# Possible Constraints on Neutron Star Internal Structure from Astronomical Observations (Postprint)

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## Abstract

Deep space pulsar detection is a key factor in constraining the neutron star equation of state and theoretical models. In particular, the detection of exotic pulsars and their systems, such as sub-millisecond pulsars, binary neutron star systems, and neutron star-black hole systems, is expected to drive innovations in stellar evolution theory, neutron star equation of state, and theoretical models. This work primarily reviews observations of macroscopic properties of pulsars—including pulsar rotation speed, mass, gravitational redshift, moment of inertia, and gravitational wave detection—and introduces a series of research advances in constraining the internal structure of neutron stars.

## Full Text

## Preamble

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### The Possible Constraints of Astronomical Observations on the Internal Structure of Neutron Stars

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## Abstract

Deep space pulsar detection is a key factor in constraining the equation of state (EOS) and theoretical models of neutron stars. In particular, the detection of exotic pulsars and their systems—such as sub-millisecond pulsars, binary neutron star systems, and neutron star-black hole systems—holds promise for advancing stellar evolution theory, neutron star EOS, and theoretical models. This work reviews observations of pulsar macroscopic properties, including pulsar rotation speeds, masses, gravitational redshifts, moments of inertia, and gravitational wave detections. It also introduces a series of research advances in constraining the internal structure of neutron stars, demonstrating how pulsar observations can impose constraints on the EOS and internal composition of neutron stars.

**Keywords:** neutron star; internal structure of neutron star; equation of state; astronomical observation
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## 1. Introduction

The Milky Way contains approximately $1.2 \times 10^5$ neutron stars, most of which are observed as pulsars [?, ?, ?]. Pulsar observations span multiple wavelengths, covering radio, infrared, optical, ultraviolet, X-ray, and gamma-ray bands, as well as very high-energy radiation [?]. The Australia Telescope National Facility (ATNF) has currently cataloged only 3,320 pulsars, representing a small fraction of the total neutron star population. As pulsar search and observation technologies continue to advance, an increasing number of pulsars will be discovered.

Neutron stars represent one of the final products of stellar evolution. They are often regarded as giant atomic nuclei, but with a crucial difference: neutron stars are gravitationally bound or self-bound at the surface under strong interactions, whereas atomic nuclei rely on nuclear forces to tightly bind nucleons. The nature of nuclear forces remains fundamentally strong interaction. Currently, the internal structure and composition of neutron stars are not well understood. In addition to traditional neutron stars composed of neutrons, protons, electrons, and muons proposed in early studies, scientists believe neutron stars may also be hyperon stars composed of hyperonic matter, quark stars composed of quark matter, hybrid stars composed of hadron-quark mixtures, or strangeon stars composed of strange quark clusters. Research into neutron star internal structure represents a major challenge in current theoretical and observational studies of neutron stars, and detailed information about their internal structure is difficult to obtain through direct means.

The equation of state (EOS) is a thermodynamic equation describing the state of matter under specific physical conditions and serves as a crucial bridge connecting neutron star internal structure with macroscopic properties. The neutron star EOS provides the relationship between pressure and energy density within the star, containing substantial information about its internal structure. Once the neutron star EOS is determined, the star' s mass and radius can be obtained by solving the hydrostatic equilibrium equations under general relativistic effects (the Tolman-Oppenheimer-Volkoff, or TOV equations) [?, ?], expressed as:

$$\frac{dP(r)}{dr} = -\frac{[P(r) + \varepsilon(r)][M(r) + 4\pi r^3 P(r)]}{r(r - 2M(r))}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

where $\varepsilon(r)$ and $P(r)$ represent the energy density and pressure of the neutron star, respectively, and $M(r)$ denotes the neutron star mass.

The EOS fundamentally depends on quarks and their strong interactions. Due to the low-energy non-perturbative effects of complex quantum chromodynamics (QCD), first-principles theoretical calculations are not feasible, necessitating research through phenomenological or microscopic nuclear many-body models combined with astronomical observations that constrain macroscopic stellar properties. Observations of pulsar macroscopic properties—such as stellar rotation, mass, radius, gravitational redshift, and gravitational wave radiation—represent the most important astronomical constraints in neutron star EOS and internal structure research. Han Jinlin, Li Di, Li Kejia, and Wang Na have conducted foundational research on pulsar searches, rotation characteristics, and radiation features, earning high international reputation [?, ?, ?, ?, ?, ?]. Shen Hong, Zuo Wei, Zheng Xiaoping, Fan Yizhong, Chen Liewen, Xu Renxin, and Li Ang have explored many fundamental issues in neutron star EOS research, achieving fruitful results [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. Notably, Shen et al. [?] proposed the Shen EOS, widely used internationally for astrophysical calculations, which incorporates contributions from exotic hadrons such as hyperons that may appear in neutron star cores and is publicly available on the EOS website. Zheng et al. [?, ?] studied the suppression of quark star thermal evolution by nucleon deconfinement heating and the role of r-mode instability in neutron star thermal evolution. Lai et al. [?] proposed the strangeon star model, a new type of quark star model suggesting that the constituent units of neutron star matter might be hadron-like "quark clusters" carrying strangeness. Li et al. [?, ?] used GW170817 to constrain quark star EOS, suggesting that GW170817 may have originated from a (superfluid) quark star, constrained quark interaction parameters based on tidal deformation observations, and used multi-messenger observations of GW170817 to constrain the maximum neutron star mass and the critical density for strange phase transitions.

