Phase Transitions of Strongly Interacting Matter in Vortex Fields: Postprint

Authors: Jiang Yin, Liao Jingfeng

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Abstract

The investigation of phase structures in strongly interacting matter constitutes a highly active frontier domain in nuclear physics, bearing significant implications for understanding relativistic heavy-ion collision experiments and neutron star observations. Traditional phase structure studies have focused on matter property variations induced by high temperatures and high densities, particularly concerning associated phase transition processes such as chiral restoration transitions, color superconductivity transitions, among others. Recent experimental and theoretical research has revealed that non-central heavy-ion collision systems carry substantial initial angular momentum, thereby generating extremely intense hydrodynamic vorticity fields. This development has precipitated a series of important questions regarding the properties of strongly interacting matter within vorticity fields, with investigations into these questions yielding numerous novel results. Specifically in the context of phase transitions in strongly interacting matter, thermal field theory calculations in rotating reference frames have been developed, and in conjunction with mean-field approximations, have facilitated in-depth studies of related phenomena including chiral phase transitions, color superconductivity transitions, and superfluid phase transitions at high isospin, thereby uncovering the influence of vorticity fields on phase boundaries and the rich phase structures they engender.

Full Text

Preamble

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Phase Transitions of Strong Interaction Matter in Vorticity Fields

Jiang Yin[1,2], Liao Jinfeng[3]
[1](Beihang University, Beijing 100083, China)
[2](Hangzhou Innovation Institute (Yuhang), Beihang University, Hangzhou 310023, China)
[3](Department of Physics, Indiana University, Bloomington, IN 47405, USA)

Abstract

Understanding the phase structures of strong interaction matter is an active frontier in nuclear physics research currently, and it will provide crucial insights into heavy-ion collision experiments as well as neutron star observations. Most studies in this area focus on the influence of extremely high temperatures and baryon densities on matter properties, especially pertaining to phase transitions such as chiral symmetry breaking and color superconductivity. Recent experimental and theoretical studies reported that in non-central heavy-ion collisions, systems carry a large initial angular momentum that becomes very strong vorticity fields in the bulk fluid. This has thus introduced several new questions regarding the properties of strong interaction matter under vorticity fields, and have led to many novel results. Thermal field theory calculations based on rotating frame and mean-field approximation have been developed to study various phase transitions under rotation, such as chiral symmetry breaking, color superconductivity and superfluidity at high isospin asymmetry. The results have demonstrated important impacts of vorticity fields on the phase boundaries of these transitions, and have also revealed nontrivial new phase structures of strong interaction matter under rotation. A new dimension of the usual QCD phase diagram has been unveiled. The study of rotation-induced phase transition extends phase transition research to a broader space. There remain more unexplored issues that merit further study.

Key words Heavy-ion collision, Quark-gluon plasma, Fluid vorticity, Spin polarization, Chiral symmetry breaking, Color superconductivity

Introduction

The vast majority of mass in the visible material world is carried by nucleons, which are composed of color-charged quarks and gluons. The strong interaction between them is described by Quantum Chromodynamics (QCD), a non-Abelian gauge field theory. QCD is one of the main components of the Standard Model and remains one of its most mysterious aspects. Perturbative calculations show that as the interaction energy scale increases, the coupling constant of chromodynamics continuously decreases—a phenomenon known as asymptotic freedom. In the low-energy region, since free quarks or gluons have never been observed, it is believed that the interaction in this region should be strong enough to tightly bind color-charged quarks and gluons within color-neutral hadrons. This conjecture is known as color confinement or color imprisonment. To date, the confinement of color charge remains an important problem that has not been mathematically proven. Regarding bound states of light quarks, it was found that the small current masses of light quarks (only 5-10 MeV) cannot explain the excessive mass of nucleons and the light mass of pions from the perspective of ordinary binding potentials. A qualitative understanding of the mass distribution in the hadron spectrum comes from the spontaneous breaking of chiral symmetry of light quarks in the vacuum and the existence of axial anomalies. Since few-body nucleon systems are finite, color confinement and symmetry breaking cause the nuclear force, as a residual effect of QCD, to be far more complicated than QCD itself. Therefore, to reveal the essence of QCD, taking multi-body systems composed of quarks and gluons as research objects and exploring the phase diagram of strong interaction matter under different thermodynamic conditions has become an important approach to further understanding quantum chromodynamics [1-5]. Taking color confinement and chiral symmetry as important entry points, studying the breaking and restoration of strong interaction symmetries at different temperatures and densities constitutes the important research content of traditional QCD phase transitions. Using non-perturbative techniques such as lattice QCD, Dyson-Schwinger equations, and functional renormalization group, it has been found that in the region of small baryon chemical potential, chiral symmetry breaking is continuously restored with increasing temperature, color confinement is broken, and quarks and gluons in nuclear matter enter a relatively free state. This phase is called quark-gluon plasma (QGP). In the low-temperature region, the existence of baryon chemical potential prevents lattice calculations, and results from other non-perturbative schemes and effective models show that the chiral symmetry of nuclear matter is still restored with increasing chemical potential, confinement is lifted, and quarks will pair to form quark pairs similar to those in traditional BCS superconductors. This state is called color superconductivity. Due to asymptotic freedom, the existence of the color superconducting state at extremely high chemical potentials can be confirmed by perturbative calculations. In the region of moderate chemical potential, model calculations suggest that the system undergoes a first-order phase transition when entering this phase. Since first-order phase transitions involve discontinuous jumps of order parameters, they are difficult to confirm from non-perturbative calculations with limited precision. Therefore, the phase structure in the moderate chemical potential region remains an unsolved problem. Relativistic heavy-ion collision experiments can produce high-temperature, low-chemical-potential QGP matter. By changing collision energy and the types of heavy nuclei collided, it is hoped to scan different chemical potential regions of the phase diagram. Figure 1 [Figure 1: see original paper] schematically illustrates the QCD phase diagram in temperature–baryon chemical potential–rotation space. If the first-order phase transition line in the low-temperature region indeed exists, then its intersection with the confirmed high-temperature continuous transition line will produce a second-order phase transition point or region, which provides a way to experimentally detect the existence of the first-order phase transition line.

Observations and phenomenological model simulations from heavy-ion collision experiments have found that QGP is the system with the highest energy density, smallest viscosity coefficient, strongest magnetic field intensity, and most violent rotation among all matter created by humans, produced by collisions of extremely high-speed heavy nuclei. In this system, the extremely high temperature causes the thermal motion of quarks and gluons—the basic components of nuclear matter—to break the confinement of hadrons, moving relatively freely in the collision region (scale ~10 fm) and forming a quark-gluon plasma that can be described by hydrodynamics. In non-central relativistic heavy-ion collision experiments, the system itself has a huge initial magnetic field and angular momentum, and the produced quark-gluon plasma often carries enormous magnetic fields and angular momentum. These background fields with definite orientation not only change the microscopic behavior of quarks and gluons but also explicitly break the original rotational symmetry of the system. Therefore, from the general theory of continuous phase transitions, it can be expected that background fields will have nontrivial effects on the phase transition process. Studying the effects of magnetic fields and vorticity in QCD phase transitions can further promote the exploration of the specific mechanisms of strong interactions.

