Dynamic Critical Fluctuations near the QCD Critical Point (Post-Print)

Authors: Wu Shanjin, Song Huichao

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Abstract

Exploring the critical point of the Quantum Chromodynamics (QCD) phase diagram is an important experimental objective of the Beam Energy Scan program in Relativistic Heavy-Ion Collisions (RHIC-BES). Preliminary experimental observations have revealed non-monotonic behavior of net-proton number fluctuations with collision energy, which is qualitatively consistent with theoretical predictions based on static models, suggesting the existence of a QCD critical point. Since relativistic heavy-ion collisions constitute a rapidly expanding system, dynamical effects can significantly alter critical fluctuations near the QCD critical point. To ultimately confirm the existence of the QCD critical point and investigate the phase structure of QCD in the finite-temperature finite-density region, a series of dynamical models near the QCD critical point have been developed. This article reviews recent progress in the search for the QCD critical point in both experimental measurements and theoretical models, with particular emphasis on the development and challenges of dynamical models in the critical point and first-order phase transition region.

Full Text

Critical Dynamical Fluctuations Near the QCD Critical Point

WU Shanjin[1,2,3], **SONG Huichao**[1,2,3]

[1](Center for High Energy Physics, Peking University, Beijing 100871, China)
[2](School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China)
[3](Collaborative Innovation Center of Quantum Matter, Beijing 100871, China)

Abstract

Exploring the critical point on the Quantum Chromodynamics (QCD) phase diagram represents a primary experimental objective of the Beam Energy Scan program in relativistic heavy-ion collisions (RHIC-BES). Preliminary measurements have revealed non-monotonic behavior in net-proton fluctuations as a function of collision energy, which qualitatively aligns with predictions from static theoretical models and hints at the existence of a QCD critical point. However, the fireball created in relativistic heavy-ion collisions undergoes rapid expansion, and dynamical effects can significantly modify critical fluctuations near the QCD critical point. To definitively establish the existence of the QCD critical point and investigate the QCD phase structure in the finite-temperature and finite-density region, researchers have developed a series of dynamical models describing the vicinity of the QCD critical point. This review examines recent progress in experimental searches and theoretical modeling of the QCD critical point, with particular emphasis on the development and challenges of dynamical models near the critical point and first-order phase transition region.

Keywords: QCD phase diagram and phase transition, Beam energy scan program in relativistic heavy-ion collisions (RHIC-BES), Dynamical models

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1. Introduction

Quantum Chromodynamics (QCD) provides the theoretical framework for describing strongly interacting matter across different energy scales. In the high-energy regime, QCD successfully explains asymptotic freedom observed in proton collision experiments, while at low energies, experimental results on vacuum symmetry and its spontaneous breaking support QCD predictions. Theoretical calculations and model studies of QCD at finite temperature and density predict a rich phase structure for nuclear matter. In the low-temperature, low-density region, the strongly interacting system consists of hadrons such as baryons and mesons, with colored quarks and gluons confined within color-neutral hadrons—this is the confinement phase. Simultaneously, chiral symmetry is spontaneously broken, placing the system in the chiral symmetry breaking phase. In the high-temperature, high-density region, colored quarks and gluons become deconfined from hadrons, forming a quark-gluon plasma (QGP). Consequently, the system enters the deconfinement phase where chiral symmetry is restored. In nature, QGP may have existed in the early universe and exists today in the interiors of neutron stars and other compact stellar objects. In the laboratory, extreme conditions created in relativistic heavy-ion collisions produce QGP, enabling systematic studies of the QCD phase diagram and phase transitions.

Currently operational relativistic heavy-ion collision experiments include the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and the Large Hadron Collider (LHC) heavy-ion program at CERN. Additional facilities under construction include Germany's FAIR (Facility for Antiproton and

Ion Research), Russia's NICA (Nuclotron-based Ion Collider fAcility), China's HIAF (High Intensity heavy ion Accelerator Facility), and Japan's J-PARC (Japan Proton Accelerator Research Complex) heavy-ion program. Decades of research have yielded substantial progress in understanding strongly interacting matter through relativistic heavy-ion collisions.

The QCD phase diagram is typically represented in the plane spanned by temperature and baryon chemical potential. Lattice QCD calculations in the low chemical potential region indicate a continuous crossover from the hadronic phase to the QGP phase at = 0 MeV, with a transition temperature around 155 MeV. In the high chemical potential region, effective QCD models such as the Functional Renormalization Group (fRG) and Dyson-Schwinger Equations (DSE) predict a first-order phase transition between the hadronic and QGP phases. Consequently, a first-order phase transition line must terminate at a critical point in the phase diagram. In the finite chemical potential region (> 0), lattice QCD faces the sign problem where the integration measure becomes complex. Various methods have been developed to address this issue, including Taylor expansion, reweighting, and imaginary chemical potential techniques. Recent lattice calculations suggest the critical point may be located at T < 140 MeV and _B > 300 MeV. Meanwhile, continuum field theory models like fRG place the critical point in the region 450 MeV < _B < 650 MeV. However, predictions from different effective models show significant dependence on model parameters, leaving the existence and location of the QCD critical point highly uncertain and requiring further investigation.

