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Postprint: Gravitational Wave Bursts from Core-Collapse Supernovae

Authors: Li Hongbo[1,2], Shao Lijing[2,3], Xu Renxin[1,2]

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Abstract

Since the advent of gravitational wave detection, core-collapse supernovae have been considered an important class of astrophysical sources of gravitational waves. This review covers the fundamentals of gravitational waves in general relativity, discusses the influence of the angular velocity of Fe core rotation and convective instabilities on gravitational wave bursts during the Fe core collapse and bounce phases of core-collapse supernovae, and examines the oscillation theory of proto-neutron stars—the final products of core-collapse supernova explosions—and the associated gravitational wave bursts. Finally, we anticipate that through coordinated observations of electromagnetic and gravitational waves, we will be able to achieve a more comprehensive understanding of the entire core-collapse supernova explosion process and thereby probe the internal structure of neutron stars.

Full Text

Preamble

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Gravitational Wave Bursts from Core-Collapse Supernovae

LI Hong-bo[1,2], SHAO Li-jing[2,3], XU Ren-xin[1,2]
(1. Department of Astronomy, School of Physics, Peking University, Beijing 100871, China;
2. Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China;
3. National Astronomical Observatory, Chinese Academy of Sciences, Beijing 100012, China)

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Abstract: Since the beginning of gravitational wave detection efforts, core-collapse supernovae have been recognized as important astrophysical sources of gravitational waves. This paper reviews the fundamentals of gravitational waves in general relativity and examines how the angular velocity of the iron core and convective instabilities affect gravitational wave bursts during the collapse and bounce phases of core-collapse supernovae. We also discuss the oscillation theory of proto-neutron stars—the final products of supernova explosions—and the resulting gravitational wave bursts. Finally, we anticipate that coincident observations of electromagnetic and gravitational waves will enable a better understanding of the complete core-collapse supernova explosion process and provide insights into the internal structure of neutron stars.

Key words: core-collapse supernovae; gravitational wave bursts; neutron stars; oscillation modes
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1 Introduction

Gravitational waves are ripples in spacetime predicted by Einstein' s general relativity, representing fluctuations of spacetime itself. According to gravitational wave theory, the power of gravitational radiation produced by matter is proportional to the square of the third time derivative of the mass quadrupole moment. Consequently, the more rapidly the mass quadrupole moment changes, the stronger the resulting gravitational waves. For a comprehensive treatment of gravitational wave theory, readers may refer to Maggiore' s monograph [?]. Ground-based gravitational wave detectors fall into two categories: resonant bar detectors and laser interferometer detectors. Due to their high sensitivity and broad frequency range, laser interferometer detectors have become the primary instruments for gravitational wave detection. On February 11, 2016, the LIGO and Virgo collaborations announced the first direct detection of gravitational waves from a binary black hole merger, the GW150914 event [?, ?].

With subsequent upgrades to the LIGO and Virgo laser interferometer gravitational wave detectors, an increasing number of gravitational wave events from binary black hole mergers have been observed [?, ?], heralding a new era of gravitational wave astronomy.

Since gravitational waves cannot be produced in laboratories at detectable levels, massive, rapidly moving astronomical objects become the primary sources. The more massive and compact the object, the greater the total energy of the emitted gravitational waves, as exemplified by binary black hole mergers [?]. Other sources include binary neutron star inspirals [?, ?, ?], supernova explosions [?, ?], and neutron star oscillations [?, ?]. Traditionally, experimentally detected gravitational wave sources are categorized into four main types: gravitational waves from the inspiral and merger of compact binaries [?, ?], continuous gravitational waves [?, ?], stochastic background gravitational waves [?, ?, ?, ?, ?], and gravitational wave bursts [?, ?, ?, ?]. Gravitational wave bursts typically arise from violent astrophysical events such as core-collapse supernova explosions [?] and pulsar glitches [?]. However, modeling stellar collapse requires consideration of hydrodynamics, neutrino transport, magnetic fields, and other complex physical processes, making numerical calculations extremely challenging. Additionally, the internal structure of neutron stars—one of the final products of core-collapse supernovae—remains poorly understood. These factors introduce significant uncertainties into theoretical predictions of gravitational wave burst waveforms, despite the certainty that core-collapse supernova explosions produce gravitational wave signals. Figure 1 [Figure 1: see original paper] illustrates a schematic waveform of a gravitational wave burst from rotating core collapse, as simulated by Ott et al. [?], though the detailed features of such waveforms require further investigation.

