Postprint: Current Status and Development of Three-Dimensional Ionospheric Research Based on Multi-Source GNSS Observations

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Abstract

In recent years, with the development of GNSS (global navigation satellite system) technology, the utilization of spatial characteristics from ground-based and space-based GNSS ionospheric sounding has formed a more comprehensive three-dimensional ionospheric observation system, yielding abundant multi-source GNSS ionospheric observation data. This has promoted progress in ionospheric theoretical models, empirical models, and assimilation models. Simultaneously, through integration with research on the neutral atmosphere and plasmasphere, it has provided a data foundation and research means for monitoring extreme weather, solar activity, magnetic storms, and other events, as well as for investigating their physical mechanisms. This paper introduces the current development status of multi-mode ground-based and satellite-borne GNSS ionospheric observation data, and discusses the progress in improving empirical models for the topside ionosphere region using space-based GNSS ionospheric observation data. It is pointed out that ionospheric empirical models can serve as the initial field for assimilation models, while ionospheric theoretical models can be employed as the dynamic component of Kalman filtering, thereby enabling the reconstruction or prediction of the ionosphere using multi-source data through assimilation algorithms. Additionally, the new requirements and developments in the construction of ionospheric assimilation models against the backdrop of increasing multi-source and multi-mode GNSS ionospheric observation data are elaborated. Finally, it is proposed that adaptive grids and machine learning could become new research directions in ionospheric assimilation algorithms to advance three-dimensional ionospheric studies.

Full Text

Preamble

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Current Status and Development of 3-D Ionospheric Research Based on Multi-Source GNSS Observation Data

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Abstract

In recent years, the development of GNSS (Global Navigation Satellite System) technology has enabled the utilization of spatial characteristics from ground-based and space-based GNSS ionospheric observations, forming a more comprehensive three-dimensional ionospheric observation system. This has yielded abundant multi-source GNSS ionospheric observation data, promoting advances in ionospheric theoretical models, empirical models, and assimilation models. Concurrently, through integration with research on the neutral atmosphere and plasmasphere, it has provided data foundations and research methods for monitoring and investigating the physical mechanisms of extreme weather events, solar activities, and magnetic storms. This paper introduces the current development status of multi-mode ground-based and spaceborne GNSS ionospheric observation data, and discusses the progress of spaceborne GNSS ionospheric observation data in improving empirical models of the topside ionosphere. It is pointed out that ionospheric empirical models can serve as the initial field for assimilation models, while ionospheric theoretical models can be used as the dynamic component of Kalman filters to reconstruct or predict the ionosphere through assimilation algorithms using multi-source data. The paper also elaborates on new requirements and developments in the construction of ionospheric assimilation models under the background of increasing multi-source and multi-mode GNSS ionospheric observation data. Finally, it proposes that adaptive grids and machine learning could become new research directions in ionospheric assimilation algorithms to promote three-dimensional ionospheric research.

Keywords: GNSS; ionosphere; empirical models; assimilation models

1 Introduction

As an important component of the solar-terrestrial space environment, the ionosphere has significant impacts on modern radio engineering systems and human space activities (for example, it is closely related to positioning errors in satellite navigation systems and seismic anomalies [?, ?]). Studying the ionosphere facilitates understanding of the ionosphere itself, helps identify ways to mitigate potential ionospheric hazards, explores methods for ionospheric utilization, and promotes theoretical and applied developments in Earth science.

In recent years, with the establishment of the Global Positioning System (GPS) and navigation constellations from various countries, as well as the implementation of regional deformation and space environment monitoring projects, a new era has emerged for applying ground-based GNSS and space-based GNSS multi-source ionospheric observation data to ionospheric research. The increase and improvement of GNSS satellite constellations, the construction of global and regional monitoring networks, and the widespread use of multi-mode GNSS receivers have led to a substantial increase in the amount of ground-based GNSS ionospheric observation data.

The distribution of ground-based GNSS ionospheric observation data is closely related to the distribution of ground monitoring stations. In oceanic regions, South America, Africa, and Russia, the sparsity of ground stations results in poor ionospheric accuracy in these areas. Navigation satellites move extremely slowly, and even with increased data sampling rates, the impact on the geometric distribution of piercing information is minimal, and the improvement in data spatial resolution is not significant.

Since low Earth orbit (LEO) satellites move faster than navigation satellites, they can conduct global-scale ionospheric detection with high dynamics. LEO satellites can generally obtain slant path total electron content (TEC) extracted from their positioning data and ionospheric occultation observations for ionospheric model construction. Adding LEO satellite ionospheric observation data can compensate for the lack of oceanic data. Currently, there are not many LEO satellites with ionospheric occultation detection capabilities. It is expected that more LEO satellite constellations will be built and equipped with ionospheric occultation antennas to construct a more complete global ionospheric observation system.

Since the ionosphere is a non-closed system, even if its theoretical model has an extremely accurate initial state, errors will accumulate over time. Although ionospheric empirical models account for the spatial characteristics and temporal periodic patterns of real observation data and can even reproduce the Weddell Sea anomaly, they are essentially statistical models that still have large errors in extreme space environments. By updating ionospheric physical or climatological models using multi-source ionospheric observation data, more refined and accurate ionospheric structures can be obtained.

These three research directions complement each other: with Kalman filtering as the core module of assimilation models, ionospheric empirical models can serve as the initial field due to their relatively high accuracy, while ionospheric theoretical models can be used as the dynamic component of the Kalman filter. Previously, ionospheric observation data mainly came from ionosondes and incoherent scatter radars. The introduction of GPS ionospheric observation data improved the temporal and spatial detection capabilities of the ionosphere. However, ionospheric observation data remained relatively scarce, making it difficult to capture anomalies in oceanic regions during magnetic storms or active ionospheric periods.

Through measured data, ionospheric climatological/empirical models can be established—data-based models describing electron density, electron temperature, ion temperature, ion composition, etc.—to avoid uncertainties generated by theoretical models (typically offering better accuracy and serving as background fields for assimilation or tomography algorithms). Current ionospheric and plasmaspheric empirical models mainly include NeQuick [?, ?], IRI (International Reference Ionosphere) \cite{5–9}, GCPM (Global Core Plasmasphere Model) [?], and IRI Plas \cite{11–16}. The space environment in the topside ionospheric F2 region is complex and has limited observation data, leading to large errors in traditional models. GNSS ionospheric occultation observation data can retrieve electron density profiles below LEO satellite orbital altitudes through inversion, which can be used to improve empirical models of the topside ionosphere.

