Advances in Lattice QCD under Strong Magnetic Fields (Postprint)

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Abstract

This article reviews recent progress in studying the properties of Quantum Chromodynamics (QCD) under strong magnetic fields using lattice QCD. It first introduces how to incorporate magnetic fields on the lattice, focusing on the chiral properties of QCD at zero temperature, the QCD phase transition temperature and inverse magnetic catalysis, as well as the QCD phase structure under strong magnetic fields, and finally provides a summary.

Full Text

Preamble

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Progress on Lattice QCD Studies in Strong Magnetic Fields

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Abstract

This article reviews recent progress in studying the properties of Quantum Chromodynamics (QCD) in strong magnetic fields using lattice QCD. We first introduce how magnetic fields are implemented on the lattice, then focus on discussing the chiral properties of QCD at zero temperature, the QCD transition temperature and inverse magnetic catalysis, and finally the QCD phase structure in strong magnetic fields, concluding with a summary.

Keywords: Strong magnetic fields, Lattice QCD, Phase structure, Chiral properties

Introduction

In recent years, the properties of hot matter in strong magnetic fields have become a new research focus in the field of high-energy heavy-ion collisions. Off-central relativistic heavy-ion collisions can generate magnetic fields with intensities of 10^{18}–10^{20} Gauss [?, ?, ?, ?], reaching magnitudes comparable to the pion mass squared or the characteristic energy scale of QCD. Similar magnetic fields may have existed in the early universe [?] and in magnetars [?]. Consequently, electromagnetic interactions between magnetic fields and quarks become non-negligible. A prominent and well-known effect induced by magnetic fields is the chiral magnetic effect [?, ?]. Due to the QCD chiral anomaly, the number of fermions of one chirality (e.g., left-handed) exceeds that of the other chirality (right-handed). Under polarization by strong magnetic fields, the total momentum of these fermions aligns with the magnetic field direction, leading to charge separation and current formation. High-energy heavy-ion collision experiments have measured signals consistent with charge separation effects, but it remains uncertain whether these signals originate entirely from the chiral magnetic effect or from other backgrounds [?, ?, ?, ?]. A significant reason for this uncertainty is the lack of clarity regarding the thermodynamic properties of QCD chiral anomalies in strong magnetic fields [?, ?] and the dynamic properties such as the survival time of magnetic fields in heavy-ion collisions [?].

Unlike non-zero baryon chemical potential, introducing external magnetic fields into first-principles lattice QCD simulations does not introduce a sign problem. Therefore, lattice QCD can directly simulate various properties of strongly interacting matter under external magnetic fields. The magnitude of the external magnetic field is determined by the simulated lattice spacing and the spatial volume of the system. Currently, lattice QCD has achieved many important advances in studying hadron properties and phase structure in QCD at both zero and finite temperatures. For recent reviews on lattice QCD studies in strong magnetic fields, please refer to [?, ?, ?].

Implementing Magnetic Fields in Lattice QCD

We generally consider a static external magnetic field $\mathbf{B} = (0, 0, B)$ pointing in the z-direction and constant in time. The external magnetic field is described by fixed $U(1)$ link factors $u_\mu(n)$, which in Landau gauge take the form [?]:

$$u_x(n_x, n_y, n_z, n_\tau) = \exp\left[iqa_x^{2BN} n_y\right], \quad u_y(n_x, n_y, n_z, n_\tau) = \exp\left[iqa_x^{2Bn}\right], \quad u_z(n_x, n_y, n_z, n_\tau) = u_\tau(n_x, n_y, n_z, n$$

where $n_\mu = 0, 1, 2, ..., N_\mu - 1$ represents coordinates in the $\mu = x, y, z, \tau$ directions, and N_μ denotes the total number of lattice points in the μ direction.

