Flavor-mixing induced by the mismatched vector interactions at finite I (Postprint)

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Abstract

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Full Text

Preamble

Flavor-mixing induced by the mismatched vector interactions at finite μ_I

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Abstract

The relation between the vector-isoscalar and vector-isovector interactions in the two-flavor Nambu-Jona-Lasinio (NJL) model is investigated under different constraints from QCD. We demonstrate that flavor-mixing can be induced by mismatched vector-isoscalar and vector-isovector interactions at finite baryon chemical potential μ and isospin chemical potential μ_I. The effect of this non-anomaly flavor-mixing on possible separate chiral transitions at nonzero μ_I is studied under the assumption of effective restoration of U(1)A symmetry. We find that for weak isospin asymmetry, the two separate phase boundaries found previously can be converted into one only if the vector-isovector coupling g_v^v is significantly stronger than the vector-isoscalar one g_s^v without the axial anomaly.

Keywords: Chiral phase transition, QCD critical point, Flavor-mixing, Isospin chemical potential
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Introduction

Mapping the QCD phase diagram at finite temperature T and quark chemical potential μ has attracted growing interest. In particular, a chiral critical endpoint is predicted by some model studies, which separates the crossover and first-order phase transition. Currently, lattice QCD computations at finite density still struggle within a limited range of μ [1], so the existence and location of the critical endpoint remain under debate due to the lack of reliable theoretical methods for non-perturbative dense QCD. Theoretically, fluctuations of conserved charges such as net-baryon, net-charge, and net-strangeness are predicted to be sensitive to the correlation length of the system [2–4] and directly connected to some susceptibilities [5, 6]. Consequently, experimental data related to these quantities can serve as powerful tools to probe the critical endpoint in heavy-ion collisions. The search for such a point is ongoing at RHIC (BES) [7–9] and will be performed at future facilities at GSI (FAIR) and JINR (NICA). Moreover, unconventional multiple chiral critical endpoints have also been proposed: it was found in [10, 11] that finite isospin chemical potential μ_I may lead to two critical endpoints, and when considering color superconductivity (CSC), low-temperature critical endpoint(s) may appear due to the interplay between chiral and diquark condensates [12–17].

Besides T and μ, the U(1)A anomaly may significantly impact the QCD phase transition [18]. This has been confirmed in model studies and Ginzburg-Landau analyses, where the U(1)A anomaly is usually incorporated by introducing the Kobayashi-Maskawa-' t Hooft (KMT) interaction [19–21]. The KMT interaction explicitly breaks the U(1)A symmetry and gives rise to flavor-mixing among light quarks: the dynamical mass of the u quark may contain contributions from both d and s quark condensates, and the diquark condensate for u-d pairing at moderate or high baryon density may contribute to the s quark mass. Consequently, the U(1)A anomaly may affect not only the properties of the traditional critical endpoint [22] but also those of unconventional ones: the separation of chiral transition due to finite μ_I [10, 11] may be removed by anomaly-induced flavor-mixing [23], and a new critical endpoint at low temperature could be induced in the presence of color-flavor-locking (CFL) CSC [13].

However, recent lattice calculations indicate that the U(1)A symmetry may be restored significantly near and above T_c for $\mu = 0$ [24, 25]. The effective restoration of U(1)A symmetry would influence the universality class and critical properties of the chiral transition [26]. Phenomenologically, model studies suggest that the properties of the conventional critical endpoint are quite sensitive to the degree of U(1)A restoration [22]. Moreover, if anomaly-related flavor-mixing is very weak near the phase boundary, the two critical endpoints due to isospin asymmetry [10, 11] could still be possible due to the decoupling of light quarks.

On the other hand, it is also probable that non-anomaly flavor-mixing can be induced by other ingredients of QCD, especially under certain conditions. The main purpose of this work is to study the possible non-anomaly flavor-mixing of light quarks and its effect on the chiral phase transition under isospin asymmetry. In particular, we will focus on the fate of the two critical endpoints due to separate chiral transitions found in [10, 11] with the assumption of effective restoration of U(1)A symmetry near the phase boundary.