In relativistic heavy-ion collisions, the baryon stopping effect causes the baryon chemical potential of the produced hot dense nuclear matter to increase as collision energy decreases. The RHIC Beam Energy Scan program covers Au+Au collision energies from 200 to 7.7 AGeV, corresponding to temperatures of 100–180 MeV and baryon chemical potentials of 50–600 MeV according to statistical model calculations. This range encompasses the predicted critical point region from multiple models, allowing exploration of the QCD phase diagram by varying collision energy. Phase I of the Beam Energy Scan (BES-I) has been completed. Analysis of higher-order cumulants of net-proton multiplicity distributions revealed that the kurtosis ratio (σ^2) shows significant deviation from baseline fluctuations and exhibits non-monotonic behavior below 39 GeV, with a minimum near 19.6 GeV. This qualitative agreement with equilibrium critical fluctuation models suggests possible critical point discovery. Phase II of the Beam Energy Scan (BES-II), launched in early 2019, will soon provide complete experimental results, necessitating refined theoretical models to interpret data and predict new observables.

2. QCD Critical Fluctuations and Experimental Signatures

The QCD critical point, as the endpoint of the first-order phase transition line, is characterized by diverging correlation length. This divergence leads to unique properties near the critical point, including divergent fluctuations, singularities, universality, and critical slowing down. The most famous example is critical opalescence: when water approaches its critical point, the correlation length increases to match the wavelength of incident light, causing strong scattering and producing the characteristic milky appearance.

In relativistic heavy-ion collisions, direct measurement of fireball properties is impossible; instead, researchers search for remnants of QCD critical signals in final-state observables. This section briefly reviews experimental signatures related to QCD critical fluctuations.

2.1 Event-by-Event Particle Number Fluctuations The order parameter for chiral phase transitions can be represented by the chiral condensate (σ field: σ ¯). At high temperature and density, chiral symmetry restoration in the QGP phase yields ¯ $= 0$, while below the transition temperature, the QGP converts to hadronic matter with broken chiral symmetry and ¯ $\neq 0$. Near the critical point, the σ field develops long-range correlations that couple to final-state particles through σNN interactions, generating long-range correlations and large event-by-event fluctuations in particle numbers. These fluctuations exhibit non-monotonic behavior with collision energy. Since conserved quantities directly relate to theoretical predictions (e.g., lattice QCD and Hadron Resonance Gas model susceptibilities) and are sensitive to correlation length , fluctuations in conserved charges such as baryon number (B), electric charge (Q), and strangeness (S) serve as sensitive probes for the QCD critical point. Net-proton fluctuations approximate net-baryon fluctuations since chiral critical effects do not affect isospin susceptibility.

Due to finite-time and finite-volume effects in the QGP fireball, correlation lengths are limited to 2–3 fm, making it difficult to extract critical information using only second-order cumulants of event-by-event particle number fluctuations. Higher-order cumulants, particularly the fourth-order cumulant which changes sign in the critical region, provide enhanced sensitivity to correlation length. Additionally, long-range correlation effects cause cumulants to increase with detector acceptance, known as acceptance dependence.

The RHIC-BES Phase I measured net-proton fluctuations (momentum range |y| < 0.5, 0.4 < p_T < 2 GeV) at collision energies of 7.7, 11.5, 14.5, 19.6, 27, 39, 54.4, and 200 GeV. Results show that the fourth-order cumulant ratio σ^2 first decreases then increases with decreasing collision energy, reaching a minimum near 19.6 GeV. Recent HADES measurements at 2.4 GeV found negative σ^2 values. The non-monotonic behavior of net-proton fourth-order cumulants generally agrees with equilibrium critical fluctuation models, and the rapid variation with detector acceptance matches theoretical predictions. However, third-order cumulant measurements disagree with theoretical predictions, necessitating consideration of dynamical critical fluctuations (discussed in §3). The low-energy region below 7.7 GeV requires further investigation, as it may involve first-order phase transitions and QGP formation remains controversial. Larger statistics and greater acceptance from BES-II and fixed-target experiments are needed.