Considering periodic boundary conditions, the magnetic field on the lattice is quantized as:

$$eB = \frac{2\pi N_b}{a^2 N_x N_y}$$

where q is the quark charge, and $N_b \in \mathbb{Z}$ is the number of magnetic flux quanta passing through a unit area in the X-Y plane ($N_x N_y a^2$ being the area of the X-Y plane). Current lattice QCD studies typically examine $2+1$-flavor QCD, considering up, down, and strange quarks, with degenerate up and down quark masses. Since the up quark has charge $+2/3e$ while the down and strange quarks have charge $-1/3e$, and the quantization condition must be satisfied for all quarks in the system, lattice QCD simulations choose the greatest common divisor of all quark charges, $q = 1/3e$. Consequently, the magnetic field strength is expressed as:

$$eB = \frac{2\pi N_b}{3a^2 N_x N_y}$$

In practical simulations, periodic boundary conditions for the $U(1)$ links apply to all directions except the X direction. Due to boundary condition constraints, N_b is restricted to the range $0 \leq N_b < N_x N_y/4$. The minimum and maximum magnetic field strengths achievable in lattice QCD simulations are therefore:

$$eB_{\min} = \frac{2\pi}{L^2}, \quad eB_{\max} = \frac{\pi}{2a^2}$$

where L is the physical length in the spatial direction. Thus, to obtain smaller magnetic field strengths requires increasing the physical length of the lattice simulation volume, while larger magnetic field strengths require decreasing the lattice spacing a.

Currently, lattice QCD studies in strong magnetic fields generally discuss either $2+1$-flavor QCD or quenched QCD. Simulations including sea quarks employ naive staggered fermions [?, ?], as well as two improved staggered fermion discretization schemes: the Highly Improved Staggered Quarks (HISQ) scheme [?] and the stout fermion scheme [?, ?]. For quenched QCD, primarily used in hadron spectroscopy studies, valence quark discretization schemes include Overlap fermions [?] and Wilson fermion discretization [?].

2.1 Chiral Properties of QCD at Zero Temperature

Hadron spectra in lattice QCD can be extracted from hadron two-point correlation functions. When the temporal extent approaches infinity, the two-point correlation function $G_H(\tau)$ relates to the hadron ground state mass M_H as:

$$G_H(\tau) \sim e^{-M_H \tau}$$

The correlation function $G_H(\tau)$ can be constructed from operators corresponding to different channels $\mathcal{O}_H^\dagger(\tau, \mathbf{x})$. For example, for pseudoscalar meson operators P^a and vector meson operators V_μ^a:

$$P^a = \bar{\psi}\gamma_5\tau^a\psi(\mathbf{x}), \quad V_\mu^a = \bar{\psi}\gamma_\mu\tau^a\psi(\mathbf{x})$$

For charged ρ mesons, due to their spin-1 nature, there are two polarization directions with corresponding operators [?]:

$$V_\mu^{a,\pm} = \frac{1}{\sqrt{2}}(V_x^a \pm iV_y^a)$$

When studying hadron spectra in magnetic fields, two-point correlation functions constructed using Wilson or Overlap discretization schemes and staggered fermion discretization are generally employed. The staggered fermion discretization has inherent disadvantages for hadron spectroscopy: it breaks taste symmetry, leading to contributions from oscillating, unphysical parity partners in hadron two-point correlation functions. Fortunately, pseudoscalar mesons do not have such unphysical oscillating states, but for studying vector and other meson mass spectra, the staggered fermion discretization becomes inadequate. Wilson or Overlap discretization schemes, however, are computationally expensive for generating lattice QCD configurations, limiting vector meson studies to the quenched approximation.

Figure 1 Figure 1: see original paper shows the mass dependence of charged pseudoscalar mesons, pions and kaons, on magnetic field strength. These results are obtained from 2+1-flavor lattice QCD using highly improved staggered fermions [?]. When the magnetic field strength is less than approximately $0.3\,\mathrm{GeV}^2$, both pion and kaon masses can be described by the Lowest Landau Level (LLL) approximation. For magnetic field strengths greater than $0.3\,\mathrm{GeV}^2$, pion and kaon masses deviate from the LLL approximation. When the magnetic field strength exceeds approximately $0.7\,\mathrm{GeV}^2$, their masses decrease with increasing magnetic field. This deviation from LLL approximation is not observed in quenched lattice QCD simulations, suggesting that charged pions and kaons can no longer be considered point particles in strong magnetic fields.