In this review, we focus on the core-collapse supernova explosion process, including gravitational wave bursts generated during core collapse, bounce, and the final neutron star formation stage, and discuss the prospects and significance of coincident electromagnetic and gravitational wave observations. Throughout this paper, we adopt a source distance of $D = 10$ kpc for all calculations.

2 Gravitational Wave Bursts from the Core-Collapse Supernova Process

Core-collapse supernovae represent explosive phenomena that occur when massive main-sequence stars exhaust their nuclear fuel. These explosions release approximately 10^{46} J of energy. For progenitor stars with masses exceeding $10M_{\odot}$, nuclear fusion in the core ultimately produces iron—the element with the highest binding energy per nucleon—which cannot undergo further nuclear reactions to release energy and support the star against self-gravity. As fusion continues, the iron core grows in mass. When its mass exceeds the Chandrasekhar limit, self-gravity overwhelms electron degeneracy pressure, causing the iron core to collapse and form a neutron star or black hole.

During collapse, when the central density exceeds nuclear saturation density $\rho_{\rm nuc} \approx 2.8 \times 10^{14}$ g cm^{-3}, a rebound shock is generated. Early studies of core-collapse supernovae suggested that the energy carried by this rebound shock would be sufficient to power the supernova explosion, a mechanism known as the prompt explosion scenario. However, subsequent research revealed that as the shock propagates outward, it dissociates iron nuclei, consuming substantial energy and ultimately failing to produce an explosion [?]. Following the supernova explosion, the newly formed neutron star—called a proto-neutron star—has an extremely high internal temperature and cools by emitting neutrinos. This process carries away approximately 10^{46} J of energy, with about 1% being absorbed by the material surrounding the proto-neutron star. Studies indicate that this energy deposition may successfully drive the supernova explosion [?, ?, ?]. Since neutrino propagation and energy transfer to the outer material require time, this mechanism is termed the delayed explosion scenario.

Based on this understanding, the core-collapse supernova process can be divided into four stages: core collapse, shock formation, shock propagation, and neutrino emission leading to the delayed explosion. Gravitational radiation is produced throughout these stages by the rotation of the pre-collapse iron core, convective instabilities during bounce, and oscillations of the proto-neutron star. We now discuss the gravitational wave signals generated by each process.

2.1 Rotating Iron Core Collapse and Bounce

Dimmelmeier et al. [**?**] investigated gravitational wave signals from rotating iron core collapse and bounce, finding that these signals depend critically on the pre-collapse core' s angular velocity Ω_c. In the Newtonian limit, the differential rotation angular velocity Ω satisfies the following relation [**?**]:

$$\Omega = \Omega_c \frac{A^2}{A^2 + d^2}$$

where $d = r \sin\theta$ is the distance from the rotation axis (θ being the polar angle) and the parameter A characterizes the degree of differential rotation. When $A \to \infty$, $\Omega/\Omega_c = 1$ (uniform rotation), and when $A \to 0$, $\Omega/\Omega_c = A^2/d^2$.

Dimmelmeier et al. [**?**] calculated the effects of angular velocity Ω_c, progenitor mass M, parameter A, and different equations of state on gravitational wave burst signals. Their results indicate that the gravitational wave signal depends primarily on the core' s angular velocity Ω_c, with progenitor mass M being secondary. They also found that differential rotation and different equations of state have minimal impact on the gravitational waves; detailed calculations can be found in Figures 3 and 11 of [**?**].

