Light Nuclei Production in Heavy-Ion Collisions and the Imprint of QCD Phase Transition

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Abstract

Searching for signals of Quantum Chromodynamics (QCD) phase transitions is a fundamental scientific goal of current heavy-ion collision experiments, and is of great significance for understanding important frontier issues such as the properties of strongly interacting matter, the evolution of the early universe, supernova explosion mechanisms, the internal structure of compact stars, and gravitational waves produced by binary neutron star mergers. When a discontinuous QCD phase transition occurs in heavy-ion collisions, the density fluctuations and correlations of strongly interacting matter triggered thereby can be probed through yield observables of light nuclei. Many-body correlations among nucleons determine the yields of light nuclei in heavy-ion collisions. In particular, calculations of light-nuclei yields based on the nucleon coalescence model indicate that the yield ratio of protons (p), deuterons (d), and tritons (t), NtNp /N 2d, is sensitive to nucleon density fluctuations and correlations, and serves as a good observable for searching for QCD phase transition signals. Furthermore, simulations based on transport models for chiral phase transitions in heavy-ion collisions have found that when the system' s phase trajectory passes through the first-order phase transition region, the light-nuclei yield ratio NtNp /N 2d exhibits a significant increase. These results provide a theoretical basis for utilizing light-nuclei production in heavy-ion collisions to search for QCD phase transitions.

Full Text

Preamble

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Light Nuclei Production and QCD Phase Transition in Heavy-Ion Collisions

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Abstract

Searching for signals of quantum chromodynamics (QCD) phase transitions represents a fundamental scientific objective of current heavy-ion collision experiments, with profound implications for understanding the properties of strongly interacting matter, the evolution of the early universe following the Big Bang, supernova explosion mechanisms, the internal structure of compact stars, and gravitational waves from binary neutron star mergers. When a non-smooth QCD phase transition occurs in heavy-ion collisions, the resulting density fluctuations and correlations in strongly interacting matter can be probed through observables related to light nuclei yields. Multi-body correlations among nucleons determine the production yields of light nuclei in these collisions. In particular, calculations based on the nucleon coalescence model indicate that the yield ratio N_tN_p/N_d^2 of protons (p), deuterons (d), and tritons (t) is sensitive to nucleon density fluctuations and correlations, making it a promising observable for detecting QCD phase transition signals. Furthermore, transport model simulations of the chiral phase transition in heavy-ion collisions reveal that when the system' s trajectory passes through a first-order phase transition region, the light nuclei yield ratio N_tN_p/N_d^2 exhibits significant enhancement. These results provide a theoretical foundation for utilizing light nuclei production in heavy-ion collisions to search for QCD phase transitions.

Keywords: Heavy-ion collisions, QCD phase transition, Light nuclei production, Coalescence model, Critical fluctuations

1. Introduction

Quantum chromodynamics (QCD), the fundamental theory describing the strong interaction between quarks and gluons, predicts that nuclear matter undergoes a deconfinement phase transition under extreme temperature and density conditions, forming a new state of matter known as the quark-gluon plasma (QGP). The thermodynamic properties of strongly interacting matter are described by the QCD phase diagram [Figure 1: see original paper] (left panel). At zero baryon density, lattice QCD calculations demonstrate that the transition from QGP to hadron gas is a smooth crossover with a transition temperature $T_c \sim 160$ MeV, where the spontaneously broken chiral symmetry in vacuum is restored at high temperatures. At finite baryon density, the fermion sign problem prevents definitive lattice QCD conclusions, though many QCD-based effective models predict that this transition becomes first-order. The QCD critical point marks the endpoint of this first-order transition line. In addition to temperature and chemical potential, strong magnetic fields also significantly influence QCD phase transition properties.

Relativistic heavy-ion collisions provide a unique experimental tool to search for the QGP and explore its properties, representing one of the most important physics endeavors of the 21st century. This research is crucial for understanding nuclear strong interactions, early universe evolution, supernova mechanisms, compact star interiors, and gravitational waves from neutron star mergers. The Beam Energy Scan program at the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory enables systematic investigation of the QCD phase diagram and the conjectured QCD critical point. While event-by-event fluctuations of conserved charges are widely recognized as a valuable probe of the QCD critical point, light nuclei production serves as another sensitive observable for QCD phase transitions in high-energy heavy-ion collisions. The density fluctuations and correlations among nucleons are inherently encoded in light nuclei production yields.

