

## A new digital Gaussian pulse shaping algorithm based on bilinear transformation (Postprint)

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### Abstract

Nuclear pulse signals need to be transformed into an appropriate pulse shape to eliminate noise and improve the energy resolution of nuclear spectrometry systems. In this paper, a novel digital Gaussian shaping method is proposed. Based on Sallen-Key analog Gaussian shaping filter circuits, the system function of the Sallen-Key analog Gaussian shaping filter is derived using Kirchhoff's laws. The system function of the digital Gaussian shaping filter based on bilinear transformation is also derived. The expression for the unit impulse response of the digital Gaussian shaping filter is obtained through inverse z-transformation. The response of the digital Gaussian shaping filter is derived from the convolution sum of the unit impulse response and the digital nuclear pulse signal. Simulation and experimental results demonstrate that the digital nuclear pulse has been transformed into a pseudo-Gaussian pulse, thereby confirming the feasibility of the novel digital Gaussian pulse shaping algorithm based on bilinear transformation.

### Full Text

## A New Digital Gaussian Pulse Shaping Algorithm Based on Bilinear Transformation

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**Abstract:** Nuclear pulse signals must be transformed into an appropriate shape to remove noise and improve the energy resolution of nuclear spectrometry systems. This paper proposes a novel digital Gaussian shaping method based on Sallen-Key analog Gaussian shaping filter circuits. The system function of the Sallen-Key analog Gaussian shaping filter is derived from Kirchhoff's laws, and the system function of the digital Gaussian shaping filter is obtained through bilinear transformation. The unit impulse response expression of the digital Gaussian shaping filter is derived via inverse z-transform, and the filter response is computed through the convolution sum of the unit impulse response and the digital nuclear pulse signal. Simulation and experimental results demonstrate that the digital nuclear pulse is successfully transformed into a pseudo-Gaussian shape, confirming the feasibility of the proposed digital Gaussian pulse shaping algorithm based on bilinear transformation.

**Keywords:** Digital shaping, Bilinear transformation, Convolution sum  
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## Introduction

In nuclear spectrometry systems, the peak amplitude of pulses from the detector is proportional to the energy of incident particles, enabling particle energy determination through pulse height measurement. A major source of inaccuracy in energy measurement is electronic noise and ballistic deficit [1]. Shaping techniques are employed to improve the signal-to-noise ratio and accurately extract pulse height information [2–12]. Gaussian pulse shaping, as a general shaping method in nuclear spectrometry, performs effectively in this regard [12, 13].

With the rapid development of digital integrated circuits and their inherent advantages in flexibility and stability, research on digital Gaussian shaping algorithms for nuclear pulse signals has become increasingly popular over the past decades [12–21]. Valentin T. Jordanov investigated digital Gaussian shaping algorithms for exponential signals [15]. Chen et al. developed digital Gaussian shaping methods based on wavelet analysis [13, 14]. Zhou et al. advanced digital Gaussian shaping algorithms based on numerical differentiation [19]. In these approaches, digital nuclear pulse signals are treated as quasi-Gaussian signals, and recursive algorithms are used to achieve digital Gaussian shaping.

This paper proposes a digital Gaussian shaping algorithm based on bilinear transformation. The unit impulse response of the digital Gaussian shaping system is derived, and Gaussian shaping is realized by computing the convolution sum of the unit impulse response and the digital nuclear pulse signal.

