Postprint: A New Method for Detecting the Chandler Wobble Period and Q-Value

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Abstract

Based on classical polar motion theory, this study innovatively proposes a mathematical model and fast algorithm for estimating the period and quality factor of the Earth' s Chandler wobble, which employs weaker assumptions regarding model errors and produces estimates with superior statistical properties. Using polar motion observation series and atmospheric and oceanic angular momentum data from January 1993 to December 2009, the new method yields point estimates of (430.8 ± 0.50) d and 62.6 ± 9.63 for the Chandler wobble period and quality factor, respectively, consistent with recent research findings; the corresponding 90% confidence intervals are (430.0, 431.6) d and (43.5, 75.7), respectively. These extremely narrow confidence intervals enhance the reliability of the point estimates and facilitate a more precise understanding of the excitation and maintenance mechanisms of the Earth' s Chandler wobble.

Full Text

Preamble

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A New Method for Detecting Chandler Wobble Period and Q Value

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Abstract

Based on classic polar-motion theory, we innovatively propose a mathematical model and fast algorithm for estimating the period and quality factor of Earth' s Chandler wobble. This method makes weaker assumptions about model errors and yields superior statistical properties for the estimates. Using polar motion observations and atmospheric and oceanic angular momentum data from January 1993 to December 2009, the new method yields point estimates of (430.8 ± 0.50) days and 62.6 ± 9.63 for the Chandler wobble period and quality factor, respectively, which are close to the latest research results. The corresponding 90% confidence intervals are (430.0, 431.6) days and (43.5, 75.7). These extremely narrow confidence intervals enhance the reliability of the point estimates and contribute to a more precise understanding of the excitation and maintenance mechanism of Earth' s Chandler wobble.

Keywords: Earth' s rotation variation; polar-motion theory; angular momentum; optimization algorithm; bootstrapping

1 Introduction

The Chandler wobble is an eigenmode of Earth' s rotation. Since its discovery in 1891, the physical mechanism causing the Chandler wobble and its two key parameters—period and quality factor—have remained hot topics in the scientific community. Space geodetic techniques have provided abundant, high-precision, and high spatiotemporal resolution observational data for detecting the Chandler wobble. Using these high-precision observations, significant progress has been made in understanding the physical mechanisms responsible for the Chandler wobble. For example, Smith and Dahlen [?], Zhu [?, ?, ?], Mathews et al. [?], and Chen et al. [?, ?] theoretically elaborated on the contributions of various geophysical factors (such as oceans, mantle viscoelasticity, and core-mantle coupling) to the Chandler wobble. Wahr [?], Eubanks [?], Barnes et al. [?], Gross [?, ?], Liao et al. [?], and Fang et al. [?] investigated the physical mechanisms of atmospheric, oceanic, and other surface fluid excitations of the Chandler wobble. Nevertheless, due to limitations in theoretical assumptions [?], uncertainties in some observational data [?], and incompleteness of observational data [?], the physical mechanism causing the Chandler wobble and the values of its period and quality factor remain inconclusive [?].

Because the Chandler wobble is related to many geophysical factors, its precise detection holds significant scientific value. Current methods for calculating and estimating the Chandler wobble period and quality factor fall into four main categories: (1) Spectral analysis methods, which directly perform spectral analysis on polar motion observation series to obtain the Chandler wobble period and quality factor [?, ?, ?]. (2) Stochastic excitation hypothesis methods, which assume that the Chandler wobble excitation mechanism is random and employ statistical methods to estimate the period and quality factor [?, ?, ?, ?, ?]. (3) Geophysical excitation theory methods, which use actual observational data to study the influence of geophysical mechanisms such as atmosphere, oceans, and terrestrial water on the Chandler wobble [?, ?, ?, ?, ?]. (4) Semi-analytical methods, which provide analytical expressions for the Chandler wobble period and quality factor based on certain theoretical models and actual observational data, such as oceans, mantle, core-mantle coupling, and triaxial ellipsoid [?, ?, ?, ?, ?, ?].

In this paper, we refer to the period and quality factor obtained from the first three categories as observational values, and those from the last category as theoretical values. Table 1 summarizes the calculation and estimation results from existing methods for the Chandler wobble period and quality factor. The table reveals that the parameter estimates for the quality factor have an extremely wide range (36, 1,000). Although recent research has narrowed its confidence interval to (56, 255) [?], this interval remains relatively large.

Building upon previous work, this paper innovatively proposes a mathematical model and fast algorithm for estimating the Chandler wobble period and quality factor. The new method makes weaker assumptions about model errors, yields superior statistical properties for the estimates, and employs a mechanism-clear optimization algorithm with higher computational efficiency.