Recent theoretical studies based on the nucleon coalescence model have established a close connection between the light nuclei yield ratio $N_t N_p / N_d^2$ and nucleon density fluctuations. Non-smooth QCD phase transitions can enhance baryon density fluctuations and correlations, thereby increasing the $N_t N_p / N_d^2$ ratio. Additionally, near the critical point, the effective interaction between nucleons becomes more attractive, potentially leading to baryon clustering effects. These clustered baryons may subsequently decay into light nuclei clusters, making the yield ratio sensitive to long-range correlations near the QCD critical point. Consequently, light nuclei production in heavy-ion collisions offers an effective means to search for QCD phase transition signals. Figure 1 schematically illustrates this approach: when the strongly interacting matter produced in relativistic heavy-ion collisions passes through the critical point or first-order phase transition region in the QCD phase diagram, significant baryon density fluctuations and correlations emerge, subsequently affecting light nuclei production in the final state.

The physics of QCD phase transitions and light nuclei production in relativistic heavy-ion collisions involves multiple energy scales. The nucleon mass $m_N \sim 1$ GeV, the QCD scale $\Lambda_{\text{QCD}} \sim 200$ MeV, the quark-hadron transition temperature $T_c \sim 150$ MeV, and light nuclei binding energies E_B are only a few MeV. The fireball radius R is $10 \sim 20$ fm, light nuclei radii $r_{d,t} \sim 2$ fm, and correlation lengths ξ near the QCD critical point are several fm. These considerations indicate that light nuclei provide spatial resolution down to approximately 2 fm, comparable to correlation lengths near the QCD critical point.

Beyond critical point signals, identifying first-order QCD phase transition signatures is equally important. As mentioned, the first-order phase transition region and critical point are intimately connected in the QCD phase diagram. When a first-order phase transition occurs in heavy-ion collisions, mechanical instabilities produce substantial density inhomogeneities and slowed system expansion, potentially affecting observables such as Hanbury Brown and Twiss (HBT) correlations, dilepton yields, radial flow, and light nuclei production. Studies have shown that nucleon density fluctuations from a first-order QCD chiral phase transition can increase the $N_t N_p / N_d^2$ ratio.

The strongly interacting matter produced in heavy-ion collisions undergoes rapid expansion, making the description of non-equilibrium QCD phase transition dynamics both essential and challenging. For critical fluctuations, several hydrodynamic approaches have been developed, including stochastic fluid dynamics and hydrokinetics. However, these models cannot yet quantitatively explain the observed non-monotonic energy dependence of net-proton multiplicity fluctuations in terms of the QCD critical point. For first-order transitions, hydrodynamic and transport model approaches have been developed.

This review focuses on recent progress in studying light nuclei production and QCD phase transitions in heavy-ion collisions. We discuss the relationship between the light nuclei yield ratio $N_t N_p / N_d^2$ and nucleon density fluctuations using the coalescence model, dynamical simulations of first-order chiral phase transitions, and their impact on light nuclei production. In the transport model, the Nambu-Jona-Lasinio (NJL) model describes the first-order chiral phase transition, assuming simultaneous deconfinement and chiral restoration transitions. By adjusting the effective quark interaction, we can investigate how equations of state with different critical temperatures affect light nuclei production. These results provide theoretical support for using light nuclei production to search for QCD phase transitions.

1.1 Coalescence Model for Light Nuclei Production

We employ the nucleon coalescence model to calculate light nuclei yields because it provides a convenient tool for studying the relationship between nucleon density fluctuations, correlations, and light nuclei production. For deuterons, the yield is determined by the neutron-proton joint distribution function $f_{np}(x_1, p_1; x_2, p_2)$ and the deuteron Wigner function $W_d(x, p)$:

$$N_d = g_d \int d_1^{3x} d_2^{3x} d_1^{3p} d_2^{3p} \, f_{np}(x_1, p_1; x_2, p_2) \times W_d(x, p)$$