Based on the magnitude of the iron core' s angular velocity Ω_c, Dimmelmeier and collaborators classified the gravitational wave signals into three categories:

1. **Slow rotation** ($\Omega_c \lesssim 1 - 1.5$ rad s^{-1}): The resulting gravitational wave amplitude is small, with a maximum value $|h_{\max}|$ below 5×10^{-22}. Numerical simulations show that for slow rotation, strong and rapid convective overturn develops in the post-shock region due to negative entropy gradients formed after the shock decelerates. We discuss convective instabilities in detail in the following section.
2. **Moderate rotation** ($\Omega_c \sim 6 - 13$ rad s^{-1}): Faster rotation of the iron core produces a larger mass quadrupole moment and increases the mass of the rotating inner core, generating stronger gravitational wave signals.
3. **Rapid rotation** ($\Omega_c \gtrsim 13$ rad s^{-1}): Increased centrifugal forces cause the core to rebound, with calculations showing maximum gravitational wave amplitudes reaching $|h_{\max}| = 7.5 \times 10^{-21}$.

Ott [**?**] summarized gravitational wave signals produced by different iron core rotation rates. Table 1 presents the relevant parameters of gravitational wave signals for various iron core rotation speeds, where $|h_{\max}|$ is the maximum characteristic gravitational wave amplitude, E_{GW} is the total radiated energy, and f_{peak} is the frequency at which the gravitational wave energy spectrum dE_{GW}/df peaks.

According to stellar evolution models and pulsar spin period calculations by Heger et al. [?, ?], ordinary massive stars with solar metallicity have core rotation angular velocities below 1 rad s^{-1}. Based on Table 1, this corresponds to a characteristic gravitational wave amplitude of 0.5×10^{-21} and total radiated energy of approximately $0.1 \times 10^{-8} M_{\odot} c^2$. If such a source were located within the Milky Way, the resulting gravitational wave signal would be detectable by Advanced LIGO. More rapidly rotating iron cores may be associated with the origin of gamma-ray bursts; relevant studies can be found in [?].

2.2 Convective Instabilities

Most astrophysical fluid dynamics problems involve instabilities. An instability arises when a fluid in hydrostatic equilibrium is perturbed, leading to changes in density, pressure gradients, and entropy behind shock waves that drive fluid overturn and mixing until thermal equilibrium is restored. This motion is called convective instability. We now discuss gravitational wave signals generated by convective instabilities during the rebound phase of core-collapse supernovae.

As the rebound evolves, the proto-neutron star radiates neutrinos that deposit energy in the net energy gain region behind the shock, creating negative entropy gradients that drive neutrino-driven convective instabilities. Woosley and Weaver [?] considered gravitational wave signals from neutrino-driven convection and the standing accretion shock instability (SASI) [?, ?].

Woosley and Weaver [?] performed two-dimensional simulations of core-collapse supernova explosions for a $15 M_{\odot}$ progenitor. The entire simulation lasted 850 ms, with the characteristic gravitational wave amplitude [?] given by:

$$h_{\mathrm{char}}(f) = \sqrt{\frac{2G}{\pi c^3 D^2} \frac{dE_{\mathrm{GW}}}{df}}$$

where D is the source distance, G is the gravitational constant, and c is the speed of light. Due to neutrino-driven convection and SASI, the total radiated gravitational wave energy is approximately $7.5 \times 10^{-12} M_{\odot} c^2$, with a corresponding frequency range of $100 - 500$ Hz.

In addition to neutrino-driven convection, proto-neutron stars are also convectively unstable due to negative radial lepton gradients. Müller et al. [?] studied convection in proto-neutron stars. In their calculations, the full core-collapse supernova process lasted 1.2 s, producing a characteristic gravitational wave amplitude of $(2 - 5) \times 10^{-23}$ and total radiated energy of approximately

$1.6 \times 10^{-10} M_{\odot} c^2$, with characteristic frequencies in the range $700 - 1500$ Hz. We present these results in Table 2 .

3 Proto-Neutron Star Oscillations

The final product of a core-collapse supernova explosion is a proto-neutron star with a very high internal temperature. As neutrinos radiate outward, the proto-neutron star gradually cools. In this section, we introduce the oscillation theory of proto-neutron stars within the Newtonian framework [**?**], analyze the restoring forces that generate various oscillation modes, discuss gravitational wave bursts dominated by g-modes, and examine the dependence of gravitational wave bursts on the neutron star equation of state.

