Determination of Binary Star Orbital Parameters from Spectroscopic and Photometric Data: Postprint

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Abstract

By cross-matching the medium-resolution data from LAMOST (Large sky Area Multi-Object fiber Spectroscopic Telescope) DR7 and partial DR8 with the WISE eclipsing binary catalog of Chen et al. [1], we obtained 151 short-period solar-type stars. We then selected 23 stars with more than 9 observations and relatively uniform phase distribution. Through radial velocity analysis, we derived orbital parameters for this sample, including orbital period, mass ratio, and orbital eccentricity. Our analysis revealed that the derived periods are essentially consistent with those provided by WISE, demonstrating the reliability of WISE periods. Subsequently, using the WISE periods as known quantities to solve for other orbital parameters, this approach yields more accurate values for the remaining orbital parameters. We selected 23 eclipsing binaries with well-fitted velocity curves, analyzed the statistical properties of these parameters, and compared them with the results of Ragavan et al. [2]. We found that they are essentially consistent with the results of Ragavan et al. [2]: for stars with periods within 10 d, the mass ratio distribution covers nearly the entire range of 0.2 1, and the orbital eccentricities are relatively small, below 0.2.

Full Text

Preamble

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Determination of Orbital Parameters of Binary Stars Based on Spectroscopic and Photometric Data

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Abstract

By cross-matching the medium-resolution data from LAMOST (Large Sky Area Multi-Object Fiber Spectroscopic Telescope) DR7 and partial DR8 with the WISE eclipsing binary catalog of Chen et al., we obtained 151 short-period solar-type stars. From these, we selected 23 stars with more than nine observations and relatively uniform phase distribution. Through analysis of their radial velocities, we derived orbital parameters including orbital period, mass ratio, and orbital eccentricity for this sample. Our analysis reveals that the periods we obtained are basically consistent with those given by WISE, confirming the reliability of the WISE periods. Subsequently, we used the WISE periods as known quantities to solve for other orbital parameters, which improved the accuracy of these parameters. We selected 23 eclipsing binaries with well-fitted velocity curves and analyzed the statistical properties of these parameters. Comparison with the results of Raghavan et al. shows that our mass ratio distribution covers nearly the full range from 0.2 to 1 for periods less than 10 days, and the orbital eccentricities are all relatively small, below 0.2.

Keywords: binary stars; LAMOST; radial velocities; orbital parameters

1. Introduction

Binary stars are common across stellar systems of various masses, from star clusters to entire galaxies, and some astronomers have even suggested that the majority of stars exist in binary systems. Binary stars can be classified into several types based on whether one or both components fill their Roche lobes, a scheme introduced by Kopal. When neither star fills its Roche lobe, the two stars evolve independently, which is considered a detached binary system. When one star fills its Roche lobe, it is a semi-detached binary, and when both stars fill their Roche lobes, they form a common envelope with strong interaction through mass transfer, constituting a contact binary system. The mass ratio, component masses, and other orbital parameters of binary stars can be determined through analysis of light curves from eclipsing binary systems and time-domain spectroscopic observations.

Shortly after the birth of modern astronomy in the 17th century, the existence of binary and multiple star systems was recognized. Kuiper first proposed in 1935 that determining the multiplicity of stars and the distribution of orbital parameters would be theoretically valuable. Stellar multiplicity is a common outcome of star formation, making the characterization of binary frequency and the statistical study of orbital parameters powerful tools for testing formation processes. Such studies are crucial for many areas of astrophysics, as binary evolution significantly impacts stellar population evolution and can explain important phenomena such as Type Ia supernovae and gravitational wave events from black hole mergers. Whether in star clusters or entire galaxies, binaries are abundant across all scales of stellar systems, and their statistical properties —described by binary frequency, mass ratio distribution, orbital period distribution, and orbital eccentricity distribution—are typically related to the age, metallicity, and primary mass of the stellar population.