Neutral pseudoscalar mesons also exhibit interesting behavior in magnetic fields. A neutral point particle without internal structure would not electromagnetically interact with magnetic fields. However, as shown in Figure 1(b), the masses of neutral π^0 and K^0 decrease with increasing magnetic field strength. Two important properties must be noted: first, π^0 and K^0 are no longer point particles, otherwise their masses would be independent of magnetic field strength; second, the masses of neutral pseudoscalar mesons are always smaller than their charged counterparts, reflecting isospin symmetry breaking in strong magnetic fields, and indicating that neutral pseudoscalar mesons rather than their charged partners should be the Goldstone bosons. These results have attracted considerable attention from phenomenological theoretical interpretations [?, ?, ?, ?, ?, ?].

Figure 2 [Figure 2: see original paper] shows the mass dependence of vector meson ρ on magnetic field strength, obtained using Wilson discretization in the quenched approximation [?]. Considering that ρ has spin-1, charged ρ mesons have different polarization directions. Red points represent positively charged ρ mesons with spin parallel to the magnetic field, blue points represent positively charged ρ mesons with spin anti-parallel to the magnetic field, and magenta points represent neutral ρ mesons. Different polarization directions of charged ρ mesons show completely different dependencies on magnetic field strength: the mass of positively charged ρ mesons with spin parallel to the magnetic field decreases with increasing magnetic field, while those with spin anti-parallel increase with magnetic field strength. For neutral ρ mesons, their mass increases with magnetic field strength, which differs significantly from neutral pseudoscalar mesons.

Decay constants of hadrons can be extracted from two-point correlation functions [?]. Figure 3 Figure 3: see original paper shows the dependence of neutral pion and kaon decay constants on magnetic field strength. Both increase with magnetic field strength. At small magnetic field strengths, the neutral pion decay constant is smaller than that of neutral kaon, but when the magnetic field strength exceeds approximately $0.35\,\mathrm{GeV}^2$, the former surpasses the latter. Figure 3(b) displays the dependence of the squares of decay constants extracted from the up-quark and down-quark components of the neutral pion correlation function on magnetic field strength. The figure shows that at strong magnetic fields, predictions from low-energy effective theory [?] (represented by solid lines) are consistent with lattice results.

At zero temperature, the chiral condensate also increases with magnetic field strength, showing a power-law relationship in weak fields (Figure 4 Figure 4: see original paper) and a linear relationship in strong fields (Figure 4(b)) [?]. Additionally, the up-quark and down-quark chiral condensates are no longer identical, indicating that isospin symmetry is broken in magnetic fields. This is primarily due to the different electric charges of up and down quarks, with the larger absolute charge of up quarks leading to a larger chiral condensate than down quarks.

Combining the available data on neutral pseudoscalar meson decay constants, masses, and chiral condensates, one can examine the Gell-Mann-Oakes-Renner (GMOR) relation [?]:

$$M_{\pi^0}^2 f_{\pi^0}^2 = (m_u + m_d)\langle\bar{\psi}\psi\rangle + \delta_\pi, \quad M_{K^0}^2 f_{K^0}^2 = (m_d + m_s)\langle\bar{\psi}\psi\rangle + \delta_K$$

where δ_H represents corrections to the GMOR relation in magnetic fields. These correction results are shown in Figure 5 [Figure 5: see original paper], representing the first lattice QCD study of magnetic field corrections to the GMOR relation. The figure shows corrections for two-flavor and three-flavor GMOR relations. Despite different ultraviolet cutoffs (λ_{UV}) yielding different chiral condensate values, the trend of magnetic field corrections to the GMOR relation with magnetic field strength shows little variation (all corrections tend to zero in strong magnetic fields). This suggests to some extent that neutral pions and kaons maintain their Goldstone boson characteristics. It should be noted that in these results, the pion mass has not reached the chiral limit, and the lattice calculations have not performed continuum extrapolation. Therefore, these corrections contain contributions not only from magnetic fields but also from finite mass effects and lattice discretization artifacts.