Among these two, research on magnetic fields started earlier and has achieved many important results [7–15]. In chiral systems, magnetic fields polarize the spin direction of fermions, thereby locking the momentum direction and producing the chiral magnetic effect, which leads to measurable charge separation phenomena. This phenomenon has been observed in condensed matter systems. Based on the chiral magnetic effect, a series of chiral-related electromagnetic effects have been discovered. In quark-gluon plasma with extremely strong magnetic fields and naturally chiral fermions, the existence of the chiral magnetic effect is almost certain, but confirming its signal from background noise requires more effort. Similar to magnetic fields, strong vorticity fields have also been observed in experiments [16–18]; in nature, vorticity is also a pseudovector field with definite orientation that polarizes angular momentum, but it is not entirely the same as magnetic fields [19–22]. First, vorticity is a purely kinematic effect without electromagnetic interaction coupling, so its polarization of angular momentum is indiscriminate without charge distinction. Second, vorticity is an important component of the system' s total angular momentum, and due to angular momentum conservation constraints, the decay of vorticity with system expansion may be slower than that of magnetic fields, thus providing more opportunities to observe vorticity effects. Finally, vorticity is actually part of the single-particle motion that constitutes the many-body system, rather than an independently existing background field like magnetic fields. This characteristic brings great theoretical difficulties, especially for interacting systems, where calculating rotational corrections to coupling vertices through standard quantum field theory approaches is a very difficult task requiring the development of more complete and general quantum field theory techniques. Therefore, in many model studies, although vorticity is introduced into the Lagrangian, the focus ultimately remains on its single-particle effects, effectively simplifying vorticity as an independent background field to concentrate on discussing the collective effects caused by its single-particle polarization energy.

Parallel to the research route on magnetic fields, researchers have used quantum field theory and gravity dual models to provide preliminary results on the effects of rotation on chiral condensation, pion condensation, color-superconducting diquark condensation, and color confinement phase transitions [23–44]. At the mean-field level, these studies focus on calculating the effects of vorticity on quark single-particle energy levels, providing corrections to condensation from rotation, and discovering that the polarization effect of vorticity will suppress quark condensation with total angular momentum 0 while enhancing spin-1 pairing modes. At the single-particle or few-body level, in addition to the transport properties of chiral fermions in vorticity backgrounds [45–48], researchers have also discussed the effects of rotating backgrounds on various hadron bound states [49–50], providing corrections to the properties of probes for QGP. For the more complex problem of rotational corrections to interaction vertices, there have recently been some attempts that do not rely on standard quantum field theory techniques, giving qualitative conclusions different from the mean field, similar to inverse magnetic catalysis, and similar results have also appeared in lattice calculations [51]. This review begins with how to introduce rotating backgrounds into models, successively introducing results on chiral, diquark, pion, and vector condensation under mean-field approximation, while demonstrating technical details specific to rotation problems, such as boundary conditions and system non-uniformity. Finally, it will introduce the effects on chiral and deconfinement phase transition temperatures after considering rotational corrections to QCD interactions. Beyond QCD phase transition content, this review will also briefly present results on parton transport, fluid dynamics, and experiments under vorticity backgrounds [37, 52–69], where most phenomenological studies focus on simulating vorticity distributions in QGP fireballs and the global or local polarization directions of hyperons [70–76]. These studies show that similar vorticity distributions and dependencies on collision conditions can be obtained using different QGP phenomenological models. The polarization of hyperons and anti-hyperons is an important probe of vorticity distribution, and while overall polarization measurements are described with considerable precision, quantitative understanding of local polarization experimental results still requires further research.

Quantum Field Theory in Non-Inertial Rotating Frames

Although intuitively vorticity fields can be regarded as background fields similar to magnetic fields, rotation is actually the overall motion of the system, and microscopically there is no interaction vertex with particles like electromagnetic fields. To introduce overall rotation into the system Lagrangian, the most direct method is to enter the co-moving reference frame that is relatively stationary with the rotating small region for research, i.e., to study the thermodynamic properties of a specified local region. Taking a system rotating uniformly around the z-axis with angular velocity Ω as an example, the transformation relationship between spacetime coordinates in the co-moving rotating frame and the laboratory stationary frame is: x' = xcosΩt - ysinΩt y' = xsinΩt + ycosΩt z' = z t' = t

The corresponding spacetime line element can be written as:

$$ds^2 = \eta_{\mu\nu} dx'^{\mu} dx'^{\nu} = g_{\mu\nu} dx^\mu dx^\nu = (1-\Omega^2 r^2)dt^2 + 2\Omega y dx dt - 2\Omega x dy dt$$

It should be noted that rotating reference frames also have several subtle issues in general relativity discussions, and there are several different choices for transformation relationships with the stationary frame. For example, in the above transformation, Lorentz contraction factors for relative motion can also be considered, making the transformation relationship more complex. In most research, people still choose the above simple form to simplify the problem. For this form, the metric in the co-moving reference frame can be read as:

$$g_{\mu\nu} = \begin{pmatrix} 1-\Omega^2 r^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Clearly, this is a non-inertial frame. According to the principle of general relativity, the Lagrangian of the system in this reference frame should be no different in form from that in flat spacetime, with the only difference being that the reference frame now is non-inertial and the metric is no longer flat. All differential operators in the flat spacetime Lagrangian should be replaced with covariant differentials under the corresponding field representation. Taking the simplest scalar field as an example, the field' s Lagrangian in a non-inertial frame is:

$$\mathcal{L}_g = \sqrt{-g}\left[|D_\mu \phi|^2 - (m^2 + \xi R)|\phi|^2\right]$$

The corresponding equation of motion is:

$$D_\mu(\sqrt{-g} g^{\mu\nu} D_\nu \phi) + (m^2 + \xi R)\phi = 0$$

Of course, for scalar fields, covariant differentiation equals ordinary differentiation. Since scalar fields do not carry spin, there is no single-particle spin polarization term, only polarization of energy levels for orbital angular momentum. This approach to rotating systems was involved in early literature [77].

In order of spin, the first nontrivial field in a rotating frame is the spinor field. For spinor representations, the connection involved in covariant differentiation is the spin connection, defined as:

$$\omega_\mu^{ij} \sigma_{ij}$$

where:

$$\omega_\mu^{ij} = \frac{1}{2} g^{\alpha\beta} e_\alpha^i (\partial_\mu e_\beta^j + \partial_\nu e_\beta^j - \partial_\beta e_\mu^j)$$

and:

$$\sigma^{ij} = \frac{1}{4}[\gamma^i, \gamma^j]$$

Since non-inertial effects are caused by rotation, it can be anticipated that the connections should all be powers of angular velocity, and actual calculations confirm this:

$$\Gamma^x_{tx} = \Gamma^y_{ty} = -\Omega^2, \quad \Gamma^y_{tx} = -\Gamma^x_{ty} = \Omega$$

In a local inertial frame, covariant derivative terms can be explicitly written as the sum of ordinary derivatives and metric curvature terms, where the metric curvature part is:

$$i\gamma^\mu \Gamma_\mu = i\gamma^0 \Omega J_z$$

where J_z is the z-component of the total angular momentum operator. The Dirac equation in a rotating frame is:

$$\left[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m\right]\psi = 0$$

Using cylindrical coordinates (t, r, θ, z), the eigenfunctions can be written as:

$$\psi_{n,k_t,k_z,s}(t, r, \theta, z) = e^{-i(k_0 t + k_z z)} e^{i(n+1/2)\theta} \begin{pmatrix} J_n(k_t r)P_+ + e^{i\phi} J_{n+1}(k_t r)P_- \\ i\gamma^1 k_t e^{-i\phi} J_n(k_t r)P_+ + i\gamma^2 k_t J_{n+1}(k_t r)P_- \end{pmatrix}$$

where $P_\pm = \frac{1}{2}(1 \pm i\gamma^1\gamma^2)$ are projection operators, and $r' = (t, r, \theta, \phi)$ are cylindrical coordinate components.