Based on function bases or pixel bases, assimilation updates to physical or climatological models through tomography algorithms [?, ?], Kalman filtering \cite{19–23}, 3D or 4D variational methods, or statistical optimization (the solution forms are equivalent [?, ?]) can yield refined and accurate ionospheric structures. Tomography algorithms can obtain 3D ionospheric information using minimal memory and computation time based on background fields, but they cannot provide error assessments for the ionosphere at the next moment and struggle to ensure spatial continuity of electron density. To obtain continuous 3D ionospheric structures, smoothing tomography results would sacrifice local fine structures.

By introducing assimilation algorithms applied in meteorology, ocean geology, and seismology, better results can be achieved in physical mechanisms, temporal evolution error evaluation, and spatial structure continuity, though assimilation models have extremely high requirements for computer memory and computational power. Huang proposed an advanced spatial filtering scheme to avoid 3D-Var's search for the inverse of the background field error covariance matrix [?], greatly reducing computational time. Schunk et al. developed the Global Assimilation of Ionospheric Measurements (GAIM) model using a physics-based ionosphere-plasmasphere model and Kalman filter as the foundation for fusing various near-real-time measurement data. The GAIM model lacks support for multi-constellation GNSS data and has poor spatiotemporal distribution of ionospheric observation data [?, ?, ?].

Courtier et al. [?] and Wang et al. [?] applied 4D variational methods to time-varying observation data, using incremental approximation methods to provide flexibility for cost-benefit trade-offs in 4D variational methods. Yue et al. [?] conducted a 3D ionospheric reanalysis from 2002 to 2011 using ground-based and space-based GNSS ionospheric observation data through an ensemble Kalman filter assimilation model, but did not use 4D variational methods or ionospheric physical models. Fu et al. [?] proposed that ionospheric assimilation needs to consider hardware delays of receivers and transmitters, and that a two-layer stepwise assimilation method can improve ionospheric assimilation model accuracy. Under the background of increasing multi-source and multi-mode GNSS ionospheric observation data, the diverse data characteristics of multi-source observations and constraints on computer memory and computational power have created new requirements and developments for ionospheric assimilation model construction.

Section 2 introduces the development status of multi-mode ground-based and spaceborne GNSS ionospheric observation data. Section 3 discusses the progress of spaceborne GNSS ionospheric observation data in improving empirical models of the topside ionosphere. Section 4 elaborates on new requirements and developments in ionospheric assimilation model construction under the background of increasing multi-source and multi-mode GNSS ionospheric observation data. Section 5 proposes that adaptive grids and machine learning could become new research directions in ionospheric assimilation algorithms. Fine 3D ionospheric structures have been greatly improved through the fusion of theoretical/empirical models with multi-source ionospheric observation data and have been applied to many aspects of research to promote 3D ionospheric studies.

2 Multi-Source GNSS Ionospheric Observation Data

Ionospheric data can be categorized into three types based on data characteristics or detection methods. The first includes various indicative data such as solar activity, magnetic storm conditions, and geomagnetic variations, including geomagnetic indices, geomagnetic disturbance amplitude/planetary indices, and sunspot numbers—these only provide macroscopic descriptions of the ionosphere and Earth's space environment.

The second includes ionospheric data obtained through direct detection means, such as data from ionosondes and incoherent scatter radars (ISR). These methods have small coverage, high detection costs, short detection durations, and low continuity, but the data obtained have high precision and can serve as information for ionospheric model construction and validation.

The third refers to ionospheric products obtained by parsing ionospheric effects from GNSS carrier phase and pseudorange measurements as microwaves pass through the ionosphere since the development of GNSS technology, such as the International GNSS Service (IGS) two-dimensional ionospheric map GIM

(Global Ionosphere Model) products and ionospheric electron density profiles (ionPrf) data provided by the COSMIC (Constellation Observing System for Meteorology, Ionosphere, and Climate) system through CDAAC/UCAR (the COSMIC Data Analysis and Archive Center of the University Corporation for Atmospheric Research). The following introduces the characteristics, quantity, and distribution of ionospheric data obtained through indirect GNSS technology detection.

Based on ground-based GNSS stations, LEO satellites, and GNSS navigation satellites, [Figure 1: see original paper] shows the observation systems for space science in the atmosphere/ocean/ionosphere. Taking the ionosphere as an example, ground-based GNSS satellite systems and ground monitoring stations constitute the ground-based GNSS ionospheric observation system; LEO satellites/GNSS satellites form the topside ionospheric observation and occultation ionospheric observation system; LEO satellites form an inter-satellite ionospheric monitoring system by mutually transmitting and receiving signals, such as COSMIC/COSMIC-2 (launched in 2006 and 2019); taking Jason-1/Jason-2/Jason-3 (launched in 2001, 2008, and 2016) as examples, ocean vertical observations form the oceanic ionospheric observation system.

[Figure 1: see original paper] Current and future neutral atmosphere/ocean/ionosphere data observation systems

2.1 Ground-Based GNSS Ionospheric Observations

As shown in [Figure 1: see original paper], the signal transmission paths between GNSS satellites and ground stations at different moments form the paths indicated by brown lines, crossing the stratosphere, troposphere, ionosphere, and plasmasphere. As satellite signals pass through different spheres, various additional effects are produced on microwave signals due to the physical characteristics of each sphere. Although this introduces many errors for GNSS positioning, it also provides conditions for retrieving physical characteristics of each sphere from GNSS data.

Currently, there are complete GNSS satellite constellations, including GPS, GLONASS (Global'naya sputnikovaya navigatsionnaya sistema), Galileo, and Beidou navigation systems. The different orbital designs and characteristics of satellites in each system ensure that each ground station can see at least a dozen satellites in the combined system. Taking the ground-based GNSS/IGS observation system as an example, 30-second sampling rate observation data can yield hundreds of thousands of observations per hour. [Figure 2: see original paper] shows the distribution of stations in 2019 for IGS, China's Crustal Movement Observation Network, Australia's GeoScience, New Zealand's GeoGoing, and the United States/Japan GeoNet regional monitoring networks.