2 Polar Motion Theory

Assuming Earth is not subject to external celestial gravitational forces, the angular momentum theorem in the Earth-fixed coordinate system can be written as the Liouville equation:

$$\frac{d\mathbf{L}}{dt} + \omega \times \mathbf{L} = 0,$$

where ω is Earth' s rotation angular velocity relative to the inertial frame. Let Ω denote Earth' s mean rotation angular velocity; then ω can be expressed as:

$$\omega = \Omega(m_1, m_2, 1 + m_3).$$

Here, $\mathbf{L}$ is Earth' s angular momentum. In the Earth-fixed coordinate system, its expression is:

$$\mathbf{L} = \mathbf{I} \cdot \omega + \mathbf{h},$$

where $\mathbf{h} = (h_1, h_2, h_3)^T$ is Earth' s relative angular momentum, and $\mathbf{I}$ is Earth' s inertia tensor, expressed as:

$$\mathbf{I} = \begin{pmatrix} A+\Delta I_{11} & \Delta I_{12} & \Delta I_{13} \\ \Delta I_{12} & A+\Delta I_{22} & \Delta I_{23} \\ \Delta I_{13} & \Delta I_{23} & C+\Delta I_{33} \end{pmatrix},$$

where A and C are Earth' s principal moments of inertia. Taking $\mathrm{diag}(A, A, C)$ as the unperturbed term of $\mathbf{I}$, the second part represents the perturbation of $\mathbf{I}$, which varies with time.

According to Wahr' s [?] equations (2.17), (2.21), and (2.24), polar motion $m(t)$ and the excitation function $\psi(t)$ satisfy the following differential equation:

$$\dot{m}(t) - i\sigma_{cw} m(t) = i\frac{C-A}{A_m}\psi(t),$$

where A_m is the principal moment of inertia of the mantle, and the relationship between A and A_m is $A = 1.12843A_m$ or $A = 1.129A_m$ [?, ?]; σ_{cw} is the Chandler frequency, which is a real number here; $m(t)$ is expressed as $m(t) = m_1(t) + im_2(t)$; and $\psi(t)$ is expressed as $\psi(t) = i\Omega^2(C-A)(\Omega\Pi(t) - i\dot{\Pi}(t))$, where $\Pi(t)$ is the angular momentum containing atmospheric, oceanic, and other factors, expressed as:

$$\Pi(t) = h(t) + \Omega\Delta I(t),$$

where $h(t) = h_1(t) + ih_2(t)$ is the relative angular momentum (also called the motion term because it relates to the velocity of material relative to the Earth-fixed frame), and $\Delta I(t) = \Delta I_{13}(t) + i\Delta I_{23}(t)$ is the perturbation of Earth' s inertia tensor (the term $\Omega\Delta I(t)$ is called the mass term because it relates only to the density distribution of material).

Considering the mantle' s anelasticity and the ocean' s non-equilibrium response, we first replace the real σ_{cw} in equation (5) with the complex number $\sigma_c = \frac{2\pi}{T_c}\left(1 + \frac{i}{2Q_c}\right)$, where T_c and Q_c represent the Chandler wobble period and quality factor, respectively, and then simplify to obtain:

$$\dot{m}(t) - i\sigma_{cm}(t) = -i\psi(t).$$

A note on equation (7): The traditional approach simplifies equation (5) first and then replaces σ_{cw} with σ_c (i.e., simplify then replace), as in equations (13) and (14) of reference [?]. This work, however, replaces first and then simplifies. Since the difference between these two approaches affects $m(t)$ by no more than 1% [?], and given the limited measurement precision of observational data, we can consider the two approaches equivalent.

Considering the relationship between polar motion $m(t)$ and observed polar motion $p(t)$, we have [?]:

$$m(t) = p(t) - \frac{i}{\Omega}\dot{p}(t),$$

where $p(t) = x(t) - iy(t)$, and $x(t)$ and $y(t)$ are polar motion observation series provided by the International Earth Rotation and Reference Systems Service (IERS) or other organizations, with $y(t)$ positive toward 90°W. Substituting equation (8) into equation (7) yields:

$$\dot{p}(t) - i\sigma_{cp}(t) = -i\Omega\Pi(t).$$

Solving this differential equation, the free term of $p(t)$ gradually decays after a long time, leaving only the forced excitation term. Therefore, the analytical expression for $p(t)$ is:

$$p(t) = -ie^{i\sigma_{ct}} \int_{-\infty}^{t} e^{-i\sigma_c\tau}\Omega\Pi(\tau)d\tau.$$

3 Estimation Algorithm

Based on equation (10), this section makes fundamental assumptions about errors and proposes a mathematical model for estimating T_c and Q_c. Simultaneously, based on these error assumptions and the new mathematical model, we introduce the bootstrap method for interval estimation of T_c and Q_c.