Section 2 provides a brief review of the scientific background, focusing on how observations of pulsar macroscopic properties—including rotation characteristics, mass measurements, surface gravitational redshift, moment of inertia, and gravitational wave detection—can constrain neutron star internal structure. Section 3 offers a summary and outlook for this work.

## 2. How Pulsar Observations Constrain Neutron Star Internal Structure

The commissioning of large radio telescopes worldwide provides powerful support for the independent observation and discovery of new and exotic pulsars. China’ s Five Hundred Meter Aperture Spherical Radio Telescope (FAST), the future 110 m Xinjiang Qitai Radio Telescope, and the 120 m Yunnan Jingdong Radio Telescope all have detection of unknown types of pulsars—such as millisecond-submillisecond pulsars (MSP-SMSPs), neutron star-black hole (NS-BH) binary systems, and gravitational wave verification—as key scientific objectives [**?**, **?**, **?**, **?**]. As important targets for radio telescope detection, observations and theoretical studies of pulsar macroscopic properties have become essential pathways for understanding stellar evolution, exploring neutron star internal structure and exotic matter composition, and investigating the formation and evolution of compact objects.

### 2.1 Pulsar Rotation Characteristics

Pulsar rotation characteristics form the foundation for other related research, with the rotation period $P$ and its time derivative generally obtained directly from astronomical observations. Pulsars are classified into normal pulsars and millisecond pulsars (MSPs) based on their evolutionary history and rotation period magnitude. Normal pulsars typically have rotation periods on the order of seconds, while MSPs generally refer to pulsars with rotation periods below 30 ms. In 1967, the first normal pulsar PSR B1919+21 was observed with $P = 1.34$ s. In 1982, the first millisecond pulsar MSP B1937+21 was discovered with $P = 1.56$ ms. On April 18, 2018, China’ s FAST discovered its first millisecond pulsar MSP J0318+0253 with $P = 5.19$ ms, which received international certification. In 2006, Hessels et al. used the Green Bank Telescope to discover MSP J1748−2446ad in the globular cluster Terzan 5, rotating at 716 Hz ($P = 1.40$ ms), making it the fastest-spinning millisecond pulsar discovered to date. If MSP J1748−2446ad has a mass less than $2M_\odot$ ($M_\odot$ denotes solar mass), its rotation speed implies a radius smaller than 16 km. Currently, MSP J1748−2446ad can be used to constrain the upper limit of neutron star rotation frequency. Due to its very low flux density and high eclipse fraction (approximately 40% of the orbit), detecting this pulsar is extremely difficult, suggesting that even faster-rotating pulsars—submillisecond pulsars (SMSPs)—may exist [**?**, **?**].

Submillisecond pulsars are rapidly rotating neutron stars with rotation periods less than 1 ms. They represent a key scientific target for FAST surveys and have become a hot research topic in astrophysics due to their potential to resolve the controversy over whether pulsars are fundamentally neutron stars or quark stars, or to verify whether substantial quark matter exists inside neutron stars. In 1998, Madsen [**?**] conducted theoretical research on the possibility that submillisecond pulsars might be quark stars. In October 1999, NASA’ s Rossi X-ray Timing Explorer first detected the low-mass X-ray binary (LMXB) XTE

J1739−285 [?]. In 2007, Kaaret et al. [?] and Zhang et al. [?] reported its rotation frequency as 1,122 Hz ($P = 0.89$ ms). However, observations in February and March 2020 by Bult et al. [?] found no evidence for this 1,122 Hz rotation frequency, leading them to conclude that XTE J1739−285 cannot have a submillisecond rotation period. In 2004, Han et al. [?] used the Parkes 64 m radio telescope to search for submillisecond pulsars but detected none. In 2009, Du et al. [?] conducted in-depth studies on the formation mechanisms and possible detection methods for submillisecond pulsars. In 2019, Han et al. [?] designed the FAST Galactic Plane Pulsar Snapshot Survey (GPPS survey) observation mode based on FAST' s key scientific research projects. Compared to the previously most sensitive Arecibo L-band Feed Array pulsar survey (PALFA), the GPPS survey is generally about an order of magnitude more sensitive. To date, FAST has discovered approximately 500 pulsars in less than two years of operation. With the continuous improvement of FAST and global radio telescope detection capabilities, the certification of exotic objects like submillisecond pulsars is expected to be achieved soon.

