PET Image Reconstruction with Highly Compressible System Matrix Based on Rotationally Symmetric Polygonal Pixel Grid (Postprint)

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Abstract

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Full Text

PET Image Reconstruction with a Rotationally Symmetric Polygonal Pixel Grid Based Highly Compressible System Matrix

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Abstract

To achieve maximum compression of the system matrix in positron emission tomography (PET) image reconstruction, we propose a polygonal image pixel division strategy that aligns with rotationally symmetric PET geometry. We established the geometrical definition and indexing rules for polygonal pixels and implemented image conversion from polygonal to conventional rectangular pixel structures using a conversion matrix. A set of test images were analytically defined in polygonal pixel structure, converted to conventional rectangular pixel-based images, and correctly displayed, verifying the correctness of the image definition, conversion description, and structural conversion. A compressed system matrix for PET image reconstruction was generated using a strip model and tested by forward-projecting three different radioactive source distributions to the sinogram domain and comparing them with theoretical predictions. On a practical small animal PET scanner, a compression ratio of 12.6:1 was achieved for the system matrix size with the polygonal pixel structure compared to the conventional rectangular pixel-based strip-mode approach. OS-EM iterative image reconstruction algorithms were developed for both polygonal and conventional Cartesian pixel grids. A hot rod phantom was detected and reconstructed using both grids with reasonable time cost, achieving 1.35 mm image resolution in both cases. We conclude that it is feasible to reconstruct and display images using a polygonal image pixel structure based on a compressed system matrix in PET image reconstruction.

Keywords: Positron emission tomography (PET), Rotationally symmetric polygonal pixel grid, Compressed system matrix, Image reconstruction

Introduction

Positron emission tomography (PET) has rapidly developed as an essential branch of molecular imaging technologies. Based on modern high-sensitivity 3D data acquisition modes and statistical algorithms, high-quality molecular and functional images can be obtained from organs to cells in the human body.

PET plays an important role in oncology, cardiology, and neurology diagnosis, including early detection, staging, and post-treatment evaluation of malignant diseases. However, it remains a great challenge to implement a fully 3D reconstruction algorithm in a clinical PET system, as a modern PET machine operating in 3D mode requires calculation of numerous lines-of-response (LORs) (approximately 10^7) [1]. Correspondingly, a huge system matrix (about 10^{12} elements) is needed for image reconstruction. To address this issue, it is critical to compress the system matrix and obtain the best trade-off between image quality and reconstruction speed.

Considering that most modern clinical PET systems employ symmetric ring-shaped geometry, a concept of leveraging this symmetry with irregular image pixel shapes instead of conventional rectangular pixels to achieve maximum system matrix compression has been proposed, which has recently become a new focus area in fully 3D PET iterative algorithm development. Compared with other methods of system matrix compression [2], compression with symmetry is direct and has proven to be efficient in achieving better compression factors.

Early approaches designed a polar grid-based system matrix structure with rotational and reflectional symmetries. For example, this principle was proposed to accelerate the PET iteration process using polar pixel structures [3,4]. In recent years, with the popularization of 3D PET technology and explosive growth of acquisition data size, more attention has been devoted to this field. In addition to the polar grid, various irregular pixel shapes have been presented in commercial or laboratory PET systems, such as in MicroPET Focus 120 [5] and LabPET [6]. A software toolkit PRESTO [7-11] was developed, which integrated generic calculation, storage, and access of rotational symmetry in a C++ library to improve the efficiency and accuracy of image reconstruction.

Compared to the original polar pixel-based geometric structure, an irregular division can overcome the unevenness of pixel area distribution across the image field of view. It has been demonstrated that a polar voxel with square-like shape produces fewer artifacts [12,13], but one usually has to design a specific division mode for each compression factor, as shown in Ref. [11]. The increased complexity of pixel shapes in the irregular division mode brings further challenges in image reconstruction and display. Since most existing PET systems have a polygonal ring shape rather than a circular one, a polygonal pixel grid may have an advantage over traditional polar grids.

This work proposes a polygonal image pixel division strategy based on the characteristics of rotationally symmetric PET geometry. It is adaptable to the structural symmetries of various types of PET systems. One can simplify the calculation of the system matrix in a compressed format by utilizing rotational symmetry of pixel structure during the PET image formation process. The proposed method was applied to a practical small animal PET system. In view of the definition of this pixel grid and calculation of the system matrix, we accomplished iterative image reconstruction of experimental data from a hot rod phantom with 30 mm diameter and length based on the practical PET scanner.