3.1 Fast Algorithm for Estimating T_c and Q_c

Assuming equally spaced sampling with interval δt, data length N, and initial epoch t_0, equation (10) yields the following first-order autoregressive form for discrete data:

$$e^{-i\sigma_c\delta t}p(t_{n+1}) - p(t_n) = -ie^{i\sigma_{ct}n} \int_{t_n}^{t_{n+1}} e^{-i\sigma_c\tau}\Omega\Pi(\tau)d\tau \quad (t_n = t_0 + n\delta t, n = 0, 1, 2, \dots, N-1).$$

Since $\Pi(t)$ is not entirely atmospheric and oceanic angular momentum but may also contain unobservable and unknown factors, in equation (6) both $h(t)$ and $\Delta I(t)$ include not only surface fluid effects but also Earth's internal effects (such as geomagnetic jerks [?]), which cannot be directly observed [?]. Therefore, we can decompose $\Pi(t)$ into three parts:

$$\Pi(t) = \Pi_{obv}(t) + \Pi_{err}(t) + \Pi_{res}(t),$$

where $\Pi_{obv}(t) + \Pi_{err}(t)$ represents the true value of the observable portion, $\Pi_{obv}(t)$ is the corresponding actual observational data, and $\Pi_{err}(t)$ is the measurement error of the observational data (i.e., observational error). $\Pi_{res}(t)$ represents other unobservable or unknown excitation sources mentioned above. Here we combine the last two terms of equation (12) and denote them as $\Pi_{mer}(t)$, referred to as model error. Finally, we assume that the discrete sequence $\Pi_{mer}(t_n)$ corresponding to model error is a random noise with zero mean function and zero first-order autocorrelation function.

Substituting $\Pi(t) = \Pi_{obv}(t)+\Pi_{mer}(t)$ into equation (11), we expand the integral concerning $\Pi_{obv}(t)$ using Simpson' s rule and the integral concerning $\Pi_{mer}(t)$ using the trapezoidal rule, and introduce the following expressions:

$$\begin{cases} \beta_1 = \exp(-i\delta t\sigma_c/2) \\ \beta_2 = \exp(-i\delta t\sigma_c) \\ X_{n,1} = \Omega\Pi_{obv}(t_{n+0.5}) \\ X_{n,2} = \frac{-i\delta t}{3}\Omega\Pi_{obv}(t_{n+1}) \\ Y_n = p(t_n) - \frac{-i\delta t}{3}\Omega\Pi_{obv}(t_n) \\ \varepsilon_n = \Omega\Pi_{mer}(t_n) \end{cases}$$

Using equations (13) and (14), we expand and simplify equation (11) to:

$$Y_n - X_{n,1}\beta_1 - X_{n,2}\beta_2 = \varepsilon_n + \varepsilon_{n+1}\beta_2 \quad (n = 0, 1, 2, \dots, N-2).$$

Introducing vectors $\mathbf{Y}$, $\mathbf{X}_1$, $\mathbf{X}_2$, ε, and $\tilde{\varepsilon}$, equation (15) can be rewritten as the regression equation:

$$\mathbf{Y} - \mathbf{X}_1\beta_1 - \mathbf{X}_2\beta_2 = \varepsilon + \tilde{\varepsilon}\beta_2.$$

Since the first-order autocorrelation function of the sequence $\Pi_{mer}(t_n)$ is zero, $\varepsilon^H\tilde{\varepsilon} = 0$ holds (superscript H denotes the conjugate transpose of a vector), and we can obtain the analytical expression for the sum of squared residuals:

$$\|\varepsilon\|^2 = \frac{\|\mathbf{Y} - \mathbf{X}_1\beta_1 - \mathbf{X}_2\beta_2\|^2}{1 + \beta_2^H\beta_2},$$

where $\|\cdot\|$ represents the vector 2-norm. To solve for β_1, we can transform equation (17) into a constrained optimization problem:

$$\min_{\beta_1,\beta_2} \|\varepsilon\|^2 \quad \text{s.t.} \quad \beta_2 = \beta_1^2.$$

Using the Lagrange multiplier method, we define the Lagrangian function $L(\beta_1, \beta_2, \lambda)$ as:

$$L(\beta_1, \beta_2, \lambda) = \|\varepsilon\|^2 - \lambda(\beta_2 - \beta_1^2).$$