Using the same method, massive vector fields can be studied [78]. Here, covariant differentiation is the usual covariant differentiation for vectors, i.e., $D_\mu A_\nu = \partial_\mu A_\nu + \Gamma^\alpha_{\mu\nu} A_\alpha$. Writing out the covariant derivative explicitly and entering the local inertial frame, i.e., projecting the vector in curved spacetime to the local flat frame through the tetrad to obtain components $A_a = e^\mu_a A_\mu$, and finally organizing the Lagrangian as a function of field quantities in the local inertial frame gives:

$$\mathcal{L}(\omega) = -\frac{1}{4} F_{ij} F^{ij} + \frac{1}{2}(\partial_i A_0 + \epsilon_{ijm}\omega_m A_j)^2$$

Since the Lagrangian and Hamiltonian of vector fields have dimensions of energy squared, unlike spinor fields, the polarization effect cannot be directly seen here. However, solving the eigenstates and eigen-energies of the equations of motion will give explicit rotational polarization energy $n\Omega$. Using canonical quantization, the corresponding Feynman propagator can be given. Due to its complex form, with the help of symbolic notation:

$$\Pi_n = J_n(\rho) J_n(r), \quad Z^\pm_n(\rho, \phi) = J_{n\pm1}(\rho) e^{\pm i\phi}$$

the propagator can be written as:

$$D_{\mu\nu}(x,x') = \sum_{n,k_t,k_z} \frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{k_0^2 - \epsilon^2_{n,k_t,k_z} + i\epsilon}$$

The result is the same as the usual vector field quantization. The "interaction term" between vorticity fields and single particles is equivalent. The tetrad is directly related to the metric; it is the transformation matrix between curved metric and local flat metric, i.e., it needs to satisfy:

$$g_{\mu\nu} = e^i_\mu e^j_\nu \eta_{ij}$$

where η_{ij} is the flat Minkowski metric. Clearly, the choice of tetrad is not unique; different choices only differ by a local coordinate rotation, so they have no effect on physical results. In a rotating frame, a simple choice is:

$$e^0_0 = 1, \quad e^1_1 = \cos\theta, \quad e^2_1 = \sin\theta$$

$$e^1_0 = -y\Omega, \quad e^2_0 = x\Omega, \quad e^3_3 = 1$$

With this choice, the Lagrangian for spinor fields is rewritten as:

$$\mathcal{L} = \bar{\psi}\left[i\gamma^0(\partial_0 + i\Omega J_z) + i\gamma^i\partial_i - m\right]\psi$$

It can be seen that the correction term introduced by rotation is very clear—polarization of spin and orbital angular momentum. Obviously, rotational symmetry around the z-axis ensures that the corresponding eigenstates are the same as the cylindrical coordinate solutions in the free case, and the eigen-energies are corrected by the polarization energy $(n+1/2)\Omega$. Directly using the standard canonical quantization scheme, the corresponding Feynman propagator can be obtained as:

$$S(x,x') = \sum_{n,k_t,k_z} e^{-ik_0(t-t')}e^{ik_z(z-z')}e^{in(\phi-\phi')}\frac{M_n^+ + M_n^-}{k_0^2 - \epsilon^2_{n,k_t,k_z} + i\epsilon}$$

From the above examples, it can be seen that for single-particle problems, regardless of the field, the field equations in a rotating frame are in the form of free field equations, so the correction terms introduced by rotation do not lead to new interactions and can be regarded as external fields like background magnetic fields. Their single-particle eigenstates are also consistent with the results in cylindrical coordinates in flat spacetime. At the mean-field level, this means that finite-temperature field theory can be directly used to analytically calculate phase transition order parameters to obtain phase structures at different rotational speeds, temperatures, and chemical potentials. However, it should be noted that when further considering interactions, to strictly solve this system, the computational logic of quantum field theory needs to be reconstructed in curved coordinates. This is because usual quantum field theory is used for scattering processes, i.e., states in the infinite past and future are free plane wave states, and interactions occur at the moment of collision. In rotation problems, such assumptions are obviously unreasonable; the eigenstates before and after interactions should be angular momentum eigenstates. The change of incoming and outgoing states implies that the framework of quantum field theory needs to be rebuilt. The work in the above literature is a preliminary attempt at this problem. In the co-moving coordinate system, single-particle eigenstates remain analytically solvable, so the usual canonical quantization method can be used to quantize them and calculate Feynman propagators. This advancement process is blocked at the interaction part. Due to the breaking of spacetime translation symmetry, vertex calculations will involve integrals of three or more Bessel functions, and interaction vertices can no longer be rewritten in simple forms using good quantum numbers (energy-momentum) of eigenstates through conservation relations. In the angular momentum representation, angular momentum conservation and energy conservation contained in interactions no longer have simple additive forms. This systematic problem requires further research.

2.1 Local Potential Approximation and Boundary Conditions

To obtain the effective potential from the mean-field Lagrangian, one needs to solve for the single-particle energy levels $\Psi_{\{\xi\}}$ of fermions. Taking a system considering chiral and pion condensation as an example, the eigenvalue equation of the system is:

$$\hat{H}\psi = \left[i\partial_t + (m_0+\sigma)\gamma^0 + i\gamma^0\gamma^5\pi\cdot\tau + i\gamma^0\gamma^i\nabla_i - \Omega J_z\right]\psi = \epsilon\psi$$

If chiral condensation σ and pion condensation π are independent of spacetime, the eigenvalue equation is analytically solvable. Then the effective potential can be written analytically as:

$$V_{\text{eff}} = \int d^{4x}\left[\frac{\sigma^2+\pi^2}{4G}\right] - \frac{1}{\beta}\sum_{\{\xi\}}\ln\left(1+e^{-\beta\epsilon_{\{\xi\}}}\right)$$

Clearly, in rotating systems, not only is the magnitude of condensation unknown, but there may also be spatial distributions. However, to obtain single-particle energy levels, the distribution of condensation must be known in turn to determine a solvable eigenvalue equation under mean-field approximation. In non-rotating systems, due to spacetime translation invariance, it is reasonable to assume that condensation is a macroscopic quantity uniformly distributed throughout space. However, practical experience shows that the steady state ultimately reached by a uniformly rotating system is often non-uniform. If one wants to solve single-particle energy levels analytically, a feasible scheme is to introduce the assumption of slowly varying condensation, i.e., assuming it changes slowly with space so that its spatial derivative terms can be neglected, focusing only on its zeroth-order component:

$$\partial_i \sigma \approx 0, \quad \partial_i \pi \approx 0$$

Intuitively, the sufficient condition for this approximation to hold is that the rotational speed is sufficiently slow. In most early work, people were more concerned with whether phase transitions occurred rather than the details of condensation distribution. This assumption can simplify the problem and give physically expected results, so it is widely adopted. In fact, discussing the overall equilibrium thermodynamics of rotating systems is a subtle issue, especially when further considering relativistic covariance and quantum corrections. Therefore, a safer definition of this problem is limited to local fluid elements that are macroscopically infinitesimal but microscopically infinite. In such small systems, as long as rotation is not particularly violent, the local potential approximation appears quite reasonable. Most qualitative studies often take such small systems as research objects and adopt the local potential approximation. For uniformly rotating systems, finite boundaries are particularly important because under the constraint of the speed-of-light limit, the velocity at the edge of the system must be less than the speed of light $v = R\Omega < 1$. Once the system' s initial rotational energy is too high, under the constraint of the speed-of-light limit, the system either evolves toward a non-uniform angular velocity distribution or breaks into multiple smaller systems forming a vortex lattice morphology. Therefore, in principle, considering the existence of boundaries and introducing explicit boundary conditions is important in uniformly rotating systems. However, if the research object is concentrated on the local fluid element mentioned above, as long as the fluid element is not exactly at the system boundary, boundary conditions are not so crucial. In some studies, since people are only concerned with qualitative behavior of phase transitions and subsequent effects caused by condensation changes, they directly choose natural boundary conditions for calculation. Comparison with other studies that consider explicit boundary conditions shows that qualitative conclusions about phase transitions at finite temperature are not significantly affected.