## 2.2 Pulsar Mass Measurements

The successive discoveries of four pulsars with masses around $2M_\odot$—PSRs J1614−2230 ($P = 3.15$ ms), J0348+0432 ($P = 39$ ms), J0740+6620 ($P = 2.89$ ms), and J2215−5135 ($P = 2.61$ ms)—provide strong evidence for the existence of massive neutron stars [?, ?, ?, ?, ?]. Additionally, in 2019, the Laser Interferometer Gravitational-wave Observatory (LIGO) and Virgo interferometer detected the binary merger gravitational wave signal GW190814, suggesting it contains a compact star with mass $2.5M_\odot \sim 2.67M_\odot$, which could be either a massive neutron star or a low-mass black hole [?, ?]. In 2021, Logoteta et al. [?] used finite-temperature extensions of the quantum many-body theory Brueckner-Bethe-Goldstone in the Brueckner-Hartree-Fock approximation to calculate structural properties of non-rotating protoneutron stars (PNSs). They proposed a viable mechanism for producing low-mass (approximately $2M_\odot$) black holes, which is significant for explaining the possibility that GW190814 might be a BH-BH merger. Since gravitational wave detection did not reveal the nature of the compact object in GW190814, whether this object is a neutron star or black hole will impose constraints on neutron star theoretical models.

Precise mass measurements of pulsars provide an effective method for constraining neutron star EOS. However, pulsar mass measurement is challenging. Mass measurements are typically obtained through observations of binary systems such as double neutron star systems (NS-NS or BNS), neutron star–white dwarf binaries (NS-WD), high-mass X-ray binaries (HMXB), low-mass X-ray binaries (LMXB), and neutron star–main sequence star systems (NS-MS). Currently, only a small fraction of pulsars in binary systems have achieved mass measurements. For isolated neutron stars (INS), the Neutron Star Interior Composition Explorer (NICER) has published mass measurements for only one pulsar, MSP J0030+0451 ($P = 4.87$ ms), along with constraints on its radius [?].

Since NICER’ s mass measurements depend heavily on accumulated observation time/data, pulse profile modeling, and source brightness, most isolated neutron star masses remain unmeasurable.

Nevertheless, with continuous development in pulsar timing, X-ray, and optical observations, the number of pulsars with credible mass measurements has grown steadily over the past decades, making it possible to statistically infer the characteristics of neutron star mass distributions. Neutron star mass distributions contain substantial physical information: neutron star formation mechanisms, evolution of compact binaries through mass accretion, and stellar EOS all leave distinct observable signatures on the mass distribution. In 1999, Thorsett and Chakrabarty [?] reviewed general issues in radio pulsar binary mass measurements. In 2005, Lattimer and Prakash [?] found that the maximum measured neutron star mass determines the upper limit of observable cold matter energy density, highlighting the significance of current and future neutron star mass measurements. In 2011, Valentim et al. [?] used Bayesian statistics to study 54 measurement points from various systems, obtained the neutron star mass distribution, and evaluated the likelihood of their proposed Gaussian peak. In 2013, Kiziltan et al. [?] reviewed radio pulsar mass measurements and provided a detailed survey enabling rigorous constraints on the underlying neutron star mass distribution. In 2018, Liu et al. [?] conducted research on neutron star mass distributions and formation mechanisms, performing statistical studies on measured masses of 72 neutron stars in 63 binary systems, suggesting two possible formation mechanisms: iron core-collapse supernova explosions and electron-capture supernova explosions. In the same year, Alsing et al. [?] used Gaussian mixture models to analyze mass distributions of neutron stars in binary systems, providing the first positive evidence for multimodality and a maximum mass cutoff in neutron star mass distributions using Bayesian models, with 74 neutron stars having reliable masses. Their inference of the maximum neutron star mass can impose strict constraints on neutron star EOS. In 2020, Shao et al. [?] updated Alsing et al.’ s statistical results, increasing the number of neutron stars with reliable masses to 103. Using methods insensitive to neutron star EOS, they investigated prospects for forming supramassive neutron stars (SMNSs) in BNS mergers, ultimately providing statistical characteristics of massive neutron stars: relatively short periods, relatively weak surface magnetic fields, older ages, and moderate distances from Earth.