where $g_d = 3/4$ is the spin statistical factor arising from combining spin-1/2 protons and neutrons into spin-1 deuterons. Following common practice in coalescence models, we adopt a Gaussian approximation for the deuteron Wigner function: $W_d = 8\exp(-x^2/\sigma_d^2 - p^2\sigma_d^2)$, with $x = x_1 - x_2$ and $p = (p_1 - p_2)/2$. The normalization condition is $\int d^{3x} d^{3p} \, W_d(x, p) = (2\pi)^3$. The width parameter σ_d relates to the deuteron root-mean-square radius r_d through $\sigma_d = \sqrt{3/8}\, r_d = 2.26$ fm, much smaller than the size of the hadronic matter produced in relativistic heavy-ion collisions.

Assuming neutrons and protons are emitted from a thermal source with mass m and temperature T, the joint distribution function takes the form:

$$f_{np}(x_1, p_1; x_2, p_2) = \rho_{np}(x_1, x_2)(2\pi m T)^{-3} \exp\left(-\frac{p_1^2 + p_2^2}{2mT}\right)$$

where $\rho_{np}(x_1, x_2)$ is the coordinate-space neutron-proton joint density distribution, expressed in terms of single-nucleon distributions ρ_n, ρ_p and the two-body correlation function $C_2(x_1, x_2)$:

$$\rho_{np}(x_1, x_2) = \rho_n(x_1)\rho_p(x_2) + C_2(x_1, x_2)$$

These considerations extend to triton production:

$$N_t = g_t \int d_1^{3x} d_2^{3x} d_3^{3x} d_1^{3p} d_2^{3p} d_3^{3p} \, f_{nnp}(x_1, p_1; x_2, p_2; x_3, p_3) \times W_t(x, x_s, p, p_s)$$

where the triton Wigner function is $W_t = 8^2 \exp(-x^2/\sigma_t^2 - p^2\sigma_t^2 - s^2/\sigma_s^2 - p_s^2\sigma_s^2)$. The coordinate transformations use $x_s = (x_1 + x_2 - 2x_3)/\sqrt{6}$ and $p_s = (p_1 + p_2 - 2p_3)/\sqrt{6}$, with width parameter $\sigma_t = r_t/\sqrt{3} = 1.59$ fm. The normalization condition is $\int d^{3x} d^{3p} d^{3s} d_s^{3p} \, W_t(x, s, p, p_s) = (2\pi)^6$. The three-body density distribution can be decomposed as:

$$\rho_{nnp}(x_1, x_2, x_3) = \rho_n(x_1)\rho_n(x_2)\rho_p(x_3) + C_2(x_1, x_2)\rho_p(x_3) + C_2(x_2, x_3)\rho_n(x_1) + C_2(x_3, x_1)\rho_n(x_2) + C_3(x_1, x_2, x_3)$$

where we neglect isospin dependence of two-nucleon correlations, and $C_3(x_1, x_2, x_3)$ is the three-nucleon correlation function. For uniform hadronic matter, C_3 depends only on relative coordinates $\tilde{x}_\alpha = x_1 - x_2$ and $\tilde{x}_\beta = x_2 - x_3$. The spatial integral yields the third-order event-by-event fluctuation of nucleon number $(\delta N)^3 = \int d\tilde{x}_\alpha d\tilde{x}_\beta C_3$. As the system approaches and moves away from the critical point, the singular part of C_3 changes sign.

1.2 Thermodynamic Limit

Based on the coalescence model assumptions, we first calculate deuteron and triton yields in a uniform, thermally equilibrated system of volume V, assuming $C_2 = C_3 = 0$. Integrating Eq. (1) yields:

$$N_d = \frac{g_d V}{(2\pi)^3}\left(\frac{2mT\sigma_d^2}{\pi}\right)^{3/2}\rho_n\rho_p = \frac{g_d}{(2\pi)^3}\left(\frac{2mT\sigma_d^2}{\pi}\right)^{3/2}N_n N_p$$

Similarly, the triton yield is:

$$N_t = \frac{g_t}{(2\pi)^6}\left(\frac{2mT\sigma_t^2}{\pi}\right)^{3}N_n^2 N_p$$