Using the above calculation method, reference [35] shows the phase diagram of chiral phase transition in the T-Ω plane in the NJL model considering rotating backgrounds (Figure 2 [Figure 2: see original paper]). It can be seen that due to the suppression effect of rotation on chiral condensation, the "critical" temperature of chiral transition decreases with increasing rotation speed, and the chiral transition becomes a first-order phase transition process when the angular velocity is about 0.63 GeV. This paper also studies color-superconducting pairing, obtaining similar conclusions for pairing with total angular momentum 0, for obvious physical reasons. Because the rotational polarization energy correction appears directly in the thermal distribution function of single particles. Since chiral condensation is scalar condensation, where two fermions have parallel spins and opposite relative orbital angular momentum, rotational polarization will inevitably suppress one part of the angular momentum while enhancing the other, thus reducing the magnitude of chiral condensation. The same analysis applies to scalar color-superconducting diquark condensation. In general, if at least one of the two component quarks and relative angular momentum has a direction opposite to rotation, the polarization energy will cause a suppression effect, thus suppressing the amplitude of this pairing condensation. Conversely, if all angular momentum parts can be aligned with the rotation direction, condensation will be enhanced. This intuitive physical picture has been confirmed in different research works.

Since rotation physically enhances vector channel condensation, considering vector interactions in the NJL model has attracted further interest. Wang, Chen, et al. [30, 40] studied the phase diagram of systems containing vector interactions $(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}i\gamma^\mu\gamma^5\psi)^2$. Their calculations confirmed previous results that angular velocity plays a role similar to baryon chemical potential in the rotating frame, suppressing chiral condensation. They also carefully compared the similarities and differences between rotational polarization and chemical potential, finding that the critical endpoint (CEP) in chiral phase transitions appears not only in the temperature-chemical potential plane but also in the temperature-angular velocity plane. Furthermore, they found that in the temperature-chemical potential plane, the existence of angular velocity only reduces the critical temperature of CEP without affecting the critical chemical potential; while in the temperature-angular velocity plane, increasing chemical potential also only reduces the critical temperature without changing the critical angular velocity. This indicates that at the mean-field level, rotation is not only very similar to chemical potential in the phase diagram structure of chiral phase transitions, but there are also many similarities in their interactions. This is closely related to the fact that rotational polarization suppresses chiral condensation. Since rotation promotes vector condensation, it can be expected that vector channel interactions will change this similarity. In their research, they found that the phase structure in the temperature-chemical potential plane is sensitive to the vector channel coupling strength, while the phase structure in the temperature-angular velocity plane is not. They also comprehensively calculated various thermodynamic quantities of the system, observing that rotational angular velocity suppresses baryon number fluctuations and promotes pressure density, energy density, specific heat, and sound speed. Zhang et al. [28] simultaneously considered chiral and pion condensation in the NJL model and introduced isospin vector interactions $(\bar{\psi}\gamma^\mu\tau\psi)^2$. They systematically studied the suppression of scalar pairing and enhancement of vector pairing under rotation at finite temperature and density, and presented phase diagrams (Figure 3 [Figure 3: see original paper]). Due to the existence of isospin chemical potential that explicitly breaks isospin symmetry, pions in the system undergo Bose-Einstein condensation to form pion condensation. The larger the isospin chemical potential, the stronger the pion condensation in the system, gradually reaching saturation in the NJL model. However, the above results show that when the system rotates as a whole, the ground state of pion condensation will be destroyed. At moderate isospin density, the emergence of rotation will suppress pion condensation until it disappears, and further increasing angular velocity will promote the formation of vector condensation, thus forming a triple point. More remarkably, even with only a small rotation, pion condensation will be destroyed at high isospin chemical potential, and the system will enter the vector condensation phase through a first-order phase transition. This new phase diagram may have important guiding significance for rotating highly asymmetric nuclear matter systems, such as neutron stars. Sun et al. [29] further extended the NJL model to three flavors, focusing on discussing the effect of rotation on quark polarization. The model considering strange quarks and calculations of quark polarization will play a positive role in understanding experimental polarization measurement results.

Under the local potential approximation, considering different boundary conditions will bring certain corrections to the phase transition process. The most significant feature is simply called "vacuum does not rotate." At the boundary, one can consider the condition that the net particle flow along the radial direction is zero:

$$\gamma^r \psi|_{r=R} = 0$$

Substituting the eigen wavefunction under this condition gives quantization of the transverse momentum quantum number along the radial direction:

$$p_{l,k} = \frac{\xi_{l,k}}{R^{-1}} \quad \text{for} \quad l = 0, 1, ...$$

where $\xi_{l,k}$ is the k-th zero of the l-th order Bessel function. If boundary conditions are not considered, the transverse momentum is a continuous quantum number from zero to infinity.

At zero temperature, the gap equation for chiral condensation becomes:

$$4G \int \frac{d^{3p}}{(2\pi)^3} \frac{m}{\sqrt{p^2 + m^2}} \theta(\sqrt{p^2 + m^2} - |\Omega j|)$$

Note that the step function will always be greater than the absolute value of the energy level shift caused by rotation when non-zero lower bounds appear in eigen-energies:

$$(\xi_{l,1} - \Omega R j)(\xi_{l,1} - j) > 0$$

Therefore, the step function will always be 1. This indicates that at zero temperature, rotation does not introduce any correction. The conclusion that the vacuum at zero temperature does not accept rotational corrections is important for phase structures near absolute zero, but for finite temperature situations, numerous studies have shown that qualitative and even quantitative results of condensation under mean-field approximation are not significantly affected by boundary conditions.

The choice of boundary conditions is not unique. Chernodub and Gongyo introduced MIT boundary conditions in [23, 26], expressed as:

$$\left[i\gamma^{\mu}n_{\mu}(\theta)+1\right]\psi|_{r=R}=0$$

Its direct result is to require that particle flow at the radial boundary is strictly zero in all directions, which is a stricter boundary condition than the above. Under MIT boundary conditions, they also gave the behavior of chiral condensation with temperature and angular velocity and the corresponding phase diagram in the plane (Figure 4 [Figure 4: see original paper]). These results have no qualitative or quantitative significant differences from known results. MIT boundary conditions can be further generalized by introducing a manually selectable chiral angle Θ to set a series of boundary conditions:

$$\left[i\gamma^{\mu}n_{\mu}(\theta)+e^{i\Theta\gamma_5}\right]\psi|_{r=R}=0$$

Researchers [23] studied this class of boundary conditions and found that for any chiral angle Θ, the chiral phase transition temperature decreases quadratically with angular velocity, and the position and slope of the phase transition line in the temperature-angular velocity phase diagram clearly depend on the chiral angle value. At the same time, it should be noted that first, for all chiral angles the phase transition temperature decreases with angular velocity; second, the span of phase transition temperatures under different chiral angles is actually not large (about 10%). This again shows that the influence of boundary conditions on phase transition behavior is very limited.

2.2 Self-Consistent Solution for Non-Uniform Condensates

The discussion of boundary conditions has triggered attempts to numerically solve non-uniform systems. Except for ideal rigid bodies, uniformly rotating systems will inevitably cause redistribution of internal matter, forming non-uniform many-body systems. This is particularly evident in the calculation of gap equations above. To obtain explicit Dirac equation eigen-energies, one needs to assume that the distribution of chiral condensation is known. In the above solutions, to simplify the problem, people assumed chiral condensation to be an unknown constant, which actually solves for chiral condensation in a small system at a certain transverse distance from the rotation axis within a macroscopically infinitesimal but microscopically infinite small region. Obviously, if chiral condensation changes slowly with radius, such an approximation is reasonable. Since the expression for condensation explicitly contains the transverse position R, the local potential approximation can also give the dependence of condensation on radius. However, this dependence is clearly inconsistent and unreliable. Under extreme conditions of large angular velocity, condensation may even jump at some radius position, which obviously completely violates the local potential approximation. At the same time, in actual quark-gluon plasma, not only is the matter distribution itself non-uniform, but the centrifugal effect of rotation can also easily cause strong matter redistribution. Therefore, exploring methods that can self-consistently solve the non-uniformity of the system is of considerable importance. Wang et al. [27] borrowed the BdG equation from cold atom systems to develop a numerical method that can self-consistently solve non-uniform condensation without relying on the local potential assumption.