Using global ground-based GNSS observations, massive amounts of all-weather, globally distributed ionospheric observation data can be obtained. As shown in [Figure 2: see original paper], ground-based GNSS observation stations are mainly distributed on continents, with a small number built on islands, resulting in very sparse ground-based GNSS ionospheric observation data in oceanic regions. To establish a global ionospheric model, observations from oceanic regions need to be supplemented.

2.2 Space-Based GNSS Ionospheric Observations

Space-based GNSS ionospheric observation data are mainly divided into three types: (1) positioning data used in satellite orbit determination can obtain slant total electron content between satellites and navigation satellites; (2) ionospheric occultation observation data can be obtained when occultation occurs between satellites and navigation satellites; and (3) altimetry satellites can obtain vertical total electron content between satellite altitude and Earth's surface.

From the perspective of ionospheric observation, occultation data based on ionospheric limb sounding have the advantages of high precision, high vertical resolution, complete global coverage, and no ionospheric bias. The COSMIC system, jointly launched by Taiwan and the United States in 2006 consisting of 6 LEO satellites, provided a large amount of occultation observation data for ionospheric research. Although the COSMIC constellation has reached its operational lifespan in recent years and can only provide limited observation data, the COSMIC-2 constellation with GPS and GLONASS dual-mode observation was launched in 2019 and will provide more space-based ionospheric observation data.

As shown in [Figure 3: see original paper], CDAAC/UCAR processes and archives data from more than 10 occultation missions in near-real-time, providing an opportunity for global 3D ionospheric electron density reconstruction and reanalysis.

[Figure 3: see original paper] Occultation data from satellite missions carrying occultation payloads between 2001–2012

Using the parameters in , [Figure 4: see original paper] simulates the distribution of ionospheric piercing points (IPP) from ground-based observations of GPS and GLONASS constellations by 328 IGS stations, and ionospheric occultation target points (OccTarget) from COSMIC six-satellite occultation observations of the GPS constellation on January 10, 2008. [Figure 4: see original paper] shows that ionospheric occultation observation data can greatly improve the lack of ground-based ionospheric observation data in oceanic regions. However, compared with numerous ground-based ionospheric observations, the ionospheric occultation observation data generated solely by the COSMIC LEO satellite constellation is relatively small, resulting in still unbalanced distribution of observation data over land and sea.

Ground-based/spaceborne ionospheric observations

Navigation constellation and satellite count	Receiver carrier and number	Cut-off elevation angle/(°)	Sampling rate/s	Observation count/h
GPS + GLO (56)	IGS (256)	10	30	~10,000
GPS (32)	COSMIC (6)	0	1	~10,000

In the past five years, companies including OneWeb, SpaceX, Boeing, Samsung, China Aerospace Science and Industry Corporation, and China Aerospace Science and Technology Corporation have designed, deployed, and plan to launch dozens to tens of thousands of commercial LEO satellite constellations to provide seamless and stable Internet services from space. Zhang and Ma [?] compiled the design parameters of various satellite constellations, as shown in . Reid et al. [?] comprehensively explored the possibility of expanding these constellations into navigation augmentation constellations from the perspective of overall system architecture, including constellation geometry, space signal ranging error, onboard atomic clock performance, and orbit determination methods, with encouraging results. With future miniaturization and cost reduction of ionospheric occultation payloads, deploying them on different navigation augmentation constellations can form more uniformly distributed and numerous ionospheric occultation events, thereby obtaining more ionospheric occultation observation data for 3D ionospheric modeling.

Summary of partially deployed or proposed commercial LEO constellations

Constellation	Satellite count	Height/km	Inclination/(°)	Build time	Country	Purpose
Iridium	66	780	86.4	1998	USA	Voice+STL
Globalstar	48	1414	52	1999	USA	Voice+STL
Iridium NEXT	66	780	86.4	2017	USA	Broadband+STL
OneWeb	882	1200	87.9	2021	UK	Broadband
SpaceX Starlink	11926	1110-1325	53-81	2027	USA	Broadband
Boeing	2956	1200	45-88	-	USA	Broadband
LeoSat	108	1400	95	-	USA	Broadband
Telesat	512	1000-1248	50.2-99.5	-	Canada	Broadband
Kepler Comunications	140	575	72.5	-	Canada	Broadband
Astrocast	64	600	97.8	-	Switzerland	M2M
Yaliny	135	600	98.4	-	Russia	Broadband
Astrome	150	1400	45	-	India	Broadband

Constellation	Satellite count	Height/km	Inclination/(°)	Build time	Country	Purpose
Samsung	4600	1500	78-84	-	South Korea	Broadband
Hongyan	320	1100	50-55	2023	China	Broadband+Navigation augmentation
Hongyun	156	1000	55	2022	China	Broadband+Navigation augmentation
CentiSpace	120	850	55	2020	China	Broadband+Navigation augmentation

3 Application of Multi-Source GNSS Data to Ionospheric Empirical Models

From the SAMI2/3 model [?, ?] to the NCAR/TIEGCM model [?, ?], and then to the GAIA (Ground-to-topside model of Atmosphere and Ionosphere for Aeronomy) model covering from the ground to the magnetosphere [?], and the WACCM-X (Thermosphere and Ionosphere extension of the Whole Atmosphere Community Climate Model) that has been/is being extended [?, ?], ionospheric theoretical models have achieved considerable development. Taking the GAIA model as an example, GAIA is a combination of three independently developed models, including the whole atmosphere (from troposphere to thermosphere) general circulation model (GCM), ionospheric model, and electrodynamic model. The core of GAIA is a "coupler" module that manages differences among the three models. Compared with other whole atmosphere models (such as TIE-GCM), only GAIA simulates the neutral atmosphere from Earth's surface to the thermosphere [?].

Relatively speaking, ionospheric theoretical models are not as accurate as ionospheric empirical models when used as the initial field for assimilation models and are often used as the dynamic component of Kalman filters. Therefore, using multi-source data to improve ionospheric empirical models is one of the current research hotspots. Due to the complex space environment and limited observation data in the ionospheric F2 layer topside region, the IRI, GCPM, and IRI Plas models have done extensive work in topside ionosphere modeling. Here, we mainly introduce the IRI model and research progress on improving IRI using occultation data \cite{42–45}.

