QCD Critical Endpoint and Baryon Number Fluctuations Postprint

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Abstract

One of the objectives of relativistic heavy-ion collisions is to search for the Quantum Chromodynamics (QCD) Critical End Point (CEP). The non-monotonic behavior of net-proton fluctuations measured in experiments hints at the existence of the CEP. This work employs the three-flavor PNJL (Polyakov-loop Nambu-Jona-Lasinio) model to calculate, along the chemical freeze-out line fitted from experimental data, the ratio of baryon number fluctuation cumulants C4/C2 as a function of collision energy. It is found that, based on the equilibrium assumption, within the collision energy range of 7.7–200 GeV, C4/C2 first decreases slowly and then increases as the collision energy decreases, which is consistent with experimental data. This also implies that the equilibrium assumption can be employed to explore the evolution behavior of the system after heavy-ion collisions, revealing that the relationship between the phase transition line and the chemical freeze-out line has a crucial influence on observables.

Full Text

Preamble

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QCD Critical End Point and Baryon Number Fluctuation

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Abstract

One of the primary objectives of relativistic heavy-ion collisions (HIC) is to search for the critical end point (CEP) of quantum chromodynamics (QCD). Experimental measurements of net-proton number fluctuations exhibit non-monotonic behavior, suggesting the existence of a CEP. This work employs a three-flavor Polyakov-loop Nambu–Jona-Lasinio (PNJL) model to calculate the ratio of baryon number fluctuation cumulants, C4/C2, as a function of collision energy along the chemical freeze-out line fitted from experimental data. Under the equilibrium assumption, the results show that C4/C2 first decreases slowly and then increases as the collision energy decreases within the range of 7.7–200 GeV, consistent with experimental observations. This implies that the equilibrium assumption can be applied to explore the evolution of the post-collision system in heavy-ion collisions, and that the relationship between the phase transition line and the chemical freeze-out line significantly influences observables.

Keywords: QCD phase diagram, Critical end point, Heavy-ion collision, Baryon number fluctuation

1. Introduction

As the standard theory describing strong interactions, quantum chromodynamics (QCD) has attracted considerable research interest in recent years regarding its phase structure. Under extreme conditions, QCD matter exhibits completely different behavior compared to vacuum. As temperature increases, QCD matter transitions from a hadron gas to a quark-gluon plasma (QGP), with degrees of freedom changing from hadrons to quarks and gluons. Lattice QCD calculations reveal that both chiral and deconfinement phase transitions are smooth crossovers at small chemical potentials [1–3]. On the other hand, symmetry analysis and chiral effective model calculations suggest that a first-order chiral phase transition occurs at high chemical potentials. According to Landau's theory of phase transitions, this implies the existence of a first-order phase transition end point at finite temperature and chemical potential—the QCD critical end point (CEP) [1–3].

These high-temperature, high-density extreme environments are strongly relevant to early universe evolution and astrophysics, with high-density conditions existing in the interiors of compact stars such as neutron stars. The equation of state of strongly interacting matter at finite temperature and density directly affects the mass-radius relationship of neutron stars. In the laboratory, heavy-ion collisions (HIC) produce high-temperature, high-density environments that serve as crucial platforms for studying the properties of strongly interacting matter under extreme conditions, including light nuclei properties [18], transport properties [19–22], fluid properties [23–24], heavy-flavor particles [25–27], magnetic fields and spin polarization [28–35], and the focus of this paper—QCD phase diagrams and the QCD critical point.

In heavy-ion collisions, if the chemical freeze-out line is sufficiently close to the phase boundary, information about the phase diagram can be revealed through fluctuations of conserved charges. These conserved charges are predicted to be highly sensitive to the correlation length [36]. Therefore, by analyzing event-by-event fluctuations of baryon number, charge, and strangeness, particularly their cumulants, one can uncover the thermodynamic properties of matter produced in heavy-ion collisions at RHIC (Relativistic Heavy Ion Collider) and LHC (Large Hadron Collider). For any conserved charge, let N be the measurement result in a single collision event. The cumulants of this conserved charge are defined as follows:

C1 = N
C2 = $(\delta N)^2$
C3 = $(\delta N)^3$
C4 = $(\delta N)^4$ − 3 $(\delta N)^{2\ 2}$

where $\delta N = N -$ N . These cumulants relate to different moments as:

M = C1, σ^2 = C2, S = C3/(C2)^(3/2), = C4/$(C2)^2$

where M, σ^2, S, and represent the mean, variance, skewness, and kurtosis of the corresponding conserved charge, respectively. In experiments, cumulants scale with system volume, so to eliminate volume effects, one typically uses ratios of cumulants, such as:

σ^2/M = C2/C1, Sσ = C3/C2, σ^2 = C4/C2

With these definitions, one can calculate various cumulants and their ratios from event-by-event net-proton distributions at a given collision energy √s_{NN}.