Our starting point is the four-quark vector interactions with different coupling strengths in the isovector and isoscalar channels. The effect of vector interactions on the chiral transition has been extensively studied in NJL-type models of QCD. A well-known result is that the chiral transition at finite μ is weakened by the vector-isoscalar interaction $g_s v(\bar{\psi}\gamma^\mu\psi)^2$ and the critical point disappears for strong $g_s v$ [12, 15, 22, 27]. It is also found that $g_s v$ in a proper range can lead to new low-temperature critical endpoint(s) [12] when considering two-flavor CSC, especially with the constraint of electric-charge neutrality [15, 17]. In general, $g_s v$ and the isovector coupling $g_v v$ are independent coupling constants under chiral symmetry. In the literature, non-anomaly flavor-mixing due to mismatched vector interactions in the vacuum has been discussed in Ref. [28] based on a three-flavor NJL model. We extend such work to finite T and μ to study the effect of non-anomaly flavor-mixing on the chiral transition with isospin asymmetry.

II. Extended NJL-Type Model with Mismatched Vector Interactions

A. The general four-quark interaction model with mismatched vector interactions under chiral symmetry

We start with the following Lagrangian of four-quark interactions for two-flavor QCD:

$$L^{(4)} = L^{(4)}_{\text{sym}} + L^{(4)}_{\text{det}}$$

where

$$L^{(4)}_{\text{sym}} = g_{s1}\sum_{a=0}^{3}[(\bar{\psi}\tau_a\psi)^2+(\bar{\psi}\tau_a i\gamma_5\psi)^2]-g_{v1}\sum_{a=0}^{3}[(\bar{\psi}\tau_a\gamma_\mu\psi)^2+(\bar{\psi}\tau_a\gamma_\mu\gamma_5\psi)^2]-g_{v2}\sum_{a=1}^{3}[(\bar{\psi}\tau_a\gamma_\mu\psi)^2+(\bar{\psi}\tau_a\gamma_\mu\gamma_5\psi)^2]-g_{v3}$$

and

$$L_{\rm det}^{(4)} = g_{s2}\{\det[\bar\psi(1-\gamma_5)\psi]+\text{h.c.}\} = g_{s2}[(\bar\psi\psi)^2-(\bar\psi\vec\tau\psi)^2-(\bar\psi i\gamma_5\psi)^2+(\bar\psi\vec\tau i\gamma_5\psi)^2].$$

Here τ_0 and $\vec\tau$ refer to the unit matrix and Pauli matrices in flavor space, respectively. The former term $L_{\rm sym}^{(4)}$ in (1) is the general Fierz-invariant form of four-quark interactions in color-singlet channels respecting the global flavor symmetries of SU(2)V SU(2)A U(1)V U(1)A [29]. The latter term $L_{\rm det}^{(4)}$ is the KMT interaction induced by gauge configurations of instantons and anti-instantons [19, 20], which only possesses the SU(2)V SU(2)A U(1)V global flavor symmetries.

As mentioned, we will focus on flavor-mixing arising from mismatched vector interactions at finite density. We see that three of the four independent coupling constants in $L^{(4)}$ are related to vector and axial-vector interactions. Generally, the nonzero sum $g_{v3}+g_{v4}$ implies that the vector coupling strength in the isovector channel differs from that in the isoscalar one. Similarly, non-vanishing $g_{v3}-g_{v4}$ indicates that the axial-vector interactions are also mismatched in the isovector and isoscalar channels. How the vector coupling difference gives rise to non-anomaly flavor-mixing at finite density will be detailed in the next section.