The International Reference Ionosphere (IRI) describes monthly average values of electron density, electron temperature, ion temperature, ion composition, and other parameters in the altitude range of 60–2,000 km based on most available and reliable observations of ground and space ionospheric plasma. Compared with IRI2012, the latest version IRI2016 has two new options for F2 layer peak height and no longer relies on the relationship with the propagation factor M(3000) to describe F2 layer peak height: one is based on ionosonde data [?], recommended for obtaining more accurate ionospheric electron density distribution fields [?]; the other is based on occultation observation data [?].

Compared with ground-based GNSS ionospheric observations, ionospheric occultation observations have the advantages of being bias-free and providing global coverage. In recent years, many researchers have conducted extensive studies \cite{48–53} to combine COSMIC ionospheric occultation observations to improve topside empirical models. Using empirical orthogonal function (EOF) analysis methods, COSMIC ionospheric occultation data from 2007–2011 were processed to reconstruct an ionospheric scale height model with a spatial resolution of 5° in geomagnetic latitude (87.5°S–87.5°N) and a temporal resolution of 2 h. Through EOF analysis, the characteristics of scale height were found to be mainly reflected in geomagnetic latitude, annual, seasonal, and diurnal variations.

As shown in [Figure 5: see original paper], by using the scaling factor q as an additional constraint to improve the topside model of IRI2007 [?], the curve fitting accuracy of this part was greatly improved [?], and this model was used to assist Abel inversion to verify its accuracy [?]. On the other hand, in 2018, the topside total electron density observations from COSMIC and mapping functions were used to obtain vertical total electron content, and then the relationship between vertical total electron content and electron density at satellite orbit altitude and topside region scale height from COSMIC ionospheric occultation data was used to establish an exponential decay model for ionospheric electron density above 800 km for evaluating ionospheric topside and plasmasphere effects or error elimination [?].

[Figure 5: see original paper] Comparison of three inversion results with retrieved COSMIC simulation profiles before and after introducing scale height constraints into IRI

Note: a) Date 2007.030, UT 10.1958, latitude 24.97, longitude 105.40; b) Date 2007.030, UT 10.2158, latitude 34.75, longitude 95.66; c) Date 2007.030, UT 10.2800, latitude 40.07, longitude 129.85. "HSC" indicates F2 layer topside scale height, "non cor" indicates before q-factor constraint correction, "cor" indicates after q-factor constraint correction.

The ionospheric scale height model established based on COSMIC ionospheric occultation electron density profile data improves the IRI topside model as an additional constraint, but constant scale height is not suitable for describing the physical characteristics of ionospheric electron density, especially underestimating electron density in the plasmasphere. Therefore, using the Vary-Chap variable scale height (VCSH) technique is promising for further improving the topside electron density model. The Vary-Chap variable scale height H(h) was first introduced into the general α-Chapman electron density distribution in 1969 [?]:

$$Ne(h) = Nm\,(1 - z\exp(-z))$$

where z is an intermediate parameter; Nm and hm represent the F2 layer peak density and height, respectively; Ne(h) represents electron density at altitude h; H(h) and Hm are the scale heights at altitude h and F2 layer peak height, i.e., Hm = H(hm), and H(h) is a function of the unknown Hm.

Using COSMIC electron density profile data from January 1, 2008, to December 31, 2013, a shape function combining linear ax-shape and parabolic bx^2-shape patterns controlled by exponential weights was obtained to fit the Vary-Chap scale height profile representing lower and higher altitudes:

$$y = ax\frac{e^x - e^{-x}}{e^x + e^{-x}} + bx^2e^{-2x}, \quad h > hm$$

where y = ln(H(h)) - ln(Hm), x = (h - hm)/Hm, and a and b are fitting parameters for linear and parabolic pattern shapes, respectively. When using weighted linear pattern ax and parabolic pattern bx^2 to represent the VCSH profile shape, the fitted VCSH and reconstructed electron density profiles agree very well with original observations; most fitting residuals are less than 10% across all studied years. Parameter a ranges from 1.0 to 2.0 (monthly average), is highly correlated with geomagnetic field line altitude, and is larger during daytime and high-activity years in low- and mid-latitude regions; while parameter b and transition height hc (the intersection of the two shape patterns) show variation patterns very similar to the equatorial ionization anomaly (EIA) [?]. The parametric fitting functions for a and b can simply represent VCSH profiles with reasonable accuracy, allowing reconstruction of electron density distribution at any given time and location combined with F2 layer peak electron density, peak height, and scale height. After careful validation, it can be introduced as another topside option into the IRI model [?].

4 Application of Multi-Source GNSS Observation Data to Ionospheric Assimilation Models

In recent years, ionospheric tomography imaging algorithms based on algebraic reconstruction techniques have been widely used to reconstruct 3D ionospheric structures [?, ?]. However, the poor spatial distribution of ionospheric observation data leads to lack of spatial continuity in tomography results. Lorenc [?] proposed three-dimensional variational assimilation using the Kalman filter method as a classic approach for modern data assimilation. [Figure 6: see original paper] shows slices of the assimilated 3D ionospheric model in longitude, latitude, and altitude, which more finely describes the characteristics of 3D ionospheric structure, particularly the irregular shapes near the ionospheric equator.

[Figure 6: see original paper] Slices of ionospheric electron density field along longitude, latitude, and altitude

Similar to tomography imaging algorithms, ionospheric assimilation algorithms use observations to update the background field: based on the covariance of the ionospheric electron density background field and observation data, the minimum value of a cost function related to observations and background field is constructed and solved to obtain assimilation results. This method can maintain the smooth physical structure of the ionosphere and produce electron density fields closer to real ionospheric distribution. Huang's proposed spatial filtering scheme avoids 3D-Var's search for the inverse of the background field error covariance matrix [?], but the increase in multi-source GNSS ionospheric observation data still imposes extremely high requirements on computer memory and computational power.

A stepwise two-layer assimilation method is used to reduce the influence of the topside ionosphere on bottomside ionosphere assimilation [?]: assimilate slant TEC information obtained from LEO satellite positioning observations to obtain the electron density distribution model above 800 km; then assimilate occultation observation data and ground GNSS ionospheric observations with the topside contribution removed to obtain the ionospheric electron density distribution model below 800 km, yielding more accurate F2 layer ionospheric structure. This approach can reduce the number of unknowns in stepwise assimilation, thereby saving computation time and computer memory.

