An investigation of the role of spectroscopic factors in the breakup reaction of11Be postprint

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Abstract

The experimental elastic cross section data of the projectile 11Be on target 12C at 49.3 MeV/nucleon energy is analysed. The calculations for the elastic scattering are performed by the phenomenological optical model. The different optical potentials to include breakup effects into the calculations, which are neutron+12C, neutron+10Be and 10Be+12C, are described with the aid of the global potentials for neutron interactions and fitted to experimental data for the core and target interaction. Also, the first analysis of the optical model for 10Be on target 12C at 39.1 MeV is done for building the interaction potential of the core and the target for 11Be. For investigating the effects of the spectroscopic factors, obtained factors from the direct capture process using the nuclear level density are compared with the previous cross section and spectroscopic factor results. Obtained results for the elastic cross section reproduce the experimental data very well and show the requirement of including spectroscopic properties such as, spectroscopic factors and density of the excited states, to explain this elastic cross section data.

Full Text

Preamble

An Investigation of the Role of Spectroscopic Factors in the Breakup Reaction of ^{11}Be

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The experimental elastic cross-section data for the projectile ^{11}Be on a ^{12}C target at 49.3 MeV/nucleon are analyzed. Calculations for elastic scattering are performed using the phenomenological optical model. Different optical potentials are employed to incorporate breakup effects into the calculations, specifically for neutron+^{12}C, neutron+^{10}Be, and ^{10}Be+^{12}C systems. These potentials are described using global parametrizations for neutron interactions and fitted to experimental data for the core-target interaction. Additionally, the first optical model analysis for ^{10}Be on a ^{12}C target at 39.1 MeV is presented in this study to construct the interaction potential between the core and target for ^{11}Be. To investigate the effects of spectroscopic factors, values obtained from the direct capture process using the nuclear level density are compared with previous cross-section and spectroscopic factor results. The obtained results for the elastic cross-section reproduce the experimental data very well and demonstrate the necessity of including spectroscopic properties such as spectroscopic factors and the density of excited states to explain this elastic cross-section data.

Keywords: Spectroscopic factors, Nuclear level density, Breakup reaction, Optical model

Introduction

Experiments with radioactive ion beams (RIBs) have opened a new era in nuclear reaction physics over the last decades [1–3]. These experiments, aimed at probing and understanding nuclear structure, have revealed unexpected properties of light exotic nuclei. One of the most intriguing attributes is the halo structure [1], consisting of a core and weakly-bound valence nucleon(s). To date, this phenomenon has been extensively investigated experimentally on various targets [4–6] and has challenged nuclear reaction theorists to reproduce the experimental data [4, 7, 8]. ^{11}Be is one of the four known one-neutron halo nuclei, together with ^{19}C [9], and the recently reported ^{31}Ne [10] and ^{37}Mg [11].

Several experiments have been conducted to understand the structure of ^{11}Be. Tanihata et al. [3] first observed a large radius for ^{11}Be compared to ^{10}Be in cross-section measurements with targets at 790A MeV, establishing the halo structure of ^{11}Be originating from its small neutron separation energy of 0.503 MeV. A few years later, Fukuda et al. [12] confirmed this conclusion through elastic scattering of ^{11}Be on C and Al targets at 33 MeV/nucleon. Since these pioneering works, ^{11}Be has been continuously studied both experimentally [6, 13–16] and theoretically [17–19].

One experimental study of ^{11}Be was performed by Cortina-Gil et al. [6], measuring the elastic scattering cross-section on ^{12}C at 49.3 MeV/nucleon incident energy. The first theoretical investigation of this measurement used an adiabatic approximation, assuming no internal motion between the valence nucleon and the core in the projectile while neglecting interactions between the valence nucleon and the target nucleus [8]. In the same year, Al-Khalili et al. [20] investigated the same reaction using the few-body Glauber model, in which the projectile particles are considered to follow straight-line paths through the target's interaction field. Additionally, the continuum-discretized coupled-channels (CDCC) method was applied to this elastic scattering by Takashina et al. [21], who used the same parameter set for the optical potentials between projectile components and target as in Ref. [20]. In this non-adiabatic method, due to the very low neutron separation energy, the continuum states of the projectile above threshold are discretized into a finite number of states using momentum bins.