The axial-vector interaction may be responsible for the deviation of the chiral magnetic effect in recent lattice calculations compared to the analytic formula, as proposed in Ref. [30]. Here we mainly study the chiral phase transition in the mean field approximation (MFA), where the axial-vector interactions in Lagrangian (1) will be ignored. Thus we only consider the following effective Lagrangian:

$$L_{\rm eff} = g_{s1}\sum_{a=0}^{3}[(\bar\psi\tau_a\psi)^2+(\bar\psi\tau_a i\gamma_5\psi)^2]+g_{s2}[(\bar\psi\psi)^2-(\bar\psi\vec\tau\psi)^2-(\bar\psi i\gamma_5\psi)^2+(\bar\psi\vec\tau i\gamma_5\psi)^2]-g_v^s(\bar\psi\gamma_\mu\psi)^2-g_v^v(\bar\psi\vec\tau\gamma_\mu\psi)^2,$$

where the independent coupling constants are reduced to four.

B. Unequal vector coupling constants in the mean field Hartree-Fock approximation

Here we stress that the vector coupling difference in the MFA can also arise from a very popular version of the NJL model [29]:

$$L^{(4)} = g_{s1}\sum_{a=0}^{3}[(\bar\psi\tau_a\psi)^2+(\bar\psi\tau_a i\gamma_5\psi)^2] - g_v\sum_{a=0}^{3}[(\bar\psi\tau_a\gamma_\mu\psi)^2+(\bar\psi\tau_a\gamma_\mu\gamma_5\psi)^2],$$

in which only one vector coupling g_v is adopted. In the Hartree approximation, there is no difference between the coupling strengths of the two vector interactions at the mean field level for Lagrangian (5). However, the effective vector couplings (in the sense of direct interaction) in the isoscalar and isovector channels will differ from each other if the Fock contribution is also considered. For a four-fermion interaction, the Fock contribution can be easily evaluated according to its Fierz transformation [29]. Taking into account the exchange terms, the effective direct four-quark interactions of Lagrangian (5) take the following form:

$$L_{\text{eff-direct}} = L^{(4)}+L^{(4)}_{\text{Fierz}} = (g_{s1}+g_{s2}+\frac{g_{s1}}{2N_c})[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]+(g_{s1}-g_{s2}-\frac{3g_{s1}}{2N_c})[(\bar{\psi}\vec{\tau}\psi)^2+(\bar{\psi}i\gamma_5\psi)^2]-(g_v-\frac{g_v}{2N_c})$$

where N_c is the color number of quarks. The effective Lagrangian (6) clearly shows that exchange terms give rise to vector coupling differences in the Hartree-Fock approximation (HFA), which are of order O(1/Nc) compared to g_{s1} and g_v. Note that a similar result in a three-flavor NJL model has been given in [28].

If both g_v and g_{s1} in (5) originate from the color current-current interaction $g(\bar{\psi}\gamma_\mu\lambda^a\psi)^2$, they fulfill the relation $g_v = g_{s1}/2$ according to the Fierz transformation. In this case, the vector coupling difference shown in (6) becomes:

$$\delta g_v = g_v^s - g_v^v = \frac{g_{s1}}{2N_c}.$$

This equation indicates that g_v^s in (4) may be larger than g_v^v, and their difference is considerable compared to g_v or g_{s1} for $N_c = 3$.

C. Constraints on the vector interactions from the lattice chiral curvatures

Even though Eq. (7) implies that the coupling g_v^v is weaker than g_v^s, it is also possible that g_v^v is stronger than g_v^s. This can be understood from the curvature difference for the chiral phase transition at finite baryon and isospin chemical potentials obtained in recent lattice calculations [31]. For small baryon and isospin densities, the chemical potential dependence of the pseudo-critical temperature for the chiral crossover can be expressed as:

$$T_c(\mu_q, \mu_i) = T_c + A_q\mu_q^2 + B_i\mu_i^2 + O(\mu_{q/i}^4, \mu_q^2\mu_i^2),$$

