Compressed Sensing and Otsu' s Method Based Binary CT Image Reconstruction Technique for Non-Destructive Testing Postprint

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Abstract

This paper addresses the problem of binary CT image reconstruction in non-destructive testing using an algorithm based on compressed sensing (CS) and Otsu' s method, which can reconstruct binary CT images of test objects from incomplete projection data. Based on the characteristics of binary CT images, we employ the Split-Bregman method with L_{1/2} regularization to solve the piecewise-constant region reconstruction problem. To improve the quality of the reconstructed images from incomplete projection data, we utilize prior knowledge and Otsu' s method as optimization constraints. In our study, we conduct numerical simulations to evaluate the proposed method and compare the reconstruction results with those from different reconstruction methods. Finally, the simulation results demonstrate that the proposed method can effectively reduce noise and suppress artifacts, and reconstruct high-quality binary images from incomplete projection data.

Full Text

Preamble

Compressed Sensing and Otsu' s Method Based Binary CT Image Reconstruction Technique in Non-Destructive Detection

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Abstract: This paper addresses the problem of binary CT image reconstruction in non-destructive detection through an algorithm based on compressed sensing (CS) and Otsu's method, which can reconstruct binary CT images of test objects from incomplete detection data. According to the characteristics of binary CT images, we employ the Split-Bregman method based on L1/2 regularization to solve piecewise constant region reconstruction. To improve reconstructed image quality from incomplete detection data, we utilize a priori knowledge and Otsu' s method as an optimization constraint. In our study, numerical simulations investigate the proposed method and compare reconstructed results from different reconstruction methods. Experimental results demonstrate that the proposed method can effectively reduce noise and suppress artifacts while reconstructing high-quality binary images from incomplete detection data.

Keywords: Non-destructive detection, Computed tomography (CT), Binary image reconstruction, Compressed sensing (CS)
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Introduction

X-ray computed tomography (CT), initially developed for clinical diagnostics, has evolved into industrial CT systems for non-destructive detection in aerospace, geology, weapons, metallurgy, and other fields [1–5]. Test objects composed of single-composition materials, such as engines, rock specimens, and teeth, are commonly imaged with CT [6,7]. The resulting CT images of these objects can be considered binary images containing only two gray values (i.e., black and white), which can be modeled as piecewise constant matrices and easily sparsified through proper orthogonal transformation during reconstruction [8,9].

Binary CT image reconstruction represents a key technique for image reconstruction from incomplete projection data using continuous tomography methods [10,11]. Additionally, discrete tomography methods exist for binary CT image reconstruction from few-view projections [12–15]. In industrial CT systems, scan times are lengthy due to large test object sizes and numerous projection views [4,5]. Reconstructing test objects from few-view projections can significantly reduce scan time. In medical CT systems [8], reconstructing biomedical samples from few-view projections helps reduce radiation dose.

Conventional reconstruction algorithms (i.e., filtered back-projection and algebraic reconstruction techniques) cannot reconstruct high-quality CT images from incomplete projection data [1]. Compressed sensing (CS) theory, however, can reconstruct high-quality images from substantially less projection data than traditionally required by Nyquist sampling theory [16–18]. The fundamental principle of CS theory states that an image can be reconstructed from a rather limited amount of data as long as the underlying image can be sparsely represented in an appropriate domain and determined from these data [19–21]. Furthermore, advanced algorithms incorporating prior information in CT image reconstruction can greatly reduce the required projection data [22,23]. Thus, reconstructing binary CT images from incomplete projection data using CS-based reconstruction algorithms with prior information is feasible.

This paper focuses on binary CT image reconstruction from incomplete projection data in non-destructive detection and proposes an algorithm based on CS and Otsu' s method. To improve reconstructed image quality, we employ the Split-Bregman method with L1/2 regularization [24], which produces sparser solutions compared to the L1 regularization method commonly used in CT image reconstruction. Meanwhile, we utilize a priori knowledge of gray value information and Otsu's method [25] as an optimization constraint during reconstruction, enabling segmentation and extraction of gray value information from binary images. The remainder of this paper is organized as follows: Section II presents our reconstruction algorithm, Section III compares images reconstructed using different methods, and the final section discusses the image reconstruction process and results.