The 3D detected data were organized into sinograms. The system matrix was calculated by strip model and compressed by rotational and axial symmetries. A Cartesian grid-based image reconstruction algorithm with the same pixel size was also generated using the strip model. Storage size of system matrix, time cost, and image quality were compared for these two methods.

2.1 Rotationally Symmetric Polygonal Pixel Based Image Expression & Display

The core concept of this paper is based on a novel rotationally symmetric polygonal pixel grid. A representative division pattern of the polygonal pixel grid is shown in Figure 1: see original paper. The structure contains 8 segments that are rotationally symmetric with respect to the image center. The arrangement of polygonal pixels is identical from segment to segment. Compared to the conventional rectangular pixel division pattern (Figure 1: see original paper), the introduction of rotational symmetry helps compress the storage space of the system matrix. Compared to existing polar pixel grid patterns (Figure 1: see original paper), the regularity of this division mode ensures simplified expression and automatic segmentation. Additionally, there are only three types of basic pixel shapes, allowing easy adjustment of each pixel type's area to minimize differences in pixel area across the image FOV. Therefore, this structure may have an edge over polar grids in reducing pixel area unevenness.

Constrained by typical image display modes of modern computers, the main challenge of this novel structure is that images expressed in this rotational symmetry polygon pixel grid must be translated into cubic ones for display. Furthermore, a new method is needed to calculate geometrical weights of the system matrix since universal algorithms such as the Siddons ray-tracing method [14] are not applicable in this particular case.

2.2 Geometrical Description of Polygonal Pixels

Based on the concept of building a rotationally symmetric geometrical structure, the definition of polygonal pixels consists of three steps: (1) segment definition, (2) layer definition inside segments, and (3) pixel definition, as illustrated in Figure 2: see original paper, (b), and (c), respectively.

Segment definition: When the compression factor is P, P rays are drawn starting from the center of the image field of view (FOV), dividing the FOV into fan-shaped segments. The angle between two adjacent rays, i.e., the vertex angle of each segment, is $= 2\pi/P$. The shape and area of every arrangement are equal to each other. Starting from the positive x-axis, which is defined as the starting boundary of the first segment, an anticlockwise index number p = 0, 1, …, P−1 is assigned to each of the P segments. The ending boundary of the p-th segment is the same as the starting boundary of the (p+1)-th segment, where p = 0, 1, …, P−2. The ending boundary of segment P−1 coincides with the positive x-axis. Since all segments are identical in shape, the arrangement of pixels can be defined only once for one segment and rotationally replicated to other segments. Therefore, the definition of layers and pixels is discussed only in segment 0.

Layer definition: The area inside one segment is divided into several regions by a group of layer-boundary lines that are perpendicular to the vertex angle bisector and intersect with the segment boundary at equal distance . The distance between adjacent parallel lines is . The area embraced by two adjacent parallel lines and the two segment-boundary rays is called a layer, except the centermost layer, which is formed by the two segment-boundary rays and one layer-boundary line closest to the vertex angle. Each layer is identified by an index number s. The layer containing the vertex is layer 0, whose adjacent layer is layer 1, and so on. The total number of layers is defined as S, which is determined by the image size.

Pixel definition: As illustrated in Figure 2: see original paper, a set of lines parallel to the vertex angle bisector are drawn, dividing each layer into one or multiple polygonal regions. Those lines are denoted as lay-intersecting lines to distinguish them from layer-boundary lines in the layer definition, and each polygonal region is defined as one pixel. The distances between neighboring layer-intersecting lines are all equal to . The s-th layer has s+1 pixels in a row. The row-index for those pixels is labeled as t. We define that in each layer, the 0-th pixel contains the ending boundary, and the s-th pixel contains the starting boundary.

With the above steps, one can establish a rotationally symmetric polygonal pixel grid and enumerate all pixels with three parameters: segment-index p, layer-index s, and row-index t. Therefore, a pixel is uniquely specified by (p, s, t). Compared to conventional rectangular-pixel based image division patterns, the proposed pixel division strategy has P-fold rotational symmetry when there are P segments, leading to 1/P storage size of the system matrix. Existing polar-pixel based methods can achieve a similar compression factor. However, considerable differences in both pixel shape and area exist in pixel division patterns. For example, in Ref. [15], the largest area ratio of pixels reaches almost 5, which may lead to non-uniform image quality distribution and even image artifacts. As listed in , in the proposed polygonal pixel division strategy, there are only three basic pixel types: triangle, right-angle trapezoidal, and rectangle, with area ratios of 2:3:4. The simple pixel shapes and reduced pixel area difference are beneficial for minimizing resolution non-uniformity and reducing image artifacts.