RHIC's Beam Energy Scan (BES) program measures cumulants of net-proton, net-charge, and net-kaon distributions in Au+Au collisions at collision energies √s_{NN} = 7.7, 11.5, 14.5, 19.6, 27, 39, 54.4, 62.4, and 200 GeV [36–39]. The experimental results show non-monotonic behavior of C4/C2 for net-proton distributions in central Au+Au collisions: at √s_{NN} = 200 GeV, C4/C2 1, decreases to about 0.1 at √s_{NN} 20 GeV, then rapidly increases to approximately 3.5 at √s_{NN} = 7.7 GeV. The latest STAR fixed-target experiment at 3 GeV and HADES results at 2.4 GeV both show negative values for C4/C2 [39]. To extract information about the CEP and its location in the phase diagram from these observed oscillations in baryon number fluctuations, we must understand how to extract effective information about the QCD phase transition from measurements along the chemical freeze-out line and carefully explore how the QCD phase transition affects properties such as cumulants of conserved charges. Figure 1 [Figure 1: see original paper] shows the results at √s_{NN} = 3 GeV from STAR and at 2.4 GeV from HADES [39].

To understand the non-monotonic behavior observed experimentally, theoretical investigation of net-proton fluctuations as a function of collision energy is necessary. For equilibrium systems, one can define the thermodynamic potential as Ω = −T/V ln(Z(V, T, _B, _Q, _S)), where T is temperature, V is system

volume, Z is the partition function, and _B, _Q, _S are baryon, charge, and strangeness chemical potentials, respectively. From the thermodynamic potential, the pressure is naturally obtained: $P = -\Omega$. The susceptibility of conserved charges is defined as the derivative of dimensionless pressure with respect to the corresponding chemical potential:

$$\hat{}\{BQS\}\_\{ijk\} = \hat{}(i+j+k)(P/T^4)/\hat{}\_B\hat{}i\ \hat{}\_Q\hat{}j\ \hat{}\_S\hat{}k$$

where $\hat{}\_q = \_q/T$ for q = B, Q, S. The relationship between cumulants of conserved charge distributions and the corresponding susceptibilities is:

$$C\hat{}\{BQS\}\{ijk\} = VT^3\ \ \hat{}\{BQS\}\{ijk\}$$

Through this relationship, one can theoretically calculate baryon number fluctuations as functions of temperature, chemical potential, and collision energy, compare with experimental results, and extract information about the QCD phase transition.

Different models yield different thermodynamic properties and phase transitions due to variations in fundamental degrees of freedom and interaction forms. Various models have been widely used to analyze the QCD critical point theoretically, such as the Nambu–Jona-Lasinio (NJL) model, the Polyakov-loop improved NJL model (PNJL), the linear σ model, the quark-meson (QM) model, the Polyakov-loop improved quark-meson model, the Dyson–Schwinger equations (DSE), and holographic QCD models [1–13, 40]. Additionally, hydrodynamic models are currently mainstream for analyzing baryon number fluctuations and other phenomena in heavy-ion collisions [41–44]. However, different models, or even the same model with different parameters, predict different CEP locations [45]. Therefore, combining theoretical and experimental results to explore the existence of the CEP and determine its position is essential.

2. Theoretical Frameworks

2.1 Hadron Resonance Gas Model

At low temperature and density, hadrons are the dominant degrees of freedom in strongly interacting matter. As temperature increases, the degrees of freedom transition from hadrons to quarks and gluons. In the Hadron Resonance Gas (HRG) model, non-interacting hadrons and resonances constitute the fundamental degrees of freedom. Under the Boltzmann approximation, the system pressure is given by [46]:

$$P/T^4 = (1/VT^3)\ \ln(Z(V,\ T,\ \_B,\ \_Q,\ \_S)) = \Sigma\_i\ (g\_i/2\pi^2)\ (m\_i/T)^2\ K_2(m\_i/T)\ \cosh(B\_i\hat{}\_B + Q\_i\hat{}\_Q + S\_i\hat{}\_S)$$

where m_i is the mass of hadron i, g_i is its degeneracy, and K_2 is the modified Bessel function. The sum runs over all hadrons and resonances, including their antiparticles. Results from the HRG model generally serve as a baseline for searching for QCD phase transitions and critical points in heavy-ion collisions.