Various methods exist for studying binary stars, including light curve analysis, radial velocity measurements, and Hertzsprung-Russell diagram methods. In binary systems, the projected radial velocities vary as the components orbit their common center of mass, and these variations can be plotted against orbital phase once the orbital period is determined. Analysis of such velocity curves yields quantities describing the orbital geometry and the masses of the two stars. Following the discovery of Algol' s orbit in 1890, many graphical methods were developed, which have been further refined and improved with the advent of computers. The first successfully analyzed light curve of an eclipsing binary was for a circular orbit system, initially using techniques developed by Russell and Shapley, later extensively used and further developed by Kopal, who defined the famous binary classification scheme. Subsequent work explored Fourier methods for analyzing light curves, particularly for systems with distinctly non-spherical stars and those affected by other interactions. Meanwhile, increasing computational power enabled researchers to model light curves produced by non-spherical stars with non-uniform surface brightness distributions rotating about their common center of mass, leading to numerical methods that could synthesize binary light curves while accounting for all known astrophysics at the time. The work of Lucy, Mochnacki and Doughty, Rucinski on contact binaries, and Hill and Hutchings, Hutchings and Hill, and Wilson and Devinney on all types of binaries revolutionized binary analysis methods. They developed numerous programs for solving binary orbital parameters, which greatly improved the precision with which we can determine the physical properties of various types of binary systems and facilitated more rigorous testing of general binary evolution models.

In recent years, large-scale astronomical surveys have provided vast amounts of high-quality photometric and spectroscopic data, such as from LAMOST, SDSS, 2MASS, and Gaia. For these large datasets, statistical methods have been developed to study binary properties without distinguishing whether each star is single or binary, instead analyzing the overall statistical properties of the sample. These methods can significantly increase sample sizes and enable investigation of how binary properties vary with metallicity, effective temperature, binary frequency, and orbital parameters.

Metallicity, age, and mass are all closely related to star formation and evolution, and recent work has shown these are also intimately connected with the statistical properties of binary stars. For example, substantial evidence now indicates that the binary frequency of close solar-type binaries clearly decreases with increasing metallicity. Moe et al. conducted a comprehensive analysis and synthesis of discordant or contradictory results to accurately measure the relationship between close binary frequency and metallicity, finding that after correcting for completeness, all different datasets for close binaries show a consistent strong anti-correlation between binary frequency and metallicity. Their study used samples of binaries with separations less than 10 AU and orbital periods less than 10^4 days. Additionally, the binary frequency of close binaries decreases with increasing effective temperature. Gao et al. estimated the binary frequency for F, G, and K-type stars by analyzing radial velocity variations in SDSS and LAMOST stellar spectra, finding a binary frequency of 43.0%$±2.0±$8.0% for the LAMOST sample, both with orbital periods within 1,000 days.

Liu studied the binary frequency of stars observed by LAMOST and Gaia in the solar neighborhood, assuming a power-law mass ratio distribution with exponent γ, and found a clear anti-correlation between binary frequency and γ. In other words, stellar populations with higher binary frequencies contain more binaries with larger mass ratios. Using $\gamma = 1.2$ as a boundary, stars with high γ have lower masses and higher metallicities, while those with low γ have higher masses and lower metallicities. The binary frequency of high-γ stars shows an anti-correlation with metallicity but no correlation with primary mass, whereas low-γ stars exhibit a clear correlation between binary frequency and primary mass but no obvious correlation with metallicity.

The binary frequency of main-sequence stars also correlates significantly with primary mass. Duchêne and Kraus summarized empirical knowledge for main-sequence stars, brown dwarfs, pre-main-sequence stars, and embedded protostars. For solar-type and low-mass stars, the orbital period distribution is unimodal, but as stellar mass decreases, the median separation and distribution width decrease sharply. Consequently, the multiplicity frequency for stars in the 1-10 AU range does not vary significantly with stellar mass, and important parameters such as binary frequency vary smoothly with mass.

Building on these findings, we cross-matched large LAMOST datasets with WISE data to solve for the orbital periods, mass ratios, and orbital eccentricities of a sample of binary stars and analyzed their distributions, comparing them with the results of Raghavan et al.

This paper is organized as follows: Section 2 describes the research methods and models used to solve for binary orbital parameters; Section 3 introduces the data; Section 4 presents the results obtained by applying these methods to the data; and the final section provides a summary and outlook.