Including breakup effects in the theoretical calculations of these methods yields almost identical results. In the present study, the elastic scattering of the halo nucleus ^{11}Be on a ^{12}C target at 49.3 MeV/nucleon [6] is investigated as a breakup reaction using the optical model with the aid of a nuclear structure model. Unlike previous studies, the optical model potential for the interaction between the core nucleus ^{10}Be and the target ^{12}C is obtained by fitting to elastic cross-section data at 39.1 MeV/nucleon. This data [13] is analyzed with the optical model for the first time in this study. For the interaction between the valence neutron of the halo nucleus and the target, the optical potential is deduced from an interpolation of different incident energies for neutron on ^{12}C using the global potential of Ref. [22]. To describe non-elastic contributions, we employ a surface potential, referred to as DPP (dynamical polarization potential) or VCP (virtual coupling potential), in our calculations. Finally, a binding potential is used for the n+^{10}Be system. Unlike similar studies, we determine the spectroscopic factor, which describes the wave function of ^{11}Be in terms of the wave function of ^{10}Be, using the method given in Ref. [23] for the direct neutron capture reaction ^{10}Be+n→^{11}Be+γ. However, we employ a new nuclear level density (NLD) model [24] that strongly depends on nuclear deformation.

This paper is organized as follows: The methodology is presented in Section II, results for the breakup reaction of ^{11}Be are given in Section III, and concluding remarks are provided in Section IV.

Theory

Since the mid-fifties, the optical model has been widely used to investigate elastic scattering cross-sections for both light and heavy ions across a wide range of incident energies. The optical model treats the projectile and target nuclei as structureless particles to avoid many-body problems in nuclear physics calculations and describes their interaction using an effective potential. In this work, since we include breakup effects, the halo projectile ^{11}Be is considered as a two-body system consisting of a ^{10}Be core and a valence neutron. Therefore, we define effective potentials between projectile components and the ^{12}C target, specifically n+^{12}C, ^{10}Be+^{12}C, and n+^{10}Be, as used in Ref. [25]:

$$U_{\text{eff}} = U_{CT} + U_{VT} + U_{CV},$$

where C, T, V correspond to the ^{10}Be core, the ^{12}C target, and the valence nucleon, respectively. An effective potential combines the following terms:

$$U(r) = V_l(r) + V_C(r) + V_{\text{Vol}}(r) + V_{\text{Sur}}(r) + V_{SO}(r). \tag{2}$$

The first term is the centrifugal potential, traditionally defined as $V_l(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2}$. The uniformly charged sphere assumption is employed for the Coulomb potential:

$$V_C(r) = \begin{cases} \frac{Z_P Z_T e^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right) & r \leq R_C \\ \frac{Z_P Z_T e^2}{r} & r \geq R_C \end{cases}$$

where the charge radius is defined as $R_C = r_C(A_P^{1/3} + A_T^{1/3})$, and the Coulomb potential parameter r_C is taken as 1.2 fm in this work. In the optical model, the volume term in an effective potential plays a crucial role and is described by its real part. However, for inelastic contributions, an imaginary part is added to the volume term to account for absorption of reaction flux from the elastic channel to inelastic channels. Conventionally, the volume term therefore consists of real and imaginary parts:

$$V_{\text{Vol}}(r) = \frac{V_0}{1 + \exp\left(\frac{r - R_v}{a_v}\right)} + i\frac{W_0}{1 + \exp\left(\frac{r - R_w}{a_w}\right)},$$

where potential depths, radii, and surface diffuseness parameters for both real and imaginary parts should be adjusted to fit elastic scattering data. Even when investigating elastic scattering, non-elastic contributions can still exist in the elastic channel. To include these contributions, a surface potential is used:

$$V_{\text{Sur}}(r) = -4V_0\frac{\exp\left(\frac{r - R_v}{a_v}\right)}{\left[1 + \exp\left(\frac{r - R_v}{a_v}\right)\right]^2} - 4iW_0\frac{\exp\left(\frac{r - R_w}{a_w}\right)}{\left[1 + \exp\left(\frac{r - R_w}{a_w}\right)\right]^2},$$

which is the derivative form of the volume term. The final term in Eq. (2) is the spin-orbit (SO) potential:

$$V_{SO}(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr}\left[\frac{V_{SO}}{1 + \exp\left(\frac{r - R_{SO}}{a_{SO}}\right)}\right]\mathbf{L}\cdot\mathbf{s},$$

where $(\hbar/m_\pi c)^2 = 2$ fm^2.

The optical potential parameters in these equations can be determined from elastic scattering data. As a first step in fitting potential parameters, the geometrical parameters are adjusted to match peak positions in the data. Afterwards, the potential depths of all optical model potentials are fitted to experimental data to minimize the ^{2} value.