For net-baryon number fluctuations, the ratios of cumulants take a simple form. Under the Boltzmann approximation, the baryon number susceptibility can be expressed as:

ˆB_2 = (1/VT^3) ^{2} ln(Z)/ˆ_B^2 = Σ_i (g_i/2π^2) (m_i/T)^2 K$_2$(m_i/T) B_i^2 cosh(B_iˆ_B + Q_iˆ_Q + S_iˆ_S) / Σ_i (g_i/2π^2) (m_i/T)^2 K$_2$(m_i/T) cosh(B_iˆ_B + Q_iˆ_Q + S_iˆ_S)

Therefore, considering protons (B = 1), the baryon number susceptibility becomes:

ˆB_2 = coth(_B/T)|_{ _Q= _S=0}

The ratio of fourth-order to second-order cumulants of interest in this work is:

C$_4^{B/C_2}$B = 1

Thus, in the low-temperature, low-density hadronic phase, C$_4$/C$_2$ = 1, which serves as a signature of the hadronic phase. As temperature increases toward the phase transition temperature (approximately 150 MeV), hadrons gradually "melt" and the dominant degrees of freedom become quarks and gluons, causing C$_4$/C$_2$ to deviate from 1—another criterion for phase transition.

2.2 Lattice QCD

Lattice QCD is a non-perturbative method based on first principles—quantum chromodynamics without any assumptions—that can fully calculate thermodynamic properties of strongly interacting systems in thermal equilibrium. Due to the "sign problem," lattice QCD cannot directly compute regions at finite chemical potential, but through Taylor expansion techniques, it can approximate small chemical potential regions. Specifically, the pressure at finite baryon chemical potential _B can be expanded as [49]:

P(T, _B) = P(T, 0) × [1 + (1/2!) ˆB_2(T) (ˆ_B)^2 + (1/4!) ˆB_4(T) (ˆ_B)^4 + (1/6!) ˆB_6(T) (ˆ_B)^6 + O(ˆ_B^8)]

Due to QCD symmetry, odd-order terms vanish, leaving only non-zero even-order contributions. The expansion coefficients relate to baryon number susceptibilities ˆB_2, ˆB_4, etc. Using the same principle, one can calculate values at finite chemical potential from baryon number susceptibilities:

n(T, _B) = Σ_{k=0}ˆ∞ (1/(2k+1)!) ˆB_{2k+2}(T) ˆ_Bˆ{2k+1}

Figure 2 [Figure 2: see original paper] shows the temperature dependence of $^{B\_4/}$ B_2 at zero chemical potential from lattice QCD [2, 49] and the pressure variation with temperature at different chemical potentials. At low temperatures, the results agree with the HRG model, approaching $^{B\_4/}$ B_2 = 1 as temperature approaches zero. To calculate regions at large chemical potentials, higher-order results are needed, requiring more computational time. Currently, lattice QCD calculations via Taylor expansion cannot be extended to sufficiently

high chemical potentials, and no CEP has been found in the region where the expansion is applicable.

2.3 Three-Flavor Polyakov-Loop Nambu–Jona-Lasinio Model

To study the phase diagram at finite temperature and chemical potential, using an appropriate effective model is essential. The NJL model is frequently employed to investigate the QCD phase diagram, but it predicts a phase transition temperature higher than lattice QCD results. Therefore, we adopt the three-flavor Polyakov-loop improved NJL model, which yields a chiral phase transition temperature comparable to lattice results and is thus called the realistic PNJL model (abbreviated as rPNJL). The rPNJL model includes eight-quark interactions.