2.2 QCD Transition Temperature and Inverse Magnetic Catalysis

Since the chiral condensate serves as the order parameter for the chiral phase transition, the degree of chiral symmetry breaking correlates with the magnitude of the chiral condensate. An increase in the chiral condensate indicates enhanced chiral symmetry breaking. Early lattice QCD studies showed that at zero temperature, the chiral condensate increases with magnetic field strength, a phenomenon known as magnetic catalysis [**?**]. Consequently, early phenomenological theoretical studies predicted that higher temperatures would be required to restore chiral symmetry in strong magnetic fields, i.e., the transition temperature would increase with magnetic field strength. However, surprisingly, lattice QCD results at the physical point and in the continuum limit show that the QCD pseudo-critical temperature decreases with increasing magnetic field (Figure 6 Figure 6: see original paper) [**?**]! Simultaneously, near the critical temperature (Figure 6(b)), the up-quark chiral condensate no longer increases monotonically with magnetic field strength as at zero temperature, but instead first increases then decreases, or even decreases monotonically with magnetic field [**?**, **?**]. This decrease in chiral condensate is generally referred to as inverse magnetic catalysis, sometimes closely linked with the simultaneous decrease in pseudo-critical temperature. Interestingly, inverse magnetic catalysis or magnetic catalysis phenomena have also been observed recently in strange quark condensates [**?**].

Recent studies indicate that the decrease in pseudo-critical temperature and the decrease in chiral condensate do not necessarily occur simultaneously. Figure 7 [Figure 7: see original paper] shows the difference between the chiral condensate at magnetic field strength $0.6\,\mathrm{GeV}^2$ and at zero magnetic field as a function of pion mass at the pseudo-critical temperature [**?**]. When the pion mass is small, e.g., less than $500\,\mathrm{MeV}$, inverse magnetic catalysis (IMC) is observed, where the difference is negative. When the pion mass exceeds $500\,\mathrm{MeV}$, the difference becomes positive, indicating magnetic catalysis (MC). Notably, the magnitude of the pion mass does not change the trend of pseudo-critical temperature variation; the pseudo-critical temperature always decreases with increasing magnetic field strength. Therefore, this demonstrates that neither magnetic catalysis nor inverse magnetic catalysis is a necessary or sufficient condition for the increase or decrease of pseudo-critical temperature, and the two phenomena should not be confused.

The decrease in pseudo-critical temperature with increasing magnetic field strength is likely related to the pion mass variation trend in the system. We can draw lessons from the zero magnetic field case: as shown in Figure 8 Figure 8: see original paper, the system's pseudo-critical temperature $T_{\rm pc}$ decreases with decreasing quark mass (system pion mass) [**?**]. In non-zero magnetic fields, the system's pion mass (Figure 1(b)) also decreases with increasing magnetic field strength. Comparing with the zero magnetic field case, this seems to explain why the system's pseudo-critical temperature decreases with increasing magnetic field strength: when the mass of the lightest Goldstone boson, the pion, decreases—whether in magnetic fields or at zero magnetic field—the system's pseudo-critical temperature always decreases.

This assertion can be further confirmed by examining the contribution of hadrons to the system's energy density. In the hadron resonance gas model, one can investigate the contributions of various hadrons to thermodynamic quantities. Figure 8(b) shows that at zero magnetic field strength, the dominant contribution to the system's energy density comes from pions [**?**]. When the magnetic field is non-zero, considering that neutral pion and kaon masses decrease while charged pion and kaon masses increase, the main contribution to the system's energy density comes from neutral pions, and their proportion becomes larger compared to the zero magnetic field case. This indicates that neutral pions dominate the thermodynamic properties of the system. On the other hand, when the mass of the lightest degree of freedom in the system becomes smaller, the system is more easily excited to another phase state. This is consistent with the decrease in the system's pseudo-critical temperature as magnetic field strength increases.