Taking three partial derivatives of $L(\beta_1, \beta_2, \lambda)$ and setting them to zero, then eliminating λ, we obtain:

$$(\mathbf{Y} - \beta_1 \mathbf{X}_1 - \beta_2 \mathbf{X}_2)^H [2\beta_1 (\mathbf{X}_2 + \bar{\beta}_2 \mathbf{Y}) + (1 - |\beta_1|^4)\mathbf{X}_1] = 0.$$

Denoting the left side of equation (20) as $f(\beta)$, we only need to find the zero point of $f(\beta)$. We use the two-point secant method from numerical analysis for iterative solution, with the iteration format:

$$\beta_{k+1} = \beta_k - \frac{f(\beta_k)}{\mathrm{Re}\left(\frac{f(\beta_k) - f(\beta_{k-1})}{\beta_k - \beta_{k-1}}\right) - i\mathrm{Im}\left(\frac{f(\beta_k) - f(\beta_{k-1})}{\beta_k - \beta_{k-1}}\right)} f(\beta_k),$$

where $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the real and imaginary parts, respectively. Here, we only need to provide initial values $\beta_0 = \exp\left(\frac{-i\pi\delta t}{430}\right)$ and $\beta_1 = \beta_0 + \epsilon$, where ϵ is a relatively small complex random number. Substituting β_0 and β_1 into equation (21) and iterating repeatedly, if the difference between β_{k+1} and β_k is smaller than a certain threshold (taken as 10^{-15} here), we exit the loop and consider $\hat{\beta} = \beta_{k+1}$. From $\hat{\beta}$, we can then calculate $\hat{T}_c$ and $\hat{Q}_c$.

In summary, this method employs a mechanism-clear algorithm to directly obtain the optimal $\hat{T}_c$ and $\hat{Q}_c$ without comparing results at each grid point within a specific region, as done by Nastula and Gross [**?**] (hereinafter referred to as the grid search method). Therefore, this method has higher computational efficiency.

3.2 Interval Estimation Method–Bootstrapping

Current interval estimations of the Chandler wobble period and quality factor are based on Monte Carlo methods [**?**, **?**], which require prior knowledge of the theoretical distribution of model error $\Pi_{mer}(t)$. As shown in equation (12), $\Pi_{res}(t)$ is unknown or unobservable, and assuming it follows any particular distribution may be unreasonable. Moreover, with improving observation accuracy, the measurement precision of observational data is gradually increasing, making the observational error $\Pi_{err}(t)$ heteroscedastic. Considering these two factors, it is difficult for theoretical models to describe the distribution function of $\Pi_{mer}(t)$, and thus interval estimation results from Monte Carlo methods may not be sufficiently accurate.

Based on this, this work adopts bootstrapping for interval estimation, whose advantage is that it does not require knowledge of the distribution function of $\Pi_{mer}(t)$. Bootstrapping is a modern nonparametric statistical method first proposed by Efron [**?**]. After decades of development, it has established a solid theoretical foundation and wide application scope [?]. The bootstrapping procedure is simple and straightforward: (1) Treat the initial dataset containing K samples as the population, draw K samples with replacement each time, and repeat this B times to obtain B new samples; (2) Perform parameter estimation on each resampled sample to obtain B estimates for each parameter; (3) Based on these B sets of estimates, provide interval estimates for the initial dataset and even study the validity of parameter estimates.

This work uses bootstrapping for interval estimation of T_c and Q_c. Following the above steps, we treat the $N-1$ equations in equation (15) as the population (this approach is described in detail in reference [?]), draw $N-1$ samples with replacement each time, and then perform parameter estimation on the resampled samples according to equations (16)-(21). Repeating this process many times yields multiple sets of parameter estimates for T_c and Q_c, from which interval estimates for T_c and Q_c are derived.

4 Data and Preprocessing

For polar motion observation series, we use the COMB2018 dataset solved by Ratcliff and Gross [?], which has very high precision after 1990. Atmospheric angular momentum (AAM) data come from the National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) dataset [?], which considers topographic effects and has higher accuracy [?]. Oceanic angular momentum (OAM) data come from the ECCO-kf080i dataset of the IERS Special Bureau for Ocean. All three datasets have a sampling interval of 1 day, and the common time span selected is January 1993 to December 2018. Before parameter estimation, we preprocess these data as follows [?, ?]:

(1) Remove annual, seasonal, and linear trend terms from the three series. Use the least squares method to remove harmonic signals with frequencies of 1, 2, 3, 4, 5, 6, 7, 8 cpy, two tidal terms with periods of 13.661 days and 13.633 days, and a first-order polynomial function from the original series.