For the halo nucleus ^{11}Be, the spectroscopic factor as a structural property is used to describe the ground state and first excited state of ^{11}Be in terms of ^{10}Be. The spectroscopic factor can be determined by fitting to experimental cross-section data from transfer or direct capture processes, and can also be obtained theoretically from shell model calculations. In the literature, many transfer processes include spectroscopic factor values for ^{11}Be from reactions such as ^{9}Be(t,p)^{11}Be [26-29], ^{10}Be(d,p)^{11}Be [30-33], and ^{11}Be(p,d)^{10}Be [34]. However, experimental data for the direct capture cross-section of ^{10}Be(n,γ)^{11}Be is not available, though direct capture cross-sections can be deduced from Coulomb dissociation [35]. The nuclear level density plays a crucial role in calculations of light-ion cross-sections, such as direct capture processes, for reproducing measured data and defining the spectroscopic factor. The relationship between direct capture cross-section and nuclear level density, which represents the number of excited levels around a given excitation energy, can be defined as [23]:

$$\sigma_{DC}(E) = \langle S \rangle \sum_{J_f,\Pi_f} \int_{S_n} \rho(E_f, J_f, \Pi_f)\sigma_{\mathrm{cont}}(E)dE_f,$$

where S represents the average spectroscopic factor and ρ is the level density function in terms of excitation energy E_f, total angular momentum J_f, and parity Π_f of the compound nucleus. In the present work, we calculate the direct neutron capture cross-section and compare it to deduced data [35] from Coulomb dissociation of ^{11}Be measured by Nakamura et al. [36]. For this calculation, a Laplace-like formula [24] is used for the energy dependence of the nuclear level density parameter in the Fermi gas model. According to this formula, the level density parameter strongly depends on nuclear deformation, and results obtained with this formula successfully describe low-lying collective levels compared to other phenomenological level density models [37]. Therefore, considering that ^{10}Be and ^{11}Be are well-deformed nuclei, we expect this formula to be suitable for explaining the neutron capture cross-section data of ^{10}Be.

Results and Discussion

To describe interactions between the projectile and target, we consider the weakly-bound nucleus ^{11}Be as ^{10}Be+n. First, we focus on the interaction between the neutron and the target. A substantial amount of experimental data exists in the 0–100 MeV energy range [38–41] for elastic scattering of neutrons on ^{12}C, which can be used to define the effective potential between the valence nucleon and target. Unfortunately, no experimental data are available for 49.3 MeV incident energy. Therefore, an interpolation of the global parametrization [22] is used. The results obtained with this global potential are shown in Fig. 1 [Figure 1: see original paper]. As seen in the figure, this interpolation of the global parametrization for n+^{12}C at 49.3 MeV incident energy is in good agreement across a wide energy range.

In contrast to ^{11}Be, the long-lived ($T_1/_2 = 1.5 \times 10^6$ y) and tightly-bound nucleus ^{10}Be has a larger neutron separation energy of 6.81 MeV. One experimental study of ^{10}Be is that of Lapoux et al. [13], who measured elastic cross-sections for ^{10}Be and ^{11}Be projectiles on proton and ^{12}C targets at 39.1 MeV/nucleon and 38.4 MeV/nucleon, respectively. This data was investigated using the microscopic Jeukenne-Lejeune-Mahaux nucleon-nucleus potential for the proton target and the folding model for the carbon target. Unlike other studies [20, 21], to be more physical and reliable, the potential parameters describing the interaction between the core and target are adjusted to the elastic scattering data at 39.1 MeV/nucleon [13]. Our obtained potential depth parameters are shallower than those in their potential. We use the experimental β_2 quadrupole deformation value of -0.6 [42] for the first (2^+) excited level of ^{12}C at 4.4 MeV. Additionally, to account for non-elastic contributions arising from surface region interactions, a surface term can be added to the effective potential. This potential is sometimes referred to as a surface term, derivative form of the Woods-Saxon potential, DPP, or VCP, and can be obtained through different methods. DPP parameters can be derived from microscopic [43–45] or phenomenological [46–52] calculations by fitting to experimental data. For ^{10}Be+^{12}C, we used a phenomenological DPP obtained from fitting to experimental data combined with a volume term. The results for this elastic scattering and the optical potential parameters used are shown in Fig. 2 [Figure 2: see original paper] and Table 1 , respectively. With the exception of the well-known phenomenon at 5°, the data are reproduced well.