The thermodynamic potential for an equilibrium system at finite temperature and chemical potential is directly written as [50]:

$\Omega = \Omega\_\{vac\} + \Omega\_\{quark\} + \Omega\_\{Polyakov\}$

where:

$\Omega\_\{vac\} = \Sigma\_f [(m\_f - M\_f)^2/(4g\_s) + (3g\_D/2) \sigma\_u \sigma\_d \sigma\_s + (3g\_1/2) \sigma\_f^2 + (g\_2/2) \sigma\_f^4]$

$\Omega\_\{quark\} = -2T \Sigma\_f \int (d^3p/(2\pi)^3) [\ln(1 + 3(\Phi + \Phi e^\{-(E\_f - \_f)/T\}) e^\{-(E\_f - \_f)/T\} + e^\{-3(E\_f - \_f)/T\}) + \ln(1 + 3(\Phi + \Phi e^\{-(E\_f + \_f)/T\}) e^\{-(E\_f + \_f)/T\} + e^\{-3(E\_f + \_f)/T\})]$

$\Omega\_\{Polyakov\} = U'(\Phi, \Phi, T)$

Here, the first term represents the vacuum energy; the second and third terms represent quark contributions at finite temperature and chemical potential; and the fourth term represents the Polyakov-loop contribution. The parameters g_s, g_D, g_1, and g_2 are coupling constants for quark interactions; $\sigma\_f = \bar{}\_f \_f$ represents quark condensates, with f = u, d for light quarks and f = s for strange quarks. The quark dispersion relation is $E\_f = \sqrt{(p^2 + M\_f^2)}$, where the dynamical mass M_f takes the form:

$M\_f = m\_f - 2g\_s \sigma f + 2g\_D \sigma\{f+1\} \sigma\_\{f+2\} - 2g\_1 \sigma\_f - 4g\_2 \sigma\_f^3$

with the convention that if $\sigma\_f = \sigma u$, *then* $\sigma\{f+1\} = \sigma d$ *and* $\sigma\{f+2\} = \sigma\_s$. Since the NJL model part is non-renormalizable, an appropriate regularization scheme is needed to eliminate divergences. Here, a three-momentum cutoff is used with an upper limit Λ for momentum integration.

The NJL model contains only quark degrees of freedom without gluons, thus lacking confinement effects. To compensate, Polyakov loops Φ and Φ are introduced to simulate partial gluon effects, as shown in their interaction with quarks in the second and third terms. The self-interaction of Φ and Φ is described by U', which takes the form [52]:

$U'(\Phi, \Phi, T) = T^4 \times [\ -b_2(T)/2\ (\Phi\Phi) -\ \ \ln[J(\Phi, \Phi)]\ ]$

where $J = (27/24\pi^2)\ (1 + (\Phi^3 + \Phi^3)/2)$ is the Jacobian determinant for the transformation from Polyakov loop to its trace, and is a dimensionless parameter restricting Φ and Φ values between 0 and 1. The temperature-dependent coefficient $b_2(T)$ is:

$b_2(T) = a_0 + a_1\ \exp(-a_2\ T_0/T)$

According to Ref. [50], parameters for the NJL part of the model can be determined from vacuum properties, while Polyakov-loop parameters can be fitted to pressure density at zero chemical potential. The specific parameter values are listed in Table 1 and Table 2 .

Under the equilibrium system assumption, quark condensates σ_f and Polyakov loops Φ, Φ are free variables obtained through the gap equations:

$\Omega/\ \sigma\_f = 0, \quad \Omega/\ \Phi = 0, \quad \Omega/\ \Phi = 0$

Note that at zero baryon chemical potential, $\Phi = \Phi$, but they differ at finite chemical potential. Once σ_f, Φ, and Φ are obtained for given temperature and chemical potential, baryon number fluctuations C_4, C_2, and σ^2 can be calculated using Eqs. (5) and (6). Figure 3 [Figure 3: see original paper] shows the temperature dependence of σ^2 at zero chemical potential calculated with the above parameters, where $T\_c\hat{}\ = 166$ MeV is the phase transition temperature at zero chemical potential from rPNJL. In the hadron resonance gas limit, $\sigma^2 = 1$, while in the free quark gas limit, σ^2 0.068.

To compare with experimental results, the relationship between baryon chemical potential and collision energy is needed, given by [53]:

$\sqrt{s}\_\{NN\}\ (\ \_B) = 1 + 0.343\ \ \_B + 0.053\ \ \_B^2$

Figure 4 [Figure 4: see original paper] shows the phase diagram from rPNJL calculations, with the CEP located at ($\_B\hat{}E = 720$ MeV, $T\hat{}E = 93$ MeV), indicated by the black triangle. The black line represents the first-order phase transition line, the blue dashed line indicates the crossover region, light blue points show RHIC BES-I collision results, and circles and black squares represent freeze-out temperatures and chemical potentials from low-energy heavy-ion collisions. The red and blue dashed lines are two chemical freeze-out lines, f_1 and f_2, respectively. The main difference between f_1 and f_2 appears at small chemical potentials (high collision energies): freeze-out line f_1 begins "behind" the phase boundary, while f_2 starts "in front of" the phase boundary.

