

## Applications of Water Vapor Radiometers in Radio Interferometers (Postprint)

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### Abstract

Radio interferometers achieve signal correlation by acquiring the time delays of signals from the same celestial object arriving at multiple radio antennas, thereby forming high angular resolution radio telescopes that play an important role in the fine mapping and high-precision positioning of celestial objects. In radio interferometer observations, the wet component of the atmosphere—water vapor—introduces additional delays unrelated to the position and structure of celestial objects, characterized by rapid variations and low regularity, making it difficult to establish precise models and thus requiring correction. Water vapor radiometers are important observational instruments for measuring atmospheric water vapor content, which can be used to correct time delays caused by water vapor and to rectify phase fluctuations in radio interferometry. Compared with other water vapor detection methods, water vapor radiometers offer higher temporal resolution. This article introduces the principle, research progress, and application status of water vapor radiometers in radio interferometers both domestically and internationally, and finally provides an outlook on future development trends.

### Full Text

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## Application of Water Vapor Radiometer in Radio Interferometer

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## Abstract

Radio interferometers achieve signal correlation by acquiring the time delays of signals from a common celestial object arriving at multiple radio antennas, thereby forming a high angular resolution radio telescope that plays an important role in fine mapping and high-precision positioning of celestial objects. In radio interferometer observations, the wet component in the atmosphere—water vapor—causes additional delay independent of the position and structure of celestial bodies. This delay exhibits characteristics of rapid variation and low regularity, making it difficult to establish an accurate model, thus requiring correction. The water vapor radiometer (WVR) is an important observational instrument for measuring atmospheric water vapor content and can be used to correct the delay caused by water vapor and reduce phase fluctuations in radio interferometry. Compared with other water vapor detection methods, WVR offers higher temporal resolution. This paper introduces the principles, research progress, and application of water vapor radiometers in radio interferometers both domestically and internationally, and concludes with a prospect for future development trends.

**Keywords:** water vapor radiometer; radio interferometry; atmospheric profile inversion; tropospheric delay

## 1 Introduction

Radio interferometer observations experience time delay as electromagnetic waves pass through the atmosphere, with dry and wet components in the troposphere being the main contributors. Among these, due to the high variability of water vapor in time and space, changes in atmospheric wet components occur significantly faster than dry components, making wet delay caused by water vapor one of the most serious errors in radio interferometry. This error has non-negligible effects on fine mapping and high-precision positioning and must be corrected in actual observations.

Wet delay is directly related to atmospheric water vapor content. The water vapor radiometer (WVR) is a microwave radiometer that measures water vapor and liquid water content by detecting atmospheric water vapor radiation, offering advantages such as high sensitivity, automated operation, and high temporal resolution, with excellent development prospects. Since the 1970s, WVR technology has been used in radio astronomy to detect atmospheric water vapor content. After years of continuous improvement, the manufacturing level of WVR hardware structures has continuously improved, instrument stability and noise resistance have become more robust, and the accuracy of inversion algo-

rithm models has gradually increased. This paper provides a brief overview of recent research on water vapor radiometers: Section 2 introduces the working principles of WVR, including commonly used model algorithms for inverting atmospheric profiles and water vapor content; Section 3 briefly reviews the development history of WVR; Section 4 details the application of WVR in radio interferometers both domestically and internationally; Section 5 summarizes the main error sources when WVR is applied to radio interferometry; and finally, the conclusion and future development directions are presented.

## 2 Principles of Water Vapor Radiometer

### 2.1 Obtaining Sky Brightness Temperature

Both dry components (such as  $O_2$ ,  $SO_2$ ,  $CO$ ,  $NO$ ,  $H_2S$ ) and wet components ( $H_2O$ ) in the atmosphere absorb electromagnetic waves, with different substances exhibiting different absorption, emission, and scattering effects. In the microwave band, the absorption by  $O_2$  and  $H_2O$  is most pronounced. Since water vapor and other dry gas components cause attenuation and transmission delay of radio signals, and they each have their own absorption spectral lines, their content can be inverted by detecting changes in radiation energy in corresponding bands.

For convenience in characterization and calculation, the physical quantity “brightness temperature” is typically used to represent the magnitude of radiation energy received by an antenna. The radiation energy of an object is expressed as the blackbody temperature with equivalent radiation—this blackbody temperature value is the brightness temperature. When radio signals pass through an absorbing medium while ignoring scattering effects, they satisfy the radiative transfer equation, whose solution for transmission in the zenith direction is:

$$T_B(v) = T_{B0}(v)e^{-\tau \sec \theta} + \sec \theta \int_0^\infty k_a(v, z)T(z)e^{-\tau \sec \theta} dz$$

where  $v$  is frequency,  $T_{B0}$  is brightness temperature before penetrating the medium,  $T_B$  is brightness temperature received by the antenna,  $k_a$  is atmospheric absorption coefficient,  $z$  is altitude,  $T$  is physical environmental temperature, and  $\tau$  is optical thickness or opacity.

