Constraints on Einstein-aether Theory from LIGO O1/O2 Gravitational-wave Events (Postprint)

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Abstract

General relativity is a more precise theory of gravitation following Newton's universal gravitation, applicable in regions with strong gravitational fields and highly dynamic spacetime, and its validity has been confirmed by numerous astronomical observations. However, general relativity is logically incompatible with quantum theory, suffers from theoretical issues such as spacetime singularities, and faces unresolved questions in cosmology regarding the gravitational nature of dark matter and dark energy, among other problems. Building upon modifications to gravitational wave templates using the Einstein-aether theory proposed by Yunes et al., we employ the Fisher Matrix parameter estimation method to investigate the parameter estimation accuracy achievable for modified gravitational templates by several typical binary systems, as well as the constraining power on modified gravity from the 11 binary merger events already discovered by LIGO and VIRGO; finally, using the match factor method, we explore the influence of modified gravity parameters on gravitational wave templates. If the Einstein-aether theory only yields a small correction to the general relativistic gravitational wave waveform, the 11 binary gravitational wave events can place constraints on three parameters of the Einstein-aether theory; if the Einstein-aether theory waveform exhibits significant deviation from the general relativistic gravitational wave waveform, testing the Einstein-aether theory will require more careful investigation of gravitational waveforms and data analysis.

Full Text

On the Test Ability of Einstein-aether Theory with Gravitational Wave Events in LIGO O1/O2

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Abstract: General relativity represents a more precise gravitational theory than Newton's universal gravitation for strong-field and highly dynamical spacetime regimes, with numerous astronomical observations confirming its validity. However, general relativity remains logically incompatible with quantum theory, suffers from theoretical issues such as spacetime singularities, and faces challenges in cosmology regarding the gravitational nature of dark matter and dark energy. Building upon the modified gravitational wave templates proposed by Yunes et al. for Einstein-aether theory, we employ the Fisher Matrix parameter estimation method to investigate the achievable parameter estimation precision for several typical binary systems and assess the constraining power of the 11 binary merger events discovered by LIGO and Virgo. Finally, using the matching factor method, we examine how modified gravity parameters affect gravitational wave templates. If Einstein-aether theory represents only a minor correction to general relativistic waveforms, the 11 binary gravitational wave events can constrain three parameters of the theory. However, if Einstein-aether waveforms deviate significantly from general relativistic predictions, testing the theory will require more careful investigation of gravitational waveforms and data analysis.

Keywords: gravitational waves; general relativity; Einstein-aether theory

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1 Introduction

General relativity succeeded Newton's theory of universal gravitation as a more accurate description of gravity in strong-field and highly dynamical spacetime regions. Many phenomena predicted by general relativity—including starlight deflection, gravitational redshift, Shapiro time delay, frame-dragging effects, and black holes—have been verified experimentally. In 2015, exactly 100 years after its publication, the Laser Interferometer Gravitational-Wave Observatory (LIGO) successfully detected gravitational waves directly on September 15. This achievement completed the experimental verification of general relativity's final major prediction and opened an entirely new observational window onto astrophysical objects and the universe.

Nevertheless, general relativity faces various unresolved issues. Dark matter and dark energy remain two major clouds obscuring our understanding. For dark energy, which drives the accelerated expansion of the universe, we must resort to effective field theories and phenomenological modifications to standard general relativity (typically by adding an extra scalar field), with k-essence theory being a prominent example. Dark matter presents a similar challenge. In 1933, Fritz Zwicky discovered excessively high velocity dispersions in the Coma galaxy cluster, initially attributing them to non-luminous matter. By the 1980s, extensive observations revealed that this non-luminous matter was not composed of known fundamental particles, posing a significant challenge to fundamental physics—a substance with strong macroscopic gravitational effects yet extremely weak local interactions with ordinary matter. Additionally, general relativity is logically incompatible with quantum theory and predicts the existence of spacetime singularities, suggesting theoretical limitations.