It can be seen that the introduction of multi-source and multi-mode GNSS ionospheric observation data has created new requirements and developments for ionospheric assimilation models: (1) To ensure the characteristics and accuracy of multi-source and multi-mode GNSS ionospheric observation data, various error sources in assimilation models require more detailed analysis and methods for elimination; (2) The improvement effects of multi-source and multi-mode GNSS ionospheric observation data in assimilation models need evaluation, and algorithm design must adapt to computational efficiency due to increased observation data.

4.1 Error Analysis in Assimilation Algorithms

In ionospheric assimilation, assumptions about background field error distribution, observation ray path distribution, observation data weighting, and sparse optimization of matrix solutions can all lead to errors in assimilation results. Classifying error sources and effectively improving them is an important means to enhance assimilation accuracy. Ionospheric error sources can be divided into algorithm errors and model assumption errors.

Although algorithm errors are difficult to eliminate, simulations have verified that empirically adjusting ionospheric background field errors and correlation coefficients, background field weights, and various observation errors can reduce assimilation result errors to 20% of the true field, with overall bias less than 5%. However, several errors arising from assimilation process assumptions cause large systematic biases between simulated observation data and fixed background field errors, though they can be improved through algorithms.

As shown in [Figure 7: see original paper], the effects of ionospheric topside and plasmasphere, ionospheric temporal variation, and grid representation errors can be effectively reduced through appropriate methods, all belonging to model assumption errors. Taking the ionospheric topside and plasmasphere effect as an example, its contribution to GPS TEC varies throughout the day, from a minimum of about 12% during equinox to a maximum of about 60% during winter nights [?]. Simulation statistics using the GCPM model show that all model assumption errors can generally reach 10%–20%, consistent with conclusions from many scholars [?, ?, ?]; after removing the ionospheric topside and plasmasphere effects, the error is reduced to about 8%.

[Figure 7: see original paper] Schematic diagram of combined effects of ionospheric topside and plasmasphere influence, ionospheric temporal variation, and grid representation errors on assimilation process

Note: "STECXX" indicates slant total electron content observation (XX indicates number), "SatelliteXX" indicates GNSS navigation satellite, "ReceiverXX" indicates ground observation station.

For ionospheric topside and plasmasphere effects, occultation data can be used in the topside region to obtain more accurate bases for topside electron density decay functions [?]. Since LEO satellites in the COSMIC project have orbital altitudes of about 800 km, the corrected total electron content near satellite orbital altitude can serve as a constraint for ionospheric assimilation, achieving consistency and continuity between topside and bottomside assimilation models. When processing observation data, the topside and bottomside regions are selected for stepwise assimilation. First, an empirical model of the topside region (such as IRI Plas [?]) is used as the background field, and podTec data is provided to obtain the assimilation model for the topside region. Then, the topside contribution is subtracted from ground observation data, which together with the ionospheric empirical model below 800 km serves as the background field for the bottomside region to obtain a high-precision 3D electron density field for the bottomside region after assimilation [?]. The assimilation results show that after removing the ionospheric topside effect, the ionospheric bias during quiet periods improved from 1.645 TECu to 1.464 TECu; when the ionosphere is active, the standard deviation decreased from 4.408 TECu to 3.536 TECu.

Although the ionosphere changes significantly during assimilation time windows, it is generally assumed to be fixed within 1 or 2 hours in ionospheric inversion, leading to increased errors in ionospheric assimilation results due to temporal variation. Reducing the time window can improve assimilation accuracy and decrease the number of observations per assimilation, thereby reducing execution time for each assimilation algorithm [?]. However, reducing observations may affect their uniform distribution in the ionospheric grid. Therefore, methods to mitigate the impact of ionospheric variation within the assimilation window need to be found.

Ignoring the equatorial ionization anomaly, the ionospheric distribution has certain temporal translational properties with the movement of the subsolar point in geomagnetic coordinates. Therefore, by using the time difference between ray time and ionospheric background field time, corrected ray coordinates can be obtained through geomagnetic longitude translation using the formula:

$$\phi'_{mag} = \phi_{mag} + (MJD_{background} - MJD_{ray}) \times 360^{\circ}$$

where ϕ_{mag} and ϕ'_{mag} are the geomagnetic longitudes before and after correction, respectively, and MJD_{ray} and $MJD_{background}$ are the Julian dates corresponding to observation data and ionospheric background field time, respectively. Statistical analysis of relative accuracy shows that correcting slant TEC at different times through geomagnetic coordinate time correction produces stable positive effects every hour throughout the day, reducing the ionospheric temporal variation effect from 9.27% to 5.62% (about 0.3 TECu).

In ionospheric gridding, the classic ray tracing method forms the entire ray path through the distance the ray passes through each grid, then represents the electron content contribution of each segment by the intercept in each grid and the electron density at the corresponding grid center. However, when the piercing path is at a grid corner or extremely close to another grid, using the electron density at the current grid center to represent the average density along this path segment is inappropriate. The electron density at the midpoint p of the piercing intercept can more accurately represent the density along this segment. The density at point p can be represented by weighting the densities at surrounding grid centers, giving the following relationship:

$$NE_{segment} \approx NE_p \approx \sum_i w_i NE_{xi}$$

where $NE_{segment}$, NE_p, and NE_{xi} are the intercept average density, intercept midpoint density, and surrounding grid densities, respectively, and w_i is the weight for each density. After statistical analysis using bilinear coefficient weighting to represent grid intercept density, the original ionospheric grid representation error of about 2.5% is reduced to 0.9% using bilinear interpolation.

[Figure 8: see original paper] simulates the spatial distribution of GPS ground-based observation data and the accuracy improvement of ionospheric density distribution models obtained by assimilating GPS ground-based observation data before and after correcting for the three model assumption errors: ionospheric topside and plasmasphere effects, ionospheric temporal variation, and grid representation errors. [Figure 8a: see original paper] shows that ground-based GPS ionospheric observation data are mainly distributed over continents and some islands. [Figure 8b: see original paper] and [Figure 8c: see original paper] indicate that compared with the ionospheric density distribution model obtained by ignoring model assumption errors and assimilating ground-based ionospheric observation data, the assimilation model initial field shows obvious global accuracy improvement, with larger errors near the equator and oceanic regions. [Figure 8c: see original paper] and [Figure 8d: see original paper] demonstrate that after correcting ground-based GPS ionospheric observation data for model assumption errors, the errors in the ionospheric density distribution model obtained by assimilating ground-based ionospheric observation data in South America and near the equator are significantly reduced.