Microwave radiation from the sky received by the antenna first passes through a signal amplifier, and a converter quantizes and samples the amplified low-frequency filtered signal to obtain a set of sampling voltages, known as brightness temperature voltage. The relationship between brightness temperature voltage  $U$  and the measured brightness temperature value  $T_B$  at the antenna aperture can be expressed by the following radiometer equation:

$$U = G_S \cdot (T_B + T_{RN})$$

where  $G_S$  is the system gain of the radiometer and  $T_{RN}$  is the system equivalent noise temperature. Therefore, to ensure accurate measurement of brightness temperature  $T_B$ , the gain coefficient and system noise temperature must first be calibrated—a process called calibration. Common calibration methods include hot-cold load calibration and zenith calibration. Taking the hot-cold load method as an example, the calibration process involves establishing the following linear equations by obtaining the radiation temperatures and output voltages of internal cold and hot sources in the receiver:

$$U_{hot} = G_S \cdot (T_{hot} + T_{RN})$$

$$U_{cold} = G_S \cdot (T_{cold} + T_{RN})$$

where  $T_{hot}$  and  $T_{cold}$  are the hot and cold source radiation temperatures (known values), while  $U_{hot}$  and  $U_{cold}$  are the corresponding output voltages (measured values). Through multiple measurements, the gain coefficient  $G_S$  and system noise temperature  $T_{RN}$  to be calibrated can be solved, and the measured brightness temperature can then be converted to brightness temperature measurement values according to Equation (2).

## 2.2 Absorption Coefficients of Water Vapor and Liquid Water

Water vapor exhibits resonance bands near both 22 GHz and 183 GHz. Under dry conditions, using the 183 GHz absorption line to invert water vapor provides better accuracy than the 22 GHz absorption line. However, when atmospheric path water vapor is abundant, the high-frequency band is prone to saturation in its effect on phase due to short wavelength, so measurement at 183 GHz frequency is only applied in special areas such as high-altitude and polar regions. Therefore, this paper focuses on inversion algorithms for water vapor absorption in the K-band (20-30 GHz). Currently, in this band, the water vapor absorption coefficient  $k_v$  model with good data fitting is the van Vleck-Weisskopf (VW) profile model:

$$k_v = 4.5671 \times 10^{-4} \frac{v^2 \rho_v}{(v - v_0)^2 + \Delta v^2} + \frac{v^2 \rho_v}{(v + v_0)^2 + \Delta v^2} e^{[2.143(1 - \frac{300}{T})]} P$$

where  $\Delta v$  is the full width at half maximum of the absorption peak, defined as:

$$\Delta v = 2.784 \times 10^{-3} + 4.8 \rho_v$$

In the equations,  $v_0 = 22.235$  GHz is the resonance frequency of water vapor,  $v$  is frequency,  $T$  is air temperature,  $P$  is dry air pressure, and  $\rho_v$  is water vapor pressure. The relationship curve between the VW model water vapor

absorption coefficient and frequency is shown in Figure 1 [Figure 1: see original paper].

*Note: The model assumes given surface meteorological parameters: air temperature of 277.05 K, pressure of 1037.6 mbar, and water vapor pressure of 3.228 mbar. The peak of water vapor absorption coefficient  $v'_0$  (pink dashed line) lies between 22–23 GHz, where the received brightness temperature is highest in this frequency range.*

### Figure 1 VVW water vapor absorption line

In addition to the water vapor absorption coefficient curve, the absorption characteristics of liquid water in clouds or fog must also be obtained. This is because liquid water absorbs large amounts of radiation—WVR receives brightness temperature from liquid water, but liquid water causes negligible change in refractive index and has negligible impact on wet delay. To avoid bias in inversion results due to liquid water interference under cloudy or foggy weather conditions, its interfering effects must be eliminated based on liquid water absorption characteristics. The liquid water absorption coefficient model is given by the following formula:

$$k_l = \frac{\rho_l \cdot 0.0122(291 - T)}{\lambda^2}$$

where  $\rho_l$  is the density of liquid water droplets and  $\lambda$  is wavelength.

### 2.3 Meteorological Parameter Conversion

From the water vapor absorption coefficient model in Equation (4), the absorption coefficient  $k_v$  is related to ambient temperature  $T$ , pressure  $P$ , and water vapor pressure  $\rho_v$ . When the external environmental water vapor pressure and temperature are known, water vapor density  $\rho_v$  (absolute humidity) can be determined from the ideal gas law:

$$\rho_v = \frac{217\rho_v}{T}$$

Then the precipitable water vapor in the atmospheric water vapor column is:

$$\omega = \int_0^{\infty} \rho_v(h)dh$$

where  $\rho_l = 10^6 \text{ g} \cdot \text{m}^{-3}$  is liquid water density.

If the ground meteorological measurement parameter is relative humidity  $R_h$ , the relationship between  $R_h$  and  $\rho_v$  can be obtained according to the definition of relative humidity:

$$R_h = \frac{\rho_v}{\rho_{vs}} \times 100\%$$

where  $\rho_{vs}$  is saturated water vapor pressure, which can be approximated using the Clausius-Clapeyron equation:

$$\rho_{vs} = 6.11 \exp\left(\frac{25.2T - 273}{T - 35.85}\right)$$

## 2.4 Inversion Models

The water vapor absorption coefficient is key to the forward calculation process. The process of estimating precipitable water vapor (PWV) involves solving the atmospheric microwave transfer equation for brightness temperature in Equation (1) to obtain parameters such as water vapor pressure in the water vapor absorption coefficient formula, and then calculating water vapor content through Equation (8). The atmospheric microwave transfer equation is a first-kind Fredholm equation. Due to insufficient information in brightness temperature data, the equation becomes ill-posed and cannot be solved analytically.

An intuitive inversion method is layered calculation: by measuring multiple groups of brightness temperatures at different elevation angles, a system of atmospheric parameter equations based on different altitudes is obtained, and then the solutions of each layer are curve-fitted to obtain atmospheric parameter profiles. Currently, popular regression algorithms and neural network algorithms can be used to solve this model equation system. Limiting factors of this method include data volume and economic cost. Statistical methods require extensive historical data for temporal and spatial matching to debug model weights and parameters. Large amounts of observations also require multiple channels to receive radiation from target areas, a process that is technically demanding and costly. Therefore, in practical applications, a single unified model can also be established by linearly simplifying Equation (1), fitting a brightness temperature correction model from measured data, and then calculating water vapor content or wet delay from the corrected linear brightness temperature atmospheric microwave transfer equation. This simplified model approach is more economical and convenient but has relatively large measurement errors. The following sections focus on introducing some common models.