2. Methods

2.1 Research Methods

For the special case of circular orbits, the orbital velocity of an object about the center of mass is constant. However, for the general case of eccentric orbits, the velocity is a continuous function of position and time, being a strong function of position in elliptical orbits, particularly for highly eccentric ones. The position of a star in an elliptical orbit is not a simple function of time, and the fundamental equation describing this position-time relationship for elliptical orbits is Kepler' s equation:

$$E - e \sin E = \frac{2\pi(t-T)}{P}$$

where the right side is determined directly from observations and the left side represents the orbital phase measured in radians. Here, E is the eccentric anomaly, e is the orbital eccentricity, P is the orbital period, t is time, and T is the integration constant. From the properties of elliptical orbits, we can easily derive:

$$r \cos\theta = a(\cos E - e)$$
$$r = a(1 - e\cos E)$$
$$\cos\theta = \frac{\cos E - e}{1 - e\cos E}$$

Once the orientation of the binary orbit relative to the observer' s line of sight is specified, the above formulas can be used to derive the radial velocity produced by orbital motion:

$$V_{\rm rad} = \frac{2\pi a \sin i}{P\sqrt{1-e^2}}[\cos(\theta+\omega) + e\cos\omega]$$

The final expression for radial velocity is typically written as:

$$V_{\rm rad} = K[\cos(\theta+\omega) + e\cos\omega] + \gamma$$

where $K = \frac{2\pi a \sin i}{P\sqrt{1-e^2}}$ is the semi-amplitude of the velocity curve and γ is the systemic velocity. Thus, knowing a set of binary parameters—orbital period P, mass ratio q, orbital eccentricity e, orbital inclination relative to the line of sight i, argument of periastron ω, phase angle θ, and primary mass m_1—allows us to plot the radial velocity curve of the binary.

With n radial velocity measurements and known primary mass m_1, we use the EMCEE software package to perform Markov Chain Monte Carlo simulations. We adopt the maximum probability value from the MCMC random draws and the 15th and 85th percentiles as the best value and its uncertainty for each orbital parameter, obtaining the orbital period P, mass ratio q, orbital eccentricity e, orbital inclination i, argument of periastron ω, and orbital zero-phase θ.

2.2 Stellar Mass Determination

We estimate each star' s mass by comparing its effective temperature (T_{eff}), surface gravity ($\log g$), and metallicity ([Fe/H]) with the PAdova and TRieste Stellar Evolution Code (PARSEC) models. We use the T_{eff}, $\log g$, and [Fe/H] provided by PARSEC models as training data and stellar mass as training labels to train an XGboost model. The training process uses root mean square error (RMSE) as the loss function. Testing on independent samples reveals an RMSE of 0.02 $M_\odot$ between true and predicted stellar masses. We then use 10-fold cross-validation to confirm that the overall uncertainty introduced by the algorithm is 0.02 $M_\odot$.

Additionally, we estimate the error for each derived stellar mass using Monte Carlo methods. Specifically, we perform 1,000 samplings, each time randomly drawing new values for T_{eff}, $\log g$, and [Fe/H] from Gaussian distributions with means equal to the measured values and standard deviations equal to their uncertainties. These sampled parameters are then used with the trained XGboost model to estimate stellar mass. After multiple measurements, we adopt the median of each set as the derived stellar mass and the standard deviation as the error. The typical uncertainty in stellar mass is 0.03 $M_\odot$ (median value).

2.3 Method Validation

First, we simulate a single star with known orbital period, mass ratio, and orbital eccentricity, then estimate its orbital parameters and compare them with the preset true values to determine the method' s accuracy. Given the star' s observation times and radial velocities, we randomly select all other parameters and run MCMC. The parameters $\log P$, q, and e converge well to specific values (see Figure 1 [Figure 1: see original paper]). The true orbital parameters of this simulated star are $\log P = 1.90$, $q = 0.7$, $e = 0.3$, $m_1 = 1M_\odot$, with 20 observations uniformly distributed over a 60-day period.

For this star, we simulated how the difference between model-derived values and true values changes with increasing observation number, as shown in Figure 2 [Figure 2: see original paper]. The observation number ranges from 2 to 20, with the vertical axis showing $\log P$, q, and e. The black dashed line represents the true values, the blue line shows model values at different observation numbers, and red lines indicate error bars. As observation number increases, model values approach true values and error bars generally decrease. Since the radial velocity equation for binaries is controlled by seven parameters, reliable fitting becomes difficult when data points are fewer than parameters; the figure shows that when observations reach 8 or 9, model values are essentially consistent with true values.

Next, we statistically analyzed 50 simulated stars, with results shown in Figure 3 [Figure 3: see original paper]. These 50 stars have orbital periods ranging from 10 to 100 days, mass ratios from 0.3 to 0.9, orbital eccentricities from 0.1 to 0.7, $m_1 = 1M_\odot$, and observation numbers from 4 to 20. The first three panels show parameter errors for P, q, and e at different observation numbers, while the last three panels show the root mean square of these errors. As observation number increases, parameter errors fluctuate around zero, and the overall RMS error decreases. Moreover, if the orbital period P can be determined and used as a known quantity in the model, estimates of other orbital parameters become more accurate.