II. Materials and Methods

Reducing scan time or radiation dose in CT imaging is of significant importance, which in turn affects detection data completeness. Currently, CS-based reconstruction algorithms more readily accommodate incomplete projection data during reconstruction and achieve better performance. We propose an algorithm based on CS theory and Otsu' s method to reconstruct binary CT images from incomplete detection data in non-destructive detection. Below, we summarize the proposed algorithm scheme.

It is widely accepted that CT image reconstruction can be modeled as a linear system $Af = b$, where $b = (b_1, \cdots, b_M) \in \mathbb{R}^M$ represents the detection data, $f = (f_1, \cdots, f_M) \in \mathbb{R}^N$ denotes the image object, and $A = (a_{ij})$ is the measurement matrix.

The CT image reconstruction method can be empowered by CS techniques to reduce the necessary datasets and improve image quality under less favorable conditions. The CS-based reconstruction method can be expressed as

$$\min E(f), \quad \text{subject to } Af = b \text{ and } f > 0,$$

where $E(f)$ is the regularization function. Using the penalty function method, we can convert this into an unconstrained optimization problem:

$$\min E(f) + \lambda \|b - Af\|_2^2.$$

In CS theory, applying the L0 norm—the most ideal regularization norm—is difficult in image reconstruction [26,27]. Thus, L0 norm is commonly replaced by L1 norm in CT image reconstruction [28,29]. Theoretically, the L1/2 norm is closer to L0 norm and can produce sparser solutions while reconstructing high-quality CT images [30,31]. We propose an algorithm based on L1/2 regularization to solve the binary image reconstruction problem [24]. Using the L1/2 norm as the regularization norm and gradient transformation as the sparse transformation, the objective function can be redefined as

$$f = \arg\min \|\phi(f)\|_{1/2}^{1/2} + \beta\|b - Af\|_2^2,$$

where $\phi(f)$ is the sparse transformation. Introducing an intermediate variable $d = \phi(f)$, this can be converted into

$$(f^{k+1}, d^{k+1}) = \arg\min \|d\|_{1/2}^{1/2} + \beta\|b - Af\|_2^2 + \rho\|d - \phi(f^{k+1}) - b^k\|.$$

The Split-Bregman method based on L1/2 regularization can reconstruct high-quality binary images from few-view projections [24].

To improve the quality of reconstructed binary images from incomplete detection data, we utilize a priori knowledge of the two gray values and a segmentation mechanism (Otsu' s algorithm) in the reconstruction process. In Otsu' s algorithm [25,34], the normalized histogram of the reconstructed image is $p[m]$, $m \in [m_{\min}, m_{\max}]$, where $[m_{\min}, m_{\max}]$ is the gray-level range. Setting a threshold $T \in [m_{\min}, m_{\max}]$ to divide the gray-level range into two classes ($[m_{\min}, T]$ and $[T+1, m_{\max}]$), the reconstructed binary image can be divided into two gray value regions. The class separability associated with T is defined as

$$S^2(T) = w_0(\mu_0 - \mu)^2 + w_1(\mu_1 - \mu)^2 = w_{0w}1(\mu_1 - \mu_0),$$

where $w_0(T) = \sum p[m](m = m_{\min} \to T)$, $w_1(T) = \sum p[m](m = T+1 \to m_{\max})$, $\mu_0(T) = \sum mp[m](m = m_{\min} \to T)$, $\mu_1(T) = \sum mp[m](m = T+1 \to m_{\max})$, and $\mu = \sum mp[m](m = m_{\min} \to m_{\max})$. The purpose of Otsu' s algorithm is to search for an optimal threshold T^* to maximize $S^2(T)$:

$$T^* = \arg\max[S^2(T)],$$

which can localize the reconstructed image structure information in non-destructive detection. In our algorithm, Otsu' s method calculates the optimal segmentation threshold T and segments the reconstructed image into two parts. Meanwhile, prior knowledge (the true gray value) is used to determine the gray value of each part. Finally, we use the segmented image as an intermediate image in the next iterative loop.

In implementation, the whole iteration process of our proposed algorithm can be summarized as follows:

Step 1: Initialize reconstructed image $f = 0$.