### 2.3 Pulsar Gravitational Redshift

Pulsar mass is difficult to measure, and radius is even more challenging. Therefore, researchers seek relationships between pulsar mass and radius. Gravitational redshift from pulsar surface radiation spectral lines is a physical quantity related to both mass and radius. Compared to radius, pulsar mass is relatively easier to measure. If astronomical observations obtain both mass and gravitational redshift measurements for a pulsar, the star’ s radius can be calculated. Consequently, measuring gravitational redshift from pulsar surface radiation spectra has become a probe for studying neutron star EOS [?, ?, ?]. General relativity gives the following relationship for stellar gravitational redshift (in geometric units where $G = c = 1$) [?, ?]:

$$z_s = \left(1 - \frac{2M}{R}\right)^{-1/2} - 1$$

Combined with the TOV equations, the surface gravitational redshift value can be obtained. Tang et al. [?] proposed in 2020 a method to estimate neutron star masses using gravitational redshift measurements, employing EOS constrained by gravitational wave data, nuclear experiments, and the maximum mass of non-rotating neutron stars to infer masses of isolated neutron stars. This constrained neutron star EOS can map a series of mass-radius data points obtained by solving the TOV equations, yielding a probability distribution of gravitational redshift versus mass, and ultimately determining the effective mass range for a given gravitational redshift. They used this method to estimate masses of three thermally emitting radio-quiet isolated neutron stars in "The Magnificent Seven" (M7) discovered by the Roentgen satellite (ROSAT): RXs J1856.5−3754, J0720.4−3125, and J1308.6+2127/RBS 1223, achieving relatively high precision. This method is therefore highly beneficial for constructing mass distributions of isolated neutron stars. In 2021, Sen analyzed the effect of temperature on stellar gravitational redshift using finite-temperature EOS for neutron star matter, with numerical results satisfying current constraints from the largest measured neutron star gravitational redshift values: $z_s = 0.35$ (EXO 07482−676), $z_s = 0.23$ (1E 1207.4−5209), and $z_s = 0.205^{+0.003}_{-0.006}$ (RX J0720.4−3125) [?]. Sun et al. [?] proposed a new set of quasi-universal relations among gravitational redshift, moment of inertia (or angular momentum), and gravitational binding energy for static, general rotational, and Keplerian rotational cases based on 11 EOS from normal and hybrid neutron star microscopic nuclear many-body theories, providing new methods for studying unobservable or difficult-to-observe neutron star properties.

### 2.4 Pulsar Moment of Inertia

Research shows that astronomical observations can determine stellar moment of inertia through pulsar timing, providing supplementary constraints on stellar EOS. Specifically, relativistic spin-orbit coupling in binary systems causes precession, and astronomical observations can obtain moment of inertia measurements by measuring periastron precession rates, with the magnitude of precession closely related to the binary system' s orbital period and compactness [?].

In neutron star internal structure models, stellar moment of inertia can be obtained by solving Einstein's field equations. For slowly rotating, axisymmetric neutron stars, the metric in spherical coordinates can be written (in geometric units where $G = c = 1$) as [?, ?]:

$$ds^2 = -e^{2\Phi(r)}dt^2 + \left(1 - \frac{2M(r)}{r}\right)^{-1} dr^2 - 2\omega r^2 \sin^2\theta dt d\phi + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where $J$ and $\Omega$ generally represent the angular momentum and angular velocity observed in the asymptotic inertial frame, and $\omega$ denotes the angular velocity in the local inertial frame. $\bar{\omega} = \Omega - \omega$ represents the local inertial frame dragging angular velocity, satisfying the following differential equation:

$$\frac{d}{dr}\left(r^4 j(r)\frac{d\bar{\omega}(r)}{dr}\right) + 4r^3\frac{dj(r)}{dr}\bar{\omega}(r) = 0$$

The boundary conditions satisfying equation (5) at the neutron star center $r = 0$ and surface $r = R$ are $\bar{\omega}(0) = \bar{\omega}_c$ ($\bar{\omega}_c$ takes any constant) and $\frac{d\bar{\omega}(r)}{dr}\Big|_{r=0} = 0$ and $\bar{\omega}(R) = \Omega - \frac{2J}{R^3}$. $j(r)$ is defined as:

$$j(r) \equiv e^{-\Phi(r)}\sqrt{1 - \frac{2M(r)}{r}} \quad (r \leq R)$$

$$j(r) = 1 \quad (r > R)$$

$\nu(r)$ represents the metric, satisfying the following equation and boundary condition:

$$\frac{d\nu(r)}{dr} = \frac{M(r) + 4\pi r^3 P(r)}{r[r - 2M(r)]}$$

$$\frac{d\nu(r)}{dr} = -\frac{1}{[P(r) + \varepsilon(r)]}\frac{dP(r)}{dr}$$

$$\nu(R) = \frac{1}{2}\ln\left(1 - \frac{2M}{R}\right)$$

The neutron star moment of inertia can be written as:

$$I \equiv \frac{J}{\Omega} = \frac{8\pi}{3}\int_0^R r^4 e^{-\Phi(r)}\bar{\omega}(r) \times \frac{[P(r) + \varepsilon(r)]}{\sqrt{1 - 2M(r)/r}}dr$$

Combined with EOS and TOV equations, the above equations can be solved to obtain the stellar moment of inertia.