**2.4.1 Mainstream Dual-Channel Inversion Models** Water vapor content is linearly correlated with path delay (see Equation (27)), so both water vapor content and path delay can serve as inversion targets for the model. Existing research provides the following mainstream dual-channel WVR linear model:

$$\begin{cases} L_v = a_0 + a_1 T_{B1} + a_2 T_{B2} \\ \omega = k \cdot \frac{L_v - T_{c1} - T_0}{T_{c2} - T_0} \end{cases}$$

where  $L_v$  is path delay,  $v_1, v_2$  are the center frequencies of the two channels,  $T_{B1}, T_{B2}$  are the brightness temperatures of the two channels,  $T_{c1}, T_{c2}$  correspond to the cosmic background noise temperatures received by the two channels after atmospheric attenuation,  $k = 1.723 \times 10^{-3} \text{ K} \cdot \text{g}^{-1} \cdot \text{m}^3$ , and  $W_m, T_0$  are related conversion functions whose expansions can be found in reference [8] and can be treated as constants for fitting to reduce computational complexity.

Water vapor content and liquid water content can also be inverted through regression methods. Guiraud et al. [9] used atmospheric data from radiosondes in Denver, USA over a 6-year period (WVR with channel frequencies of 20.6 GHz and 31.6 GHz) and provided the following coefficient corrections based on this model:

$$\omega = -0.19 + 0.118T_{B1} - 0.0560T_{B2}$$

$$l = -0.018 - 0.00114T_{B1} + 0.0284T_{B2}$$

where  $\omega$  is water vapor content (in cm),  $l$  is liquid water content (in cm), and  $T_{B1}$  and  $T_{B2}$  correspond to brightness temperatures at 20.6 GHz and 31.6 GHz channels, respectively.

Based on sufficient historical meteorological data, to improve inversion accuracy, relevant coupling parameters can be added to enhance the model:

### (1) Adding Ground Parameter Coupling

Since atmospheric profile parameters may have certain correlations with ground atmospheric parameters, the following multivariate linear regression equation can be established on the basis of the above method to incorporate the influence of ground environmental parameters [10]:

$$\omega = a_0 + a_{1T_{B1}} + a_{2T_{B2}} + a_{3P}P_0 + a_{4H}H_0 + a_{5T}T_0$$

where  $P_0, H_0, T_0$  are ground pressure, humidity, and temperature, respectively,  $\omega$  is water vapor content in the line-of-sight direction, and  $a_i$  ( $i=1, 2, 3, 4, 5$ ) are coefficients.

### (2) Adding Higher-Order Term Coupling

In addition to linear relationships, atmospheric profile parameters and brightness temperature are more likely to have complex nonlinear relationships. Huang and Wang [11] added quadratic terms and cross terms for each brightness temperature channel to the basis of Equation (14) to obtain a nonlinear regression model:

$$\omega = a_0 + a_{1T_{B1}} + a_{2T_{B2}} + a_{3T_{B1}}^2 + a_{4T_{B2}}^2 + a_{5T_{B1}}T_{B2} + a_{6P}P_0 + a_{7H}H_0 + a_{8T}T_0$$

where  $a_i$  ( $i=1, 2, 3, \dots, 8$ ) are coefficients.

### (3) Introducing Atmospheric Opacity

Some models use atmospheric opacity  $\tau$  as an inversion variable. The relationship between  $\tau$  and brightness temperature  $T_B$  at frequency  $\nu$  is:

$$\tau(\nu) = \ln \frac{T_{mr} - T_{B0}}{T_{mr} - T_B(\nu)}$$

where  $T_{mr}$  is the atmospheric mean radiation temperature, which can be considered constant at 270 K in certain seasons [12], and  $T_{B0} = 2.9$  K is the cosmic background radiation. The wet delay model can then be written as:

$$L_\nu = a_0 + a_1\tau_1 + a_2\tau_2$$

where  $a_i$  ( $i=1, 2, 3$ ) are coefficients.

**2.4.2 Multi-Channel WVR Inversion Models** Multi-channel WVR inversion methods can still adopt the dual-channel model concept by establishing linear relationships. In recent years, the development of frequency-sweeping and even full-sampling WVRs has enabled more abundant brightness temperature data through broader frequency bands. Tahmoush and Rogers [13] introduced a frequency-sweeping WVR and established the following brightness temperature correction model based on the absorption properties of water vapor and liquid water to improve the accuracy of separating water vapor from liquid water:

$$T_{sky}(\nu) = a + b\nu^2 + ch_\nu$$

where  $a, b, c$  are parameters to be fitted, and  $h_\nu$  is the normalized value of  $k_\nu$  centered at 22.2 GHz. Coefficient  $a$  is used to fit the influence of instrument gain. From the liquid water absorption coefficient model represented by Equation (6), it can be seen that  $k_l \propto \nu^2$ , so the second term  $b\nu^2$  in the model is used to fit the influence caused by liquid water.

For a thin atmospheric layer at a certain altitude, considering constant temperature and pressure conditions and substituting the brightness temperature correction results from Equation (18), the thin-layer path wet delay  $d$  can be calculated using the following formula:

$$d = 3.776 \times 10^5 \frac{p_\nu T_{sky}}{k_\nu T^3}$$

where  $p_\nu$  is water vapor pressure, and its derivation process can be found in Section 2.5.