3.1 Theoretical Framework

In 1988, McDermott et al. [**?**] studied non-radial oscillations of neutron stars in the Newtonian framework. When a fluid element inside a neutron star experiences a small perturbation, its state is called the perturbed state, and the physical quantities can be expressed as the equilibrium values plus small perturbation variables. For example, density, pressure, and gravitational potential are:

$$\rho = \rho_0 + \delta\rho, \quad p = p_0 + \delta p, \quad \Phi = \Phi_0 + \delta\Phi$$

where f_0 denotes equilibrium quantities and δf represents perturbation variables ($f \in \{\rho, p, \Phi\}$). Fluid motion can be described using either the Lagrangian or Eulerian approach, with the following transformation relation between them [**?**]:

$$\delta f = f' + \delta \mathbf{r} \cdot \nabla f_0$$

where δf and f' are the Lagrangian and Eulerian perturbation variables, respectively.

Before presenting the fundamental equations for linear, adiabatic, non-radial oscillations of neutron stars, McDermott et al. [**?**] decomposed the displacement vector $\delta\mathbf{r}$ into radial and horizontal components: $\delta\mathbf{r} = \xi_r \mathbf{e}_r + \xi_h$. In spherical coordinates, the fundamental equations for linear, adiabatic, non-radial oscillations are:

Continuity equation:

$$\frac{\delta\rho}{\rho} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \xi_r) + \nabla_h \cdot \xi_h = 0$$

Equation of motion:

$$\frac{\partial^2 \xi_r}{\partial t^2} = -\frac{1}{\rho}\frac{\partial p'}{\partial r} + \frac{\rho'}{\rho^2}\frac{dp}{dr} - \frac{\partial \Phi'}{\partial r}$$

$$\frac{\partial^2 \xi_h}{\partial t^2} = -\frac{1}{\rho}\nabla_h p' - \nabla_h \Phi'$$

Adiabatic equation of state:

$$\frac{\delta p}{p} = \Gamma_1 \frac{\delta \rho}{\rho}$$

Poisson equation:

$$\nabla^2 \Phi' = 4\pi G \rho'$$

Here, the adiabatic index is defined as $\Gamma_1 \equiv (d\ln p/d\ln\rho)_{\rm ad}$, and we have $\nabla_h \equiv \mathbf{e}_\theta \frac{1}{r}\frac{\partial}{\partial\theta} + \mathbf{e}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$.

McDermott et al. [**?**] reformulated this system into equations containing only the variables $p'(r)$, $\Phi'(r)$, and ξ_r:

$$\frac{d\xi_r}{dr} = -\frac{2}{r}\xi_r + \frac{1}{\rho c_s^2}\left(\frac{\omega^2 - N^2}{\omega^2}\right)p' - \frac{l(l+1)}{r^2\omega^2}\Phi' + \frac{l(l+1)}{r^2\omega^2}\frac{dp}{dr}\frac{\xi_r}{\rho}$$

$$\frac{dp'}{dr} = \rho(\omega^2 - N^2)\xi_r + \frac{\rho N^2}{g}p' - \rho\frac{d\Phi'}{dr}$$

$$\frac{d\Phi'}{dr} = 4\pi G\frac{p'}{c_s^2} - \frac{l(l+1)}{r^2}\Phi' - 4\pi G\rho\frac{N^2}{g}\xi_r$$

where $g \equiv -\nabla\Phi$, the adiabatic sound speed c_s satisfies $c_s^2 = dp/d\rho = \Gamma_1 p/\rho$, the buoyancy frequency (Brunt-Väisälä frequency) N satisfies $N^2 = g\left(\frac{1}{\Gamma_1}\frac{d\ln p}{dr} - \frac{d\ln\rho}{dr}\right)$, and the acoustic frequency (Lamb frequency) S_l satisfies $S_l^2 = l(l+1)c_s^2/r^2$. We have also used $\nabla_h^2 Y_{lm}(\theta,\phi) = -l(l+1)Y_{lm}(\theta,\phi)$, where $Y_{lm}(\theta,\phi)$ are spherical harmonics and l is the number of nodes in the angular direction. For convenience, we omit the subscript "0" for equilibrium quantities.