In 2001, Li et al. [?] pointed out that neutron star moment of inertia depends strongly on the incompressibility coefficient and symmetry energy strength coefficient of nuclear matter, but not significantly on nucleon effective mass. Therefore, astronomical measurements of pulsar moment of inertia can constrain these coefficients and consequently the stellar EOS. In 2003, Bejger and Haensel studied the moment of inertia of the Crab pulsar, inferring its lower limit as $(97 \pm 38)M_\odot \cdot \mathrm{km}^2$, and roughly estimated the Crab pulsar' s mass and radius using correlations between moment of inertia, mass, and radius, with results depending on the Crab Nebula' s mass [?, ?]. In 2005, Lattimer and Schutz estimated the moment of inertia of star A in the double pulsar system PSR J0737−3039, noting that after several years of observation, the moment of inertia precision could reach about 10%, which would enable accurate estimation of stellar radius and pressure near $1 \sim 2$ times saturation density, thereby providing powerful constraints on neutron star EOS and internal physics. Through calculations with multiple EOS, Lattimer and Schutz also provided empirical formulas relating moment of inertia to mass and radius: when $M/R$ is greater than approximately $0.07M_\odot \cdot \mathrm{km}^{-1}$, the stellar moment of inertia can be approximated as:

$$I \simeq (0.237 \pm 0.008)MR^2 \left(1 + 4.2\frac{M\ \mathrm{km}}{M_\odot R}\right)$$

In 2010, Zhao and Zhang [?] calculated the moment of inertia of PNSs considering strange mesons based on relativistic mean field theory, noting that strange mesons reduce PNS moment of inertia. In 2018, Hong and Ren [?] provided theoretical values for the moment of inertia and gravitational redshift of the massive pulsar PSR J0348+0432 based on relativistic mean field theory with strange mesons. In the same year, Landry and Kumar [?] derived a relationship between moment of inertia and tidal deformation parameters using constraints on neutron star tidal deformation from the gravitational wave signal GW170817, calculating the moment of inertia of MSP J0737−3039A ($P = 22.6$ ms) as $I = 1.15^{+0.38}_{-0.24} \times 10^{45}$ g · cm$^2$. In 2019, Lim et al. [?] calculated the moment of inertia of MSP J0737−3039A using Bayesian analysis of nuclear energy density functionals under constraints from chiral effective field theory and finite nuclear matter properties. With a measured mass of $(1.3381 \pm 0.0007)M_\odot$, the moment of inertia at 95% confidence level falls within $(1.04 \sim 1.51) \times 10^{45}$ g·cm$^2$, with the most likely value being $I = 1.36 \times 10^{45}$ g · cm$^2$. They noted that for neutron stars in the $(1.3 \sim 1.5)M_\odot$ mass range, the crust contributes approximately $1\% \sim 6\%$ of the total moment of inertia. Assuming pulsar timing measurements of MSP J0737−3039A' s moment of inertia with 10% precision, the moment of inertia and tidal deformation parameters can strongly constrain each other, yielding an approximate functional relationship that can constrain Bayesian modeling of neutron star EOS.

### 2.5 How Gravitational Wave Detection Constrains Neutron Star EOS

Gravitational waves cause spacetime distortion in their propagation path. When gravitational waves pass between Earth and a pulsar, the path of the pulsar’s radio waves is periodically compressed and stretched, causing the radio signals received by telescopes to arrive periodically early or late. Long-term monitoring of pulse arrival times may capture gravitational wave signals. But how can gravitational wave signals be used to study neutron star internal structure? Like other celestial bodies, neutron stars deform under external tidal fields. For neutron star internal structure research, important gravitational wave sources primarily come from merger signals of BNS and NS-BH systems [**?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**]. From the late inspiral, merger, and ringdown gravitational waveforms of BNS or NS-BH systems, the dimensionless parameter $\Lambda = (2/3)k_2(GM/c^{2R})^{-5}$ describing their deformation can be extracted, which relates to $M$, $R$, and $k_2$. The Love number $k_2$ depends on $M/R$ and $y_R$, where $y_R$ requires solving the following differential equation [**?**]:

$$\frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^{2Q}(r) = 0$$

where $F(r)$ and $Q(r)$ are functions of $\varepsilon(r)$, $P(r)$, and $M(r)$, expressed as:

$$F(r) = \frac{r - 4\pi r^3[\varepsilon(r) - P(r)]}{r - 2M(r)}$$

$$Q(r) = \frac{4\pi r[5\varepsilon(r) + 9P(r) + \frac{\varepsilon(r)+P(r)}{\partial\varepsilon/\partial P}]}{r - 2M(r)} - \frac{6}{r^2}$$

The Love number $k_2$ is expressed as:

$$k_2 = \frac{8C^5}{5}(1-2C)^2[2+2C(y_R-1)-y_R]\times\{2C[6 - 3y_R + 3C(5y_R - 8)] + 4C^3[13 - 11y_R + C(3y_R - 2) + 2C^2(1 +$$

where $C \equiv M/R$. Equations (10)-(13) must be solved combined with EOS, TOV equations, and boundary conditions $y(0) = 2$, $P(0) = P_c$, $M(0) = 0$. Therefore, $\Lambda$ can be used to constrain neutron star EOS, mass, radius, sound speed, gravitational redshift, moment of inertia, and other properties.

