Postprint: Covariance Analysis of the Impact of Different VGOS Frequency Band Combinations on Group Delay Precision

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Abstract

The VLBI Global Observing System (VGOS) is a new-generation geodetic VLBI system capable of freely selecting up to four frequency bands within the 2-14 GHz range for observations. Unlike conventional geodetic VLBI systems that eliminate ionospheric effects through S/X dual-frequency observations, the substantial frequency span of VGOS ultra-wideband observations necessitates the simultaneous extraction of group delay observables and differential ionospheric total electron content from interference fringes across multiple bands. By incorporating ionospheric effects into the interferometric phase model, this study performs quantitative calculations of group delay precision using covariance analysis methods and investigates the impact of different VGOS frequency band combinations on group delay precision. Statistical analysis of group delay precision using international VGOS observational data demonstrates that the measured precision aligns with theoretical predictions, thereby validating the effectiveness of the proposed delay precision calculation methodology. This approach can be applied to the selection and optimization of VGOS ultra-wideband frequency sequences.

Full Text

Impacts of Various Frequency Band Combinations on the Uncertainties of VGOS Group Delays Based on Covariance Analysis

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Abstract

The VLBI Global Observing System (VGOS) represents a new generation of geodetic VLBI systems, capable of observing at up to four frequency bands within the 2–14 GHz range. Unlike conventional geodetic VLBI systems that employ S/X dual-frequency observations to eliminate ionospheric effects, VGOS' s ultra-wideband frequency coverage requires simultaneous extraction of group delay observables and differential ionospheric total electron content from interference fringes across multiple bands. By incorporating ionospheric effects into the interferometric phase model, we performed a quantitative calculation of group delay uncertainties using covariance analysis and investigated how different VGOS frequency band combinations affect group delay precision. Statistical analysis of international VGOS observational data demonstrates that the measured group delay uncertainties are consistent with theoretical predictions, thereby validating our computational method. This approach can be applied to the selection and optimization of ultra-wideband frequency sequences for VGOS observations.

Key words: VLBI; VGOS; ultra-wideband; group delay; dTEC

1 Introduction

In the late 1960s, radio astronomers proposed the Very Long Baseline Interferometry (VLBI) technique. After decades of development, VLBI station positioning accuracy has improved from meter-level to sub-centimeter level. This advancement is primarily attributed to bandwidth synthesis technology, calibration systems for instrumental delay correction, and dual-frequency observation strategies for eliminating ionospheric effects.

With technological developments in electronic equipment, global fiber networks, and low-cost antennas in the early 21st century, the International VLBI Service for Geodesy and Astrometry (IVS) proposed the concept of a next-generation geodetic VLBI system, which was officially named the VLBI Global Observing

System (VGOS) in 2012 to meet future high-precision requirements for astrometry and geodesy.

In September 2005, the IVS Working Group 3 (WG3) established VGOS objectives: (1) continuous monitoring of station positions and Earth orientation parameters (EOP) on a global scale; (2) rapid data processing within 24 hours; and (3) station position accuracy of 1 mm. To achieve these goals, VGOS implements measures to reduce both systematic and random errors in delay observables. To reduce random errors, VGOS employs antennas with approximately 12-meter apertures to shorten observation time per radio source and increase all-sky coverage of reference sources. Within the 2–14 GHz frequency range, up to four bands can be selected for observation to increase effective bandwidth. The expected broadband group delay precision is 4 ps.

In geodetic VLBI observations, group delay precision depends on effective bandwidth and signal-to-noise ratio (SNR), while SNR is determined by factors including the number of channels, single-channel bandwidth, integration time, system equivalent flux density (SEFD) of stations, and correlated flux density of radio sources. For a specific baseline, target source, and observation duration, frequency setup is the primary factor affecting group delay precision. Due to the use of ultra-wide bandwidth, dispersive ionospheric effects must also be addressed, requiring simultaneous fitting of differential total electron content (dTEC) along with group delay extraction. Broadband group delay and dTEC exhibit strong correlation, which must be considered when analyzing VGOS broadband group delay precision.

By introducing the dTEC parameter into the interferometric phase model, we have developed a covariance analysis method for calculating group delay uncertainties. This method was validated through statistical analysis and testing using international VGOS observational data. It enables comparison of different frequency combinations among various pre-selected frequency setup schemes, providing guidance for VGOS observation frequency selection and configuration.