These results show that deuteron and triton yields depend on system temperature, volume, and nucleon density. Considering the yield ratio:

$$\frac{N_t N_p}{N_d^2} = \frac{g_t}{g_d^2}\frac{1}{(2\pi)^3}\left(\frac{\sigma_t}{\sigma_d}\right)^6\frac{1}{N_p}$$

Statistical or thermal models give consistent results. For neutrons (protons) with chemical potential μ_n (μ_p), the density is:

$$\rho_{n,p} = \frac{1}{(2\pi)^3}4\pi T m^2 K_2\left(\frac{m}{T}\right)\sinh\left(\frac{\mu_{n,p}}{T}\right)$$

where K_2 is the modified Bessel function of the second order. Assuming deuterons and tritons are in chemical equilibrium with nucleons, their yields are:

$$N_d = \frac{g_d V}{(2\pi)^3}4\pi T(2m)^2 K_2\left(\frac{2m}{T}\right)\sinh\left(\frac{\mu_n+\mu_p}{T}\right)$$

$$N_t = \frac{g_t V}{(2\pi)^3}4\pi T(3m)^2 K_2\left(\frac{3m}{T}\right)\sinh\left(\frac{2\mu_n+\mu_p}{T}\right)$$

This yields the simple ratio:

$$\frac{N_t N_p}{N_d^2} = \frac{g_t}{g_d^2} \frac{K_2(3m/T)K_2(m/T)}{[K_2(2m/T)]^2} \frac{\cosh(\mu_n/T)\cosh((2\mu_n+\mu_p)/T)}{\cosh^2((\mu_n+\mu_p)/T)}$$

Remarkably, this ratio is a constant independent of system temperature and density, providing an excellent baseline for studying its energy dependence in relativistic heavy-ion collisions. However, the strongly interacting matter produced in these collisions undergoes rapid expansion with finite volume and non-uniform density, causing the $N_t N_p/N_d^2$ ratio to deviate from $1/(2\sqrt{3})$. Detailed background contributions are analyzed in Ref. [123].

1.3 Effects of Density Inhomogeneity on Light Nuclei Production

In general, the density of strongly interacting matter produced in relativistic heavy-ion collisions is non-uniform. Assuming nucleon densities $\rho_{n,p}$ depend on spatial position but neglecting nucleon correlations ($C_2 = C_3 = 0$), the deuteron coalescence yield approximates to:

$$N_d \approx g_d \int d_1^{3x} d_2^{3x} d_1^{3p} d_2^{3p}\, \rho_n(x_1)\rho_p(x_2)(2\pi mT)^{-3} \exp\left(-\frac{p_1^2+p_2^2}{2mT}\right) W_d(x,p)$$

Using the gradient approximation:

$$\int d_1^{3x} d_2^{3x}\, \rho_n(x_1)\rho_p(x_2)e^{-(x_1-x_2)^2/2\sigma_d^2} \approx \int d^{3X} d^{3x}\, \rho_n(X+\frac{x}{2})\rho_p(X-\frac{x}{2})e^{-x^2/2\sigma_d^2}$$

Expanding to second order:

$$\approx \int d^{3X} \rho_n(X)\rho_p(X) + \frac{\sigma_d^2}{6}\int d^{3X} \nabla\rho_n(X)\cdot\nabla\rho_p(X)$$

When density fluctuation scales exceed light nuclei sizes, the second term becomes negligible, giving:

$$N_d \approx \frac{g_d}{(2\pi)^3}\left(\frac{2mT\sigma_d^2}{\pi}\right)^{3/2} \int d^{3x}\, \rho_n(x)\rho_p(x)$$