In addition, some multi-channel WVRs applied to radio interferometer arrays use another inversion approach. Instead of solving for the path wet delay of individual antennas, they directly calculate the path wet delay difference between every two antennas, and then determine their phase difference using Equation (28). This can also achieve the purpose of solving phase fluctuations between different antennas in the interferometer array, but requires that each antenna in the array be equipped with WVRs of the same configuration. Interferometer arrays using this processing method include VLA (Very Large Array), ALMA (the Atacama Large Millimeter/submillimeter Array), and ATCA (Australia Telescope Compact Array). Taking ATCA as an example, each antenna is equipped with a 4-channel WVR (filter=1, 2, 3, 4), and the wet delay  $\Delta L_v$  between two antennas can be calculated as:

$$\Delta L_v = \sum_{\text{filter}=1}^4 C_{W;\text{filter}} \cdot \frac{\Delta T_{\text{filter}}}{K_{\text{filter}}}$$

where  $C_{W;\text{filter}}$  are weighting coefficients and  $K_{\text{filter}}$  are correction factors, detailed in reference [15].

## 2.5 Path Delay and Phase Correction

The electromagnetic wave signal received by a single antenna in a radio interferometer passing through the atmosphere is shown in Figure 2 [Figure 2: see original paper]. The path difference along the troposphere is:

$$\Delta L = L - G = \int_s n ds - G = \int_s (n - 1) ds + S - G$$

where  $n$  is refractive index,  $S$  is refracted path,  $G$  is path without refraction effect, and  $L = \int_s n ds$  is the optical path. Defining atmospheric refractivity  $N = 10^6(n - 1)$ . For zenith direction paths, the geometric delay term ( $S - G$ ) is negligible, yielding:

$$\Delta L_{\text{zenith}} = 10^{-6} \int_0^{\infty} N ds$$

The wet atmospheric refractivity component caused by water vapor dipole moment can be given by empirical formula  $N_v = 3.776 \times 10^5 \frac{p_v}{T^2}$  [14]. Therefore, the zenith path wet delay term in Equation (22) can be written as:

$$\Delta L_{\text{zenith}} = 0.3776 \int_0^{\infty} \frac{p_v}{T^2} ds$$

For a very thin atmospheric layer, temperature and pressure can be treated as constants. The path wet delay then becomes:

$$\Delta d_{\text{zenith}} = 0.3776 \cdot \frac{p_v}{T^2} \cdot \Delta s$$

From Equation (1), the attenuation exponential term is a higher-order term and can be neglected. The radiative transfer equation for thin-layer brightness temperature in the zenith direction is:

$$\Delta T_B = k_v T \cdot \Delta s$$

From Equations (24) and (25), the path wet delay at the thin layer can be expressed as:

$$\Delta d_{\text{zenith}} = 3.776 \times 10^5 \frac{p_v \Delta T_B}{k_v T^3}$$

Without considering integration and assuming constant atmospheric environmental temperature of 280 K, the commonly used approximate estimate can be obtained from Equations (7), (8), and (23):

$$L_v \approx 6.3\omega$$

The phase change caused by path delay is:

$$\Delta\phi = \frac{360}{\lambda} \Delta L_v$$

where  $\lambda$  is wavelength and  $\Delta\phi$  (in degrees) is the corrected phase.

### 3 Brief Development History of Water Vapor Radiometer

Starting from the 1940s, van Vleck's [16, 17] research on the absorption characteristics of water vapor and oxygen in the microwave band laid the foundation for atmospheric microwave radiation theory. In 1965, Layson and Martin [18] systematically studied microwave refraction correction theory for radiometers and provided the Marcor (microwave atmospheric range correction) technique for inverting water vapor delay. In the 1970s, the Jet Propulsion Laboratory (JPL) in the United States demonstrated the feasibility of applying water vapor radiometers to geodesy. This passive remote sensing method for calculating wet delay by receiving electromagnetic signals also began to be applied in radio astronomy [1]. In 1979, Wu [8] proposed the optimal frequency pair for passive microwave radiometers for tropospheric path length correction, providing sufficient theoretical support for dual-channel WVR design. Subsequent dual-channel WVR frequency bands were mostly selected according to this method and widely used in radio astronomy and other atmospheric science fields. To this

day, many observatories continue to use dual-channel radiometers. The Onsala Space Observatory in Sweden established two dual-channel WVRs, Astrid and Konrad. Astrid conducted its first comparative measurement with radiosondes at Gothenburg-Landvetter Airport in May 1980, while Konrad first operated at Kiruna, Esrange Space Center in August 2000. In 2017, Forkman et al. [20] demonstrated through testing that even after years of service, the zenith wet delay inversion results of these two instruments over four years had root mean square differences from GPS data of 0.92 cm and 0.75 cm, respectively, still maintaining good consistency.

To further improve WVR accuracy, multi-channel design will gradually replace traditional dual-channel design and become the mainstream for current and future development. The MP series radiometers designed by Radiometrics Corporation in the United States and the RPG series radiometers designed by Radiometer Physics Company in Germany are currently widely used products. The ground-based WVR represented by the MP-3000 series is a commonly used product in China [21], featuring 35 frequency channels [22]—21 channels in K-band (22–30 GHz) and 14 channels in V-band (51–59 GHz)—with serial frequency modulation detection at lower cost. The latest model of the RPG series is the fifth-generation RPG-HATPRO-G5, using 14 frequency channels, with 7 channels (51–58 GHz) for temperature profile inversion and 7 channels (22.24–31.4 GHz) for water vapor profile inversion. Notably, it also provides 183 GHz band channels for extremely cold weather regions and high-altitude areas to improve instrument performance in low PWV regions. The RPG-HATPRO-G5 adopts parallel direct detection, improving equipment reliability and anti-interference capability. Figure 3 [Figure 3: see original paper] briefly outlines the development history of water vapor radiometers.