2 Method for Calculating Group Delay Precision with dTEC Parameter in the Interferometric Phase Model

In VLBI observations, data from multiple frequency channels are typically recorded and combined through bandwidth synthesis to obtain baseline group delay observables. For VGOS ultra-wideband observations, ionospheric dispersive effects cannot be neglected, necessitating the introduction of the dTEC observable during group delay fitting. The interferometric phase model can be expressed as:

$$\phi(f) = \tau_g(f - f_0) + \phi_0 - 1.3445\, dTEC\, f^{-1}$$

where ϕ is the phase (in cycles), f is the frequency (in GHz), f_0 is the reference frequency (in GHz), τ_g is the group delay (in ns), ϕ_0 is the phase at the reference frequency (in cycles), and $dTEC$ is the differential ionospheric total electron content (in TECU).

According to linear least-squares principles, the parameter solution coefficient matrix A_{ij} can be expressed as:

$$A_{ij} = \sum_{k=1}^{N} \rho_k \frac{\partial \beta_i}{\partial x_k} \frac{\partial \beta_j}{\partial x_k}$$

where k denotes the channel number, ρ_k represents the weight of the k-th channel, and parameters β_1, β_2, and β_3 correspond to τ_g, ϕ_0, and $dTEC$, respectively. Their partial derivatives are:

$$\frac{\partial \tau_g}{\partial x_k} = f_k - f_0, \quad \frac{\partial \phi_0}{\partial x_k} = 1, \quad \frac{\partial dTEC}{\partial x_k} = b\, f_k^{-1}$$

where $b = -1.3445$. Therefore, A_{ij} can be expressed as:

$$A_{ij} = \begin{pmatrix} \sum \rho_k (f_k - f_0)^2 & \sum \rho_k (f_k - f_0) & b \sum \rho_k (f_k - f_0) f_k^{-1} \\ \sum \rho_k (f_k - f_0) & \sum \rho_k & b \sum \rho_k f_k^{-1} \\ b \sum \rho_k (f_k - f_0) f_k^{-1} & b \sum \rho_k f_k^{-1} & b^2 \sum \rho_k f_k^{-2} \end{pmatrix}$$

The parameter uncertainties σ_i are given by:

$$\sigma_i = \sigma_0 \sqrt{(A^{-1})_{ii}}$$

where σ_1, σ_2, and σ_3 represent the uncertainties of τ_g, ϕ_0, and $dTEC$, respectively.

3 Theoretical Analysis

For theoretical calculations to validate our method, we selected the four frequency bands from actual VGOS observation vo0009 (3 GHz, 5 GHz, 6 GHz, and 10 GHz) and combined them into five frequency setup modes. Each band contains eight frequency channels with 32 MHz bandwidth per channel. The sky frequencies are shown in . Sky frequencies represent the RF frequencies read from the observation schedule file (vex file). VGOS observations use the lower sideband (LSB). For example, the first channel with sky frequency 3032.40 MHz indicates a frequency range of 3000.40–3032.40 MHz. We denote bands A, B, C, and D as 3 GHz, 5 GHz, 6 GHz, and 10 GHz, respectively.

The five frequency combination modes are presented in . Mode 1 includes all four bands (A, B, C, D); Mode 2 includes B, C, and D (missing band A); Mode 3 includes A, C, and D (missing band B); Mode 4 includes A, B, and D (missing band C); and Mode 5 includes A, B, and C (missing band D). Mode 1 comprises 32 frequency channels, while Modes 2–5 each have 24 channels.