## II. System Function of the Analog Gaussian Shaping System

Analog Gaussian shaping in nuclear spectrometry systems is often implemented using a Sallen-Key filter, as shown in Fig. 1 [Figure 1: see original paper], which

can shape a nuclear pulse signal into a pseudo-Gaussian waveform. According to Kirchhoff's laws, we obtain the following equation:

$$[f(t) - y_1(t)]/R = C \frac{dy_2(t)}{dt} + C \frac{d(y_1(t) - y(t))}{dt}$$

with the relationships:

$$\begin{aligned} y_2(t) &= R \\ y_1(t) &= y_2(t) + RC \frac{dy_2(t)}{dt} \end{aligned}$$

Thus, the relationship between the input signal  $f(t)$  and output signal  $y(t)$  can be expressed as:

$$R^2C^2y''(t) + RCy'(t) + y(t) = 2f(t)$$

Let the Laplace transforms of  $f(t)$  and  $y(t)$  be  $F(s)$  and  $Y(s)$ , respectively. Using the differential properties of Laplace transform, the transforms of  $y'(t)$  and  $y''(t)$  become  $sY(s)$  and  $s^2Y(s)$ , yielding:

$$R^2C^2s^2Y(s) + RCsY(s) + Y(s) = 2F(s)$$

Therefore, the system function of the analog Gaussian shaping system is:

$$H(s) = \frac{Y(s)}{F(s)} = \frac{2}{R^2C^2s^2 + RCs + 1}$$

### III. System Function of the Digital Gaussian Shaping System

In the process of transforming the Gaussian shaping system from analog to digital domain, bilinear transformation is employed for conversion to maintain the basic properties of the system function unchanged while overcoming spectrum aliasing. The principle of bilinear transformation is a two-step mapping method, as illustrated in Fig. 2 [Figure 2: see original paper].

The mapping process includes three steps:

1. The  $s$ -plane is mapped onto a strip-like region from  $-\pi/T$  to  $\pi/T$  of the  $s_1$ -plane through the conversion:

$$s = \frac{2 \tan(s_1 T/2)}{T} = \frac{1 - e^{-s_1 T}}{1 + e^{-s_1 T}} \left[ \frac{2}{T} \right]$$

2. The  $s_1$ -plane is mapped onto the  $z$ -plane with the conversion relationship:

$$z = e^{s_1 T}$$

3. Combining steps (1) and (2), the direct transform equation between the  $s$ -domain and  $z$ -domain is obtained:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \left[ \frac{2}{T} \right]$$

This establishes a one-to-one mapping relationship from the  $s$ -plane to the  $z$ -plane, thereby eliminating spectrum aliasing. The system function of the digital Gaussian shaping system can be obtained from Eqs. (6) and (9) by letting  $M = RC/T$ :

$$H(z) = \frac{2 + 4z^{-1} + 2z^{-2}}{4M^2 + 2M + 1 + (2 - 8M^2)z^{-1} + (4M^2 - 2M + 1)z^{-2}}$$

Let  $a = 4M^2 + 2M + 1$ ,  $b = 2 - 8M^2$ , and  $c = 4M^2 - 2M + 1$ . Performing inverse  $z$ -transform on Eq. (10) yields the unit impulse response of the digital Gaussian shaping system:

$$h(n) = \frac{2k_1}{a} \left\{ \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right\}^n u(n) + \frac{2k_2}{a} \left\{ \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right\}^n u(n) + \frac{2k_3}{a} \left\{ \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right\}^{n-1} u(n-1) +$$

where:

$$k_1 = \frac{b - 4a}{2\sqrt{b^2 - 4ac}} + \frac{1}{2}$$

$$k_2 = \frac{-b + 4a}{2\sqrt{b^2 - 4ac}} + \frac{1}{2}$$

$$k_3 = -\frac{a}{\sqrt{b^2 - 4ac}}$$

$$k_4 = \frac{a}{\sqrt{b^2 - 4ac}}$$

Substituting  $z = e^{j\omega}$  into Eq. (10) gives the discrete-time Fourier transform of  $h(n)$ :

$$H(e^{j\omega}) = \frac{2 + 4e^{-j\omega} + 2e^{-2j\omega}}{a + be^{-j\omega} + ce^{-2j\omega}}$$

The amplitude spectrum of  $H(e^{j\omega})$  is shown in Fig. 3 [Figure 3: see original paper]. The spectral width of the digital Gaussian shaping system increases as  $M$  decreases, which reduces high-frequency noise filtration.