The successful verification of all general relativistic predictions both demonstrates its triumph and provides opportunities to identify physical conditions where it might break down. Newtonian gravity's applicability to weak-field, low-velocity regimes is defined relative to general relativity's strong-field predictions near black holes and the speed of light limit. Analogously, general relativity's domain of validity should be understood relative to characteristic physical scales of a new theory. Consequently, the optimal approach for identifying general relativity's limitations involves comparing it with alternative gravitational theories and using experimental data for theory selection. Gravitational wave experiments probe precisely the strong-field, highly dynamical spacetime regimes, testing general relativity with unprecedented precision while ruling out numerous alternative theories. This scientific progress undoubtedly deepens our understanding of fundamental gravitational physics.

This paper uses Einstein-aether theory as a case study to explore how gravitational waves can test theories beyond general relativity. The gravitational action of Einstein-aether theory can be written as:

$$S = -\frac{1}{16\pi G_{EA}} \int d^{4x} \sqrt{-g} \left[R + c_1 \nabla_\mu U^\nu \nabla^\mu U_\nu + c_2 \nabla_\mu U^\mu \nabla_\nu U^\nu + c_3 \nabla_\mu U^\nu \nabla_\nu U^\mu + c_4 U^\mu U^\nu \nabla_\mu U^\alpha \nabla_\nu U_\alpha\right]$$

where U^μ is the aether four-velocity, $g_{\mu\nu}$ is the spacetime metric, R is the Ricci scalar, and δ^μ_ν is the Kronecker delta. G_{EA} is the effective gravitational constant in Einstein-aether theory, distinct from Newton's constant, while c_1, c_2, c_3, c_4 are the four parameters of the theory. When all four parameters equal zero, Einstein-aether theory reduces to general relativity. The parameter combinations are:

$$\begin{cases} c_- \equiv c_1 - c_3 \\ c_{123} \equiv c_1 + c_2 + c_3 \\ c_+ \equiv c_1 + c_3 \\ c_{14} \equiv c_1 + c_4 \\ \alpha_{ppN} \equiv -\frac{8(c_3^2 + c_{1c}4)}{2c_1 - c_+ c_-} - \frac{(c_1 + 2c_3 - c_4)(2c_1 + 3c_2 + c_3 + c_4)}{(2 - c_{14})c_{123}} \end{cases}$$

Observations of binary pulsars and cosmological constraints yield:

$$|\alpha_{ppN}| < 10^{-4}, \quad c_+ < 10^{-2}, \quad c_- < 10^{-3}$$

The remainder of this paper is organized as follows: Section 2 presents our post-Einsteinian gravitational waveform for circular-orbit binary systems in Einstein-aether theory; Section 3 introduces the Fisher Matrix parameter estimation method; Section 4 presents constraints on Einstein-aether theory parameters; and Section 5 discusses our conclusions.

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2 Post-Einsteinian Gravitational Waveform for Circular-Orbit Binaries in Einstein-aether Theory

Based on calculations from Ref. [18] and the TaylorF2 waveform model for circular-orbit binaries [19, 20], we express the post-Einsteinian gravitational waveform in Einstein-aether theory as [21]:

$$h(f) = -\mathcal{A}\eta^{1/5} f^{-7/6} e^{i\Psi_1(f)} + \mathcal{A} f^{-3/2} e^{i\Psi_2(f)}$$

where the phase functions are:

$$\Psi_1(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta} u_1^{-5} \left[1 + \sum_{n=0}^{7} (\beta_{-1}\eta^{2/5} u_1^{-2} - \beta_0 + 1)(p_n + l_n \ln u_1)\right]$$

$$u_1 = (2\pi M f)^{1/3}$$

$$\Psi_2(f) = 2\pi f t_c - 2\phi_c - \frac{\pi}{4} + \frac{3}{128\eta} u_2^{-5} \left[1 + \sum_{n=0}^{7} (\beta_{-1}\eta^{2/5} u_2^{-2} - \beta_0 + 2)(p_n + l_n \ln u_2)\right]$$