Expressing the spatially dependent nucleon density as $\rho_n(x) = \bar{\rho}_n + \delta\rho_n(x)$ and $\rho_p(x) = \bar{\rho}_p + \delta\rho_p(x)$, the yields simplify to:

$$N_d \approx \frac{g_d}{(2\pi)^3}\left(\frac{2mT\sigma_d^2}{\pi}\right)^{3/2} N_p\bar{\rho}_n(1+C_{np})$$

$$N_t \approx \frac{g_t}{(2\pi)^6}\left(\frac{2mT\sigma_t^2}{\pi}\right)^{3} N_p\bar{\rho}_n^2(1+2C_{np}+\Delta\rho_n)$$

where C_{np} characterizes the correlation between neutron and proton density inhomogeneities, and $\Delta\rho_n$ describes fluctuations in neutron number density in coordinate space [41]. The yields thus depend on temperature, average nucleon density, and nucleon density fluctuations. The ratio becomes:

$$\frac{N_t N_p}{N_d^2} \approx \frac{g_t}{g_d^2}\frac{1}{(2\pi)^3}\left(\frac{\sigma_t}{\sigma_d}\right)^6 \frac{1}{N_p}(1+\Delta\rho_n)$$

assuming C_{np} is small. This shows that the light nuclei yield ratio depends only on nucleon density fluctuations, making it an ideal observable for detecting first-order QCD phase transition signals [40].

1.4 Effects of Long-Range Correlations on Light Nuclei Production

Thus far we have considered only density inhomogeneity effects. Near the QCD critical point, many-body correlations become crucial and C_2 contributions cannot be neglected. For a thermally equilibrated system near the critical point, the critical fluctuations dominate, and C_2 can be generically expressed as [42]:

$$C_2^{\text{critical}}(x_1,x_2) = \lambda\frac{e^{-|x_1-x_2|/\xi}}{|x_1-x_2|}$$

where ξ is the correlation length and λ characterizes the correlation strength, depending on the specific interaction. Integrating gives the deuteron yield:

$$N_d = \frac{g_d}{(2\pi)^3}\left(\frac{2mT\sigma_d^2}{\pi}\right)^{3/2} N_p\bar{\rho}_n\left[1+C_{np}+\frac{\lambda}{\bar{\rho}_n}G\left(\frac{\xi}{\sigma_d}\right)\right]$$

where the enhancement function $G(z) = \frac{1}{\sqrt{2\pi}}\int d^{3r}\, e^{-r^2/2}\frac{e^{-r/z}}{r} = \text{erfc}\left(\frac{1}{\sqrt{2}z}\right)e^{1/(2z^2)}$ depends on the correlation length as shown in FIGURE:2.

Similarly, the triton yield is:

$$N_t = \frac{g_t}{(2\pi)^6}\left(\frac{2mT\sigma_t^2}{\pi}\right)^3 N_p\bar{\rho}_n^2\left[1+\Delta\rho_n+2C_{np}+\frac{2\lambda}{\bar{\rho}_n}G\left(\frac{\xi}{\sigma_t}\right)\right]$$

and the yield ratio becomes:

$$\frac{N_tN_p}{N_d^2}\approx\frac{g_t}{g_d^2}\frac{1}{(2\pi)^3}\left(\frac{\sigma_t}{\sigma_d}\right)^6\frac{1}{N_p}\left[1+\Delta\rho_n+\frac{2\lambda}{\bar{\rho}_n}\left(G\left(\frac{\xi}{\sigma_t}\right)-G\left(\frac{\xi}{\sigma_d}\right)\right)+O(G^2)\right]$$

The ratio increases monotonically with correlation length, demonstrating that N_tN_p/N_d^2 can serve as a probe for QCD critical fluctuations [42].

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2. Transport Model Study of QCD First-Order Phase Transition Effects on Light Nuclei Production

The previous calculations assumed that density inhomogeneities and long-range correlations from QCD phase transitions survive until the final stage of hadronic gas expansion. To quantitatively study these effects, constructing microscopic models that describe QCD phase transition dynamics is essential. Due to non-perturbative effects, solving finite-density finite-temperature QCD dynamics is extremely difficult. The NJL model, a low-energy effective theory of QCD, is widely used to describe chiral symmetry breaking and restoration. We employ a microscopic transport model based on the NJL model to describe the dynamical evolution of relativistic heavy-ion collisions and investigate first-order chiral phase transition effects on light nuclei production.