2.3 Pixel Indexing and Addressing

To store a digital image in computer memory, a pixel indexing rule is needed to map each pixel to a memory unit. Likewise, a pixel addressing method is needed to calculate the pixel coordinate given its memory address. In an image expressed by rotationally symmetric polygonal pixels, each pixel is uniquely represented by its three indices (p, s, t). When the image is stored in computer memory, it occupies a continuous space. For each pixel (p, s, t), a unique address i needs to be determined.

[Figure 4: see original paper] shows a representative scheme for assigning memory addresses to image pixels in an 8-segment, 5-layer image. The pixel indices are determined with a layer-by-layer sequence. The pixel indices inside the p-th segment range from 15p to 15p+14. Inside a segment, pixel indices are increasingly assigned from smaller layers to larger layers and from smaller rows to larger rows within each layer. According to these rules, the mapping relationship between i and (p, s, t) is:

$$i = \frac{S(S+1)}{2} \times p + \frac{s(s+1)}{2} + t \quad (1)$$

where $\frac{S(S+1)}{2}$ is the total number of pixels in one segment. Meanwhile, the coordinate (p, s, t) of the i-th pixel is determined as:

$$p = \left\lfloor \frac{i}{\frac{S(S+1)}{2}} \right\rfloor, \quad s = \left\lfloor \frac{i - p \times \frac{S(S+1)}{2}}{\text{something}} \right\rfloor$$

2.4 Image Pixel Structure Conversion

To view an image stored in the rotationally symmetric polygonal pixel structure on a computer screen, it must first be converted to a conventional rectangular pixel structure. The conversion process can be expressed as:

$$\text{Image}_M = Q_{MN} \times \text{Image}_N \quad (5)$$

where Image_M is the converted image in rectangular pixel-based structure, Image_N is the original image in rotationally symmetric polygonal pixel structure (both organized in column vector format), and Q_{MN} is the conversion matrix.

As illustrated in [Figure 5: see original paper], the value of the m-th rectangular pixel in Image_M can be expressed as a weighted sum of the values of multiple polygonal pixels overlapping it. The overlap area of the n-th pixel in polygonal structure and the m-th pixel in rectangular structure is denoted as S_{mn}. The weight is considered as the ratio of the intersection area and the total area of the cubic pixel: $q_{mn} = S_{mn}/S_m$.

2.5 Verification of Image Expression and Structure Conversion

To verify that an image is correctly stored in a rotationally symmetric polygonal structure and converted to a rectangular pixel structure, a set of test images are analytically defined:

$$a(p, s, t) = 0, \text{ where } s \text{ is odd} \quad (7)$$
$$a(p, s, t) = 1, \text{ where } s \text{ is even}$$

By selecting different segment numbers, a set of images A_n is numerically defined. [Figure 6: see original paper] illustrates the test images for 8, 9, and 12 segments, respectively. These images are defined with rotationally symmetric polygonal structures, converted to conventional rectangular pixel structure, and displayed on a computer screen for examination. If the structure is defined correctly and the transformation matrix works effectively, one should see images corresponding to the predefined ones in various rotationally symmetric polygonal grids.

3 Generation and Verification of System Matrix

3.1 Generation of a Compressed System Matrix

In the calculation of the system matrix, each line-of-response (LOR) is considered as a strip with certain width. As shown in [Figure 7: see original paper], each LOR is uniquely identified by its projection angle ϕ and distance to the center of FOV r. Therefore, the j-th LOR is denoted as $j(\phi, r)$. Each element $c_{i,j}$ in the system matrix records the probability that the j-th LOR detects a photon emitted from the i-th pixel, denoted as $i(p, s, t)$. In this study, physical effects such as positron range, photon non-collinearity, attenuation, and scattering are ignored, and pure geometric detector response is modeled. Thus, $c_{i,j}$ is considered proportional to the overlapping area of the i-th pixel and j-th LOR.