2.2 Light Nuclei Yield Ratios Near the critical point, nucleons constituting light nuclei exhibit large fluctuations. Studies show light nuclei production correlates with nucleon density fluctuations in coordinate space. Recent STAR measurements of the yield ratio N_t N_p/N_d^2 in central Au+Au collisions at mid-rapidity ($|y| < 0.5$) revealed non-monotonic energy dependence. However, light nuclei production is influenced by many factors due to their small binding energies. Existing hydrodynamics-based coalescence models struggle to reproduce experimental data, while numerical models have examined critical point effects on light nuclei yields and final-state resonance contributions. To isolate critical point signals from background effects, researchers have investigated sensitivity to higher-order phase-space distribution cumulants in coalescence models. By constructing combinations of light nuclei yields that suppress background effects and scale with correlation length, new observables have been proposed that predict double-peak structures in the critical region. Higher-statistics data including more nucleon-rich light nuclei are needed to confirm such signatures.

3. Dynamical Models Near the QCD Critical Point

The rapidly expanding fireball in relativistic heavy-ion collisions experiences only a finite duration in the critical region. Critical slowing down prevents physical quantities like correlation length from following the equilibrium growth, resulting in non-equilibrium critical fluctuations. To account for these effects, dynamical critical fluctuation models have been developed. The dynamics near the critical point are governed by dynamical universality classes, determined by order parameter symmetry, dimensionality, conservation laws, and Poisson brackets between conserved quantities. Finite-temperature QCD is believed to belong to Model H, which includes conserved order parameters, conserved momentum density, and their couplings. Due to numerical complexity, simplified models have been developed as alternatives.

3.1 Model A: Non-Conserved Order Parameter A simple model considers only the evolution of a non-conserved order parameter field (Model A). The equation of motion is a relaxation equation formally expressed as a Langevin equation. The QGP system's other degrees of freedom act as a heat bath interacting with the σ field, appearing as noise terms. Early studies showed critical slowing down suppresses correlation length growth and significantly alters the time evolution of cumulants, potentially reversing their signs compared to equilibrium predictions. Universal scaling behavior of the σ field and net-protons from Langevin dynamics has been investigated.

3.2 Model B: Conserved Order Parameter A simple coupling scheme considers only the order parameter field coupling to conserved charges. Near the critical point, conserved charges become slower modes, so evolution can be described by stochastic diffusion equations for conserved modes while treating other modes as a heat bath (Model B). This approach reveals typical critical slowing down phenomena and a competition between correlation length growth and diffusion effects, leading to non-monotonic acceptance dependence of conserved charge fluctuations.

3.3 Non-Equilibrium Chiral Fluid Dynamics (N FD) A more sophisticated approach treats the order parameter as an additional degree of freedom in hydrodynamics, coupled as a source term—Non-equilibrium chiral Fluid Dynamics (N FD). The order parameter evolves according to a Langevin equation with effective potentials from QCD-based models like (P)QM and linear σ models. Numerical simulations show enhanced variance and kurtosis of net-baryon fluctuations near the critical point compared to crossover regions.

3.4 Hydro+ Framework For self-consistent coupling between order parameter and fluid equations, the hydro+ framework evolves slow modes near the critical point. The ratio of entropy to baryon number (s/n) two-point functions serves as additional slow degrees of freedom. Numerical simulations for simple flow systems show minimal impact on fluid dynamics, with non-equilibrium effects contributing only ~0.1% to entropy. Extensions like hydro++ incorporating other slow mode effects are under development.

3.5 Fluctuating Hydrodynamics A natural approach for quantitative description of experimental data like net-proton fluctuations is relativistic hydrodynamics with stochastic noise. This effective model treats conserved quantities (energy-momentum tensor T^ and baryon current N^) as degrees of freedom, with the chiral order parameter acting as a heat bath coupled through σNN interactions, contributing noise terms determined by the fluctuation-dissipation theorem. While analytical calculations in 0+1D boost-invariant expansion show enhanced correlation functions due to increased thermal conductivity near the QCD critical point, numerical simulations face challenges with spatial noise dependencies and grid-size effects, requiring renormalized equations of state and transport coefficients.

3.6 Hydrodynamics-Kinetic Theory To circumvent numerical difficulties, deterministic hydrodynamic equations have been developed to analytically evolve two-point correlation functions (hydrodynamics-kinetic approach). This framework naturally includes renormalized transport coefficients and equations of state while resolving grid-spacing dependencies. It reproduces long-time tails from fluctuating hydrodynamics and estimates critical fluctuation correlation scales. Extensions to higher-order correlation functions (three- and four-point) have also been developed.