2.1 NJL Model

We use a three-flavor NJL model to describe parton interactions, with Lagrangian density [124]:

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_S+\mathcal{L}_V+\mathcal{L}_{\rm det}$$

$$\mathcal{L}_0=\bar{\psi}(i\gamma^\mu\partial_\mu-\hat{m})\psi$$

$$\mathcal{L}_S=G_S\sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\psi)^2+(\bar{\psi}i\gamma_5\lambda_a\psi)^2\right]$$

$$\mathcal{L}_V=-g_V\sum_{a=0}^{8}\left[(\bar{\psi}\gamma^\mu\lambda_a\psi)^2+(\bar{\psi}\gamma^\mu\gamma_5\lambda_a\psi)^2\right]$$

$$\mathcal{L}_{\text{det}} = K \left\{ \det \left[\bar{\psi}(1+\gamma_5)\psi \right] + \det \left[\bar{\psi}(1-\gamma_5)\psi \right] \right\}$$

where $\psi = (u, d, s)^T$ is the quark field, $\hat{m} = \text{diag}(m_u, m_d, m_s)$ is the current quark mass matrix, G_S is the scalar coupling constant, λ_a ($a = 1, ..., 8$) are Gell-Mann matrices, and $\lambda_0 = \sqrt{2/3}I$. The Kobayashi-Maskawa-t' Hooft (KMT) term $\mathcal{L}_{\text{det}}$ breaks $U(1)_A$ symmetry. The determinant in flavor space gives a six-point interaction.

Using mean-field approximation [127], the Lagrangian becomes:

$$\mathcal{L}_{\text{MF}} = \bar{u}(\gamma^\mu iD_\mu - M_u)u + \bar{d}(\gamma^\mu iD_\mu - M_d)d + \bar{s}(\gamma^\mu iD_\mu - M_s)s + \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{2G_S} + 4K\phi_u\phi_d\phi_s + g_V(j_u^\mu + j_d^\mu + j_s^\mu)^2$$

where the in-medium quark masses are:

$$M_u = m_u - 4G_S\phi_u + 2K\phi_d\phi_s$$

$$M_d = m_d - 4G_S\phi_d + 2K\phi_u\phi_s$$

$$M_s = m_s - 4G_S\phi_s + 2K\phi_d\phi_d$$

with quark condensates $\phi_u = \langle \bar{u}u \rangle$, $\phi_d = \langle \bar{d}d \rangle$, $\phi_s = \langle \bar{s}s \rangle$ and net quark vector currents $j_u^\mu = \langle \bar{u}\gamma^\mu u \rangle$, etc. The covariant derivative is $iD_\mu = i\partial_\mu - A_\mu$ with $A_\mu = 2g_V(j_u^\mu + j_d^\mu + j_s^\mu)$. The quark energy is $E_i = \sqrt{M_i^2 + p^2}$ and effective chemical potential is $\mu_i^* = \mu_i - 2g_V(\rho_u + \rho_d + \rho_s)$.

2.2 Equation of State

The thermodynamic properties of the three-flavor quark system are obtained from the partition function $\mathcal{Z} = \text{Tr}\, e^{-\beta(\hat{H} - \mu\hat{N})}$. The thermodynamic potential for quark matter of volume V is:

$$\Omega = \Omega_u + \Omega_d + \Omega_s + \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{2G_S} + 4K\phi_u\phi_d\phi_s - g_V(\rho_u + \rho_d + \rho_s)^2 - \Omega_0$$

where $\Omega_i = -2N_c T \int \frac{d^{3p}}{(2\pi)^3} \left\{ \ln\left[1 + e^{-(E_i - \mu_i^*)/T}\right] + \ln\left[1 + e^{-(E_i + \mu_i^*)/T}\right] \right\}$ and $\rho_i = 2N_c \int \frac{d^{3p}}{(2\pi)^3} [n_i - \bar{n}_i]$ with distribution functions $n_i = (e^{(E_i - \mu_i^*)/T} + 1)^{-1}$ and $\bar{n}_i = (e^{(E_i + \mu_i^*)/T} + 1)^{-1}$.