[Figure 8: see original paper] Daily average horizontal piercing information and vertical TEC mean error distribution

Note: a) GPS ground-based observation data horizontal piercing information; b) Initial field vertical TEC mean error; c) Vertical TEC mean error before assumption error correction; d) Vertical TEC mean error after assumption error correction.

4.2 Use of Multi-Source and Multi-Mode Data

Based on ground station ionospheric total electron content observations from global navigation satellite systems, ionospheric radio occultation observations from CHAMP (Challenging Minisatellite Payload), GRACE (Gravity Recovery and Climate Experiment), COSMIC, SAC-C (Satellite de Aplicaciones Científicas-B), Metop-A (Meteorological Operational Satellite-A), and TerraSAR-X satellites, and vertical total electron content measurements from Jason-1/Jason-2, a global 3D ionospheric data assimilation model for 2002–2011 was constructed in 2012 using the International Reference Ionosphere IRI2007 model and Kalman filter technology [**?**]. Meanwhile, COSMIC topside slant total electron content has also been used to establish topside vertical total electron content models or to build ionospheric topside and plasmasphere electron density models through assimilation and tomography [**?**, **?**].

The COSMIC operational lifespan is ending, but the COSMIC-2 constellation plan (with 6 satellites at 500 km altitude with 24° inclination and 6 satellites at 800 km altitude with 72° inclination) can serve as a better replacement. By designing and simulating occultation observation data and based on the empirical International Reference Ionosphere model and Kalman filter, a global ionospheric data assimilation model with altitude range of 80–3,000 km was established. Assimilation results verified that simulated COSMIC-2 occultation observation data supplements existing ground-based GNSS observation networks, while 24° and 72° inclination satellites can complement each other to optimize global ionospheric modeling [?].

The COSMIC-2 plan with GPS and GLONASS dual-mode observation was launched in 2019, but due to funding issues, it consists only of 6 LEO satellites with 24° inclination covering low and mid-latitudes, providing space services only for Taiwan. Based on the parameters in for GNSS ionospheric observation data simulation, [Figure 9: see original paper] shows the comparison of spatial distribution between single GPS constellation ground-based observation data and multi-mode ground-based and occultation observation data, as well as accuracy improvement from data assimilation. [Figure 9a: see original paper] and [Figure 9b: see original paper] show that multi-mode ground-based and occultation observation data have greater quantity and more comprehensive coverage than single GPS ground-based ionospheric observation data. [Figure 9c: see original paper] and [Figure 9d: see original paper] indicate that the ionospheric model obtained by assimilating single GPS constellation ground-based ionospheric observation data shows obvious global accuracy improvement compared with the assimilation model initial field, though oceanic regions still have larger errors due to relatively fewer observations. [Figure 9d: see original paper] and [Figure 9e: see original paper] demonstrate that compared with assimilation results from GPS ground-based observation data, multi-mode ground-based and occultation observation data assimilation results have smaller errors in oceanic regions, though oceanic regions still have relatively large errors and require more GNSS occultation observation data.

[Figure 9: see original paper] Daily average horizontal piercing information and vertical TEC mean error distribution

Note: a) GPS ground-based observation data horizontal piercing information; b) Multi-mode ground-based and occultation observation data horizontal piercing information; c) Initial field vertical TEC mean error; d) Vertical TEC mean error from GPS ground-based observation data assimilation results; e) Vertical TEC mean error from multi-mode ground-based and occultation observation data assimilation results.

When observation data increase, more time and memory are required for gridding to construct observation operators and solve equations in the assimilation process. Many works use supercomputers with the Community Earth System Model (CESM) or DART (Data Assimilation Research Testbed) to perform assimilation analysis on ionospheric models using structured grids or the MPAS (Model for Prediction Across Scales) unstructured grid ionospheric model [?, ?]. Many researchers reduce the extreme memory and computational requirements in ionospheric assimilation by using sparse matrices and sparse equation solving methods [?, ?, ?]. To obtain main variation patterns or anomalous distribution characteristics of 3D ionosphere, constructing ionospheric assimilation models based on non-uniform ionospheric grid division can also reduce computational demands while ensuring accuracy in regions of interest [?, ?].

5 Ionospheric Applications and Prospects

Through the fusion of theoretical/empirical models with multi-source ionospheric observation data, obtaining fine 3D ionospheric structures and variations has numerous applications and research aspects. Studies on the coupling of troposphere, thermosphere, plasmasphere, and ionosphere \cite{59, 64–70} can reveal the physical processes of ionospheric storms \cite{71–75}, the role of lightning during thunderstorms in the ionosphere and stratosphere [?], the relationship between ENSO (El Niño–Southern Oscillation) [?] and TEC, and monitoring ionospheric coseismic effects [?].

Ionospheric research remains a diverse and thriving field. From the deep coupling of physical and observation data in meteorological data assimilation to the numerical simulation convergence of high-dynamic fluids through fluid dynamics finite element analysis methods, these will provide significant guidance for ionospheric data assimilation. On the other hand, the introduction of new algorithms and computing power enables us to obtain more refined spatiotemporal scale characteristics of the ionosphere.

5.1 Use of Adaptive Grids in Ionospheric Assimilation

Ionospheric tomography or assimilation requires gridding of ionospheric space. According to the vertical distribution pattern of electron density in the ionosphere, finer division can be used in regions with large electron density height gradients, while sparser division can be used in regions with small gradients. This division method has been applied to the new version of IRI Plas [?] and Wu et al.'s work on topside ionosphere and plasmasphere assimilation [?] to obtain details of electron density vertical distribution. MPAS is a collaborative project aimed at developing atmospheric, oceanic, and other Earth system simulation components for climate, regional climate, and weather research. It allows quasi-uniform discretization and local refinement of the sphere through unstructured Voronoi grids and C-grid discretization, and can predict normal velocity components at grid edges, making it particularly suitable for high-resolution, mesoscale atmospheric and oceanic simulations [?, ?].

Both methods struggle to account for the impact of varying ionospheric peak height on ionospheric modeling. If adaptive mesh algorithms from finite element methods can be used, ionospheric grids can be planned according to the spatial distribution structure of electron density, enabling optimization of spatial resolution and computation time for ionospheric assimilation. By controlling the details of relatively large and rapidly changing parts of the ionosphere through details and correlations of electron density fields at different times, 3D correlation and spatial overall continuity of assimilation results can be achieved.