where T_c is the chiral pseudo-critical temperature at zero quark chemical potential (in this subsection, μ_q and μ_i are used to refer to the quark baryon and isospin chemical potentials, respectively). Note that $T_c(\mu_q, \mu_i)$ is an even function of $\mu_{q/i}$ [32]. So at order $\mu_{q/i}^2$, we can expand $T_c(\mu_{q/i}^2)$ as:

$$T_c(\mu_{q/i}^2) = T_c(1 - \kappa_{q/i}\frac{\mu_{q/i}^2}{T_c^2}),$$

where the two chiral curvatures are defined as:

$$\kappa_{q/i} = -\frac{1}{T_c}\frac{dT_c(\mu^2)}{d(\mu^2/T_c^2)}\bigg|_{\mu=0}.$$

The lattice QCD simulation in [31] suggests that the curvature κ_q is about 10% greater than κ_i.

Recently, the role of g_v^s on the determination of κ_q has been studied in a Polyakov-loop enhanced three-flavor NJL model [22]. It is found that κ_q decreases with g_v^s, and to reproduce the lattice κ_q, g_v^s must maintain a relatively larger value compared to g_s. The authors of Ref. [22] then propose that the lattice κ_q can be used as a useful constraint on g_v^s. We can directly extend this idea to determine κ_i by replacing μ_q with μ_i. As will be demonstrated in the next section, the coupling g_v^v influences the curvature κ_i in a similar way as g_v^s does on κ_q. In particular, κ_i and κ_q obtained at the MFA of the two-flavor NJL model will take the same value for $g_v^s = g_v^v$. In other words, the lattice curvature difference between κ_i and κ_q can be regarded as useful evidence for unequal vector coupling strengths. Following the spirit of Ref. [22], our numerical study suggests that g_v^v is about 10% larger than g_v^s near the chiral phase boundary for zero and small quark chemical potential. Note that this conclusion is quite different from the estimation given in (7).

D. Constraints on the vector interactions from the couplings of vector mesons to nucleons and lattice susceptibilities

In Ref. [33], it is argued that the ratio of the couplings of ω and ρ mesons to nucleons can be used as a constraint on the vector coupling difference. In the chirally broken phase, the empirical value for this ratio is given by $g_{\omega NN}/g_{\rho NN} \approx 3$, whereas in the chirally symmetric phase it is expected to be one. It is then proposed that the ratio g_v^s/g_v^v is located in the range from 1/3 to 1.

In addition, another quite similar estimation is given in Ref. [34], where the vector coupling difference is expressed as a function of two susceptibilities χ_q and χ_I under some assumptions. Using lattice data for these susceptibilities as input, it is found that g_v^v is always less than g_v^s: their difference is quite large below T_c but approaches zero rapidly above T_c for zero chemical potential.

All the arguments given in the above subsections suggest that the vector interactions are repulsive (namely, g_v^s and g_v^v are all positive), but the relation between g_v^s and g_v^v remains uncertain. In the following study, g_v^s and g_v^v will be treated as free parameters due to these uncertainties.

III. Vector-Interaction Induced Flavor-Mixing and the Thermal Dynamical Potential at Finite Baryon and Isospin Chemical Potentials

In this section, we shall demonstrate that vector coupling differences can lead to non-anomaly flavor-mixing at finite baryon and isospin densities.

The full Lagrangian of the two-flavor NJL model with interaction (4) reads:

$$L = \bar{\psi}\left[i\partial_\mu \gamma^\mu + \gamma^0 \hat{\mu} - \hat{m}_0\right]\psi + L^{(4)},$$

where the quark chemical potentials are introduced and $\hat{m}_0 = \mathrm{diag}(m_u, m_d)$ is the current quark mass matrix. We shall adopt isospin-symmetric quark masses with $m_u = m_d \equiv m_0$. The $\hat{\mu}$ in Lagrangian (11) is the matrix of quark chemical potentials which takes the form:

$$\hat{\mu} = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix} = \begin{pmatrix} \mu - \mu_I & 0 \\ 0 & \mu + \mu_I \end{pmatrix},$$

where μ_B (μ_I) is the baryon (isospin) chemical potential. Note that the definition of the isospin chemical potential at the quark level differs from that at the nucleon level. For more details on the role of isospin symmetry energy in nuclear matter, the reader can refer to [35–38] and references therein.