The construction of the BdG equation is relatively easy to understand. Since the eigenvalue equation cannot be solved forward, a complete set of basis functions is chosen to expand the unknown eigenvalue equation and gap equation. Based on symmetry, this set of basis functions can be directly chosen as solutions in flat spacetime in cylindrical coordinates. Under strict boundary truncation conditions $\psi(r=R)=0$, the wavefunction expansion is:

$$\psi(r,\theta)=\sum_{n,j,l}\begin{pmatrix} c^{\uparrow}_{n,j}\phi_{j,l}(r)e^{i(l+1)\theta}+d^{\uparrow}_{n,j}\xi_{j,l}(r)e^{il\theta} \\ c^{\downarrow}_{n,j}\chi_{j,l}(r)e^{i(l+1)\theta}+d^{\downarrow}_{n,j}\phi_{j,l}(r)e^{il\theta} \end{pmatrix}$$

where the chosen complete basis functions are $\phi_{j,l}(r)=c_{jl}J_l(k_t r)$. Boundary conditions give quantization of radial transverse momentum $k_t R=\alpha_{j,l}$, where $\alpha_{j,l}$ is the j-th zero of the l-th order Bessel function. This transforms continuous differential-integral equations into simple algebraic equations, and the eigenvalue problem of differential equations into matrix eigenvalue problems. Further, solving the eigenvalue equation and gap equation simultaneously can self-consistently obtain condensation varying with space. After discretization, the corresponding Hamiltonian is a block-diagonal matrix arranged by angular momentum quantum numbers:

$$H=\begin{pmatrix} \ddots & & & \\ & H_l & & \\ & & H_{l+1} & \\ & & & \ddots \end{pmatrix}$$

where each block H_l contains matrix elements:

$$S^l_{jj'}=\int rdr\sigma(r)\phi_{j,l}(r)\phi_{j',l}(r)$$

$$K^l_{jj'}=\int rdr\phi_{j,l}(r)\left(\partial_r+\frac{l+1/2}{r}\right)\phi_{j',l}(r)$$

In such systems, not only can the condensation distribution of the overall rotating system be obtained, but more realistic non-uniform angular velocity distributions can also be introduced, such as angular velocity distributions where vorticity is maximum at intermediate radii like QGP, or systems where only the middle part rotates at high speed. Numerical calculation results show that non-uniform angular velocity distributions will cause two types of effects: first, the angular velocity itself suppresses chiral condensation, i.e., chiral condensation at positions with large angular velocity is significantly smaller than at positions with small angular velocity; second, the gradient of angular velocity enhances chiral condensation, i.e., chiral condensation in regions where angular velocity drops rapidly is significantly greater than in regions where angular velocity remains stable (Figure 5 [Figure 5: see original paper]).

2.3 Combined Effects with Magnetic Fields

Magnetic fields and rotation have many similarities: they are both pseudovector fields and both polarize angular momentum. However, they are not entirely the same, most notably in that background magnetic fields exist as independent objects with real QED interaction vertices with charged particles in the system, while rotation is collective motion of the system without microscopic interaction mechanisms. Classical electrodynamics shows that charges moving in circles can produce magnetic fields, and magnetic fields can drive charges moving perpendicular to them to move in circles. Therefore, there is also mutual transformation, competition, or promotion between rotation and magnetic fields. Studying systems with both background magnetic fields and rotation has become a research hotspot. In quark models, especially the NJL model, simultaneously considering contributions from both is relatively simple. In addition to introducing the metric of non-inertial reference frames, magnetic fields can be introduced through the usual minimal coupling. The motion equation satisfied by free fermions is:

$$\left[i\gamma^{\mu}(\partial_{\mu} + iqA_{\mu} + \Gamma_{\mu}) - m\right]\psi = 0$$

When considering s-wave interactions, the rotating metric does not introduce more corrections, and the interaction term is the same as in ordinary flat spacetime. Chen et al. [34] considered scalar and pseudoscalar interaction channels and obtained the effective potential with both background magnetic field and rotation using the same technique:

$$V_{\text{eff}}(m) = \frac{(m - m_{\text{current}})^2}{4G} - N_c \sum_{q=\pm} \int \frac{dp_z}{2\pi} \sum_{n,l,s_z} \frac{|qB|}{2\pi} \ln\left[1 + e^{-\beta(\epsilon + q\Omega j)}\right]$$

where $\epsilon = \sqrt{p_z^2 + 2|qB|(n + 1/2 - s_z) + m^2}$.

The paper discussed two phase transition curves under different coupling constant values (Figure 6 [Figure 6: see original paper]). Researchers found that the behavior of chiral phase transition along the angular velocity direction is similar in both strong and weak coupling cases, i.e., it decreases with increasing angular velocity. This is very similar to the effect of temperature. When along the magnetic field direction, the situation is completely different for different couplings. In the weak coupling case, chiral condensation increases with increasing magnetic field, showing the usual magnetic catalysis effect, and the critical angular velocity of chiral phase transition therefore also increases with increasing magnetic field. In the strong coupling case, chiral condensation shows a decrease with increasing magnetic field, while the behavior of critical angular velocity is also opposite, decreasing with increasing magnetic field, showing characteristics similar to "inverse magnetic catalysis." This behavior can be understood through analysis of Landau levels. In weak coupling, lower Landau levels participate in interactions, which carry smaller angular momentum. Therefore, at a given angular velocity, the polarization effect of angular velocity is also relatively small. The qualitative behavior of the system is similar to the case without rotation but with magnetic field, showing normal magnetic catalysis characteristics. When coupling becomes strong, Landau levels with high angular momentum gradually begin to participate in interactions, and the rotational suppression effect on these levels is more obvious at a given angular velocity. Therefore, when magnetic field is further increased, chiral condensation decreases instead of increasing, showing behavior similar to "inverse magnetic catalysis." In fact, the "inverse magnetic catalysis" picture here is similar to the famous puzzle in the temperature-magnetic field plane. However, to solve the problem in the temperature–magnetic field plane, it is necessary to truly understand the variation of coupling constants and the suppression effect of different Landau levels at fixed temperature. Obviously, the first point cannot be answered by quark effective models.

Cleverly using the combined action of magnetic field and rotation to promote physical phenomena is particularly evident in references [24-25]. In this work, charged $\pi^{\pm}$ systems are considered. Since this is a system composed of scalar particles, in principle no obvious physical effect would occur in a rotating background. However, considering that once a magnetic field parallel to the angular velocity direction is applied, charged scalar mesons will rotate and acquire nonzero orbital angular momentum, which can be polarized by rotation. Therefore, Liu and Zahed studied in detail the spatial distribution of $\pi^{\pm}$ in this system and found that π^{+} will tend to accumulate at the edge while π^{-}, due to overall charge conservation, will accumulate in the central region (Figure 7 [Figure 7: see original paper]). Macroscopic accumulation of bosons will trigger Bose-Einstein condensation. The BEC corresponding to $\pi^{\pm}$ is pion superfluid, i.e., due to the combined action of magnetic field and rotation, condensation distributions of different charge components of pions may be produced in systems with zero overall isospin chemical potential.