The rotational symmetric structure in the image domain can be utilized to compress the system matrix. As shown in [Figure 8: see original paper], considering two image pixels i_0 and i, when they have the same layer index s and row index t, and when the angle from LOR j_0 to j is equal to $p\theta$, i.e., the angle from segments p_0 to p, the overlapped area between pixel i and LOR j is rotationally symmetric to that between pixel i_0 and LOR j_0. Thus, the corresponding system matrix element has the same value, i.e., $c_{i,j} = c_{i_0,j_0}$. The relationship between i and i_0 is shown as:

$$i = i_0 + p \times \frac{S(S+1)}{2} \quad (8)$$

The relationship between j and j_0 can be expressed through relationships of r and r_0 as well as ϕ and ϕ_0:

$$\phi = \phi_0 + p\theta, \quad r = r_0, \quad \phi = \phi_0 + p\theta + \pi, \quad r = -r_0 \quad (9)$$

3.2 Verification of System Matrix

The verification studies are based on a simulated typical small animal PET system. As shown in [Figure 9: see original paper], the detector consists of 6 rings, each with 32 detector blocks evenly distributed. Every block is composed of 8$×$8 LYSO crystals. The crystal size is 1.59 mm × 1.59 mm × 10 mm. The detector ring diameter is 147.2 mm, and the FOV size is 30 mm in diameter and 76.32 mm in axial length. The radial center of the entire detector field of view is a circle with a radius of about 30 mm.

A rotationally symmetric polygonal pixel structure with P = 12,   = 0.5 mm is defined. The layer-height   is chosen to be much smaller than the desired PET system resolution (i.e., 1.6 mm). To cover the 30 mm FOV, the image has 60 layers. The acquired 2D PET data are organized into a sinogram with 155 angular samplings over 180 degrees and 60 radial samplings over 30 mm trans-axial FOV, and the width of the LOR strip is the same as the layer-height.

By defining different source distributions in the image structure, the corresponding sinogram data were generated by projecting the source image into LOR space using the pre-calculated system matrix. The sinograms are visually examined and compared to theoretical expectations to verify the correctness of the system matrix and projection model.

Data for image reconstruction were acquired with an experimental small animal PET system. The detector consists of 5 rings, each with 24 detector blocks evenly distributed. Every block is composed of 9$×$9 LYSO crystals. The crystal size is 2.11 mm × 2.11 mm × 10 mm. The inner detector ring diameter is 76.33 mm, and the FOV size is 50 mm in diameter and 95 mm in axial length.

A rotationally symmetric polygonal pixel structure with P = 12,   = 0.33 mm was defined. The layer-height   was chosen to be much smaller than the desired PET system resolution. To cover the 50 mm FOV, the image had 79 layers. The acquired 3D PET data were organized into a sinogram with 108 angular samplings over 180 degrees and 70 radial samplings over 46.76 mm trans-axial FOV, and the width of the LOR strip was the same as the layer-height.

After establishing the system matrix, the projection data acquired from the real PET scanner were reconstructed based on the rotationally symmetrical structure. Using the image transformation matrix defined in Eq. (5), the reconstructed images were transformed and displayed for visual examination. We also conducted image reconstruction based on a Cartesian pixel grid with 142$×$142 pixels of 0.33 mm × 0.33 mm area. The system matrix was calculated using the same strip model for LORs.

The commonly used OS-EM algorithm [16,17] was chosen for PET image reconstruction. The OS-EM algorithm is based on maximum likelihood estimation of Poisson-distributed signals. The objective is to maximize a cost function defining a logarithmic likelihood between the projection of the image estimate and actual projection values. An iterative expectation maximization algorithm is used to optimize the cost function. The projection views are grouped into L subsets, and the image is updated after each subset is considered. The iterative formula is:

$$f_i^{k+1} = \frac{f_i^k}{\sum_{j \in S_s} c_{i,j}} \sum_{j \in S_s} c_{i,j} \frac{p_j}{\sum_{i'} c_{i',j} f_{i'}^k}$$

where p_j is the simulated detected photon number in the j-th LOR, f_i^k is the image value of the i-th pixel after the k-th iteration using projection belonging to the s-th subset, and $c_{i,j}$ is a system matrix element recording the probability that the j-th LOR detects a photon emitted from the i-th pixel.

5.1 Structural Testing for Rotationally Symmetric Polygonal Pixel Grid

As defined in Eq. (7), three test images were defined with different segment numbers: P = 8, P = 9, and P = 12. These images were defined with rotationally symmetric polygonal structures and converted to conventional rectangular pixel structures as described in Eq. (5). The pixel size in the converted images is approximately 1/5 of the original images. The three converted images are shown in [Figure 10: see original paper]. No image quality loss is observed after conversion, demonstrating the correctness of the image structure conversion process.