At finite densities, the quark chemical potentials are shifted by the vector interactions. Here we use μ' to denote the modified quark chemical potential. Note that the u quark density differs from the d quark density under isospin asymmetry. The shifted quark chemical potentials take the form:

$$\mu'_u = \mu_u - 2g_v^s(\rho_u + \rho_d) - 2(g_v^s - g_v^v)\rho_d,$$

$$\mu'_d = \mu_d - 2g_v^s(\rho_u + \rho_d) - 2(g_v^s - g_v^v)\rho_u,$$

with the quasi-particle energy $E_f = \sqrt{\vec{p}^2 + M_f^2}$. The Λ in Eq. (20) is the three-momentum cutoff parameter in the NJL model. We see that besides the modified chemical potential μ'_f, the flavor-mixing due to vector coupling differences is also explicitly demonstrated in Eq. (19) via the direct coupling between ρ_u and ρ_d.

Minimizing the thermal dynamical potential Eq. (19), the equations of motion for the mean fields ϕ_u, ϕ_d, ρ_u, and ρ_d are determined through the coupled equations:

$$\mu'_u = \mu_u - 2g_v^s(\rho_u + \rho_d) - 2(g_v^s - g_v^v)\rho_d,$$

$$\mu'_d = \mu_d - 2g_v^s(\rho_u + \rho_d) - 2(g_v^s - g_v^v)\rho_u,$$

where $\rho_{u(d)} = \langle \psi^\dagger_{u(d)} \psi_{u(d)} \rangle$ is the u (d) quark number density. Eq. (14) clearly shows that due to vector coupling differences, not only ρ_u but also ρ_d contributes to the effective chemical potential of the u quark, and vice versa. This implies that flavor-mixing arises from vector interactions. As mentioned, this mixing has nothing to do with the axial anomaly. The modified chemical potentials can also be rearranged as:

$$\mu'_B = \mu - 2g_v^s(\rho_u + \rho_d), \quad \mu'_I = \mu_I - 2g_v^v(\rho_u - \rho_d),$$

which indicates that μ and μ_I are shifted by the isoscalar and isovector vector interactions, respectively.

Formally, the non-anomaly flavor-mixing shown in (14) for the modified chemical potentials is quite similar to the anomaly flavor-mixing for the constituent quark masses induced by instantons:

$$M_{u(d)} = m_0 - 4g_{s1}\phi_{u(d)} - 4g_{s2}\phi_{d(u)},$$

where $\phi_{u(d)} = \langle \bar{\psi}_{u(d)} \psi_{u(d)} \rangle$ is the u (d) quark condensate.

Using conventional techniques, the mean-field thermal dynamical potential of Lagrangian (11) is expressed as:

$$\Omega(T, \mu_u, \mu_d) = \Omega_0(T, \mu'_f; M_f) + 2g_{s1}(\phi_u^2 + \phi_d^2) + 4g_{s2}\phi_u\phi_d - (g_v^s)(\rho_u^2 + \rho_d^2) - 2(g_v^s - g_v^v)\rho_u\rho_d,$$

where $\Omega_0(T, \mu'_f; M_f)$ is the contribution of a quasi-particle gas of flavor f which takes the form:

$$\Omega_0(T, \mu'_f; M_f) = -2N_c T \int \frac{d^3p}{(2\pi)^3} \left\{ \ln[1 + \exp(-(E_f - \mu'_f)/T)] + \ln[1 + \exp(-(E_f + \mu'_f)/T)] \right\} - 2N_c \int \frac{d^3p}{(2\pi)^3} E$$

IV. Fate of the Separate Chiral Transitions with Non-Anomaly Flavor-Mixing

As mentioned, the separate chiral transitions due to finite μ_I [10, 11] can be removed by flavor-mixing induced by the axial anomaly [23]. Since the instanton density may be suppressed significantly near the phase boundary, we revisit this problem by taking into account non-anomaly flavor-mixing due to mismatched vector interactions.