2.4 Gluon Contributions and Lattice QCD Calculations

During the period when magnetic effects received great attention, lattice QCD calculations showed that the "critical" temperature of chiral transition was completely opposite to the trend estimated based on the usual magnetic catalysis effect, i.e., the higher the magnetic field, the lower the phase transition temperature. This phenomenon is called inverse magnetic catalysis, which is a typical non-perturbative effect that can be numerically reproduced by non-perturbative techniques but cannot give an intuitive physical picture. The reason can be roughly understood as: although gluons that transmit interactions between chiral-paired quarks are not charged, the self-energy loops of gluons have large quark fluctuations, and these quarks are affected by magnetic fields. Therefore, magnetic effects will affect gluons through this secondary quantum effect, thereby correcting quark interactions and changing the qualitative behavior of chiral transition temperature. In effective models, the inverse magnetic catalysis phenomenon can be reproduced by introducing magnetic field-dependent four-quark interaction vertices, thus partially verifying the correctness of the above understanding from a physical picture. In the background vorticity case, will gluons also bring similar anti-trend behavior? Considering that gluons themselves carry spin and can be directly polarized by vorticity without needing to obtain corrections through secondary quantum effects like magnetic fields, the effect of vorticity polarization on gluons may be relatively obvious for the impact on chiral transition.

To study rotation-dependent interaction effects in effective models, it is necessary to first obtain the behavior of NJL coupling constants varying with angular velocity $G(\Omega)$. In usual quantum field theory, corrections to coupling constants require first calculating four-point correlation functions of fields, then obtaining interaction vertex corrections through amputation operations. However, in rotating reference frames, spacetime translation invariance is broken, which leads to not only many integrals or sums of Bessel functions that cannot be analytically calculated remaining in four-point correlation functions, but also more integrals appearing during amputation. These calculations do not have good analytical results, and numerical calculations cannot proceed smoothly due to involving high multiplicities of integrals and sums. Therefore, other approaches are needed to obtain qualitative or semi-quantitative relationships of coupling constants depending on angular velocity. Nielsen et al. [79] developed a method to extract first-order quantum corrections to coupling constants by calculating the energy of gauge fields in background color magnetic fields. This method can not only reproduce the famous one-loop QCD asymptotic freedom result in vacuum but also successfully give inverse magnetic catalysis results when their magnetic field-dependent coupling constants are placed in the NJL model. The reason this scheme works is that first-order quantum correction calculations are realized by transmitting a gauge boson. From a quantum mechanics perspective, this process can be equivalent to external line particles being in a potential field formed by this intermediate boson, and the quantum correction caused by this potential field is contained in the energy level distribution of external line particles. Obviously, for higher-order quantum correction processes, this picture is incorrect. Therefore, it is not a method that can systematically calculate quantum corrections level by level like standard loop calculations, but it is a clever and intuitive method for first-order corrections. Most importantly, it avoids direct calculation of interaction vertices and can obtain first-order corrections to coupling constants through summation of single-particle energy levels. In reference [82], this method was applied to rotating systems. For example, taking a pure gluon system to calculate the running of QCD coupling constants with rotation, the specific calculation scheme is to first introduce a constant background color magnetic field, calculate the magnetization energy under the background color magnetic field by summing all eigen-energies in the rotating gluon system:

$$\Delta E = \sum_{n,i} \frac{1}{2}\epsilon_{n,i} = \frac{VB^2}{2}\left[1 + 4\pi\chi(B,\Omega)\right]$$

Thus giving the magnetic susceptibility $\mu(B,\Omega) = 1 + 4\pi\chi(B,\Omega)$ dependence on the background color magnetic field. Using the relationship between vacuum magnetic susceptibility and vacuum permittivity $\mu\epsilon = 1$, and considering the connection between vacuum permittivity and coupling constant $\alpha_{\text{eff}} = \alpha/\epsilon$, the dependence of effective coupling constant on background color magnetic field can be obtained. Similar to ordinary magnetic field effects, introducing a color magnetic field in the T_8 component in QCD will give gluon Landau levels:

$$\epsilon_{n,k_3,s_3} = \sqrt{k_3^2 + 2gB(n + 1/2 + s_3)}$$

Here it can be seen that the magnitude of energy levels is determined by the size of the background color magnetic field, so $2gB$ can be equated to the interaction energy scale and replaced with k^2 in standard textbooks. In rotating systems, using the same approach to solve gluon energy levels and through appropriate approximations, it can be obtained that the strong interaction coupling constant in rotating systems will decrease with increasing angular velocity (Figure 8 [Figure 8: see original paper]). Thus, NJL models can also introduce coupling constants $G(\Omega)$ with the same qualitative behavior to recalculate the critical temperature of chiral phase transitions. Under the local potential approximation, numerical results show that indeed, like inverse magnetic catalysis, after considering the effect of rotation on coupling constants, the chiral phase transition temperature will show a trend completely opposite to the constant coupling case.

As early as 2013, Yamamoto and Hirono examined the fermion action part of lattice QCD in rotating backgrounds [33]. They found that although rotational polarization is similar to the contribution of fermion chemical potential, it does not actually cause a sign problem, which is similar to isospin chemical potential. In this study, they did not actually perform lattice simulations but only discussed the form of the lattice action, so their estimation of this problem was overly optimistic. In recent lattice calculations [51], Braguta et al. conducted systematic studies on QCD rotation simulations. They found that when rotational corrections are considered, a sign problem appears in the gluon field part:

$$S_g = \int d^{4x} \left[\frac{1}{4}(1 - x^2\Omega^2)F^a_{xy}F^a_{xy} + \frac{1}{4}(1 - y^2\Omega^2)F^a_{xz}F^a_{xz} + \frac{1}{4}(1 - x^2\Omega^2)F^a_{yz}F^a_{yz} + \frac{1}{2}F^a_{x\tau}F^a_{x\tau} + \frac{1}{2}F^a_{y\tau}F^a_{y\tau} + \frac{1}{2}F^a_{z\tau}F^a_{z\tau}\right.$$

This is a very surprising result because in most cases the sign problem is caused by fermion chemical potential terms. Researchers also calculated pure gluon and light quark systems. To avoid the sign problem, researchers introduced imaginary angular velocity to simulate the deconfinement phase transition. The results showed that in pure gluon systems, the phase transition critical temperature decreases with increasing imaginary angular velocity, while in pure quark systems the opposite is true. When both gluons and quarks in the system are considered simultaneously, the contribution of gluons is more significant, i.e., the overall phase transition temperature of the system decreases with increasing imaginary angular velocity. Analytic continuation to real angular velocity indicates that the overall phase transition temperature of QCD systems may increase with increasing real angular velocity. Considering that in most lattice studies, the phase transition temperatures of deconfinement and chiral phase transitions are often very close, this suggests that the conclusions of the above independent NJL model studies may be consistent with lattice research results. Of course, due to the existence of the sign problem, the credibility of lattice results is also greatly reduced. When considering rotational corrections to interactions, how the phase transition temperature of the system depends on angular velocity due to contributions from the gluon field remains an unanswered question.

2.5 AdS/QCD and Deconfinement Phase Transition

Quark models are very convenient for studying quark pairing and chiral condensation but are not easy for studying deconfinement phase transitions. Gravity dual models are relatively powerful research tools. Chen et al. [31] adopted the Einstein-Maxwell-Dilaton system with the model action:

$$S = \frac{1}{16\pi G_5} \int d^{5x} \sqrt{-g} \left[R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{f(\phi)}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

The metric in this spacetime is written as:

$$ds^2 = \frac{L^2}{z^2} e^{2A(z)} \left[-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right]$$

Background rotation is introduced through the corresponding coordinate transformation relations:

$$t \to t + \frac{l^2\omega^2}{2}(t + l^2\omega\phi), \quad \phi \to \phi + \omega t$$

Comparing with the previous coordinate transformation relations, it can be found that Lorentz factors are considered here. Using this model to study QCD systems, it was found that increasing angular velocity reduces the critical temperature of deconfinement phase transition, which is similar to the research results of chiral phase transition under mean field, but different from lattice calculation expectations. In the temperature-chemical potential plane, researchers found that angular velocity has obvious effects on the CEP of deconfinement phase transition (Figure 9 [Figure 9: see original paper]). When angular velocity increases, CEP moves toward smaller chemical potential and lower temperature.