### Figure 3 Timeline of water vapor radiometer development

## 4 Application of Water Vapor Radiometer in Radio Interferometers

The following sections introduce WVR applications in radio interferometers from two aspects: connected interferometers and Very Long Baseline Interferometry (VLBI). Connected interferometers refer to interferometer arrays composed of multiple antenna systems connected by cables for relatively short-distance (e.g., several kilometers) interferometric measurements. VLBI refers to instruments that use several antennas spaced much farther apart (e.g., thousands of kilometers) for coordinated interferometric measurements without cable connections, instead using independent atomic clocks for frequency calibration and time synchronization, recording separately and then performing correlation processing for long-distance interferometric measurements. The WVR configurations for each interferometer are shown in Table 1. Before measurement tasks, radiometers must determine measurement accuracy by analyzing gain value stability and determining system noise temperature. Brightness temperature measurement accuracy becomes the main factor affecting instrument performance,

and evaluating instrument performance is essential work that researchers at most observatories must complete. Based on stable instrument performance, WVR time delay correction results are compared with other correction methods to improve wet delay correction effectiveness.

**Table 1 Operating frequency bands and channel numbers of WVRs in radio interferometers**

Interferometer/Antenna	Operating Frequency Band	Channels
Connected Interferometer		
VLBI		
Effelsberg	183 GHz	

*Note: The operating frequency bands in the table refer to the frequency bands near the two water vapor absorption peaks at 22 GHz and 183 GHz.*

#### 4.1 Connected Interferometers

**4.1.1 VLA** The Very Large Array (VLA) is a large comprehensive aperture radio telescope located on the Plains of San Agustin in New Mexico, USA, which began operation in 1980. It consists of 28 radio telescopes with 25 m diameter—one serves as backup while the other 27 are arranged in a Y-shaped configuration that can be moved along railway tracks to change their formation. The telescope array currently has four configurations (A, B, C, D) with different maximum and minimum baseline distances to meet various scientific needs. Early VLA antennas used three-channel WVRs for short-distance interferometric phase fluctuation data correction, with channels at 21.0 GHz and 23.5 GHz (750.00 MHz bandwidth) and at 22.23 GHz (1,000.00 MHz bandwidth). The three channels had similar frequencies. To facilitate research on the relationship between water vapor fluctuations and multiple brightness temperatures, Butler [19] defined the observable  $\Delta T$ :

$$\Delta T = \omega_1 T_1 + \omega_2 T_2 + \omega_3 T_3$$

where  $\omega_1 = -0.5$ ,  $\omega_2 = 1.0$ ,  $\omega_3 = -0.5$  are weights for the three channels, and  $T_1, T_2, T_3$  are brightness temperatures of the three channels. The positions of the three channel frequencies are shown in Figure 4 [Figure 4: see original paper].

#### Figure 4 VLA three-channel water vapor radiometer

In 2000, Butler conducted evaluation analysis of the WVR system [25] and briefly discussed potential issues in three aspects: filter stability, liquid water factors, and beam divergence. For a system temperature of 50 K, VLA radiometer gain stability  $\Delta g/g$  must be better than approximately 0.001 K. However,

test results showed gain stability between 0.002–0.001 K, failing to meet target requirements. Interference from liquid water in clouds was very obvious, requiring reasonable processing of three-channel measurement data. The beam divergence angle between the radiometer and antenna was about  $1.4^\circ$ , causing mismatch between actual propagation paths and instrument measurement reception paths, resulting in path errors.

In a 2004 test, VLA antenna WVR gain stability successfully achieved the expected target. Chandler et al. [26] tested three-channel WVRs at two VLA antennas and presented the results. For given phase correction expectations, prior calculations indicated that with system temperatures between 50–100 K, system gain stability should be  $(2-4) \times 10^{-4}$ . Tests showed that within 103 s correction time, measured gain stability  $\Delta g/g$  was  $(4-8) \times 10^{-5}$ , meeting measurement target requirements.

Due to demands for higher resolution and sensitivity, faster imaging, and broader frequency coverage, the 20-year-old VLA underwent functional expansion and upgrade to EVLA (the Expanded Very Large Array). EVLA's increased band bandwidth supports radiometers with more channel systems. Chandler et al. [27] proposed an improvement project for the original three-channel WVR serving VLA, planning to design and produce a Compact Water Vapor Radiometer (CWVR) for receiving EVLA antenna K-band signals. To improve instrument inversion accuracy, CWVR was upgraded to a five-channel system, with internal component design changed to integrated multi-chip modules that effectively reduce WVR internal space, making CWVR easier to install in antennas.

In 2018, Gill et al. [24] analyzed and reported laboratory test results of VLA array CWVR. CWVR uses monolithic microwave integrated circuits to replace original discrete circuit components, reducing its volume for a more compact body and adding functionality to distinguish between liquid water and water vapor. VLA's K-band frequency range is 21.40–24.40 GHz, while EVLA's K-band frequency range is 18.00–26.50 GHz with increased band bandwidth, making five-channel CWVR feasible. Regarding instrument stability, the report indicated that one channel had large output fluctuations requiring maintenance and correction. After temperature correction, gain stability of all channels was less than  $2 \times 10^{-4}$ , meeting gain stability requirements for application in ngVLA (the next generation Very Large Array).

**4.1.2 ALMA** Located in Chile, ALMA (the Atacama Large Millimeter/submillimeter Array) is a large astronomical observation facility built through international cooperation, situated on the Chajnantor plateau at 5,000 m altitude. Due to unique geographical conditions that satisfy dry characteristics, 183 GHz frequency water vapor radiometers are used to eliminate phase fluctuation errors (see Section 2.2).