For comparison, we follow the notations in Ref. [23] and introduce two parameters α and g_s which are defined as:

$$g_{s1} = (1-\alpha)g_s, \quad g_{s2} = \alpha g_s,$$

where α represents the ratio of the KMT interaction in the scalar-pseudoscalar channel, treated as a free parameter in the following calculations. The other model parameters–namely the current quark mass m_0, the scalar coupling constant g_s, and the three-momentum cutoff Λ–are all adopted from [23].

A. Fate of separate chiral transitions under weak isospin asymmetry without the axial anomaly

The role of mismatched vector interactions on the separation of chiral transition at finite T-μ under weak isospin asymmetry is investigated by switching off the KMT interaction. We focus on whether the two critical endpoints found previously could be ruled out by non-anomaly flavor-mixing without the help of the axial anomaly.

We first study cases with $g_v^v > g_v^s$ for a fixed small coupling $g_v^s = 0.2g_s$ under weak isospin asymmetry $\delta\mu = -20$ MeV (the typical value of $\delta\mu$ in heavy-ion collisions may be within this range, as estimated in [23]). The T-μ phase diagrams for varied g_v^v are shown in Fig. 1 [Figure 1: see original paper]. For $g_v^v = 2.0g_v^s$, Fig. 1(a) shows two separate first-order phase boundaries corresponding to chiral transitions for u and d quarks. For $g_v^v = 0.6g_s$, Fig. 1(b) shows that only one first-order chiral boundary emerges at low temperature, but it splits into two lines at relatively higher temperature, so there are still two critical endpoints. Further increasing g_v^v to $0.68g_s$, Fig. 1(c) displays only one phase boundary. Thus we observe that the two separate phase boundaries can be changed into one by non-anomaly flavor-mixing induced by mismatched vector interactions.

The above calculation for $\delta\mu = -20$ MeV is further extended to a fixed moderate coupling $g_v^s = 0.4g_s$. The phase diagrams for varied $g_v^v > g_v^s$ are shown in Fig. 2 [Figure 2: see original paper], which remains analogous to Fig. 1. In contrast to Fig. 1, a stronger g_v^v is required for conversion of the two phase transitions into one due to the enlarged g_v^s. Fig. 2 also shows that the chiral transition is first softened and then strengthened with g_v^v. By comparison, the chiral transition is always weakened with increasing g_v^s. Thus for weak isospin asymmetry, Figs. 1 and 2 demonstrate that chiral transition separation can be removed by mismatched vector interactions even without instanton-induced flavor-mixing. All the phase diagrams in Figs. 1 and 2 are quite similar to Figs. 2 in Ref. [23] obtained by changing α. In this sense, non-anomaly flavor-mixing due to vector coupling differences plays a similar role as the KMT interaction.

However, Figs. 1 and 2 indicate that g_v^v must be much stronger than g_v^s to turn the two chiral transitions into one: g_v^v must be at least twice as strong as g_v^s to remove the separation. Of course, the fate of separate chiral transitions depends not only on the vector coupling difference but also on the magnitudes of g_v^s and g_v^v. We do not show results for $g_v^v < g_v^s$ since in this case only crossover transition appears.

On the contrary, we do not find coincidence of detached phase boundaries for $g_v^s > g_v^v$. In Fig. 3 [Figure 3: see original paper], we show phase diagrams for $\delta\mu = -20$ MeV with varied g_v^s and fixed coupling $g_v^v = 0.2g_s$. We see that the two separate phase boundaries get farther rather than closer with increasing $|\delta g_v|$ for $g_v^s > g_v^v$, which is quite different from what is shown in Figs. 1 and 2.