The weights in the equation primarily depend on channel SNR, which is related to the correlated flux density of the radio source, SEFD of both stations, single-channel bandwidth, and integration time. For the same baseline observing the same source with identical single-channel bandwidth and integration time, the effects of correlated flux density and station SEFD on different frequency channels are complex and difficult to quantify. Therefore, under the assumption of equal single-channel SNR, we set the weights ρ_k in the equation to equal values (equal weighting). Based on the equations, we calculated the uncertainties of τ_g, ϕ_0, and $dTEC$ for the five frequency setup modes under equal weighting conditions, as shown in . Column 2 lists the theoretical effective bandwidth for each mode, column 3 shows the number of channels, column 4 gives the group delay uncertainty, column 5 provides the reference phase uncertainty, column 6 presents the $dTEC$ uncertainty, and column 7 shows the correlation coefficient between group delay and $dTEC$. The effective bandwidth is calculated as:

$$\Delta f_{eff} = \sqrt{\sum_{i=1}^{N} (f_i - \bar{f})^2}$$

where N is the number of channels, f_i is the sky frequency of the i-th channel, and $\bar{f}$ is the mean sky frequency.

The results demonstrate that group delay uncertainties vary among different frequency setup modes, indicating that observation frequency configuration affects group delay precision. Mode 1 yields the smallest group delay error and highest fitting precision. The ascending order of group delay errors is: Mode 4, Mode 3, Mode 2, and Mode 5. This means that for VGOS observations, the bands influencing group delay precision most significantly are, in order: D, A, B, and C. Although Modes 3 and 4 have slightly larger effective bandwidths than Mode 1, their group delay uncertainties remain slightly larger due to SNR loss from reduced channel count. Column 7 shows the correlation between broadband group delay and $dTEC$. The correlation is strongest when band A is missing (Mode 2), followed by when band D is missing (Mode 5), and weakest for the four-band combination (Mode 1).

It is important to note that we assumed equal SNR for all channels, setting channel weights to equal values. Therefore, the theoretical group delay precision estimates are intended only for relative comparison among different frequency setup modes, not for absolute assessment of group delay precision. The purpose is to analyze the relative impact of different frequency configurations on group delay precision to guide frequency selection. However, the results in Section 4 are based on measured SNR values and incorporate all information including channel count, bandwidth, integration time, station SEFD, and source correlated flux density, enabling calculation of actual measurement precision.

4 Processing of Real Observational Data

Observation vo0009 was a 24-hour VGOS session conducted from January 9, 2020, 18:00 UTC to January 10, 2020, 18:00 UTC, observing 71 radio sources. The six participating stations were: GGAO12M (Gs) and Westford (Wf) in the USA, WETTZ13S (Ws) in Germany, ISHIOKA (Is) in Japan, and ONSA13NE (Oe) and ONSA13SW (Ow) in Sweden, as shown in [Figure 1: see original paper].

[Figure 1: see original paper]

We processed the raw data using the correlator platform at Shanghai Astronomical Observatory, Chinese Academy of Sciences, and performed post-correlation processing for all baselines using the HOPS software to obtain group delay observables and uncertainties for each scan. The four-band frequency setup matched , with 32 MHz bandwidth per channel.

We processed data for all five frequency setup modes from . Considering factors such as low signal amplitude in some channels, poor phase calibration (PCAL) signal quality, and poor fringe fitting, we only included observables with quality factors above 5 in the fringe fitting process.

Group delay precision depends on the correlated flux density of radio sources, which is related to total source flux density and baseline length. Longer baselines result in smaller correlated flux density, lower SNR, and poorer group delay precision, while shorter baselines yield larger correlated flux density, higher SNR, and better group delay precision. We processed all baselines from vo0009. To illustrate different baseline lengths, we present statistical results for the longest baseline Gs-Is (9,593 km) and the short baseline Gs-Wf (600 km).

4.1 Case Study of the Longest Baseline Gs-Is

The VGOS design target is group delay fitting precision equal to or better than 4 ps. We processed the Gs-Is baseline data from vo0009, obtaining group delay uncertainties for all scans across different frequency setup modes. The statistics are presented in . The Gs-Is baseline yielded 503, 530, 492, 491, and 511 valid observables for Modes 1, 2, 3, 4, and 5, respectively. For consistent comparison, we analyzed 480 common observables across all five modes.

The table presents statistical results for the Gs-Is baseline: column 3 shows the percentage of observables with group delay uncertainty below 4 ps; column 4 gives the median group delay uncertainty; column 5 provides the mean ratio of group delay uncertainties relative to Mode 1; and column 6 shows the standard deviation of this ratio time series.