ALMA radiometers have a relatively complete processing system. ALMA re-

searchers developed the “wvrgcal” program for convenient offline batch processing of water vapor data observed by WVR, providing detailed software design and program descriptions [28]. The phase correction principle based on WVR and algorithms used in wvrgcal are provided in ALMA Memo 590 [29].

In 2012, Matsushita et al. [30] used this WVR to correct atmospheric phase and studied the effectiveness of WVR phase correction under different frequencies, baseline lengths, and weather conditions. In this study, they confirmed that phase stability of all baselines met ALMA specifications, with phase stability typically improving by 2–3 times and sometimes up to 7 times. From the perspective of different weather conditions, when PWV (precipitable water vapor) was 0.5 mm, 0.7 mm, 1.1 mm, and 2.9 mm, the 2-point Allan standard deviation (ASD) values of total phase errors under different baseline lengths all decreased. However, when PWV was less than 0.3 mm, there was no significant difference before and after WVR phase correction. They speculated that under extremely dry conditions with low PWV, dry gases might become the dominant factor in atmospheric phase fluctuations, causing WVR measurement results to be biased. Additionally, even under good weather conditions or after WVR phase correction, the RMS of phase fluctuations increased with baseline length, with poor WVR correction effects for long baselines.

Due to the high-altitude location of the interferometer array itself, WVR performance correction under dry conditions is a focus of ALMA. Under clear sky conditions with PWV less than 2 mm, RMS phase should be as low as about 20 m, but in reality only very few data reached this standard, with residual error terms considered to be caused by dry atmospheric environments [31].

Maud et al. [32] attempted to add empirical scaling factors to the wvrgcal program to eliminate measurement errors by scaling WVR raw data results. Using ALMA radio interferometer array to observe the structural characteristics of HL Tau star, the comparison between standard WVR correction and WVR correction with scaling factors is shown in Figure 5 [Figure 5: see original paper]. After adding scaling correction, the ring structure becomes clearer. From the flux profile in Figure 5(c), peak flux increases, and the corrected local area becomes sharper, meaning the contrast between bright rings and gaps increases and signal-to-noise ratio improves. Through summarizing multiple data sets, it was found that data with lower PWV (less than 1 mm) corresponded to larger scaling factor values, and using the scaling factor method produced considerable improvement effects. This indicates the method is effective for correcting measurement data under dry conditions.

**Figure 5 Comparison of standard WVR correction and scaling factor correction for ALMA observations of HL Tau protoplanetary disk structure**

*Note: (a) Standard WVR correction result; (b) WVR correction result after adding scaling factor; (c) Profile extraction along  $(0.3, -0.3)$  to  $(-0.3, 0.3)$ , red line shows profile after WVR scaling factor correction; (d) Difference image.*

In 2015, Hunter et al. [33] used high-precision point source observations to determine geometric antenna positions of ALMA array. They used water vapor radiometers to calibrate wet delay above each antenna and separated water vapor partial pressure from total pressure in measured data. However, corrected results still could not well offset vertical position deviations of antennas. Hunter et al. speculated this was due to the overly simple water vapor inversion model used, which could not reflect daily wet delay variations.

In 2018, Nikolic et al. [34] described the entire ALMA WVR system design, parameter selection, and data processing strategy. They conducted preliminary tests on the WVR system, showing significant improvement in phase stability compared to previous systems, with phase correction coefficients (the relationship between sky brightness temperature changes and electrical path changes) as high as  $40 \text{ K} \cdot \text{mm}^{-1}$ . Tests identified two factors affecting WVR accuracy at ALMA—liquid water (cloud/fog) and residual phase errors uncorrelated with water vapor signals. Liquid water influence is currently a key challenge for WVR. The source of residual errors remains unclear, but tests show: the drier the external conditions, the more obvious the residual errors; the longer the baseline, the more obvious the residual errors.

**4.1.3 Other Connected Interferometers** ATCA (Australia Telescope Compact Array) installed WVRs at six antennas to measure 22.2 GHz water vapor radiation changes for calculating phase delay. These WVRs have four channels at 16.5, 18.9, 22.9, and 25.5 GHz with 1 GHz bandwidth. In 2013, Indermuehle et al. [15] provided detailed introduction to WVRs at ATCA array. By establishing model calibration coefficients, phase differences between each antenna pair were obtained through weighted brightness temperature differences. They compared WVR phase correction with interpolation method correction (see Table 2). Here,  $\epsilon$  (correlation efficiency) is a measure of the standard deviation  $\sigma$  of interpolation residual phase versus WVR residual phase. It can be seen that for short baselines numbered 1-10, the two methods show little difference; while for long baselines numbered 11-15, WVR correction of residual phase is superior to the interpolation method.

**Table 2 Comparison of residual phase correction for each ATCA baseline**

No.	Baseline Length (m)	WVR Correction	Interpolation Correction	$\epsilon$
1		-0.02		
2		-0.05		
...				

SMA (Submillimeter Array) uses 183 GHz three-channel WVRs to measure water vapor at each antenna for correcting short-distance interferometric phase

fluctuations. In 2002, instrument inspection [35] showed stable internal electronic components, with WVR performance factors mainly originating from gain instability caused by mixers and amplifiers and thermal noise from receiver temperature. Martina attempted different smoothing algorithms to reduce effects from instrument noise and gain bias [36]. In 2015, during VLBI observation missions networked by SMA, APEX (the Atacama Pathfinder Experiment) telescope, and SMT (Submillimeter Telescope), WVRs were tasked with measuring and correcting atmospheric opacity at each station [37].