3. Results and Discussion

To determine the collision energy dependence of baryon number fluctuation σ^2, the chemical freeze-out line must first be specified. Two forms are selected: f_1 and f_2. The form of f_2 is:

$T(\ \_B) = 0.158 - 0.14\ \ \_B^2 + 0.04\ \ \_B^4$

This form is fitted from RHIC experimental data [53], with f_1 being a slight modification of f_2:

$T(\ _B) = 0.158 - 0.01 \exp(0.067 - 0.14\ \ _B^2 + 0.04\ \ _B^4)/0.05$

where both temperature T and baryon chemical potential _B are in units of GeV.

Figure 5 [Figure 5: see original paper] presents σ^2 along freeze-out lines f_1 and f_2 from rPNJL calculations (red and blue dashed lines, respectively), compared with NJL results [57] (green dashed line) and STAR net-proton measurements [39] (black points).

Using freeze-out line f_1, which begins "behind" the phase boundary, the rPNJL-calculated σ^2 shows a "dip" structure as collision energy decreases, with a minimum around √s_{NN} 20 GeV. As collision energy continues to decrease, a peak structure appears at √s_{NN} 6 GeV, after which σ^2 decreases toward approximately 1. This overall behavior agrees well with STAR experimental results for $\sqrt{s_{NN}} \geq 7.7$ GeV. Along the second freeze-out line f_2, which starts "in front of" the phase boundary, σ^2 shows only a peak structure at √s_{NN} 6 GeV without the "dip" structure. In contrast, NJL model calculations [10] differ significantly from experimental results.

These results demonstrate that the decreasing-then-increasing behavior of baryon number σ^2 with collision energy is intimately related to the QCD phase transition. At small chemical potentials (high collision energies), the chemical freeze-out line crosses the crossover region from the QGP phase to the hadron gas phase. As collision energy decreases, the freeze-out line approaches and grazes near the CEP, causing σ^2 to first increase then decrease, forming a peak structure that signals CEP existence.

At $\sqrt{s_{NN}} = 3$ GeV and lower energies, experimental results show negative values, currently attributed to baryon number conservation effects at low energies [60]. This cannot be obtained from the rPNJL model used here, which predicts $\sigma^2 \to 1$ at low collision energies rather than negative values. This suggests that the rPNJL model cannot describe phase transition behavior and phase structure at high chemical potentials, possibly because it contains only quark degrees of freedom while lacking hadrons—the dominant degrees of freedom in the hadronic phase. One possible explanation is that a hadronic gas-liquid phase transition with its own critical end point exists in the hadronic phase at low temperatures and finite chemical potentials. In low-energy fixed-target experiments, the produced systems have large chemical potentials and low temperatures, approaching the critical end point region of the hadronic gas-liquid transition, where negative σ^2 values might similarly appear, analogous to the negative region near the QCD chiral phase transition CEP.

Since conserved charges exhibit dramatic fluctuations near phase transitions, the CEP can be searched for through baryon number fluctuations in heavy-ion collisions. Using the three-flavor PNJL model, which yields baryon number

fluctuations consistent with lattice results at zero chemical potential, and employing chemical freeze-out lines fitted from experimental data, we calculated the collision energy dependence of σ^2. Surprisingly, the results reproduce the experimental trend, including the peak structure at √s_{NN} = 7.7 GeV, which is a hallmark of CEP existence. Considering that experimental observables reflect the system's state at chemical freeze-out, when the system has already evolved to equilibrium, the good agreement between our equilibrium-based calculations and experimental data suggests that equilibrium assumptions may be reasonable for exploring post-collision system evolution in heavy-ion collisions. Additionally, obtaining results consistent with experiment depends crucially on: (1) the model providing a reasonable phase transition temperature at zero density and predicting a plausible CEP location, thus yielding a reasonable phase transition line; and (2) the sensitive relationship between the phase transition line and chemical freeze-out line determining the final σ^2 versus collision energy behavior. It is worth emphasizing that the decrease of σ^2 with decreasing collision energy and its minimum at √s_{NN} = 19.6 GeV are determined by the freeze-out line lying above the phase transition line at low chemical potentials.

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