$$u_2 = (\pi M f)^{1/3}$$

The expansion coefficients for the gravitational wave phases Ψ_1 and Ψ_2 are:

$$\begin{aligned}
p_0 &= 1, \quad l_0 = 0 \\
p_1 &= 0, \quad l_1 = 0 \\
p_2 &= -\frac{16\pi}{5}, \quad l_2 = 0 \\
p_3 &= -\frac{116761}{3675}, \quad l_3 = 0 \\
p_4 &= \frac{34103}{175} - \frac{6848\gamma}{105}, \quad l_4 = 0 \\
p_5 &= \frac{255\pi}{8}, \quad l_5 = \frac{1025\pi}{8}\left[1 - \ln(\pi M f_{\text{lso}})\right] \\
p_6 &= -\frac{15737765635}{3048192} - \frac{127825\gamma}{672} + \frac{6848\ln 2}{63}, \quad l_6 = -\frac{6848}{63} \\
p_7 &= \frac{14809\pi}{175}, \quad l_7 = 0
\end{aligned}$$

where γ is the Euler constant, $M = m_1 + m_2$ is the total mass, $\eta = m_{1m}2/M^2$ is the symmetric mass ratio, $f_{\rm lso}$ is the frequency at the last stable orbit, and f is the gravitational wave frequency.

Our waveform model differs slightly from Ref. [21]. While that work focused on post-Newtonian approximations and used chirp mass in all formulas (including Eqs. (7), (9), (20), and (27)), we prioritize waveform accuracy and employ total mass. Our model strictly reduces to the TaylorF2 waveform used in LIGO data analysis when $\alpha = \beta_{-1} = \beta_0 = 0$. Phenomenologically, our model can be readily extended to the state-of-the-art IMRPhenom waveform models in the LIGO data analysis software library [22, 23].

The correspondence between the ψ_n and $\psi_n^{(l)}$ parameters in Ref. [24] (Eq. (2) of Ref. [25]) and our adopted parameters is:

$$\begin{cases} \psi_{-2} = 0, \quad \psi_0 = \eta^{-1} \\ \delta\psi_{-2} = -\frac{3}{128}\beta_{-1}\eta^{2/5}(\pi M)^{-7/3} \\ \delta\psi_0 = -\frac{3}{128}\beta_0(\pi M)^{-(n-5)/3} \\ \delta\psi_n = 0, \quad \delta\psi_n^{(l)} = 0 \end{cases}$$

Given the precise measurement of gravitational wave speed from GW170817 multi-messenger astronomy (with precision $\sim 10^{-15}$) [5, 10-12, 26], we set the gravitational wave speed equal to the speed of light. Combining this with Ref. [18], we obtain:

$$\alpha = \gamma_b F_b \sin\psi + \gamma_L F_L \sin\psi + \gamma_{X1} F_X \cos\psi + \gamma_{X2} F_X \sin\psi + \gamma_{Y1} F_Y + \gamma_{Y2} F_Y \sin\psi$$

$$\beta_{-1} = \beta_0 = 0$$

where ψ is the gravitational wave polarization angle, F_b, F_L, F_X, F_Y are the detector response functions for corresponding polarization modes, and $\gamma_b, \gamma_L, \gamma_{X1}, \gamma_{X2}, \gamma_{Y1}, \gamma_{Y2}$ are the mode amplitude coefficients in Einstein-aether theory. These functions and coefficients can be expressed in terms of the Einstein-aether parameters c_1, c_2, c_3, c_4 (see Ref. [18] for details). The parameters ϵ_x and κ_3, also functions of c_1, c_2, c_3, c_4, are defined in Eqs. (4.6) and (6.7) of Ref. [18], with general relativity corresponding to $\epsilon_x = 0$ and $\kappa_3 = 1$. Below we describe how gravitational wave observations can constrain α, β_{-1}, and β_0.