## 4.2 Very Long Baseline Interferometry

Currently, VLBI networks have not yet implemented collocated water vapor radiometers at all stations. WVR applications in VLBI observations are only studied at some radio telescope sites as follows.

**4.2.1 Effelsberg Telescope** The Effelsberg 100 m radio telescope, located in the Ahr Hills, Germany, is one of the world's largest fully steerable radio telescopes. The telescope is equipped with a frequency-sweeping WVR from 18 GHz to 26 GHz that rotates with the target source for correcting tropospheric delay and opacity during high-frequency VLBI observations. In 2004, Roy et al. [38] detailed this radiometer and its performance for opacity correction. In time series test data of atmospheric opacity, WVR processing results were compared with 100 m radio telescope processing results (see Figure 6 [Figure 6: see original paper]), where the solid line shows opacity measured by WVR at fixed elevation angles and dots show opacity calculated by Effelsberg telescope pointing at light sources at two different elevation angles using specific programs. The two methods show good trend consistency, indicating WVR data can be used for opacity correction.

In 2007, Roy et al. [39] presented a 7-minute time series segment when measuring VLBI phase on the Effelsberg to Pico Veleta baseline. WVR correction reduced path delay RMS from 1.0 mm to 0.47 mm, improving correlation from 0.45 to 0.86 at 240 s scales. They also compared zenith wet delays solved by WVR, radiosondes, and GPS, with errors within 15 mm. This indicates WVR results can eliminate short-term path delay fluctuations and improve geodetic accuracy, but are not yet sufficient for independent tropospheric delay correction in VLBI.

In 2012, Cho [40] used WVR observations from Effelsberg radio telescope to correct path wet delay in VLBI observations. Combining numerical weather models from the European Centre for Medium-Range Weather Forecasts, five GPS stations near Effelsberg telescope, and radiosonde data, he compared them with zenith wet delay corrected by WVR. Comparison of observation results from different periods showed WVR zenith wet delay differed from GPS and numerical model results by 10–50 mm. Cho compared four different schemes: raw data, WVR processing results, WVR mean-smoothed results, and WVR mean-smoothed results with offset correction. The last scheme showed the smallest variance and mean bias. Overall, Effelsberg WVR observations still cannot fully

correct propagation delay caused by atmospheric wet components in VLBI data, mainly due to imperfect instrument calibration.

**4.2.2 Chinese VLBI Network** Each station of the Chinese VLBI Network (CVN) is equipped with dual-channel water vapor radiometers: Shanghai She-shan 65 m, Beijing Miyun 50 m, Kunming Fenghuangshan 40 m, and Urumqi Nanshan Base 25 m radio telescopes. The purpose is to achieve wet delay calibration in engineering projects with high real-time requirements such as deep space exploration. In 2018, Dou et al. [41] compared GPS and WVR solutions for zenith wet delay (ZWD) at four CVN collocated stations, finding that Kunming and Urumqi stations have relatively open WVR installation areas with obvious systematic measurement data differences, making GPS calibration easy. Shanghai station is located in a humid area where WVR-inverted ZWD results fluctuate dramatically. Beijing station has many obstacles near WVR installation, resulting in large standard deviations between GPS and WVR results. WVR data at Shanghai and Beijing stations show no obvious systematic differences, making GPS calibration difficult. Using GPS zenith wet delay data from 2015–2016 at four CVN stations as reference to calibrate WVR-inverted zenith wet delay data at four collocated stations, systematic biases between WVR and GPS data at each station were eliminated (Shanghai station example shown in Figure 7 [Figure 7: see original paper]). Additionally, they tested WVR stability at Shanghai and Urumqi Nanshan stations. Shanghai station WVR brightness temperature oscillation amplitude of two channels could reach 2 K. Urumqi Nanshan station WVR measurement of blackbody physical temperature showed maximum deviations of 20 K between two channels' brightness temperature values and actual blackbody temperature values over 10–20 h duration, indicating poor observation stability requiring improved manufacturing level.

In 2020, CVN antennas introduced new WVRs. Miyun, Nanshan, and Kunming stations were equipped with new domestic dual-channel WVRs with improved radome design to prevent water accumulation. Compared with originally equipped WVRs, brightness temperature error technical specifications improved from 1.5 K to 1 K. Shanghai Tianma station installed a German WVR using 14 channels to invert temperature and humidity profiles, providing four calibration methods for each channel: blackbody calibration, liquid nitrogen calibration, noise injection calibration, and zenith scanning angle calibration. Increased channel numbers improve measurement efficiency, and data volume can satisfy neural network algorithms for determining regression model coefficients, theoretically significantly improving measurement performance.

**4.2.3 Other Antennas** DSN (Deep Space Network) antennas use AWVR (the Advanced Water Vapor Radiometer) to assist VLBI measurement tasks. It has three frequency channels at 22.2 GHz, 23.8 GHz, and 31.4 GHz with 400 MHz bandwidth. In 2001, Linfield [42] introduced WVRs mounted on DSN antenna sub-reflectors. This is a WVR that rotates coaxially with the antenna,

effectively reducing pointing errors. The article mainly studied the impact of beam width on correction performance. Results showed performance ranking from low to high as: 4° FWHM (full width at half maximum) WVR 50 m from antenna, coaxial 4°, 2°, and 1° FWHM WVRs. The best-performing 1° FWHM WVR could reduce tropospheric wet delay calibration Allan variance to at 10 s time scales and to at 100 s time scales.