In equations (11)-(13), when the radial node number $|n|$ and horizontal node number l are large, the gravitational potential perturbation term can be neglected ($\Phi' = 0$). This approximation is known as the Cowling approximation [**?**]. Under this approximation, equations (11)-(13) reduce to a second-order differential equation for ξ_r alone (detailed derivations can be found in [**?**, **?**]):

$$\frac{d^2\xi_r}{dr^2} = -K_s(r)\xi_r$$

where $K_s(r)$ is given by:

$$K_s(r) = \frac{\omega^2 - N^2}{c_s^2} - \frac{l(l+1)}{r^2}\left(1 - \frac{N^2}{\omega^2}\right)$$

Equation (14) adequately describes the global properties of oscillation modes and yields the frequency ranges for various modes. To better analyze different oscillation modes, we rewrite equation (14) as:

$$\frac{d^2\xi_r}{dr^2} + K_s(r)\xi_r = 0$$

The local behavior of ξ_r depends on the sign of $K_s(r)$. When $K_s(r) > 0$, ξ_r can be approximated as:

$$\xi_r \approx \frac{C}{\sqrt{K_s(r)}} \cos\left(\int K_s^{1/2}(r)dr + \phi\right), \quad K_s(r) > 0$$

where ϕ is a phase determined by boundary conditions. This shows that ξ_r is an oscillatory function of r in regions where $K_s(r) > 0$.

When $K_s(r) < 0$, ξ_r can be approximated as:

$$\xi_r \approx \frac{C'}{|K_s(r)|^{1/4}} \exp\left(\pm \int |K_s|^{1/2} dr\right), \quad K_s(r) < 0$$

showing that ξ_r grows or decays exponentially with r.

Based on the value of ω, we consider two cases:

1. $|\omega| > |N|$ and $|\omega| > S_l$
2. $|\omega| < |N|$ and $|\omega| < S_l$

This yields two classes of oscillation modes: high-frequency modes satisfying case (1), whose restoring force is pressure (called p-modes), and low-frequency modes satisfying case (2), whose restoring force is buoyancy (called g-modes). An additional mode similar to p-modes is the f-mode. Although its restoring force is also pressure, its frequency lies between those of p- and g-modes. Detailed discussions of f-modes can be found in McDermott et al. [**?**]. For more comprehensive coverage of stellar oscillation modes, see [**?**, **?**].

3.2 Gravitational Wave Bursts Dominated by Proto-Neutron Star g-Modes

McDermott et al. [**?**, **?**] calculated the gravitational radiation from various oscillation modes of proto-neutron stars under linear, adiabatic conditions, along with their oscillation periods. In 2006, Ott et al. [**?**] studied two-dimensional core-collapse supernova simulations, examining gravitational wave bursts from core collapse, bounce, and proto-neutron star g-modes. They selected three models: s11WW [?], m15b6 [?], and s25WW [?], with progenitor masses of $11M_{\odot}$, $15M_{\odot}$, and $25M_{\odot}$, respectively. The corresponding iron core masses were $1.37M_{\odot}$, $1.47M_{\odot}$, and $1.92M_{\odot}$.

Ott et al. [?] found that for the s11WW and m15b6 models, gravitational waves were completely dominated by g-modes when the evolution reached 400 ms. Their calculations revealed that g-mode-dominated gravitational waves are the strongest throughout the supernova explosion, producing the largest characteristic amplitudes. For the s25WW model, the progenitor structure differed significantly from the other two models, and its larger iron core mass produced different gravitational waveforms. In the s25WW model, g-mode-dominated gravitational waves appeared at 500 ms (see Figures 1 and 2 in [?] [Figure 2: see original paper]), and stronger signals emerged at 900 ms. Ott et al. [?] attributed these later signals to higher-order g-modes, with characteristic amplitudes reaching 5×10^{-20}.