For baselines longer than 9,000 km, Mode 1 observations achieve 64% of group delay uncertainties within 4 ps, with a median uncertainty of 3.4 ps, meeting the VGOS design target. Mode 5 observations show only 2% of scans within 4 ps, with a median uncertainty of 14.4 ps, far below VGOS expectations. Mode

5 (missing band D) shows 11% of scans within 4 ps, with a median uncertainty of 10.2 ps, also failing to meet expectations. Modes 3 and 4 achieve 50% and 54% of scans within 4 ps, respectively, with median uncertainties of 4.0 ps and 3.9 ps, satisfying VGOS requirements. [Figure 2: see original paper] visually displays the cumulative count of observables across different delay uncertainty intervals for the five modes, with the x-axis showing group delay uncertainty and the y-axis showing cumulative counts.

[Figure 2: see original paper]

Analysis of the actual data in and [Figure 2: see original paper] reveals the ascending order of group delay uncertainties: Mode 1, Mode 4, Mode 3, Mode 2, and Mode 5. The corresponding descending order of band influence on precision is: D, A, B, and C. This matches the theoretical analysis in Section 3.

4.2 Case Study of the Short Baseline Gs-Wf

Similarly, we processed the Gs-Wf baseline data from vo0009, obtaining group delay uncertainties for all scans across the five frequency setup modes. The statistics are presented in , with column definitions identical to .

The Gs-Wf baseline yielded 684, 680, 682, 681, and 686 valid observables for Modes 1, 2, 3, 4, and 5, respectively. For consistent comparison, we analyzed 674 common observables across all five modes.

For the Gs-Wf baseline, Mode 1 achieves 99% of group delay uncertainties within 4 ps, with a median uncertainty of 1.5 ps, meeting VGOS design targets. Modes 3 and 4 achieve 98% and 96% within 4 ps, respectively, with median uncertainties of 1.6 ps and 1.8 ps, showing slightly reduced performance compared to Mode 1. Mode 2 achieves only 45% within 4 ps, with a median uncertainty of 4.3 ps, approaching VGOS expectations. Mode 5 achieves only 30% within 4 ps, with a median uncertainty of 5.5 ps, failing to meet expectations. [Figure 3: see original paper] shows the cumulative count distribution for the Gs-Wf baseline across the five modes.

[Figure 3: see original paper]

The results in and [Figure 3: see original paper] show the same ascending order of group delay uncertainties: Mode 1, Mode 4, Mode 3, Mode 2, and Mode 5, with band influence ranking as D, A, B, and C. This pattern matches the Gs-Is results and the theoretical analysis in Section 3.

4.3 Other Baselines

We also processed other baselines from vo0009. Due to space limitations, detailed results are not presented here. Notably, statistical results for other baselines follow the same pattern as Gs-Is and Gs-Wf, consistent with the theoretical analysis in Section 3.

5 Conclusions and Discussion

Conventional geodetic VLBI uses S/X dual-frequency observations to eliminate ionospheric effects. Due to VGOS' s large frequency span, multi-band interference fringes must be used to simultaneously extract group delay observables and differential ionospheric delay. This paper presents a covariance analysis method based on VGOS ultra-wideband frequency sequences, investigating the impact of different frequency combinations on group delay precision by incorporating ionospheric parameters into the interferometric phase model.

Using international VGOS observation vo0009, we designed five frequency combination modes and theoretically calculated their relative impacts on broadband group delay precision. We then processed all baselines from this observation. Due to space constraints, we presented only long and short baseline results. Statistical analysis of broadband group delay uncertainties across the five modes shows excellent agreement with theoretical predictions, validating our method.

VGOS observations typically employ four bands with fixed frequency setups, but adjustments are sometimes necessary for actual conditions, such as avoiding radio frequency interference at certain stations, accommodating stations that can only record three bands, or conducting hybrid observations with traditional VLBI stations. Our method provides valuable guidance for frequency selection and configuration in such scenarios.

Channel SNR depends on factors like correlated source flux density and station SEFD, which are too complex to quantify precisely. Therefore, our theoretical calculations ignored their effects on fringe SNR across frequency channels. Additionally, we did not consider the impact of secondary sidelobe peaks in the delay resolution function on group delay ambiguity. Future work will incorporate these factors to further optimize VGOS frequency setup methods.

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Note: Figure translations are in progress. See original paper for figures.