JCMT (James Clerk Maxwell Telescope) uses 183 GHz water vapor radiometers for auxiliary correction. In 2001, Wiedner et al. [43] used an interferometer composed of JCMT telescope and CSO (Caltech Submillimeter Observatory) for observations at 0.85 mm wavelength and performed phase correction through two 183 GHz WVRs. Results showed poor phase correction effects for channels 1 and 3, but optimal correction for channel 2, reducing phase fluctuations from 141 m to 61 m. In 2008, Dempsey and Friberg [44] used WVR for atmospheric calibration optimization, comparing opacity curves calculated by HARP (Heterodyne Array Receiver Program) with WVR opacity curves at high elevation angles (see Figure 8 [Figure 8: see original paper]), showing good agreement. WVR can assist in optimizing atmospheric opacity calculations on short measurement time scales, improving wet delay correction performance in VLBI tasks.

**Figure 8 Atmospheric opacity measured at different elevation angles using HARP skydip on May 9, 2008**

*Note: Calibrated results are shown as red plus signs, solid line shows standard WVR output atmospheric opacity results for the same period.*

## 5 Error Sources

Current mastery of WVR technology is still not mature, and data inverted by water vapor radiometers for path wet delay often contain significant errors. Reasons for insufficient data accuracy may include the following aspects:

1. **Impact of liquid water.** Tropospheric wet delay comes from atmospheric water vapor, while liquid water has minimal effect on signal propagation refractive index, thus causing minimal path difference. However, when WVR receives microwave radiation from the sky, liquid water absorbs large amounts of brightness temperature radiation, increasing path delay in subsequent inversion algorithms. Only by completely separating brightness temperature radiation from water vapor and liquid water can the influence of liquid water be resolved. This is also a major challenge in WVR inversion measurement methods.
2. **Instrument noise.** As shown in Equation (2), WVR instrument system temperature noise  $T_{RN}$  and gain coefficient  $G_S$  determine the accuracy of received brightness temperature. Under relatively dry conditions with low brightness temperature values, system temperature noise dominates; gain coefficient bias causes proportional error factors. Both reduce brightness temperature measurement accuracy and cause larger final corrected path

delay bias.

3. **Insufficient accuracy of atmospheric models.** Errors exist in tropospheric temperature and humidity vertical distribution models, liquid water models are not accurate enough, and effects of dry gases on refractive index are not considered. Path changes caused by refractive index variations of dry gas components due to different temperatures and pressures may account for 30% of path changes, as proposed by Rogers [45].
4. **Beam mismatch.** At the same signal wavelength, different antenna apertures correspond to different resolutions and different antenna half-power beam widths. Figure 9 [Figure 9: see original paper] shows the impact of antenna beam mismatch. Tahmouh and Rogers [13] analyzed that for a 10 m diameter radio antenna receiving 3 mm wavelength, assuming WVR antenna aperture of 60 cm, WVR beam only matches the antenna completely at 400 m in the same pointing direction. Beam mismatch reduces correlation between WVR-estimated wet delay path changes and actual propagation path changes, causing an average delay error of 0.3 mm.
5. **Target source pointing error.** When WVR installation position does not rotate with the antenna, WVR pointing must be consistent with antenna pointing. Even under relatively uniform atmospheric conditions, jitter in pointing can cause noticeable path length changes. At 20°C physical environmental temperature, WVR elevation pointing of 20°, and radiation brightness temperature of 30 K at 22.2 GHz line center, a 0.1° elevation error causes 0.6 mm path length change [13].
6. **Inversion algorithm model errors.** Currently widely used statistical analysis regression models and iterative algorithm models have varying degrees of error and cannot truly reflect the relationship between brightness temperature and actual atmospheric profile curves. Using more flexible neural network model algorithms to invert vertical distribution of water vapor can reduce model errors [46], but requires large amounts of historical data.
7. **Other interference-induced errors.** Such as interference from dew on chassis tops; target sources being close to the Sun, susceptible to solar radiation interference; target sources at low elevation angles, prone to receiving ground radiation, etc.

## 6 Conclusion

This paper reviews the development of WVR instrument design, summarizes inversion algorithms, and discusses applications in radio interferometers both domestically and internationally. WVR has been developed for over 50 years, offering convenience and advantages in real-time measurement, high automation, high sensitivity, and independence from geographical location—features not possessed by other water vapor detection technologies. Currently, most water vapor

radiometers are developing toward more frequency channels to obtain more accurate brightness temperature measurements. Overall, water vapor radiometers will remain a research hotspot for eliminating tropospheric wet delay errors and have excellent development potential and prospects.

The article introduces the theory of inverting path delay from principles. However, in practical applications, complex meteorological conditions cause numerous error factors, and inversion models are often used for calculation. When installing WVR at observatories, extensive routine historical data is needed to debug instruments and models. Therefore, different stations usually correspond to different model parameters with large regional differences. Due to accuracy bias and instability, WVR currently cannot be used for active correction of VLBI tropospheric wet delay but mainly serves to assist in improving wet delay results. According to recent WVR research, no suitable method has been found to solve long-baseline interferometer error problems, with the main development bottleneck being liquid water interference. Hardware improvements can increase channels or scanning frequencies to enhance fitting accuracy, such as the frequency-sweeping WVR at Effelsberg telescope, and the latest MP and RPG series WVR products also have multiple channels. From the perspective of inversion function fitting, neural networks can be used to automatically summarize parameters that may affect results, such as adding seasonal tags or day-of-year parameters, adding ground meteorological data, etc. With large amounts of historical data, network weight fitting can be achieved. With the rapid development of artificial intelligence, algorithm improvements may bring more breakthroughs in WVR practice. Additionally, Kawaguchi [47] proposed measuring spectra around water vapor resonance, fitting resonance spectra to theoretical values, and then estimating total water vapor content to separate water vapor and liquid water. New digital water vapor spectrometers have implemented this method, but their effectiveness in solving WVR liquid water interference problems remains to be tested with future measured data.

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