Minimizing the thermodynamic potential with respect to effective masses and chemical potentials ($\delta\Omega/\delta M_i = \delta\Omega/\delta\mu_i^* = 0$) yields the quark condensates and net quark densities. The NJL model is non-renormalizable, requiring a momentum cutoff Λ. The energy density is:

$$\varepsilon = \sum_{i=u,d,s} \int_0^{\Lambda} \frac{d^{3p}}{(2\pi)^3} E_i(n_i+\bar{n}_i)+2G_S(\phi_u^2+\phi_d^2+\phi_s^2)-4K\phi_u\phi_d\phi_s+g_V(\rho_u+\rho_d+\rho_s)^2+\varepsilon_0$$

with parameters $m_u = m_d = 5.5$ MeV, $m_s = 140.7$ MeV, $G_S\Lambda^2 = 1.835$, $K\Lambda^5 = 12.36$, and $\Lambda = 602.3$ MeV [124,128,129].

FIGURE:3 shows the quark matter phase diagram in the temperature-density plane without vector interactions. The dot-dashed line represents the chiral restoration/breaking coexistence line, while the solid line encloses the mechanically unstable region $(\partial P/\partial\rho_q)_T < 0$. The critical point occurs at $T \approx 67$ MeV and net quark density ≈ 0.85 fm^{-3}. With vector coupling $g_V = G_S$, the critical point disappears and the chiral transition becomes a smooth crossover.

2.3 Transport Equations

To describe non-equilibrium properties during phase transitions, we derive the time evolution equation for quark phase-space distribution functions based on the NJL model [131,132]:

$$\frac{\partial f^{\pm}}{\partial t} + \mathbf{v}\cdot\nabla_r f^{\pm} \pm \mathbf{E}^*\cdot\nabla_p f^{\pm} = \left(\frac{\partial f^{\pm}}{\partial t}\right)_{\text{coll}}$$

where the effective fields are $\mathbf{E}^* = -\nabla_r A_0$ and $\mathbf{B} = \nabla_r \times \mathbf{A}$, with A_μ defined in Eq. (28). The in-medium parton energy is E^* and velocity is $\mathbf{v} = \mathbf{p}/E^*$. The collision integral is:

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \int \frac{d_2^{3p} d_3^{3p} d_4^{3p}}{(2\pi)^9} d\Omega|\mathbf{v}_{12}|\frac{d\sigma}{d\Omega}\delta^{(3)}(\mathbf{p}+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4)\times\{f(\mathbf{r},\mathbf{p},t)f(\mathbf{r},\mathbf{p}_2,t)[2N_c - f(\mathbf{r},\mathbf{p}_3,t)][2N_c - f(\mathbf{r},\mathbf{p}_4,t$$

The transport equation is solved using the test-particle method [133], where each test particle undergoes motion in the mean field and scattering with other partons. The equations of motion are:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{p}}{dt} = \pm\mathbf{E}^*$$

Partons with momentum exceeding the cutoff Λ are excluded from scalar and vector density calculations and mean-field interactions. In our calculations, we use 100 test particles per real particle, with a grid size of 1 fm $\times$ 1 fm $\times$ 1 fm for density calculations. To reduce statistical fluctuations in the mean field, scalar and vector densities are averaged over neighboring $3 \times 3 \times 3$ grids. The time step is $\Delta t = 0.2$ fm.

2.4 Box Calculations

We first examine the dynamics of a first-order phase transition in infinite nuclear matter before studying finite heavy-ion collision systems. We simulate infinite quark matter in a 20 fm $\times$ 20 fm $\times$ 20 fm box with periodic boundary conditions. The initial net quark density and temperature are 0.7 fm^{-3} and 40 MeV, respectively. As shown in FIGURE:4, at $t = 0$ fm/c, quarks are uniformly distributed with only statistical fluctuations. As time evolves, because the quark matter lies in the mechanically unstable region of the phase diagram, initial statistical fluctuations are amplified, leading to cluster formation and large density fluctuations. By $t = 100$ fm/c, clustering is clearly visible.

FIGURE:5 shows the time evolution of the scaled second-order density moment $y_2 = \langle(\delta\rho)^2\rangle/\langle\rho\rangle^2$ [54] and temperature difference. The y_2 value increases slowly initially, then rapidly reaches ~ 1.15. As clustering develops, latent heat is released, increasing the temperature by at most 3 MeV—an effect likely difficult to observe experimentally.