3. Data

LAMOST is a novel large-field and large-aperture telescope, also known as the Guo Shoujing Telescope. Employing active optics with thin mirrors and segmented mirrors, LAMOST can observe objects as faint as 18 mag within 1.5-hour exposures, making it the world' s largest optical telescope in terms of combined aperture and field of view. Using parallel controllable fiber positioning technology to place 4,000 fibers in a 5° field of view, LAMOST can simultaneously obtain spectra of 4,000 objects, giving it the highest spectral acquisition rate in the world.

We use medium-resolution data from LAMOST DR7 and partial DR8, employing blue-end observation times, velocities, and velocity errors, cross-matched with the WISE eclipsing binary catalog of Chen et al. We selected stars with LAMOST spectral signal-to-noise ratios greater than 10 and more than 8 observations, yielding 151 stars. We input these 151 stars into our model, with primary masses m_1 determined by the method described above, and compared the resulting periods with those from WISE. Figure 4 [Figure 4: see original paper] shows that the distribution of $P_{\mathrm{WISE}}/P_{\mathrm{LAMOST}}$ values is mostly around 1, with a few at 1/2, 2, and 3, indicating that our model-derived orbital periods are basically consistent with WISE values, though some are twice or three times the WISE period. We therefore consider the WISE period values to be fundamentally reliable.

Since LAMOST medium-resolution data provide relatively few radial velocity observation points, the derived periods lack reliability. Consequently, we use the WISE periods based on light curves as known quantities for determining binary orbital parameters. From the 151 stars, we selected 23 with relatively broad and uniform radial velocity phase distributions, small velocity errors, and well-fitted velocity curves. Figure 5 [Figure 5: see original paper] shows the distributions of effective temperature (T_{eff}), surface gravity ($\log g$), and metallicity ([Fe/H]) for these 23 stars from LAMOST. The effective temperatures are mainly concentrated around 5,000–7,500 K, surface gravities around 3.8–4.5, and metallicities around -0.5 to 0.5.

4. Results

We input the observation times and radial velocities of the 23 sample stars from LAMOST into our model, using the WISE orbital period P as a known quantity and determining primary masses m_1 as described above. The resulting orbital parameters were found to be somewhat inaccurate due to relatively large dispersion in LAMOST radial velocities, with orbital eccentricities e being generally too large and inconsistent with reality. We therefore used the corrected LAMOST radial velocities from Zhang et al. (submitted), which were obtained through cross-correlation with reference stars. Fitting the radial velocity curves yielded orbital parameters for these 23 stars: orbital period P, mass ratio q, and orbital eccentricity e. Table 1 lists these orbital parameters. The orbital periods are taken from WISE light-curve-based values, mass ratios q are as shown with an average uncertainty of about 0.1, and orbital eccentricities e have an average uncertainty of about 0.02. Figure 6 [Figure 6: see original paper] shows the model fits to the radial velocity curves for these 23 stars.

The distributions of orbital period P, mass ratio q, and orbital eccentricity e are shown in Figure 7 [Figure 7: see original paper]. The orbital periods, provided by WISE, are mostly less than 1 day, indicating short-period close binaries with a power-law distribution trend. The mass ratio distribution is relatively flat. Due to orbital circularization by dynamical tides, close binaries with short periods have small orbital eccentricities, and our model-derived eccentricities are indeed small as expected, all below 0.2.

We compared our results with those of Raghavan et al. Figure 8 [Figure 8: see original paper] shows the relationships between orbital period and mass ratio and orbital eccentricity, with the horizontal axis being $\log P$ (base 10) and vertical axes showing q and e. For orbital periods in the range $-0.6 \leq \log P \leq 0.2$, the mass ratio q covers nearly the entire space from 0.1 to 0.9, generally tending toward larger values, while orbital eccentricities e are all less than 0.2, mostly concentrated around 0.1.

We then compared our orbital period-mass ratio and orbital period-eccentricity relationships with those of Raghavan et al. Figure 9 [Figure 9: see original paper] shows the period-mass ratio relationship, where open circles represent our 23 short-period sample stars, and plus signs, solid triangles, and open squares represent binary, triple, and quadruple systems from Raghavan et al., respectively. Our results are basically consistent with Raghavan et al., with mass ratios covering nearly the full parameter space.