Many authors have analyzed the elastic scattering of the halo nucleus ^{11}Be on a ^{12}C target at this incident energy using different theoretical models [8, 20, 21]. However, none of these studies incorporated nuclear structure to explain the data. On the other hand, including nuclear structural information in reaction calculations for such weakly-bound systems as halo nuclei has proven crucial for achieving agreement between predictions and experimental data. The spectroscopic factor as a nuclear structure property is one of the most important ingredients for theoretical cross-section calculations of both light and heavy ions. There are many methods for determining spectroscopic factor values. The simplest method is to fit spectroscopic factor values directly to experimental cross-section data, but the traditional approach estimates the spectroscopic factor using the shell model, where it is defined as the square of the normalization of the overlap integral between the wave function of the valence nucleon in the target nucleus state and the residual nucleus. The spectroscopic factor is also a key ingredient in direct capture processes, for which the related cross-section often dominates the total cross-section at the very low energies of astrophysical interest. The direct capture process can be used to obtain the spectroscopic factor and is known to play a notable role in light exotic nuclei systems where few or no resonant states are available. Although many works containing spectroscopic factors derived from transfer processes exist for the halo nucleus ^{11}Be, direct neutron capture cross-section data for ^{10}Be forming ^{11}Be are not available in the literature. However, direct capture cross-sections can be obtained from Coulomb dissociation data.

In obtaining the spectroscopic factor from direct capture cross-section calculations, the most important component is the nuclear level density. Generally, level density models are distrusted in such calculations due to their insufficient agreement with experimental observables and their treatment of collective effects. To overcome these challenges, we recently introduced a new Laplace-like formula [24] for the NLD parameter to improve predictive power for describing low-lying collective levels, which are vital for the direct capture process. With this formula, good agreement is achieved with experimental observables. Therefore, the direct neutron capture cross-section calculation based on this level density model for the ^{10}Be(n,γ)^{11}Be process is shown in Fig. 3 [Figure 3: see original paper]. Although the data could not be reproduced below 0.5 MeV, the same behavior is well explained in the rest of the energy range. The average spectroscopic factor value is determined as 1.48 from the least chi-square fit. The parameter values used in our level density calculation are 1.345 for the asymptotic level density parameter $\tilde{a}$ and 0.285 for the deformation parameter β, obtained from fitting to discrete levels of ^{11}Be taken from our previous study [24].

Considering ^{11}Be as a two-body projectile, all optical potential parameter values are given in Table 1 for the n+^{12}C, n+^{10}Be, and ^{10}Be+^{12}C interactions. The potential depth parameter values for ^{10}Be+^{12}C at 39.1 MeV incident energy are 46.3 MeV and 13.8 MeV for the real and imaginary parts, respectively. The same procedure yields 9.820 MeV and 3.661 MeV for the surface potential. Additionally, to include non-elastic contributions for ^{11}Be+^{12}C, a surface potential is added to the effective potential. Moreover, to compare our results, we performed another calculation using the spectroscopic factor obtained from transfer reactions by Schmitt et al. [53], which gives 0.71 for the ground state and 0.62 for the first excited state. The results of this calculation are shown in Fig. 4 [Figure 4: see original paper] with a dashed blue line. In our calculations, the average spectroscopic factor value is taken as the spectroscopic factor of the ground state. Since the spectroscopic factor of the first excited state has less effect on the results, this factor is taken as 1.0. Finally, our prediction for the elastic scattering cross-section of ^{11}Be on ^{12}C is shown in Fig. 4 with a solid red line. The inclusion of the nuclear level density with the Laplace-like formula in the reaction calculations has a positive effect on reproducing the cross-section data. Additionally, our fitting method for the optical potential parameters—adjusting geometrical parameters to peak positions and depths to minimize ^{2}—positively contributes to the agreement.

Conclusions

In summary, we have investigated the elastic scattering cross-section data for the projectile ^{11}Be on a ^{12}C target at 49.3 MeV/nucleon [6]. To include breakup effects in the calculations, different optical potentials for n+^{12}C, n+^{10}Be, and ^{10}Be+^{12}C are described. The present study also contains the first analysis of the phenomenological optical model for 39.1 MeV incident energy for the projectile ^{10}Be on a ^{12}C target. The obtained results are in better agreement with experimental data compared to the microscopic study of Lapoux et al. [13], which is the first and only study of this reaction.

Not only were the effects of including the spectroscopic factor in the calculations found to be significant for the breakup reaction of ^{11}Be, but adjusting the geometrical parameters to peak positions and the depths to minimize ^{2} also positively contributed to reproducing the scattering data.

The theoretical framework used to obtain the spectroscopic factor via nuclear level density calculations of the direct neutron capture cross-section is employed for the first time in breakup reaction calculations for ^{11}Be. Moreover, the nuclear level density is used for the first time as a spectroscopic tool in reactions induced by light exotic nuclei. Consequently, besides the success of the nuclear level density with the Laplace-like formula for the level density parameter [24] as a structure model, the results show that this new method appears appropriate for performing reaction calculations.

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Note: Figure translations are in progress. See original paper for figures.