The usual AdS/QCD correspondence technique is very suitable for studying deconfinement. Braga et al. [32] used this technique, selecting a rotating cylindrically symmetric black hole model to discuss the thermodynamics of rotating systems at zero chemical potential. In cylindrical coordinates, the spacetime and black hole adopt the following metrics:

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + l^{2d}\phi^2 + \sum_{i=1}^{2} dx_i^2 + \frac{dz^2}{f(z)} + 2l^2\omega d\phi dt\right]$$

In this research, two boundary conditions were considered: "hard wall" and "soft wall" :

$$z_0 < z_h : \Delta E(\omega l, T) = \kappa^2 \frac{1}{L^3 \pi z_h^3} \frac{\omega^2 l^2}{1-\omega^2 l^2}$$

$$z_0 > z_h : \Delta E(\omega l, z_h) = \kappa^2 \frac{1}{L^3 \pi z_h^3}\left[\frac{\omega^2 l^2}{1-\omega^2 l^2} + c^2 z_h^4 \gamma(\omega l)\right]$$

Researchers found that the critical temperature of deconfinement phase transition is quite sensitive to rotation dependence. When the angular velocity is only 20 MeV, the critical temperature drops to 0.87 of the non-rotating case. The sensitive dependence of phase transition on angular velocity may bring obvious effects to final-state observables in relativistic heavy-ion collisions. For studies with different boundary conditions, the deconfinement temperature decreases with increasing angular velocity, which is consistent with the conclusions of the above EMD (Einstein-Maxwell-Dilaton) model and the trend of chiral phase transition under NJL model mean-field approximation. In this literature, researchers also noted the above lattice calculation results, but obviously, the conclusions of gravity dual models are opposite to lattice expectations. Lattice can only give rotation suppression results from the fermion part, while when both gluon and quark effects are considered, the overall system shows a catalytic effect of rotation on phase transition temperature. Of course, due to the existence of sign problems, lattice results should also be questioned. To determine whether it is catalysis or suppression under non-perturbative conditions, independent calculations using other non-perturbative methods are needed.

Transport and Experimental Observations in Vorticity Fields

From model studies and lattice calculations, it can be seen that the angular velocity magnitude that can trigger chiral or deconfinement phase transitions is about tens to hundreds of MeV. Simulation results based on parton transport AMPT (A Multi-Phase Transport), UrQMD (Ultra-relativistic Quantum Molecular Dynamics) models, and hydrodynamics show [22, 55, 84–85] that in relativistic heavy-ion collisions, only at a few local vorticity peaks can this magnitude be reached for extremely short times, while the overall average angular velocity is much smaller. Therefore, it is unrealistic to expect that rotation-induced phase transition corrections can give significant observable effects in current heavy-ion collision experiments. In vorticity backgrounds, more explicit effects are that vorticity polarizes partons or final-state hadrons, thereby affecting transport processes. Obviously, this process will ultimately affect the momentum distribution of final-state particles. Conversely, using polarization measurement results, the magnitude and distribution of vorticity inside QGP can be reconstructed.

Similar to the case of magnetic fields, relative to the energy range of heavy-ion collisions, light quarks can be regarded as natural approximate chiral fermions, with momentum and spin directions completely locked. Any polarization effect on spin will directly change the momentum direction, thus changing the momentum distribution of final-state hadrons. In systems with both vector and axial chemical potentials, vorticity will induce vector and axial currents. This transport law where particle flow is driven by fields with opposite parity is very abnormal and is therefore classified as anomalous transport phenomena. Such transport phenomena related to the existence of chiral charge are called chiral vortical effects [45–48]:

$$\mathbf{J}_V = \frac{\mu_A}{2\pi^2}\Omega, \quad \mathbf{J}_A = \frac{\mu_V}{2\pi^2}\Omega$$

Considering the combined effect of both transports will produce chiral vortical waves. Vorticity waves will cause differences in elliptic flow between particles and antiparticles and provide possible observable effects. Since charged hadrons will also show similar phenomena due to anomalous magnetic transport effects under magnetic field influence, when detecting vorticity effects, neutral hadron distributions are often used as probes to minimize magnetic field effects. Obviously, chiral transport phenomena are first-order quantum corrections to ordinary transport processes. Can quantum corrections to transport laws be systematically calculated order by order like in quantum field theory? The quantum Wigner function is an excellent way to achieve this kind of quantization. In references [56, 86], researchers systematically derived the chiral vortical effect within this framework:

$$f_s^{(0)}(x,p) = \frac{1}{2}p^\rho \tilde{\Omega}_{\rho\alpha} p^\alpha s f_s \delta(p^2)$$

$$f_s^{(1)}(x,p) = \frac{1}{2p^2}\tilde{F}_{\rho\lambda} p^\lambda s f_s \delta(p^2)$$

Significant neutral hyperon polarization has been observed in relativistic heavy-ion collision experiments [16–18, 90]. The polarization rates of hyperons and anti-hyperons increase with collision centrality and decrease with collision energy. This result is qualitatively consistent with the simulation results of local vorticity in QGP using the AMPT model. Therefore, it is considered evidence that QGP has strong overall non-zero vorticity. Overall polarization data can be explained by different models. Ayala et al. [60–61, 65] used the core-corona model to calculate the global polarization of Λ and its anti-particle. This is a statistical model whose main idea is to consider that Λ sources are respectively the high-density "core" and low-density "corona" of the fireball. Studies show that the overall properties of the polarization function are related to the relative abundance of Λ from the core and corona. For low-energy collisions, the former is more abundant; for higher energies, the latter becomes more abundant. The main consequence of this relative abundance competition is that the polarization peak is reached at a collision energy of about 10 GeV. Therefore, it is predicted that the global polarization of Λ should peak in the energy range of NICA and HADES. More detailed research uses the parton transport model AMPT to simulate parton polarization in background vorticity fields and give final-state hyperon distributions [67, 91–93]. This work also gives detailed results on the distribution of local polarization and its dependence on collision conditions. Li et al. [53] calculated the global polarization of Λ and its anti-particle in non-central heavy-ion collision events, considering spin-vorticity and spin-magnetic field coupling, and extracting strong coupling QGP fluid vorticity and magnetic field as input using AMPT. It can be seen that the calculated global polarization results match the STAR measurement data quite well (Figure 10 [Figure 10: see original paper]). Simulation calculations considering different fluid inputs and microscopic polarization details show that global polarization results are relatively easy to reproduce [57, 64, 94–102]. Richer experimental measurement data show that the polarization of hyperons and their anti-particles is not completely identical. Since they are composite particles, they are inevitably affected by electromagnetic interactions. Guo et al. [58] carefully studied global polarization when both background magnetic field and vorticity exist. Studies show that using the magnetic field in the fireball to explain the data is feasible, and the difference in polarization between hyperons and anti-hyperons depends sensitively on the lifetime of the magnetic field.

When further studying the spatial distribution of vorticity, people have found discrepancies between theory and experiment. Like measuring vorticity mean values, measuring vorticity spatial distribution boils down to measuring the momentum angular distribution of neutral vector particles. Wei et al. [59] summarized and systematically studied local polarization distributions in Au+Au collisions at different collision energies (Figure 11 [Figure 11: see original paper]). The article summarized the differences between current experiments and theory, i.e., in the central rapidity region, the theoretical results of longitudinal spin polarization azimuthal distribution are completely opposite to the trend of recent experimental data; the polarization measurements of vector mesons ϕ and K^{*0} also contradict theoretical predictions. This is a very abnormal result because overall polarization data can be fitted and predicted ideally by various models, indicating that existing phenomenological models at least partially correctly grasp vorticity-related physical processes. The completely opposite results of local polarization may come from definition deviations in the correspondence between theory and experiment.