5.2 Use of Machine Learning in Ionospheric Assimilation

Based on various ionospheric detection methods, massive multi-source ionospheric data have been collected through years of observation. Deeply mining information from ionospheric data and flexibly applying it to ionospheric modeling and prediction will deepen understanding of the ionosphere and provide substantial reference information for ionospheric anomaly forecasting and hazard control. Preliminary studies have been conducted or initiated on using long-term slant total electron content, GIM, and ionPrf data for ionospheric electron density, anomaly classification, parameter prediction, or GIM distribution prediction through artificial intelligence algorithms \cite{81–84}.

Of course, artificial intelligence algorithms are not only used for classifying, fitting, or predicting observation data or products (such as GIM and electron density), but also offer many advantages when using deep learning frameworks in 3D ionospheric model construction algorithms: in ionospheric assimilation algorithms, through existing integration of deep learning frameworks (such as TensorFlow [**?**] and GraphBLAS [**?**]) with OpenCL (Open Computing Language) [**?**] or CUDA (Compute Unified Device Architecture) [**?**], algorithm computation speed can be greatly improved. By constructing cost functions through deep learning frameworks and using various optimization strategies, ionospheric assimilation algorithms can be greatly improved. Parallel algorithms and sparse matrices can greatly save physical memory and improve computational efficiency for ionospheric data simulation and inversion.

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References

[1] Bychkov V L, Golubkov G V, Nikitin A I. *The Atmosphere and Ionosphere: Dynamics, Processes and Monitoring*. Netherlands: Springer, 2010: 387
[2] Ding M, Qiao S, Zhang B, et al. *Progress in Geophysics*, 2017, 32(2): 460
[3] Nava B, Coïsson P, Radicella S M. *Journal of Atmospheric and Solar-Terrestrial Physics*, 2008, 70(15):
[4] Radicella S M. *Annals of Geophysics*, 2009, 52(3-4): 417
[5] Bilitza D. *Radio Science*, 2001, 36(2): 261
[6] Bilitza D, Altadill D, Zhang Y, et al. *Journal of Space Weather and Space Climate*, 2014, 4: 7
[7] Bilitza D, Altadill D, Truhlik V, et al. *Space Weather*, 2017, 15(2): 418
[8] Bilitza D, McKinnell L A, Reinisch B, et al. 2011, 85(12): 909
[9] Arikan F, Sezen U, Gulyaeva T L. *Journal of Geophysical Research: Space Physics*, 2019, 124(10): 8092
[10] Gallagher D L, Craven P D, Comfort R H. *Journal of Geophysical Research: Space Physics*, 2000, 105(A8):
[11] Alçay S, Oztan G, Selvi H Z. *Annals of Geophysics*, 2017, 60(5): 7311
[12] Sezen U, Cilibas O, Arikan F, et al. *Advances in Space Research*, 2013, 55(8): 2106
[13] Gulyaeva T, Huang X, Reinisch B. *Advances in Space Research*, 2002, 29(6): 825

[14] Gulyaeva T, Arikan F, Stanislawska I. *Earth, Planets and Space*, 2011, 63(8): 929
[15] Gulyaeva T L, Huang X, Reinisch B W. *Advances in Space Research*, 2002, 29(6): 825
[16] Maltseva O A, Zhbankov G A, Mozhaeva N S. *Advances in Radio Science*, 2013, 11: 307
[17] Raymund T, Austen J, Franke S, et al. *Radio Science*, 1990, 25(5): 771
[18] Hong J, Kim Y, Chung J K, et al. *Journal of Astronomy and Space Sciences*, 2017, 34(1): 7
[19] Wu M J, Guo P, Xu T L, et al. *Radio Science*, 2015, 50(11): 1131
[20] Yue X, Schreiner W S, Kuo Y H, et al. *IEEE Transactions on Geoscience and Remote Sensing*, 2014, 52(7):
[21] Yue X, Schreiner W S, Pedatella N, et al. *Space Weather*, 2014, 12(11): 616
[22] Yue X, Wan W, Liu L, et al. *Radio Science*, 2007, 42(6): 1
[23] Yue X, Schreiner W S, Kuo Y H, et al. *Journal of Geophysical Research: Space Physics*, 2012, 117(A6):
[24] Schunk R W, Scherliess L, Sojka J J, et al. *Radio Science*, 2004, 39(1): 2794
[25] Lorenc. *J. Met Soc Japan*, 1997, 75: 339
[26] Huang. *Mon. Wea. Rev.*, 2000, 128: 2588
[27] Schunk R W, Scherliess L, Sojka J J. *Advances in Space Research*, 2003, 31(4): 819
[28] Thompson D, Scherliess L, Sojka J, et al. *Journal of Atmospheric and Solar-Terrestrial Physics*, 2006, 68(9): 947
[29] Courtier P, Thepaut J N, et al. *Quart. J. Roy. Meteor. Soc.*, 1994, 120: 1367
[30] Wang C, Hajj G, Pi X, et al. *Radio Sci.*, 2004, 39(1): 6
[31] Fu N, Guo P, Wu M, et al. *Remote Sensing*, 2019, 11(10): 1172
[32] Zhang X, Ma F. *Acta Geodaetica et Cartographica Sinica*, 2019, 48(9): 1073
[33] Reid T, Neish A, Walter T, et al. *Leveraging Commercial Broadband LEO Constellations for Navigating*. https://www.ion.org/publications/abstract.cfm?articleID=14729
[34] Huba J D, Sazykin S. *Geophys. Res. Lett.*, 2014, 41: 8208
[35] Huba J D, Krall J, Drob D. *Geophysical Research Letters*, 2019, 46(14): 7937
[36] Cai Y, Yue X, Wang W, et al. *Journal of Geophysical Research: Space Physics*, 2019, 124(12): 10708
[37] Qian L, Burns A G, Emery B A, et al. *Modeling the Ionosphere-Thermosphere System*, 2013, 201: 73
[38] Jin H, Miyoshi Y, Fujiwara H, et al. *Journal of Geophysical Research: Space Physics*, 2011, 116(1): 1
[39] Liu J, Liu H, Wang W, et al. *Journal of Geophysical Research: Space Physics*, 2018, 123(2): 1534
[40] Verronen P T, Andersson M E, Marsh D R, et al. *Journal of Advances in Modeling Earth Systems*, 2016, 8(2): 954
[41] Yamamoto M, Otsuka Y, Jin H, et al. *Progress in Earth and Planetary*