Figure 10 [Figure 10: see original paper] shows the period-eccentricity relationship, with the same symbols. The dashed curve represents an eccentricity boundary: to its left, orbits are so eccentric that at periastron one companion comes within 1.5 $R_\odot$ of the other, likely leading to collision. Our sample stars all have small eccentricities below 0.2, falling within this boundary.

5. Summary and Outlook

We used medium-resolution data from LAMOST DR7 and partial DR8 to obtain 23 sample stars and determined their orbital parameters through radial velocity analysis. The parameters generally meet expectations and are listed in Table 1. However, some issues remain. Although all orbital eccentricities e are within reasonable ranges, they are somewhat large, particularly in the second-to-last panel of the first column in Figure 6. This may be caused by errors in radial velocity measurements or non-uniform sampling phases in observation times, and we hope to improve this through higher data processing precision and increased observation numbers in the future. This method could be extended to become a hierarchical Bayesian model that can estimate orbital properties without determining parameters for individual stars.

References

[1] Chen X D, Wang S, Deng L C, et al. APJS, 2018, 237: 28
[2] Raghavan D, McAlister H A, Henry T J, et al. ApJS, 2010, 190: 1
[3] Heintz W D. AJ, 1969, 74: 768
[4] Abt H A, Levy S G. ApJS, 1976, 30: 273
[5] Duquennoy A, Mayor M. A&A,1991, 248: 485
[6] Kopal Z. AnAp, 1955, 18: 379
[7] Kopal Z. LIACo, 1959, 9: 74
[8] Kuiper G P. PASP, 1935, 47: 15
[9] Kuiper G P. PASP, 1935, 47: 121
[10] Henry T J, McCarthy D W Jr. ApJ, 1990, 350: 334
[11] Fischer D A, Marcy G W. ApJ, 1992, 396: 178
[12] Mason B D, Gies D R, Hartkopf W I, et al. AJ, 1998, 115: 821
[13] Mason B D, Henry T J, Hartkopf W I, et al. AJ, 1998, 116: 2975
[14] Moe M, Kratter K M, Badenes C. ApJ, 2019, 875: 61
[15] Gao S, Liu C, Zhang X B, et al. ApJ, 2014, 788: 37
[16] Liu C. MNRAS, 2019, 490: 550
[17] Duchêne G, Kraus A. ARA&A, 2013, 51: 269
[18] Lucy L B. ApJ, 1968, 151: 1123
[19] Lucy L B. ApJ, 1968, 153: 877
[20] Mochnacki S W, Doughty N A. MNRAS, 1972a, 156: 51
[21] Mochnacki S W, Doughty N A. MNRAS, 1972b, 156: 243
[22] Rucinski S M. Acta Astr, 1973, 23: 79
[23] Hill G, Hutchings J B. ApJ, 1970,162: 265.
[24] Hutchings J B, Hill G. ApJ, 1971, 166: 373
[25] Hutchings J B, Hill G. ApJ, 1971, 167: 137
[26] Wilson R E, Devinney E J. ApJ, 1971, 166: 605
[27] Cui X Q, Zhao Y H, Chu Y Q, et al. RAA, 2012, 12: 1197
[28] York D G, Adelman J, Anderson J E, et al. AJ, 2000, 120: 1579
[29] Stoughton C, Lupton R H, Bernardi M, et al. AJ, 2002, 123: 485
[30] Skrutskie M F, Cutri R M, Stiening R, et al. AJ, 2006, 131: 1163
[31] Lindegren L, Hernandez J, Bombrun A, et al. A&A, 2018, 616: 2
[32] Foreman-Mackey D, Hogg D W, Lang D, et al. PASP, 2013, 125: 306
[33] Zhao G, Zhao Y H, Chu Y Q, et al. RAA, 2012, 12: 723
[34] Bressan A, Marigo P, Girardi L, et al. MNRAS, 2012, 427: 127
[35] Chen Y, Girardi L, Bressan A, et al. MNRAS, 2014, 444: 2525
[36] Chen T, Guestrin C. 2016, arXiv:1603.02754

[37] Hilditch R W. *An Introduction to Close Binary Stars*, Cambridge: Cambridge University Press, 2001: 38
[38] Hilditch R W. *An Introduction to Close Binary Stars*, Cambridge: Cambridge University Press, 2001: 42

Note: Figure translations are in progress. See original paper for figures.