figure 2: see original paper] Comparison between observational data and theoretical models

Note: The optimal parameter values are taken from the maximum likelihood, with left and right errors corresponding to the 16% and 84% percentiles.
[Figure 3: see original paper] One- and two-dimensional posterior distributions of parameters obtained using the Monte Carlo optimization algorithm

Note: The central temperature and pressure are 10 K and 5.14×10^{-11} Pa, respectively.

[Figure 4: see original paper] Normalized temperature and pressure distributions as functions of core radius from the model

Substituting the best-fit parameter values into Equations (5) and (6) and combining with Equation (1) yields the temperature and pressure structures of prestellar core B68. [Figure 4: see original paper] shows the normalized radial distributions of temperature and pressure, with central values of 10 K and 5.14×10^{-11} Pa, and boundary values of 6.7 K and 1.3×10^{-12} Pa, respectively. Compared with the previous isothermal model [4], the boundary temperature from this model better matches the observational result of 6–7 K [17]. The measured dust temperature at the boundary is higher than the gas temperature, primarily due to heating by the interstellar radiation field [5, 6, 20]. The current model does not include the effects of the interstellar radiation field and therefore cannot explain the measured dust temperature. Additionally, B68 is located near the Loop 1 superbubble. X-ray observations from the ROSAT observatory indicate that the pressure of the Loop 1 superbubble is approximately 1.2×10^{-12} Pa [21], which is very close to the boundary pressure of 1.3×10^{-12} Pa derived from our model. Finally, using Equation (8), we obtain a mass of 0.8 M for prestellar core B68. Assuming that 30% of the gas mass is ultimately converted into stellar mass [22], a star of 0.24 M with spectral type M5 could form through self-gravitational collapse [23, 24].

Compared with the model of Li et al. [25], our adopted model is simpler, has fewer free parameters, and is more widely known as a fundamental equation for studying stellar structure [18]. Moreover, the model proposed by Li et al. [25] is a newly developed and relatively complex model with stringent conditions, requiring the existence of a radial linear velocity field, which represents a very strict limitation for its application.

4 Summary and Outlook

In a spherical coordinate system, we have fitted the dust extinction data of B68 using a polytropic static gas model. During the fitting process, we employed a Monte Carlo optimization algorithm to explore the posterior distributions of the free parameters, thereby obtaining the best-fit parameters and their uncertainties. Unlike previous isothermal static models, the polytropic static gas model adopted here not only reproduces the observed extinction data but also reasonably explains the gas temperature and pressure structures. Additionally, the model yields a mass of 0.8 M for prestellar core B68. Assuming a gas-to-star conversion efficiency of 30%, a star of 0.24 M with spectral type M5 could form through self-gravitational collapse.

The Atacama Large Millimeter/submillimeter Array (ALMA) has sufficient resolution and sensitivity to probe the structure of prestellar cores in distant cluster-forming regions in the Galactic plane. Due to the complexity of environments in cluster-forming regions, isothermal models may have limitations. Future work will attempt to use the polytropic static gas model to study the density distributions of prestellar cores in such environments.

References

[1] Larson R B. MNRAS, 1969, 145: 271
[2] Shu F H. ApJ, 1977, 214: 488
[3] McKee C F, Tan J C. Nature, 2002, 416(6876): 59
[4] Alves J F, Lada C J, Lada E A. Nature, 2001, 409(6817): 159
[5] Nielbock M, Launhardt R, Steinacker J, et al. A&A, 2012, 547: A11
[6] Roy A, André P, Palmeirim P, et al. A&A, 2014, 562: A138
[7] Ho P T P, Townes C H. ARA&A, 1983, 21: 239
[8] Lai S P, Velusamy T, Langer W D, et al. AJ, 2003, 126(1): 311
[9] Hotzel S, Harju J, Juvela M. A&A, 2002, 395: L5
[10] Tafalla M, Myers P C, Caselli P, et al. ApJ, 2002, 569(2): 815
[11] Bergin E A, Alves J, Huard T, et al. ApJ, 2002, 570(2): L101
[12] Hotzel S, Harju J, Juvela M, et al. A&A, 2002, 391: 275
[13] Di Francesco J, Hogerheijde M R, Welch W J, et al. AJ, 2002, 124(5): 2749
[14] Laurie E U C, Klaus H. arXiv:2106.08333, 2021
[15] Maret S, Bergin E A, Tafalla M. A&A, 2013, 559: A53
[16] Bohlin R C, Savage B D, Drake J F. ApJ, 1978, 224: 132
[17] Bergin E A, Maret S, van der Tak F F S, et al. ApJ, 2006, 645(1): 369
[18] Chandrasekhar S. An introduction to the study of stellar structure. Chicago: the University of Chicago press, 1939
[19] Möller T, Bernst I, Panoglou D, et al. A&A, 2013, 549: A21
[20] Bergin E A, Tafalla M. ARA&A, 2007, 45(1): 339
[21] Breitschwerdt D, Freyberg M J, Egger R. A&A, 2000, 361: 303
[22] Alves J, Lombardi M, Lada C J. A&A, 2007, 462(1): L17
[23] Morgan W W, Keenan P C, Kellman E. An atlas of stellar spectra, with an outline of spectral classification. Chicago: the University of Chicago press, 1943
[24] Johnson H L, Morgan W W. ApJ, 1953, 117: 313
[25] Li D, Lou Y Q, Esimbek J. MNRAS, 2018, 473(2): 2441

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.