(2) Remove low-frequency terms. In addition to linear trend terms, low-frequency signals in polar motion series are generally considered to originate from Earth' s interior, and these excitation sources cannot be directly observed. Accordingly, we design a high-pass filter to remove signals with periods longer than 2 years from the three series.

The preprocessed data are shown in Figure 1 [Figure 1: see original paper]. From this figure, we can see that the Chandler wobble amplitude has been continuously modulated and generally decreasing. During 2010-2019, the Chandler wobble amplitude has decreased to a very significant extent (this can also be verified in Figure 8 [Figure 8: see original paper] of Wang et al. [?]). For the preprocessed angular momentum series, they are very similar to stationary random noise, with the real part amplitude slightly smaller than the imaginary part amplitude.

Note: The first row shows the Chandler term in the preprocessed polar motion series p, and the second row shows the preprocessed angular momentum series Π_{obv}, which is the sum of atmospheric angular momentum (AAM) and oceanic angular momentum (OAM).

Figure 1: Preprocessed data series

5 Results

To more comprehensively evaluate the stability and reliability of the new method, this work selects three different time periods for analysis and discussion: 1993-2010, 2000-2019, and 1993-2019. For each time period, we perform 1,000 bootstrap resamplings, obtaining 1,000 sets of $\hat{T}_c$ and $\hat{Q}_c$ for each period. The statistical histograms of these 3,000 sets of estimates are shown in Figure 2 [Figure 2: see original paper], and their means, variances, and 90% confidence intervals are presented in Table 2 .

Figure 2: Statistical histograms of 3,000 sets of period and quality factor parameter estimates

Table 2: Means, variances, and 90% confidence intervals of 3,000 sets of period and quality factor parameter estimates

Time Period	T_c (days)	Q_c
	Point Estimate	Standard Deviation
1993-2010	430.8	0.50
2000-2019	432.2	0.60
1993-2019	430.7	0.50

The statistical results from Figure 2 and Table 2 show that the interval estimates of T_c and Q_c from the first time period (1993-2010) and the third time period (1993-2019) almost coincide, with very small variances for $\hat{T}_c$. For Q_c estimation, it is more sensitive to data selection, which is a common problem with existing estimation methods (as shown in Table 1). Comparing Tables 1 and 2 reveals that the new method yields smaller variances for Q_c estimates, demonstrating superior statistical efficiency.

The statistical results also reveal that the second time period (2000-2019) shows larger differences in parameter estimates compared to the other two periods. This may be related to the gradually decreasing amplitude of the Chandler wobble in recent years, and the estimation results for this period may have some systematic bias, which we will further investigate in future research.

Based on the above results, we can determine that the parameter estimates from the first time period (1993-2010) are more reasonable, with point estimates of T_c = 430.8 days and Q_c = 62.6, and 90% confidence intervals of (430.0, 431.6) days and (43.5, 75.7), respectively. The confidence intervals for this period are wider, having a greater probability of containing the true value of Q_c, representing a more conservative estimate. Moreover, these results are very close to those of Mathews et al. [?] and Nastula and Gross [?], further validating the reliability of the new method.

6 Summary and Discussion

This paper presents a new method for estimating the Chandler wobble period and quality factor based on classic polar-motion theory. The new method makes weaker assumptions about errors, employs the more reasonable bootstrapping method for interval estimation, and yields parameter estimates with superior statistical properties. Additionally, the new point estimation algorithm uses a mechanism-clear two-point secant method to directly obtain the optimal solution, which has higher computational efficiency compared to traditional grid search methods.

The optimal point estimates for Earth' s Chandler wobble period and quality factor are 430.8 days and 62.6, respectively, with standard deviations of 0.50 days and 9.63. The optimal interval estimates are (430.0, 431.6) days and (43.5, 75.7), respectively. These results are very close to those of Mathews et al. [?] and Nastula and Gross [?], and our parameter estimates have smaller variances, again demonstrating the reliability and superiority of the new method.

Since we currently use only atmospheric and oceanic angular momentum data, the optimal estimates for Earth' s Chandler wobble period and quality factor may still have some bias. In the future, we will consider the effects of terrestrial water or global hydrological angular momentum to further improve the estimation accuracy of the Chandler wobble period and quality factor. Additionally, with gradually improving understanding of the Chandler wobble and increasing observational data precision, the new method proposed in this work may enable precise prediction of Earth' s Chandler wobble.

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Note: Figure translations are in progress. See original paper for figures.