## IV. Gaussian Shaping of the Nuclear Pulse Signal

### A. Mathematical Description of the Nuclear Pulse Signal

The structure of a nuclear spectrometer system with digital shaping is shown in Fig. 4 [Figure 4: see original paper]. If the charge generated by an impinging particle is instantaneously collected by the detector, the output signal from the preamplifier can be approximated by a continuous-time exponential signal:

$$f(t) = B(e^{-t/\tau_1} - B_1 e^{-t/\tau_2})u(t)$$

where  $B$  is the signal magnitude,  $u(t)$  is the unit step signal,  $\tau_1$  and  $\tau_2$  are time constants of the exponential signal, and  $B_1$  is a proportionality coefficient. To simplify calculations, let  $B_1 = 1$ . If the ADC sampling period is  $T$ , the sampled discrete-time signal becomes:

$$f(n) = B(e^{-nT/\tau_1} - e^{-nT/\tau_2})u(n)$$

### B. Digital Gaussian Shaping of the Simulated Double Exponential Signal

The response of the input signal  $f(n)$  after passing through the digital Gaussian system is given by the convolution sum:

$$y(n) = f(n) * h(n) = \sum_{m=0}^{\infty} f(m)h(n-m)$$

where  $h(n)$  is the unit impulse response of the system and  $y(n)$  is the zero-state response.

Using a double exponential Gaussian signal as input to the digital Gaussian shaping system with sampling period  $T = 0.01 \mu\text{s}$ ,  $B = 1.6$ , and  $\tau_1 = 0.23 \mu\text{s}$ , Fig. 5 [Figure 5: see original paper] shows the response  $y(n)$  calculated and plotted using MATLAB for  $\tau_2 = 0.4$  and  $1.8 \mu\text{s}$  with different  $M$  values. The shaping waveform width increases with  $M$ , and symmetry improves with larger  $M$ . A key advantage of the digital Gaussian shaping system is that only the  $M$  parameter value needs to be changed rather than adjusting hardware components.

### C. Digital Gaussian Shaping of the Real Sampled Nuclear Pulse Signal

Signals from a  $^{60}\text{Co}$  source were measured using a NaI(Tl) detector and an Agilent DSO-X 2012A oscilloscope. After storing the digital nuclear pulse signals sampled at 200 MHz, the response  $y(n)$  of the digital Gaussian shaping system was calculated using Eq. (15) in MATLAB. Fig. 6 [Figure 6: see original paper] shows the measured pulse signal and its corresponding digitally shaped waveform at  $M = 80$  and  $M = 160$ . The shaped waveform at  $M = 160$  is wider and exhibits better symmetry compared to  $M = 80$ , producing a more desirable Gaussian shape. Decreasing the  $M$  value narrows the shaping waveform in the time domain, but a low shaping time reduces noise filtration and results in poor energy resolution. At  $M = 0.5$  (Fig. 7 [Figure 7: see original paper]), significant noise becomes visible in the shaping waveform.

### V. Discussion and Conclusion

Pulse shaping digitization is the key to nuclear instrument digitization. After deriving the unit impulse response of the digital Gaussian shaping system through bilinear transformation, the convolution sum between the digital nuclear pulse signal and the derived system's unit impulse response can be performed. The digital nuclear pulse signal is shaped into a pseudo-Gaussian form while noise is removed.

A higher  $M$  value yields better symmetry of the shaped waveform, a wider pulse, and improved noise filtering, resulting in a flat-top pulse with small ballistic deficit. However, a wider pulse increases pulse pile-up, which represents a major drawback in high-count-rate nuclear spectrometry systems. Narrowing the pulse width can reduce pile-up distortion, but low shaping time degrades noise filtration and leads to poor energy resolution. In practical applications, the shaping parameter values can be conveniently adjusted to meet different shaping requirements without hardware modifications.

This study provides a new method for achieving digital Gaussian shaping of nuclear pulse signals.

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