[Figure 10: see original paper] displays results of the test images defined in Eq. (7) converted to Cartesian coordinates. The segment numbers P for Figs. 10(a-c) are 8, 9, and 12, respectively, and the predefined images are shown in [Figure 6: see original paper]. The layer-height of the rotationally symmetric structure is 3 mm. The pixel size of the rectangular pixel structure is 1 mm, and the final image size is 128$×$128.

5.2 Validation of the Generated System Matrix Through Forward-Projection Process

In this test, three different distributions of radioactive sources were defined: (1) a point source located at the center of FOV; (2) a point source located radially 29.6 mm off the center of FOV; and (3) a volumetric source uniformly filling the entire FOV. Three corresponding sinograms were generated by forward-projecting the source images with the generated system matrix, shown in [Figure 11: see original paper].

In Figure 11: see original paper, the sinogram is expected to be a straight line (r = 0), and the generated sinogram agrees well with this expectation. When the point source is 29.6 mm off-center, the expected sinogram is a sinusoidal curve with 29.6 mm amplitude. In the generated sinogram, the measured amplitude is 29.5 mm. In Figure 11: see original paper, the covered range in the r direction is 29.5 mm. Since the FOV size is 30 mm, the covered range well matches the FOV size. Therefore, the sinograms generated by the system matrix correspond to theoretical expectations, confirming that the system matrix is calculated correctly.

5.3 Image Reconstruction Results

A hot rod phantom with six sections of hot rods (0.75 mm, 1.0 mm, 1.35 mm, 1.7 mm, 2.0 mm, and 2.4 mm in diameter) and 30 mm cross-sectional size was scanned using a practical animal PET system. 2D sinograms (108 angular bins and 70 radial bins) were generated from the list-mode event data. Images reconstructed with OS-EM iterations using polygonal and cubic grids are shown in Figs. 12(a) and (b), respectively.

Figure 12: see original paper shows reconstructed images based on a rotationally symmetric polygonal structure with 12 segments and 79 layers. The layer height was = 0.33 mm. After reconstruction, the images were converted to conventional rectangular pixel structure with 0.1 mm × 0.1 mm pixel size. Typical rotational polygonal pixel-shaped artifacts are visible in the lower middle part of the zoomed image section. No image quality loss was observed after converting the polygonal pixel structure image to a conventional one. The right column shows reconstructed images based on a cubic pixel grid with 142$×$142 pixels of 0.33 mm × 0.33 mm size.

In both Figs. 12(a) and (b), the 1.35 mm hot rod sections can be distinguished. Line profiles in [Figure 13: see original paper] illustrate that the desired 1.35 mm image resolution can be achieved with the proposed rotationally symmetric polygonal structure.

Based on this small animal PET system, the memory size of the system matrix can be compressed to 46.1 MB when elements are stored as 32-bit float numbers using this polygonal pixel grid. In comparison, when using a conventional rectangular pixel grid, the required memory size reaches 581.5 MB. The compression ratio of the system matrix size is 12.6:1. It took 2804 s to run 54 iterations on the polygonal pixel grid using a PC with a 2.5 GHz Intel(R) Core(TM) i5-3210M CPU and 4 GB memory. The iterations conducted on the cubic grid cost 2440 s. Although the polygonal grid requires 1.88 times more computation than the regular one, the slight increase in reconstruction time is acceptable considering the total pixel number.

6 Conclusion

In this work, we designed a novel rotationally symmetric polygonal pixel structure for PET image reconstruction. The structure can be correctly converted to conventional rectangular pixels for image display when the area of rectangular pixels is 1/5 of the polygonal ones. Three different distributions of radioactive sources were forward-projected to sinograms, verifying the correctness of the system matrix. Images were successfully reconstructed based on 3D list-mode sinogram data from a hot rod phantom using a practical small-animal PET system. The image resolution of the reconstructed phantom image is 1.35 mm. A compression ratio of 12.6:1 for the system matrix size was achieved on this small animal PET compared to the conventional rectangular pixel-based approach. With acceptable time cost, the image resolution based on the polygonal pixel structure is as good as that based on the cubic pixel structure.

In further studies, we will expand our investigation to optimization of the division mode for rotationally symmetric structures and seek solutions for unevenness of pixel shapes and areas. We can also apply this structure to 3D system matrix calculation to establish foundations for practical applications in full 3D PET.

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Note: Figure translations are in progress. See original paper for figures.