3.7 Equation of State with Critical Point Constructing equations of state containing the QCD critical point is crucial input for these models. Due to the sign problem in finite-density lattice QCD and parameter dependence in effective models, researchers construct parametric equations of state by mapping the non-universal relationship between finite-temperature QCD and the 3D Ising model, which share the same static universality class. The background contribution comes from lattice QCD extrapolated to = 0, patched with critical contributions from the 3D Ising model. By comparing hydrodynamic calculations with final-state observables, these parameters can be constrained using Bayesian methods, though this remains in early development stages.

3.8 Microscopic Transport Models Microscopic transport models based on the Boltzmann equation, such as AMPT, UrQMD, and JAM, provide alternative dynamical descriptions. By incorporating critical point equations of state to modify particle interactions, these models have studied various observables including proton fluctuations, collective flow, phase diagram trajectories, and HBT correlations. Critical fluctuations have been incorporated through linearized phase-space distributions and coupling between order parameter fields and Boltzmann equations.

3.9 Universal Scaling Laws A key feature of critical points is universal scaling. Near the critical point, diverging correlation length leads to scale invariance and self-similarity. The static critical universality class is determined by order parameter, symmetry, and dimensionality, with QCD and the 3D Ising model believed to belong to the same class. For heavy-ion collisions, finite-size scaling effects must be considered. Dynamical scaling introduces an additional critical exponent z, resulting from competition between expansion and relaxation effects. Critical slowing down prevents the system from reaching equilibrium, leading to a non-equilibrium correlation length known as the Kibble-Zurek scale, determined when expansion and relaxation rates become comparable. This mechanism allows construction of universal observables independent of evolution trajectory, with redefined higher-order cumulants of net-proton fluctuations showing suppressed oscillations and convergence to a universal curve.

4. QCD First-Order Phase Transition and Dynamical Models

First-order phase transitions exhibit rich physical processes including supercooling and superheating. A key characteristic is instability: when the system enters the phase coexistence region, it separates into two phases. According to phase transition theory, this proceeds via two mechanisms: nucleation (non-linear instability requiring formation of critical-sized bubbles to overcome free energy barriers) and spinodal decomposition (linear instability where small order parameter fluctuations grow uniformly when free energy curvature is negative). Classical nucleation theory was established in the 1930s and has seen extensive development.

In relativistic heavy-ion collisions, nucleation rates for hadronic bubbles in QGP have been estimated, showing strong dependence on barrier height, transport coefficients, and expansion rate. Model A and N FD studies reveal supercooling and reheating effects on observables. In rapidly expanding fireballs, systems can enter the spinodal decomposition region, leading to separation of confined and deconfined phases with experimental signatures like enhanced density distribution moments.

Despite progress, understanding first-order phase transitions remains limited. The non-equilibrium nature challenges hydrodynamic applicability, and phenomenological studies face difficulties. The first-order transition likely corresponds to low collision energies where initial conditions become increasingly important and QGP formation becomes marginal. Measurements showing negative elliptic flow v_2 below 3 GeV and negative σ^2 at 2.4 GeV suggest hadronic degrees of freedom dominate, requiring development of hadron-based phase transition models. The high-density equation of state also needs further refinement due to lattice QCD sign problems and model parameter dependencies.

5. Summary and Outlook

Relativistic heavy-ion collisions provide a crucial tool for studying nuclear matter under extreme conditions and represent a major focus of high-energy nuclear physics. Investigating the QCD phase diagram is a key objective, with collision energy scans probing a wide region. Non-monotonic behaviors in net-proton fluctuations and light nuclei ratios have been observed, prompting development of non-equilibrium critical fluctuation models. Preliminary results suggest QCD critical point existence.

Definitive confirmation requires improved dynamical models. Given hydrodynamics' success at RHIC and LHC energies, developing hydrodynamics-based critical fluctuation models is essential, including extensions like hydro++ with additional slow modes. As BES energies decrease, initial and final-state effects become more significant, necessitating comprehensive treatment of all evolution stages. Improved inputs like equations of state and transport coefficients are urgently needed.

Beyond refining phenomenological models, constructing critical-point-sensitive observables that suppress background effects will aid confirmation. The complex fireball system makes quantitative agreement challenging, but theoretically predicted characteristic behaviors, when matched with precise experimental data, will provide crucial support for critical point identification.

Experimentally, RHIC BES-II will provide higher-precision, larger-acceptance net-proton fluctuation data to reduce statistical errors below 20 GeV and confirm predicted non-monotonic behavior. Fixed-target experiments at lower energies will explore larger regions of the QCD phase diagram. Comparison with improved theoretical models will deepen understanding of QCD phase structure at high temperature and density.

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Author Contributions

WU Shanjin was responsible for manuscript writing; SONG Huichao provided guidance and review.

References

[References remain unchanged from the original text]

Note: Figure translations are in progress. See original paper for figures.