Advanced LIGO has conducted three observing runs since September 2015, with continuously improving sensitivity. The detector's noise power spectral density can be approximated as [25]:

$$S_n(f) = \begin{cases} \infty, & f < 10 \text{ Hz} \\ S_0 \left[x^{-4.14} - 5x^{-2} + 111 \left(1 - x^2 + \frac{x^4}{2}\right) \frac{1}{1+x^2/2} \right], & f \geq 10 \text{ Hz} \end{cases}$$

where $x = f/215$ Hz and $S_0 = 10^{-49}$.

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3 Fisher Matrix Parameter Estimation Method

Gravitational wave detector data $d(t)$ contains both signal $s(t)$ and noise $n(t)$: $d(t) = s(t)+n(t)$. Under ideal conditions, detector noise is stationary and Gaussian, meaning $d(t) - s(t)$ is a random process with time-independent Gaussian joint probability distributions.

Based on detector sensitivity, we define the inner product between two time series $h_1(t)$ and $h_2(t)$ as:

$$\langle h_1(t), h_2(t) \rangle \equiv 4\Re \int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}_1^*(f)\tilde{h}_2(f)}{S_n(f)} df$$

where tildes denote Fourier transforms, asterisks denote complex conjugation, $\Re$ denotes the real part, and $(f_{\min}, f_{\max})$ corresponds to the detector's sensitive frequency band. For stationary Gaussian noise $n(t)$, the inner product follows a Gaussian distribution, yielding the conditional probability:

$$p(d|h) \propto e^{-\langle d-h, d-h \rangle/2}$$

In gravitational wave analysis, we seek the probability of signal $h(t)$ given data $d(t)$, i.e., the posterior probability $p(h|d)$. Bayes' theorem gives:

$$p(h|d) = \frac{p(d|h)p(h)}{\int p(d|h)p(h)dh}$$

where the denominator serves as a normalization factor. In Bayesian statistics, $p(h|d)$ is the posterior, $p(h)$ is the prior, and $p(d|h)$ is the likelihood function.

For specific gravitational wave sources like binary systems, waveforms depend not only on time but also on source parameters such as masses, spins, and orbital eccentricity. Denoting these parameters as $\vec{\lambda}$, we have $h(\vec{\lambda}, t)$. The probability distributions $p(h|d)$ and $p(h)$ become functions over parameter space.

The posterior probability maximum corresponds to the most probable parameters $\vec{\lambda}_0$, which in matched-filtering data analysis represents the best-matching parameter set [1]. Assuming strong signals (high signal-to-noise ratio) where noise n is negligible, we have $d \approx h(\vec{\lambda}_0)$ and $\langle n, n\rangle \approx \langle n, h\rangle \approx \langle n, \partial_i h\rangle \approx \langle n, \partial_i \partial_j h\rangle \approx 0$. With a uniform prior $p(h) \propto 1$, we obtain:

$$p(h(\vec{\lambda})|d) = Ce^{-\langle d-h, d-h\rangle/2}$$

where C is a normalization constant. Expanding $\ln p(h(\vec{\lambda})|d)$ around $\vec{\lambda}_0$:

$$\ln p(h(\vec{\lambda})|d) \approx \ln C - \frac{1}{2}(\lambda_i - \lambda_i^0)\langle \partial_i h, \partial_j h\rangle(\lambda_j - \lambda_j^0)$$

The matrix $\langle \partial_i h, \partial_j h\rangle$ is precisely the Fisher Matrix. Under high signal-to-noise conditions, the posterior probability distribution near the most probable parameters approximates a Gaussian with covariance matrix Σ_{ij} equal to the inverse Fisher Matrix:

$$\langle \partial_i h, \partial_j h\rangle = \Sigma_{ij}^{-1}$$

We use the Fisher Matrix method to estimate parameter measurement errors $\Delta\lambda_i = \sqrt{\Sigma_{ii}}$.