Step 2: Input measured data b, and calculate intermediate image f_{ART} using the ART algorithm:

For $i = 1, 2, \dots, N_{\text{angle}}$:

$$f^{k+1} = f^k + \lambda \frac{b_i - A_i f^k}{\|A_i\|^2}, \quad k = 0, 1, \dots;$$

Step 3: Apply positivity constraint for f_{ART}:

$$f_{i,j} = \begin{cases} f_{i,j}, & f_{i,j} \geq 0 \\ 0, & f_{i,j} < 0 \end{cases}$$

Step 4: Update the intermediate image f_{SB} using the Split-Bregman method based on L1/2 regularization.

For $k = 1, 2, \dots, N_{SB}$:

$$f^{k+1} = \arg\min \beta \|b - Af\|_2^2 + \rho \|d^k - \nabla f - b^k\|$$

$$d^{k+1} = \min \|d\|_{1/2}^{1/2} + \rho \|d - \nabla f^{k+1} - b^k\|$$

$$b^{k+1} = b^k + (\nabla f^{k+1} - d^{k+1});$$

Step 5: Update the intermediate image f_{Otsu} using Otsu’ s method, and calculate optimal segmentation threshold T^*:

$$T^* = \arg\max\{w_0(T) w_1(T)(\mu_1(T) - \mu_0(T))\};$$

Step 6: Initialize next iteration image f:

$$f = (1 - \delta) f_{SB} + \delta f_{Otsu}, \quad 0 < \delta < 1;$$

Step 7: Return to Step 2 until the stopping criterion is met.

In our algorithm, key parameters are selected according to experimental analysis. The constraint factor λ in the ART method is determined to accomplish initialization reconstruction. Two important parameters, β and ρ, in the Split-Bregman method are determined based on L1/2 regularization to realize optimization reconstruction. Finally, the scale factor δ is determined to combine the Split-Bregman method based on L1/2 regularization with Otsu’ s method. For better reconstruction results, reconstruction errors are calculated to obtain optimal experimental parameters.

III. Results

To demonstrate the feasibility of our proposed method in non-destructive detection, numerical simulations were performed with three binary phantoms: the mandible, turbine blade, and limestone phantoms shown in Fig. 1 [Figure 1: see original paper]. The pixel size is 256×256. The binary mandible phantom was derived from a real mandible CT image, containing teeth and bone regions. The binary turbine blade phantom, having two gray levels (0 and 1) and containing turbine blade and background regions, was derived from a turbine blade CT image. The binary limestone phantom, derived from a rock specimen CT image, has two gray levels (0 and 1) and contains surrounding rock region, internal porosity region (air), and background region (air).

A typical parallel-beam geometry of the CT system is assumed. To compare and analyze reconstruction results, algebraic reconstruction technique (ART) [35], total variation based algebraic reconstruction technique (ART-TV) [28], Split-Bregman method based on L1/2 regularization (SB-L1/2) [24], and our method (Split-Bregman method based on L1/2 regularization and Otsu' s method, SB-Otsu) were used to reconstruct the three phantoms, respectively, with 200 iterations for all reconstruction processes.

The ART was implemented using Eq. (12). The ART-TV was implemented in two loops: the outer loop executed ART to reduce data discrepancy, while the inner loop minimized image TV. In the inner loop, the gradient descent method was used:

$$\|\nabla f\|_1 = \sum g_{i,j}, \quad g_{i,j} = \sqrt{(f_{i,j} - f_{i+1,j})^2 + (f_{i,j} - f_{i,j+1})^2},$$

where $\|\nabla f\|_1$ denotes TV of f, $f_{i,j}$ is the pixel value of the discrete 2D image, and $g_{i,j}$ is the discrete gradient.

$$f^{(m+1)} = f^{(m)} - \gamma \frac{\omega \upsilon}{|\upsilon|},$$

where γ is the gradient descent control coefficient, $\omega = \|f^{(m+1)} - f^{(m)}\|_2$ is the scaling coefficient of the gradient descent, and $\upsilon = (\partial \|\nabla f\|_1 / \partial f_{i,j})|_{f_{i,j}=f_{i,j}[n,m]}$ is the gradient direction when $f_{i,j} = f_{i,j}[n,m]$. The Split-Bregman method based on L1/2 regularization was implemented primarily using Eqs. (14)-(16). Finally, we analyzed key parameters of the reconstruction algorithms, with optimal parameter values being $\lambda = 0.5$, $\beta = 1000$, $\rho = 1$, $\delta = 0.2$, and $\gamma = 0.2$.