Science, 2018, 5(1): 26
[42] Wu M J, Guo P, Fu N F, et al. *Journal of Geophysical Research: Space Physics*, 2019, 124(7): 5995
[43] Wu M J, Guo P, Fu N F, et al. *Journal of Geophysical Research: Space Physics*, 2018, 123(11): 9815
[44] Wu M J, Guo P, Fu N F, et al. *Journal of Geophysical Research: Space Physics*, 2016, 121(6): 5675
[45] Wu M, Guo P, Chen Y, et al. *Journal of Geophysical Research: Space Physics*, 2020, 125(3): 637
[46] Altadill D, Magdaleno S, Torta J M, et al. *Advances in Space Research*, 2013, 52(10): 1756
[47] Shubin V N. *Advances in Space Research*, 2015, 56(5): 916
[48] Prol F D S, Hernández-Pajares M, Camargo P D O, et al. *Journal of Geophysical Research: Space Physics*, 2018, 123(3): 2104
[49] Wang S, Huang S, Fang H. *Radio Science*, 2015, 2018, 50(9): 866
[50] dos Santos Prol F, Themens D R, Hernández-Pajares M, et al. *Surveys in Geophysics*, 2019, 40(2): 277
[51] Hu A, Carter B, Currie J, et al. *Journal of Geophysical Research: Space Physics*, 2019, 124(6): 4926
[52] Li Q, Liu L, Jiang J, et al. *Journal of Geophysical Research: Space Physics*, 2019, 124(3): 2083
[53] Stankov S M, Jakowski N. *Journal of Atmospheric and Solar-Terrestrial Physics*, 2006, 68(2): 134
[54] Bilitza D, Reinisch B. *Advances in Space Research*, 2008, 42(4): 599
[55] Rishbeth H, Eds. O K G. *Introduction to Ionospheric Physics.* Amsterdam: Elsevier, 1969: 47
[56] Lorenc A C. *Quarterly Journal of the Royal Meteorological Society*, 1986, 474(112): 1177
[57] Balan N, Otsuka Y, et al. *Earth Planets Space*, 2002, 54(1): 71
[58] Chen C H, Lin C H, Matsuo T, et al. *Journal of Geophysical Research: Space Physics*, 2015, 121: 5708
[59] Pinto Jayawardena T, Chartier A, Spencer P, et al. *Journal of Geophysical Research A: Space Physics*, 2016, 121(1): 817
[60] Liu H L. *Modeling the ionosphere and thermosphere system*, Geophysical Monograph Series (chap. 16, pp. 181–200). Washington, DC: American Geophysical Union, 2014: 59
[61] Yue X, Schreiner W S, Kuo Y H, et al. *IEEE Transactions on Geoscience and Remote Sensing*, 2014, 52(7):
[62] He J, Yue X, Le H, et al. *Space Weather*, 2020, 18(3): 410
[63] Wu M, Guo P, Xu T, et al. *Radio Science*, 2015, 50(11): 1131
[64] Crowley G, Hackert C, Meier R, et al. *Journal of Geophysical Research: Space Physics*, 2006, 111(10):
[65] Cummer S A, Li J, Han F, et al. *Nature Geoscience*, 2009, 2: 607
[66] Oliveira D. *Revista Brasileira de Ensino de Fisica*, 2014, 36(1): 1
[67] Zhang R, Liu L, Liu H, et al. *Geophysical Research Letters*, 2020, 47(6): 78
[68] Dhadly M S, Emmert J T, Drob D P, et al. *Journal of Geophysical*

Research: Space Physics, 2019, 124(12):
[69] Ridley A, Deng Y, Tóth G. *Journal of Atmospheric and Solar-Terrestrial Physics*, 2006, 68(8): 839
[70] Sun Y Y. *Geoscience Letters*, 2019, 6(1): 14
[71] Meng X, Mannucci A J, Verkhoglyadova O P, et al. *Space Weather*, 2020, 18(2): 352
[72] Astafyeva E, Zakharenkova I, Förster M. *Journal of Geophysical Research A: Space Physics*, 2015, 120(10):
[73] Ngwira C M, Habarulema J B, Astafyeva E, et al. *Journal of Geophysical Research: Space Physics*, 2019, 124(8): 7123
[74] Ghodpage R, Patil P, Gurav O, et al. *Advances in Space Research*, 2018, 62(3): 624
[75] Zolotukhina N, Polekh N, Kurkin V, et al. *Journal of Geophysical Research: Space Physics*, 2017, 122(2):
[76] Shultz D. *Charging Thunderclouds Affect Ionospheric Conductivity.* https://eos.org/research-spotlights/charging-thunderclouds-affect-ionospheric-conductivity
[77] Sun Y Y, Liu H, Miyoshi Y, et al. *Earth, Planets and Space*, 2019, 71(1): 35
[78] Goto S I, Uchida R, Igarashi K, et al. *Journal of Geophysical Research: Space Physics*, 2019, 124(11):
[79] Judt F. *Journal of the Atmospheric Sciences*, 2020, 77(1): 257
[80] Michaelis A C, Lackmann G M, Robinson W A. *Geoscientific Model Development*, 2019, 12(8): 3725
[81] El-Naggar A M. *Alexandria Engineering Journal*, 2013, 52(3): 425
[82] Gross N C, Cohen M B. *Journal of Geophysical Research: Space Physics*, 2020, 125(1): 135
[83] Kim M, Kim J. *International Journal of Aeronautical and Space Sciences*, 2016, 17(1): 64
[84] Orus Perez R. *Advances in Space Research*, 2019, 63(5): 1607
[85] Martín A, Ashish A, Paul B, et al. *TensorFlow: Large-Scale Machine Learning on Heterogeneous Systems.* http://tensorflow.org/. Software available from tensorflow.org, 2020
[86] Davis T A, Aznaveh M, Kolodziej S. 2019 IEEE High Performance Extreme Computing Conference (H-PEC), USA: IEEE, 2019: 1
[87] The Khronos Group. *OpenCL 3.0 (Provisional) Reference Guide.* https://www.khronos.org/opencl/, 2020
[88] The NVIDIA. *CUDA Toolkit—NVIDIA.* https://developer.nvidia.com/cuda-toolkit, 2020

Note: Figure translations are in progress. See original paper for figures.