On the other hand, whether inverse magnetic catalysis occurs in the system is also closely related to the properties of neutral pions. At zero magnetic field, the well-known Ward-Takahashi identity connects two quantities related to chiral properties: the chiral condensate and the two-point correlation function of Goldstone bosons. This identity also holds in non-zero magnetic fields [**?**] and can be extended from neutral pions to other neutral pseudoscalar mesons such as kaons and η. The specific expressions of these identities are:

$$(m_u+m_d)\chi_{\pi^0} = \langle\bar{\psi}_u\psi_u\rangle+\langle\bar{\psi}_d\psi_d\rangle, \quad (m_d+m_s)\chi_{K^0} = \langle\bar{\psi}_d\psi_d\rangle+\langle\bar{\psi}_s\psi_s\rangle, \quad (m_u+m_s)\chi_\eta = \langle\bar{\psi}_u\psi_u\rangle+\langle\bar{\psi}_s\psi_s\rangle$$

where χ is the spacetime sum of the corresponding hadron's correlation function, i.e., $\chi_H = \sum_z G_H(z)$. $G_H(\tau)$ and $G_H(z)$ represent the hadron's two-point temporal and spatial correlation functions, respectively.

Figure 9 [Figure 9: see original paper] provides numerical verification from lattice QCD for up, down, and strange quark condensates and the corresponding neutral pion, kaon, and η at pion masses of 140 MeV and 281 MeV [?]. Notably, unlike the GMOR relation, the Ward-Takahashi identity applies at arbitrary temperature, magnetic field, and quark mass, whereas the GMOR relation only applies at zero temperature and sufficiently small quark mass.

Thus, using the Ward-Takahashi identity, magnetic catalysis or inverse magnetic catalysis observed through chiral condensates can be connected to the two-point correlation functions of neutral pions and kaons. Through the exponential decay of hadron spatial two-point correlation functions in spatial directions, one can define the following hadron screening mass M_H:

$$G_H(z) \sim e^{-M_H z}$$

Note that the hadron ground state masses in §3.1 are extracted from temporal two-point correlation functions, whereas here we examine screening masses extracted from spatial two-point correlation functions at finite temperature.

Figure 10 Figure 10: see original paper shows the ratio of neutral pion screening masses at non-zero magnetic fields to their values at zero magnetic field [?]. At temperatures close to zero, this ratio decreases with increasing magnetic field strength and remains below 1 across the entire magnetic field range. As temperature gradually increases, the decreasing trend of the ratio with magnetic field strength becomes less pronounced, and even at $T = 140\,\mathrm{MeV}$, non-monotonic behavior appears—first decreasing then increasing. When temperature rises to 169 MeV, which is the transition temperature of the system at zero magnetic field in this lattice simulation, the ratio changes from below 1 to above 1 and increases with magnetic field strength. At $T = 211\,\mathrm{MeV}$, the ratio becomes smaller but remains above 1, still increasing with eB though more slowly than at 169 MeV. At the highest temperature of 281 MeV, the ratio becomes smaller than 1 again, decreasing with eB when $eB < 1\,\mathrm{GeV}^2$, but essentially independent of eB when $eB > 1.5\,\mathrm{GeV}^2$. This complex behavior varying with magnetic field and temperature is consistent with the behavior of chiral condensates, because screening masses correspond to the long-distance behavior of two-point correlation functions: when screening masses decrease, long-distance contributions increase, and vice versa. Similar behavior appears in neutral kaon screening masses, as shown in Figure 10(b), though the variation amplitude is smaller than for neutral pions.