Based on hydrodynamic language, Becattini et al. [100] studied vorticity definitions satisfying different transformation properties. As mentioned above, there is some ambiguity in the relativistic transformation properties of theoretical treatment for rotating systems. As a local equilibrium thermodynamic system, relativistic covariance in each fluid element of QGP is broken. Therefore, when comparing with experiments, how to correctly understand and characterize vorticity in experiments is a question worth thinking about. In the above literature, researchers conducted comparative studies on three vorticity definitions, including ordinary vorticity, thermal vorticity, and T-vorticity (Figure 12 [Figure 12: see original paper]):

$$\omega_{\mu\nu} = \partial_\nu u_\mu - \partial_\mu u_\nu$$

$$\Omega_{\mu\nu} = -\partial_\mu \left(\frac{u_\nu}{T}\right) - \partial_\nu \left(\frac{u_\mu}{T}\right)$$

$$\Omega^T_{\mu\nu} = \partial_\mu (T u_\nu) - \partial_\nu (T u_\mu)$$

Clearly, thermal vorticity and T-vorticity have completely opposite temperature dependence, while ordinary vorticity does not depend on temperature at all. Considering the characteristic temperature distribution of QGP itself, the spatial distributions of these three will be very different. Therefore, correctly understanding and using the corresponding vorticity definition may contain the key to solving the local polarization puzzle.

Of course, the difficulty of local polarization may also be caused by existing phenomenological models generally missing some physical mechanism. In summary, this contradiction has aroused widespread interest. From parton transport models to fluid models, people have carried out a large number of simulation works trying to understand this obviously opposite trend [52]. For example, Wei et al. [59, 68] used the MUSIC viscous fluid model combined with AMPT to study hyperon spin polarization. The model uses UrQMD hadron cascade for QGP fireball evolution. This hybrid model can well describe various soft hadrons at different RHIC-BES energies, such as rapidity distributions, transverse momentum spectra, and elliptic flow of all charged hadrons, but the obtained local polarization is still completely opposite to experimental trends. Sun et al. [69] also used similar models to study the dependence of hyperon polarization on freeze-out temperature. Li et al. [103–104] considered the feed-down effect of strange baryons on Λ hyperon spin polarization by calculating two-body decay processes of strong, electromagnetic, and weak interactions, and systematically calculated the global and local polarization of hyperons including Λ, Ξ^-, and Ω^-. This contribution reduces the original Λ polarization but is not enough to qualitatively reverse the current theoretical prediction about the azimuthal distribution of Λ local polarization. Therefore, the completely opposite local polarization trend in experiments remains unexplained. Wu et al. [83, 105] temporarily solved the contradiction by replacing ordinary vorticity with temperature vorticity (T-vorticity). When temperature vorticity is used as the background vorticity field, the temperature distribution introduces new contributions, and theory can obtain trends similar to experimental measurements.

Recently, theoretical progress by Fu et al. [105-106] shows that local polarization may include effects from fluid thermal shear tensors beyond vorticity. When this new contribution is included in phenomenological simulations, experimental results can be well described. Becattini et al. [105-106] considered the effect of hadron freeze-out temperature on hadron polarization and found that the assumption of isothermal freeze-out will bring new spin polarization contributions, further correcting the difference between theory and experiment for local polarization.

In addition to neutral hyperons, Wei et al. [107] proposed a scheme to detect vorticity using the difference in dilepton polarization. The article shows that rotation enhances the spectral function of dileptons, and the effect is more significant for low invariant mass dileptons. By studying the dependence of measurable dilepton yield and elliptic flow on rotation, the article provides the possibility of detecting vorticity using dilepton-related signals. In phenomenology, people have further discussed many observables including lepton spectral functions, yields, and elliptic flow, as well as bound state dissociation dependence on rotation [50, 108-109]. In real QGP, polarization does not occur instantaneously but requires a certain relaxation time, while QGP formation and disappearance are also very rapid. The competition between these two times will also affect hyperon polarization observation results [62-63]. When considering the local production process of hyperons, the overall polarization results can still be well fitted, indicating that the intensity of overall polarization is more significantly related to the average vorticity of the system and hadron yield.

From the above introduction, it can be seen that for QCD many-body systems, rotation will produce effects at multiple levels, and its quantitative effect directly corresponds to the magnitude of rotational angular velocity. Since rotation does not have direct interactions with matter fields like magnetic fields, its effect on single particles is relatively clear—polarizing angular momentum and shifting single-particle energy. Therefore, when vorticity is only a few MeV, only light quarks with masses closest to this value are significantly affected. As approximate chiral fermions, rotational polarization will significantly modify the spin direction of light quarks and, through the spin-momentum locking property of chiral fermions, affect their momentum direction and thus change transport processes. This mechanism closely related to chiral fermions cannot drive particle flow in chirally balanced systems but can drive chiral flow in systems with matter-antimatter asymmetry. Through parity analysis, it can be known that anomalous transport phenomena depending on the existence of chiral fermions may appear in any system with background pseudovector fields. These anomalous transports lead to changes in final-state particle momentum distributions that are very likely to exist in QGP evolution, but clearly detecting them requires more sophisticated experimental measurements and theoretically more sensitive signals.

As the system temperature further decreases, chiral symmetry is broken, and approximate chiral fermions no longer exist in the system. The quark mass is about the constituent mass of 300 MeV. In such systems, overly small polarization energy will no longer have observable effects on system properties. Only when the rotational angular velocity increases to tens to hundreds of MeV will the phase transition temperatures of chiral and deconfinement phase transitions be significantly corrected. Effective model mean-field approximation and gravity dual model studies show that chiral and deconfinement phase transitions will occur more easily in rotating systems. However, there are also large numbers of gluons in QCD systems, which not only have small masses but also strong self-interactions. When considering the effect of rotation on gluons, both effective model and lattice calculation results suggest that although chiral condensation is suppressed by rotation, the phase transition temperature may be increased in reverse, forming a situation of rotational catalysis of symmetry breaking. Due to the unexpected sign problem in lattice calculations, these results cannot be fully confirmed. In regions where baryon and isospin chemical potentials are non-zero, effective model calculations show that rotation will also suppress the generation of color-superconducting and pion-superfluid pairing ground states. As the rotational angular velocity further increases, the system will tend toward macroscopic condensation states of vector bosons. Due to the speed-of-light limit, such large rotational angular velocities are difficult to exist in large ranges. Boundary conditions or system non-uniformity are issues that must be considered in more realistic calculations. Effective model calculations also found that boundary conditions greatly weaken the effect of rotation near zero temperature, but at finite temperature, the contribution of rotation is not significantly different from when natural boundary conditions are considered. Due to centrifugal effects of rotation, quark pairing condensation will show spatial non-uniformity. Self-consistent solution methods similar to the BdG equation give the radial distribution of chiral condensation. These results not only confirm the chiral suppression effect caused by the mean value of rotation itself but also reveal that changes in angular velocity along the radial direction enhance condensation in that local region. Various phenomenological model simulations show that although the average vorticity in QGP fireballs produced by relativistic heavy-ion collisions only reaches the magnitude that triggers chiral transport, it may locally reach large values due to fluctuations. Whether such extremely non-uniform vorticity fields will affect QGP evolution due to corrections in phase transition temperature is a meaningful question that can only be explored under the premise of determining vorticity field distribution. People attempt to use experimental results on overall and local hyperon polarization to help determine the spatial distribution of vorticity fields. The differences between experiment and theory in local polarization have promoted more detailed theoretical research, and people have gradually gained further understanding of quantum transport processes, fluid viscous effects, and subsequent hadron production processes in QGP. In summary, the discovery of QGP vorticity has not only brought a large number of novel and interesting physical effects but also triggered thinking about essential issues such as quantization and spacetime characteristics of this special system. It is believed that in-depth exploration in this direction will reveal more mysteries of QCD phase transitions, relativistic heavy-ion collisions, and promote discussion of more general theoretical physics problems.

Author Contributions

Jiang Yin was responsible for literature collection, initial draft writing, and later revisions; Liao Jinfeng was responsible for article review and revision. Both authors participated in article conceptualization.

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