Our Einstein-aether waveform model for circular-orbit binaries includes parameters $(\mathcal{A}, M, \eta, t_c, \phi_c, \alpha, \beta_{-1}, \beta_0)$: amplitude (related to luminosity distance and orientation), total mass, symmetric mass ratio, merger time and phase, and post-Einsteinian parameters. We find $\mathcal{A}$ is weakly correlated with other parameters; including or excluding it yields similar errors for the remaining parameters. Table 1 shows parameter estimation errors for typical binary systems using Advanced LIGO sensitivity. Our results agree with Ref. [21], though minor quantitative differences arise from our more accurate waveform model (equivalent to 3.5 post-Newtonian order versus their 2 post-Newtonian order), yielding slightly smaller errors.

Table 1: Parameter estimation results for typical binary systems using the Fisher Matrix method with Advanced LIGO sensitivity

m_1	m_2	$\Delta M/M$	$\Delta\eta/\eta$	Δt_c (ms)	$\Delta\phi_c$ (rad)	$\Delta\alpha$	$\Delta\beta_{-1}$	$\Delta\beta_0$
1.4	1.4	3.84×10^{-2}	3.86×10^{-2}	0.164	1.40	2.79×10^{-5}	3.47×10^{-5}	3.29×10^{-5}
10	10	3.98×10^{-2}	3.94×10^{-2}	0.152	1.61	2.99×10^{-5}	-	-

Note: Assumes signal-to-noise ratio of 10, with masses in solar units.

For parameters t_c and ϕ_c, we also explored using the F-statistic [19]. Whether or not the F-statistic is employed makes essentially no difference for constraining

Einstein-aether parameters. All results presented below were obtained without using the F-statistic.

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4 Constraining Einstein-aether Theory

The LIGO Scientific Collaboration's first observing run (O1), led by Advanced LIGO, ran from September 12, 2015 to January 19, 2016, discovering three binary black hole mergers: GW150914, GW151012, and GW151226. The second observing run (O2) extended from November 30, 2016 to August 25, 2017, with Advanced Virgo joining on August 1, 2017. O2 yielded eight additional binary merger events: GW170104, GW170608, GW170729, GW170809, GW170814, GW170817, GW170818, and GW170823.

Table 2 summarizes parameter estimation results for Einstein-aether theory using these 11 events. Since the GstLAL pipeline detected all events, we use its signal-to-noise ratios (SNR) and component masses m_1 and m_2 [27]. By substituting LIGO-Virgo's measured parameters into Eqs. (5)-(8), we generate simulated waveforms including Einstein-aether parameters and apply the Fisher Matrix method to assess constraining power. The results show that lower-mass binary systems provide tighter constraints, as more of their signal falls within the Advanced LIGO/Virgo frequency band, enabling more effective tests of Einstein-aether theory. For higher-mass systems where merger frequencies lie near the detector band, our waveform model cannot utilize the short merger signal, resulting in weaker constraints.

Table 2: Parameter estimation results for Einstein-aether theory from 11 binary merger events

Event	SNR	m_1	m_2	$\Delta\alpha$	$\Delta\beta_{-1}$	$\Delta\beta_0$
GW150914	24.4	35.6	30.6	1.95×10^{-2}	7.77×10^{-3}	1.26×10^{-3}
GW151012	9.5	23.2	13.6	1.59×10^{-2}	7.48×10^{-4}	9.96×10^{-2}
GW151226	13.0	13.7	8.8	8.07×10^{-6}	3.44×10^{-2}	4.67×10^{-2}
GW170104	13.0	31.0	20.1	-	-	-
GW170608	15.0	11.0	7.6	-	-	-
GW170729	10.8	50.6	34.3	-	-	-
GW170809	12.7	35.2	23.8	-	-	-
GW170814	16.3	30.5	25.3	-	-	-
GW170817	32.4	1.46	1.27	-	-	-
GW170818	11.9	35.4	26.7	-	-	-
GW170823	11.3	39.5	29.0	-	-	-

Note: The last three columns show our Fisher Matrix analysis results.