The reason can be traced back to Eqs. (14) and (15). First, according to Eq. (15), $|\mu_I'|$ is explicitly less than $|\mu_I|$ since the signs of μ_I and $-2g_v^v(\rho_u - \rho_d)$ in μ_I' are different for $g_v^v > 0$. So for $g_v^v > g_v^s$, increasing g_v^v implies not only enhancement of flavor-mixing but also reduction of $|\mu_I'|$. This is why the two phase boundaries approach each other with g_v^v, as shown in Figs. 1 and 2. Second, near the left side of the right phase boundary, ρ_d is remarkably larger than ρ_u due to significant suppression of the d quark mass, but around the left side of the left phase boundary, the difference between ρ_d and ρ_u is relatively small. So for $g_v^s > g_v^v$, the flavor-mixing term $-(g_v^s - g_v^v)\rho_u$ in μ_u' impacts the right phase boundary more significantly than the corresponding term $-(g_v^s - g_v^v)\rho_d$ does on the left phase boundary, according to Eq. (14). This is why the right phase boundary moves more rapidly towards higher μ with g_v^s compared to the left one, as shown in Fig. 3.

If g_v^s and/or g_v^v are strong enough, the first-order chiral transition will change into crossover and there would be no critical point. Due to vector interactions, it is possible that one of the two phase boundaries first disappears while the other remains with changes in vector interactions (in contrast, the two critical endpoints always appear at the same temperature in Refs. [10, 11]). Such a case is observed in Fig. 3 for very strong vector interaction $g_v^s = 1.0g_s$. Our numerical study suggests that emergence of only one critical endpoint via this manner does not require very strong vector interaction when weak KMT interaction is included.

Here only weak isospin asymmetry is considered because $|\mu_I|$ is small in heavy-ion collisions. On the other hand, quark matter may appear in the core of neutron stars, where the magnitude of the difference between u and d quark chemical potentials may be as large as 100 MeV due to charge neutrality constraints. For strong isospin asymmetry with relatively large $|\mu_I|$, we find that separation of chiral transition cannot be removed by non-anomaly flavor-mixing without considering the axial anomaly. However, a similar phase diagram to Fig. 3 is still observed for proper choices of g_v^v and g_v^s.

V. Discussion and Conclusion

We have studied the influence of vector interactions with different coupling constants in the isoscalar and isovector channels on possible separation of the chiral transition under isospin asymmetry in a two-flavor NJL model, where U(1)A symmetry is assumed to be effectively restored near the phase boundary.

We first show that, besides the argument from empirically different nucleon-vector-meson couplings [33], the one-gluon exchange type interaction can also give rise to unequal vector interactions with $g_v^s > g_v^v$ in the MFA when including the Fock contribution. On the other hand, by extending the work [24] to finite μ_I, we obtain quite different vector coupling differences with $g_v^v > g_v^s$ from constraints of lattice chiral curvatures at zero/small quark chemical potentials. We demonstrate that, similar to mass-mixing induced by the KMT interaction, density-mixing of two flavors is produced due to mismatched vector interactions.

The role of non-anomaly flavor-mixing on the chiral phase transition is investigated under conditions of weak isospin asymmetry. We find that to convert the two separate chiral transitions into one, g_v^v must be significantly stronger than g_v^s without the axial anomaly. In this situation, non-anomaly flavor-mixing induced by vector interactions impacts the separation of chiral phase transitions similarly to anomaly-induced mixing: the two detached phase boundaries first get closer and then coincide with enhanced flavor-mixing.

Note that recently the Polyakov-loop extended NJL model has been extensively used to investigate thermal and dense properties of QCD. We stress that introducing Polyakov-loop dynamics does not qualitatively change our main conclusions. Additionally, our study can be directly extended to the quark-meson model of QCD by incorporating quark-vector-meson couplings. Especially, it is interesting to investigate the role of non-anomaly flavor-mixing on possible quark-meson transitions in neutron stars [42].

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Note: Figure translations are in progress. See original paper for figures.