In 2015, Bauswein and Stergioulas [**?**] pointed out that for a given BNS merger event, their classification scheme could estimate the total binary mass, threshold mass for black hole formation, and maximum neutron star mass. In 2017, Chatziioannou et al. [**?**] studied the EOS of colliding neutron stars, with constraints complementing tidal deformation measurements of neutron stars during the late inspiral phase. In 2018, Zhu et al. [**?**] proposed a quark-level neutron star EOS, the QMF18 model, which can well describe observational constraints from free-space nucleons, nuclear matter saturation properties, massive pulsars, and the gravitational wave signal GW170817. In 2020, Kanakis-Pegios et al. [?] proposed a method linking pre-merger tidal deformation measurements of BNS systems to maximum neutron star mass and possible upper limits on sound speed based on recent observations of INS and BNS systems. They noted that effective upper limits on tidal deformation favor soft EOS, while measured masses of massive pulsars favor stiff EOS. Under constraints from gravitational wave signals GW170817 and GW190425, they parameterized neutron star EOS using sound speed, indicating that future gravitational wave detections can constrain neutron star EOS more strictly. In 2021, Tang et al. [?] used Bayesian parameter inference with phenomenological neutron star EOS models based on gravitational wave signal GW170817 and mass-radius observations of MSPs J0030+0451, J0437−4715, and 4U 1702−429 to search for potential first-order phase transitions inside neutron stars. They noted that current observational data remain insufficient to support or exclude phase transitions, as models with and without phase transitions have comparable evidence. In the same year, Han et al. [?] proposed a new non-parametric method, FFNN (feed-forward neural network), to reconstruct neutron star EOS based on multi-messenger data. Using this method to analyze mass-tidal deformation measurement data from GW170817 and NICER’ s mass measurement and radius constraints for MSP J0030+0451, they found for canonical neutron stars at 90% confidence level: $\Lambda_{1.4} = 329^{+322}_{-163}$, $R_{1.4} = 11.87^{+1.21}_{-1.06}$ km. Also in 2021, Xie and Li [?] inferred posterior probability distributions (PDFs) for first-order hadron-quark phase transitions in dense neutron-rich matter and parameterized correlations for nine stellar EOS based on LIGO/Virgo observations of GW170817 and astronomical observations of canonical neutron star radii reported by NICER and Chandra telescopes. They simultaneously analyzed the mass fraction of quark matter (QM) in canonical neutron stars and stellar radii, predicting their values in more massive neutron stars, and noted that based on all neutron star EOS derived from existing astronomical observations and theoretical and nuclear physics constraints, substantial quark matter formation can be inferred inside canonical neutron stars. In 2021, Kanakis-Pegios et al. [?] first studied the effects of temperature on neutron star tidal deformability and gravitational wave signals before neutron star system mergers, noting that even at temperatures below $10^{10}$ K, temperature effects on tidal deformability cannot be ignored.

## 3. Summary and Outlook

In summary, future detection of exotic pulsars and their systems, along with gravitational wave observations, will be crucial for solving challenging problems in neutron star internal structure research. Observations of pulsar macroscopic properties—rotation characteristics, mass, gravitational redshift, moment of inertia, and gravitational wave detection—represent one of the most important research methods for constraining neutron star EOS and revealing internal structure in the future. For instance, due to their extremely high rotation frequencies, sub-millisecond pulsars may play a key role in screening neutron star EOS and verifying structural evolution and mechanisms in rapidly rotating neutron stars. Studies of sub-millisecond pulsar macroscopic properties can provide important clues about whether quark matter exists inside them and whether their mass range relates to possible mass gaps between neutron stars and black holes. Constraining the maximum neutron star mass can provide effective research approaches for questions about whether quark matter exists inside neutron stars, what fraction of stellar mass it occupies if present, and whether hadron-quark phase transitions occur within neutron stars. Discoveries of exotic pulsars and their systems can promote astronomical observations to find effective methods for distinguishing neutron stars from quark stars. Gravitational redshift measurements provide potential methods for constraining or estimating neutron star mass and radius. Future astronomical observations will obtain more radiation from neutron stars across various wavelengths, making verification of pulsar gravitational redshift possible. Future gravitational wave signal detection will extract more data usable for constraining pulsar macroscopic properties. Additionally, when neutron star–neutron star or neutron star–black hole systems merge, they are often accompanied by short gamma-ray bursts and kilonova radiation. Observations of these phenomena, through the link of numerical relativity simulations, will help us constrain neutron star internal states and greatly advance our understanding of neutron star internal structure [**?**, **?**]. Discussions on how short gamma-ray bursts and kilonovae constrain neutron star internal structure will be gradually supplemented and improved in future research.