Table 3 presents the parameters of gravitational wave signals from proto-neutron star g-modes, where the characteristic amplitude h_{char} is given by equation (2). The s11WW and m15b6 models produce characteristic amplitudes of approximately 1.3×10^{-21}, while the s25WW model reaches 5×10^{-20}. The total radiated gravitational wave energies for the three models are $1.4 \times 10^{-8} M_{\odot} c^2$, $1.6 \times 10^{-8} M_{\odot} c^2$, and $8 \times 10^{-5} M_{\odot} c^2$, respectively. These results demonstrate that larger progenitor and iron core masses produce more energetic gravitational waves. Ott [?] concluded from various models that neutron star oscillation modes may be the primary mechanism for gravitational wave generation throughout core-collapse supernova events.

3.3 Equation-of-State Dependence of Gravitational Wave Bursts

Finally, we discuss the dependence of gravitational wave bursts on the equation of state (EOS). The EOS serves as a crucial link between the microscopic properties and macroscopic structure of stellar matter. It describes the relationship between pressure p and density ρ. If pressure increases rapidly with density, the EOS is considered "stiff" ; otherwise, it is "soft." Research shows that different internal compositions and structures, or even different interaction theories for the same structure, yield different EOSs.

Marek et al. [?] simulated the complete core-collapse supernova process using a two-dimensional $15M_{\odot}$ progenitor model [?], employing two different nuclear EOSs: the Lattimer & Swesty EOS [?] (L-S model) and the Hillebrandt et al. EOS [?] (H-W model). The primary difference is that the H-W EOS is stiffer than the L-S model. Marek et al. found that the L-S model produces a characteristic gravitational wave amplitude of 5×10^{-22} with frequencies in the range $600 - 800$ Hz. In contrast, the H-W model yields a smaller characteristic amplitude of 3×10^{-22} with frequencies in the range $300 - 600$ Hz. These results indicate that the softer L-S EOS produces more compact neutron stars and stronger gravitational radiation.

4 Summary and Outlook

In this paper, we have reviewed the influence of iron core rotation and convective instabilities on gravitational wave bursts, and analyzed various oscillation modes of neutron stars, including g-, f-, and p-modes. Building on this foundation, we discussed the characteristic amplitudes, frequencies, and total radiated energies of gravitational wave signals from g-modes. Compared to iron core rotation and convective instabilities, gravitational waves primarily originate from proto-neutron star oscillation modes.

Beyond these oscillation modes, Andersson and Kokkotas [**?**] discovered that the r-mode of rotating neutron stars is unstable when internal dissipation mechanisms are neglected. Building on this work, Owen and Lindblom [**?**] found that gravitational waves from r-mode instabilities could be detectable by next-generation gravitational wave detectors.

Although core-collapse supernovae represent important astrophysical sources of gravitational waves, ground-based detectors have yet to make a direct detection [**?**, **?**, **?**, **?**]. This is primarily due to the complex physical processes involved in supernova explosions and the uncertain internal structure of neutron stars. Consequently, even theoretically derived waveform templates may lack precision. The GW170817 event [**?**] provides a potential solution through multi-messenger astronomy. This gravitational wave event, generated during the inspiral phase of a binary neutron star system, was accompanied by multi-wavelength electromagnetic observations. By applying the same approach to core-collapse supernovae, coincident electromagnetic and gravitational wave observations will enable a better understanding of the complete explosion process and facilitate investigation of gravitational wave burst characteristics.

Finally, we note that on December 10, 2020, at 04:14 UTC, the Gravitational Wave High-Energy Electromagnetic Counterpart All-sky Monitor (GECAM) satellite was successfully launched [**?**]. GECAM’ s scientific objectives include continuous all-sky monitoring of high-energy phenomena such as gravitational wave gamma-ray bursts and fast radio burst high-energy radiation. We anticipate that joint observations by GECAM and ground-based gravitational wave detectors will provide deep insights into core-collapse supernova explosion mechanisms and neutron star internal structure.

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