The cause of inverse magnetic catalysis is currently believed to be related to sea quarks in the system. To separate valence and sea quark effects, valence quark condensates and sea quark condensates have been defined. In the former calculation, magnetic fields exist only in propagators, not in the system's partition function; in the latter, the opposite is true—magnetic fields exist only in the partition function, not in propagators. It has been found that valence quark condensates always increase with magnetic field strength (showing only magnetic catalysis), while sea quark condensates decrease with magnetic field strength (showing inverse magnetic catalysis) [?]. Similarly, one can define valence and sea quark-dominated hadron two-point correlation functions and extract screening masses from them [?]. Figure 11 [Figure 11: see original paper] shows the ratio of screening masses extracted from valence and sea quark neutral pion two-point correlation functions to their zero-field values at different temperatures as functions of magnetic field strength [?]. The ratio of valence quark screening masses is always less than 1 and decreases monotonically with magnetic field strength, while the ratio of sea quark screening masses is always greater than or equal to 1 and increases with magnetic field strength when temperature exceeds 120 MeV. Additionally, at a fixed magnetic field strength (e.g., $2\,\mathrm{GeV}^2$), the valence quark screening mass ratio always increases with temperature, while the sea quark screening mass ratio shows very pronounced non-monotonic behavior, reaching its maximum at $T = 169\,\mathrm{MeV}$. These findings are consistent with the behaviors of valence and sea quark condensates and also indicate that the decrease in pseudo-critical temperature is primarily caused by sea quark effects, since valence quarks are merely probes of the system and do not affect the system's thermodynamic properties such as pseudo-critical temperature.

2.3 QCD Phase Structure in Strong Magnetic Fields

QCD in strong magnetic fields may exhibit a rich phase structure [?]. At zero magnetic field, useful physical quantities for studying QCD phase structure are the correlations and fluctuations of conserved charges—baryon number (B), electric charge (Q), and strangeness (S) [?, ?, ?, ?, ?, ?, ?]. These conserved charge correlations and fluctuations are defined as Taylor expansion coefficients of pressure P near zero baryon (μ_B), charge (μ_Q), and strangeness (μ_S) chemical potentials [?, ?]:

$$\chi_{BQS}^{ijk} = \frac{\partial^{i+j+k}(P/T^4)}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k}\bigg|_{\mu_B=\mu_Q=\mu_S=0}$$

Since Eq. (11) is calculated at zero chemical potential, there is no sign problem. Considering that system pressure P depends on magnetic field eB, one can obtain fluctuations of conserved charges in the system at non-zero magnetic fields [?, ?].

At zero magnetic field, the system has isospin symmetry, meaning $\chi_{uu}^{11} = \chi_{dd}^{ss}$ and $\chi_{us}^{11} = \chi_{ds}^{11}$, from which we can derive:

$$\frac{2\chi_{BQ}^{11}}{\chi_S^{11}} = 1, \quad \frac{2\chi_{QS}^{11}}{\chi_B^{11}} = 1$$

However, in non-zero magnetic fields, since up and down quarks have different electric charges, isospin symmetry is clearly broken. As shown in Figure 4, the up and down quark chiral condensate values differ. Consequently, Eqs. (11) and (12) no longer hold in magnetic fields.

As shown in Figure 12 [Figure 12: see original paper], $(2\chi_{BQ}^{11})/\chi_S^{11}$ and $(2\chi_{QS}^{11})/\chi_B^{11}$ both equal 1 at zero magnetic field. Once the magnetic field is turned on, they deviate from 1, indicating isospin symmetry breaking. At the same magnetic field strength, lower temperatures show more severe isospin symmetry breaking [?]. Since in the high-temperature, strong-field free limit, the physical quantities in Figure 12(a) approach 1, they all exhibit non-monotonic behavior at different temperatures: first decreasing with increasing magnetic field strength, deviating from the isospin-symmetric value of 1, then rising again at strong magnetic fields and approaching the high-temperature, strong-field free limit value of 1. Unlike the quantities in Figure 12(a), which all approach the high-temperature, strong-field limit from below, the quantities in Figure 12(b) approach this limit from below only at the two highest temperatures.

Figure 13 [Figure 13: see original paper] directly shows the second-order baryon number fluctuation χ_B^2 as a function of temperature at different magnetic fluxes (i.e., different magnetic field strengths) [?]. At zero magnetic field strength ($N_b = 0$), χ_B^2 increases monotonically with temperature. When $N_b \geq 12$, χ_B^2 exhibits non-monotonic behavior with temperature—first increasing then decreasing—showing a peak shape. As magnetic field strength increases, the peak becomes more pronounced and its height grows, showing a diverging trend. The peak position also shifts to lower temperatures with increasing magnetic field strength, consistent with the decrease in pseudo-critical temperature with magnetic field.