We believe future pulsar searches and deep space exploration will advance research using pulsar macroscopic properties as constraints on neutron star EOS. We anticipate that detection results for exotic pulsars and their systems will ultimately reveal neutron star internal structure, providing effective solutions for screening neutron star EOS and improving the accuracy and reliability of neutron star theoretical models, achieving the optimal model of cross-validation between theoretical models and astronomical observations.

## References

[1] Tauris T M, Manchester R N. MNRAS, 1998, 298: 625
[2] Faucher-Gigu‘ere C A, Kaspi V M. ApJ, 2006, 643: 332
[3] 刘鹏, 王培, 李药, 等. 天文学进展, 2018, 36: 173
[4] Zhi Q J, Bai J T, Shang L H, et al. JHEAP, 2021, 32: 71
[5] Oppenheimer J R, Volkoff G M, Phys Rev, 1939, 55: 374
[6] Tolman R C. Phys Rev, 1939, 55: 364
[7] Luo R, Wang B J, Meng Y P, et al. Nature, 2020, 586: 693
[8] Lin L, Zhang C F, Wang P, et al. Nature, 2020, 587: 63
[9] Zhu W W, Li D, Luo R, et al. ApJ, 2020, 895: L6
[10] Lei Q, Pan Z C, Li D, et al. SCPMA, 2019, 62: 4
[11] Lu J G, Peng B, Xu R X, et al. SCPMA, 2019, 62: 14
[12] Zhang B. Nature, 2020, 587: 45
[13] Wang Y N, Shen H. PRC, 2010, 81: 025801

[14] Dong J M, Lombardo U, Zuo W. PRC, 2013, 87: 062801
[15] Shen H, Fan J, Hu J N, et al. ApJ, 2020, 891: 148
[16] Zheng X P, Zhou Xia, Yu Y W. MNRAS, 2006, 371: 1659
[17] Yang S H, Zheng X P, Pi C M, et al. MNRAS, 2010, 403: 2007
[18] Wen D H, Li B A, Chen L W. PRL, 2009, 103: 211102
[19] Zhang D R, Jiang W Z, Wei S N, et al. EPJA, 2016, 52: 142
[20] Zhen Y Z, Li A, Hu J N, et al. PRC, 2016, 94: 045803
[21] Xu Y, Huang X L, Zhang X J, et al. MNRAS, 2018, 474: 3576
[22] 李昂, 胡金牛, 鲍世绍, 等. 原子核物理评论, 2019, 36(1): 1
[23] 来小禹, 徐仁新. 物理, 2019, 48(9), 554
[24] Fan Y Z, Jiang J L, Tang S P, et al. ApJ, 2020, 904: 119
[25] Shao D S, Tang S P, Sheng X, et al. PRD, 2020, 101: 063029
[26] Zhou E P, Zhou X, Li A. PRD, 2018, 97: 083015
[27] Li A, Miao Z Q, Han S, et al. ApJ, 2021, 913: 27
[28] 徐仁新. 天文学报, 2021, 62(01): 10
[29] 南仁东. 中国科学: 物理学力学天文学, 2005, 35(5): 449
[30] 王娜. 中国科学: 物理学力学天文学, 2014, 44(8): 783
[31] 南仁东, 张海燕, 张莹, 等, 天文学报, 2016, 57(6): 623
[32] Hessels J W T, Ransom S M, Stairs I H, et al. Science, 2006, 311: 1901
[33] Urbanec M, Miller J C, Stuchlik Z. MNRAS, 2013, 433: 1903
[34] Madsen J. PRL, 1998, 81: 3311
[35] Markwardt C B, Marshall F E, Swank J H, et al. IAUC, 1999, 7300: 1
[36] Kaaret P, Prieskorn Z, in' t Zand J J M, et al. ApJ, 2007, 657: L97
[37] Zhang C M, Yin H X, Zhao Y H, et al. PASP, 2007, 119: 1108
[38] Bult P, Altamirano D, Arzoumanian Z. ApJ, 2021, 907: 79
[39] Han J L, Manchester R N, Lyne A G, et al. https://arxiv.org/pdf/astro-ph/0310640.pdf
[40] Du Y J, Xu R X, Qiao G J, et al. MNRAS, 2009, 399: 1587
[41] Han J L, Wang C, Wang P F, et al. RAA, 2021, 21: 107
[42] Demorest P B, Pennucci T, Ransom S M. Nature, 2010, 467: 1081
[43] Fonseca E, Pennucci T T, Ellis J A, et al. ApJ, 2016, 832: 167
[44] Antoniadis J, Freire P C C, Wex N. Science, 2013, 340: 448
[45] Cromartie H T, Fonseca E, Ransom S M, et al. Nat Astron, 2020, 4: 72
[46] Kandel D, Romani R W. ApJ, 2020, 892: 101
[47] Godzieba D A, Radice D, Bernuzzi S. ApJ, 2021, 908: 122
[48] Fattoyev F J, Horowitz C J, Piekarewicz J, et al. PRC, 2020, 102: 065805
[49] Logoteta D, Perego A, Bombaci I. A&A, 2021, 646: A55
[50] Riley T E, Watts A L, Bogdanov S, et al. ApJ, 2019, 887: L21
[51] Thorsett S E, Chakrabarty D. ApJ, 1999, 512: 288
[52] Lattimer J M, Prakash M. PRL, 2005, 94: 111101
[53] Valentim R, Rangel E, Horvath J E. MNRAS, 2011, 414: 1427
[54] Kiziltan B, Kottas A, De Yoreo M, et al. ApJ, 2013, 778: 66
[55] 刘鹏, 张洁, 支启军, 等. 天文学报, 2018, 59(3): 27
[56] Alsing J, Silva H O, Berti E. MNRAS, 2018, 478: 1377
[57] Shao D H, Tang S P, Jiang J L, et al. PRD, 2020, 102: 063006
[58] Zhao X F. ChJPh, 2020, 63: 240