In binary mandible phantom reconstruction, angular scanning ranged from 0° to 180° in 30° steps, producing 6 projections. We added 0.5% Gaussian noise to projection data, and reconstructed results using different methods are shown in Fig. 2A [Figure 2: see original paper]. In binary turbine blade phantom reconstruction, angular scanning ranged from 0° to 180° in 18° steps, producing 10 projections. We added 0.5% Gaussian noise to projection data and used the

10 projections to reconstruct the turbine blade phantom (Fig. 2B). In binary limestone phantom reconstruction, angular scanning ranged from 0° to 180° with 22.5° steps, producing 8 projections. We added 0.5% Gaussian noise to projection data, and reconstructed results using different methods are shown in Fig. 2C.

The root mean square error (RMSE) in Eq. (21) was used to quantify reconstruction results:

$$\mathrm{RMSE} = \left[\frac{\sum(f_{i,j} - \hat{f}_{i,j})^2}{N_f}\right]^{1/2},$$

where $\hat{f}_{i,j}$ is the reconstructed pixel value, $f_{i,j}$ is the true pixel value, and N_f is the pixel number of the phantom.

RMSE values for the three reconstructed phantoms using different methods are summarized in Table 1 , and iterative process curves for these reconstruction algorithms are shown in Fig. 3 [Figure 3: see original paper].

From Fig. 2, reconstructed images using ART methods contain substantial noise and artifacts, while images reconstructed using ART-TV, SB-L1/2, and SB-Otsu methods exhibit clearer edges. From Table 1, the SB-Otsu method performed better in binary CT image reconstruction from severely incomplete projection data than the other three methods, proving more effective in treating noise and artifacts. From Fig. 3, the SB-Otsu algorithm can reconstruct higher quality binary CT images with the same number of iterations.

IV. Discussions and Conclusion

In CS theory, an image can be reconstructed from a rather limited amount of detection data as long as it can be sparsely represented in an appropriate domain and determined from these data. Results from Section III demonstrate that CS-based methods effectively treat noise and artifacts in reconstructed images from incomplete detection data. However, CS-based algorithms are not omnipotent; reconstructed images may suffer loss of some detail information due to severely incomplete detection data. To improve reconstructed image quality from severely incomplete detection data, prior knowledge of gray value information and a segmentation mechanism (Otsu' s method) are introduced into binary CT image reconstruction.

Several issues warrant further discussion. First, the amount of projection data used in reconstruction depends on the structure of the reconstructed image in our study. More complex structures require more projections to reconstruct high-quality images. Second, the correlation of projection vectors also determines reconstructed image quality; weaker correlation yields higher quality reconstructed images. Third, our proposed algorithm combines the Split-Bregman method based on L1/2 regularization with Otsu' s method. During reconstruction, we must set a proper weighting coefficient δ for Otsu's algorithm implementation. If the weighting coefficient δ is too large, reconstructed image quality will be lowered. In our study, we selected the weighting coefficient δ value based on experimental analysis.

Currently, we analyzed three binary image phantoms: a mandible phantom, a turbine blade phantom, and a limestone phantom. The limestone and turbine blade phantoms were tested by industrial CT, and our proposed method can reconstruct these two phantoms from few-view projections to reduce scan time. The mandible phantom was scanned by a medical CT system, and our proposed method can reconstruct the mandible phantom from few-view projections to reduce radiation dose. This initial methodological study focuses primarily on phantom simulation analysis; follow-up studies will address more general settings for industrial and biomedical applications.

In conclusion, we proposed a binary CT image reconstruction algorithm based on compressed sensing and Otsu' s method to reduce scan time or radiation dose in non-destructive detection and investigated the feasibility and potential of the proposed method. Experimental results demonstrated that our proposed method is very effective at reducing noise and suppressing artifacts in binary CT image reconstruction from incomplete projection data. In future work, we will analyze real data reconstruction, though a systematic study is beyond the scope of this initial investigation.

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Note: Figure translations are in progress. See original paper for figures.