Combining results from all 11 events yields the constraints:

$$\alpha < 0.166, \quad \beta_{-1} < 8.07 \times 10^{-6}, \quad \beta_0 < 7.79 \times 10^{-2}$$

These limits assume general relativistic templates can perfectly match detected signals. The α parameter is poorly constrained because it affects waveform amplitude, whereas β_{-1} and β_0, which affect phase, are well-constrained—matched filtering is inherently phase-sensitive.

To assess how well general relativistic templates match Einstein-aether signals, we introduce the fitting factor FF [28, 29]. For waveforms $h_1(t)$ and $h_2(t)$:

$$FF \equiv \frac{\langle h_1(t), h_2(t)\rangle}{\sqrt{\langle h_1(t), h_1(t)\rangle \cdot \langle h_2(t), h_2(t)\rangle}}$$

When approximating one waveform with another, the signal-to-noise ratio degrades by factor FF. If this reduction is too large, the approximation fails.

Using GW170817's binary parameters, we examine matches between general relativistic and Einstein-aether waveforms. Since matched filtering is insensitive to amplitude, we focus on FF versus β_{-1} and β_0, shown in Figure 1 [Figure 1: see original paper]. Deviations of β_{-1} and β_0 from zero at the 10^{-6} and 10^{-3} levels, respectively, make Einstein-aether waveforms completely incompatible with general relativity. Thus, if GW170817's signal matches general relativity, Einstein-aether's β_{-1} and β_0 can be tightly constrained.

A crucial question remains: Could GW170817 actually be an Einstein-aether signal where parameter degeneracy with chirp mass allows general relativistic templates to match it? We investigate this by fixing $\beta_{-1} = 6 \times 10^{-5}, \beta_0 = 0$ and $\beta_{-1} = 0, \beta_0 = 0.002$, then varying chirp mass to study FF, shown in Figure 2 [Figure 2: see original paper]. Indeed, adjusting chirp mass can make general relativistic waveforms match Einstein-aether predictions, revealing parameter degeneracy between β_{-1}, β_0, and chirp mass. Therefore, without certainty that detected signals are described by general relativity, we cannot claim strong constraints on Einstein-aether theory—we can only state that general relativity excellently explains all observed gravitational wave signals [2].

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5 Conclusions and Discussion

Since LIGO's first direct detection in 2015, Advanced LIGO and Advanced Virgo have completed three observing runs, detecting dozens of gravitational wave events from binary black hole, binary neutron star, and black hole-neutron star mergers, establishing gravitational wave astronomy as a powerful observational tool.

Gravitational wave astronomy enables studies of fundamental gravitational physics [22, 24, 30]. GW170817 has been particularly successful in testing gravitational theories, placing strong constraints on numerous alternatives to general relativity.

Building upon Ref. [18] and the TaylorF2 waveform model, we developed a gravitational waveform model for binary systems in Einstein-aether theory. Using the Fisher Matrix method with publicly available data from LIGO's first two observing runs (11 binary systems), we constrained Einstein-aether theory. The Fisher Matrix method is valid only for small parameter deviations—when Einstein-aether theory provides minor corrections to general relativistic waveforms. In this regime, Einstein-aether templates would yield results similar to general relativity, allowing perturbative analysis of posterior probability distributions. This justifies using LIGO's measured parameters as inputs. If small deviations do not hold, the Fisher Matrix method becomes invalid.

We employed the matching factor method to explore scenarios where Einstein-aether waveforms significantly deviate from general relativity and their impact on data analysis.

Our results show that if Einstein-aether theory represents only a minor modification (in waveform sense), we can constrain its parameters to $\alpha < 0.166$, $\beta_{-1} < 8.07 \times 10^{-6}$, and $\beta_0 < 7.79 \times 10^{-2}$. However, based solely on gravitational wave detections, we cannot completely exclude the possibility of significant deviations, as Einstein-aether parameters exhibit degeneracy with chirp mass. In future work, we plan to employ Bayesian model selection to investigate this degeneracy.

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