[59] 喻传赞. 科学通报, 1980, 16: 50 (doi:CNKI:SUN:KXTB.0.1980-16-017)
[60] 陈家麟, 季沛勇. 上海大学学报 (自然科学版), 2013, 19(2): 176
[61] Lindblom L. ApJ, 1984, 278: 364
[62] Lackey B D, Nayyar M, Owen B J. PRD, 2006, 73: 024021
[63] Tang S P, Jiang J L, Gao W H, et al. ApJ, 2020, 888: 45
[64] Sun W J, Wen D H, Wang J. PRD, 2020, 102: 023039
[65] Sen D. JPhG, 2021, 48: 025201.
[66] Greif S K, Hebeler K, Lattimer J M, et al. ApJ, 2020, 901: 155
[67] Bao G, Engvik L, Hjorth-Jensen M, et al. NPA, 1994, 575: 707
[68] Worley A, Krastev P G, Li B A. ApJ, 2008, 685: 390
[69] 李文飞, 张丰收, 陈列文. 高能物理与核物理, 2001, 25(10): 1006
[70] Bejger M, Haensel P. A& A, 2002, 396: 917
[71] Bejger M, Haensel P. A& A, 2003, 405: 747
[72] Lattimer J M, Schutz B F. ApJ, 2005, 629: 979
[73] 赵先锋, 张华. 四川大学学报, 2010, 47(4): 825
[74] Hong B, Ren Z Z. CPC, 2018, 42: 084105
[75] Landry P, Kumar B. ApJ, 2018, 868: L22
[76] Lim Y, Holt J W, Stahulak R J. PRC, 2019, 100: 035802
[77] Daniel A G, Rossella G, David R, et al. PRD, 2021, 103: 063036
[78] Baiotti L, Rezzolla L. RPPh, 2017, 80: 096901
[79] Debojoti K, Ritam M, Prasad R, et al. 2021, arXiv:2104.00422v1
[80] Torres-Rivas A, Chatziioannou K, Bauswein A, et al. PRD, 2019, 99: 044014
[81] Lackey B D, Kyutoku K, Shibata M, et al. PRD, 2012, 85: 044061
[82] Lackey B D, Kyutoku K, Shibata M, et al. PRD, 2014, 89: 043009
[83] Kumar P, Pürrer M, Pfeiffer H P. PRD, 2017, 95: 044039
[84] Weih L R, Hanauske M, Rezzolla L. PRL, 2020, 124: 171103
[85] Radice D, Perego A, Hotokezaka K, et al. ApJ, 2018, 869: 130
[86] Radice D, Perego A, Bernuzzi S, et al. MNRAS, 2018, 481: 3670
[87] Zhu Z Y, Zhou E P, Li A. ApJ, 2018, 862: 98.
[88] Bauswein A, Stergioulas N. PRD, 2015, 91: 124056
[89] Chatziioannou K, Clark J A, Bauswein A, et al. PRD, 2017, 96: 124035
[90] Kanakis-Pegios A, Koliogiannis P S, Moustakidis Ch C. PRC, 2020, 102: 055801
[91] Tang S P, Jiang J L, Gao W H, et al. PRD, 2021, 103: 063026
[92] Han M Z, Jiang J L, Tang S P, et al. ApJ, 2021, 919: 11
[93] Xie W J, Li B A. PRC, 2021, 103: 035802
[94] Kanakis-Pegios A, Koliogiannis P S, Moustakidis Ch C. 2021, arXiv:2110.10270
[95] 俞云伟. 物理, 2019, 48(9): 581
[96] 周恩平. 物理, 2019, 48(9): 573

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