Early predictions suggested that a QCD critical point might exist in extremely strong magnetic fields [?]. Referring to the critical behavior of baryon number fluctuations at zero magnetic field and replacing the external field from quark mass to magnetic field, the critical behavior of the second-order baryon number fluctuation peak in magnetic fields is:

$$\chi_{B,\max}^2 = b(eB_c - eB)^{-\alpha/\beta\delta} + d$$

where eB_c is the magnetic field strength at the critical point, and α, β, δ are critical exponents of the $Z(2)$ universality class. Fitting the peaks of χ_B^2 in Figure 13(a) using Eq. (13) yields a critical magnetic field strength of approximately $7\,\mathrm{GeV}^2$. Note that this result is obtained from lattice QCD simulations with non-physical pion masses. Recent results at the physical point and small magnetic fields can be found in [?].

On the other hand, it should be pointed out that both Figure 13 and Figure 12 demonstrate that strong magnetic fields significantly affect fluctuations of conserved charges. Since the magnetic field strength produced in the early stage of non-central heavy-ion collisions is stronger than that in central collisions, one can construct a centrality-dependent physical quantity related to conserved charge fluctuations to probe the imprint of magnetic fields in heavy-ion collisions [?, ?, ?, ?, ?, ?, ?].

Figure 14 [Figure 14: see original paper] shows a schematic diagram of the QCD phase diagram in strong magnetic fields [?]. Recent lattice QCD studies have directly simulated results at very strong magnetic field strengths ($eB = 4\,\mathrm{GeV}^2$ and $9\,\mathrm{GeV}^2$) with physical pion masses [?]. By examining the dependence of chiral condensates and their susceptibilities on magnetic field, temperature, and volume, it was found that at $eB = 4\,\mathrm{GeV}^2$, the transition from the hadronic phase to the quark-gluon plasma phase remains a rapid crossover without any singular behavior. The temperature of this crossover transition is estimated to be 98 MeV (Figure 14). When the magnetic field strength increases to $9\,\mathrm{GeV}^2$, metastable states appear in the time history of chiral condensates, indicating a first-order phase transition with an estimated critical temperature of 63 MeV. Consequently, a QCD critical point must exist between the crossover and first-order phase transition. This QCD critical point should obey $Z(2)$ universality. Currently, the location of this QCD critical point has not been determined, and its study will also provide insights for searching for the QCD critical point in the temperature-chemical potential plane. On the other hand, the critical temperature of the first-order phase transition at $eB = 9\,\mathrm{GeV}^2$ has already decreased to 63 MeV. A plausible speculation is whether a sufficiently strong magnetic field could still induce a phase transition in the vacuum at zero temperature. This is also worth investigating in future studies.

Conclusion

Lattice QCD studies at finite temperature and density began in the 1970s, while lattice QCD studies in strong magnetic fields started only around 2010 but have already achieved many important results. This article mainly reviewed chiral properties of QCD at zero temperature, inverse magnetic catalysis, QCD transition temperature, and QCD phase structure studies in ultra-strong magnetic fields. In addition, lattice QCD has been used to study magnetic susceptibility of QCD matter [?], the QCD equation of state in strong magnetic fields [?, ?, ?], electrical conductivity [?], heavy quark chemical potential and string tension [?], QCD phase transitions at finite baryon chemical potential and strong magnetic fields [?], and baryon masses in strong magnetic fields [?], QCD phase structure at small quark masses and strong magnetic fields [?], among others. Given that lattice QCD studies in strong magnetic fields have only been conducted for just over a decade, much physics remains to be explored.

Author Contributions

Ding Hengtong, Li Shengtai, and Liu Junhong contributed equally to this work.

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Note: Figure translations are in progress. See original paper for figures.