2.5 Heavy-Ion Collision Simulations

We now consider first-order QCD phase transition effects on light nuclei production in heavy-ion collisions. The initial quark and antiquark distributions follow a Woods-Saxon profile:

$$\rho(r) = \frac{\rho_0}{1 + \exp((r - R)/a)}$$

with $R = 6$ fm, surface thickness $a = 0.6$ fm, and central net quark density $\rho_0 = 1.5$ fm^{-3}. The initial temperature is 70 MeV with $\mu_u = \mu_d$ and $\mu_s = 0$. We compare two interaction scenarios: $g_V = 0$ and $g_V = G_S$.

For $g_V = 0$ (critical temperature ≈ 67 MeV), FIGURE:6 shows the net quark density distribution and its second moment y_2 evolving in time. During the partonic phase, y_2 increases dramatically as the system expands, indicating large density inhomogeneities. In contrast, for $g_V = G_S$, FIGURE:7 shows y_2 remains nearly constant at 1.25, with no significant density inhomogeneity development.

FIGURE:8 displays the time dependence of the net-baryon density second moment y_2. The solid and dash-dotted lines correspond to $g_V = 0$ (first-order transition) and $g_V = G_S$ (crossover), respectively. Hadronization occurs at the star-marked times. For the first-order case, y_2 exceeds 2 and only decreases weakly during hadronic evolution, demonstrating that density fluctuations survive the fireball expansion—analogous to how quantum fluctuations in the early universe survived cosmic expansion to produce observed cosmic microwave background anisotropies. For the crossover case, density inhomogeneities show no significant change throughout the evolution.

FIGURE:9 shows the phase-space trajectory evolution. Points represent average temperature and net quark density, weighted by quark number in each cell. For $g_V = 0$, at $t = 4$ fm/c the system enters the spinodal instability region, where $\sim 65\%$ of quarks have masses below 200 MeV. As the system expands, density fluctuations grow, reaching $y_2 = 2$ at $t = 15$ fm/c with average quark mass ~ 270 MeV ($\sim 70\%$ of vacuum mass). At this point, $\sim 70\%$ of quarks have masses exceeding 200 MeV and the temperature is ~ 30 MeV. For $g_V = G_S$, the system passes through the phase transition region in ~ 8 fm/c, with $\sim 70\%$ of quarks having masses above 200 MeV.

Using the final-state nucleon phase-space distributions, we calculate light nuclei yields via the coalescence model. FIGURE:10 shows the $N_t N_p / N_d^2$ ratio dependence on the equation of state. For $g_V = 0$, the trajectory passes through the first-order phase transition region, developing large density inhomogeneities and yielding $N_t N_p / N_d^2 \approx 0.50$. For $g_V = G_S$ and $g_V = 2G_S$ (crossover transitions), the ratio is ≈ 0.38. The first-order phase transition thus increases $N_t N_p / N_d^2$ by $\sim 25\%$. Unlike the analytical derivation, these coalescence calculations contain no approximations.

2.6 Preliminary Results on Energy Dependence of $N_t N_p / N_d^2$

Our model calculations demonstrate that first-order QCD phase transitions in heavy-ion collisions significantly enhance the $N_t N_p / N_d^2$ ratio. Different collision energies probe different regions of the phase diagram, enabling energy scan experiments to search for first-order phase transition signals. However, the NJL model' s critical temperature is only ~ 70 MeV, making it unsuitable for RHIC energy scan studies.

Recently, by introducing scalar-vector coupling interactions [134], we developed an extended NJL model-based parton transport model [135] with critical temperatures up to ~ 150 MeV, appropriate for RHIC energy scans. Calculations show that for $\sqrt{s_{NN}} > 5$ GeV, different equations of state give consistent $N_t N_p / N_d^2$ values. For $\sqrt{s_{NN}} < 5$ GeV, the ratio increases with higher critical temperatures. When the critical temperature exceeds 140 MeV, the predicted $N_t N_p / N_d^2$ values match experimental measurements.

Model calculations without critical or first-order effects (e.g., JAM+COAL [140], AMPT+COAL [136], UrQMD+COAL [137], MUSIC+UrQMD+COAL [138]) show nearly flat energy dependence. Recent STAR data [139] for $\sqrt{s_{NN}} >$ 20 GeV exhibit possible non-monotonic behavior, though no model currently describes this quantitatively. One possibility is that long-range correlations near the critical point enhance $N_t N_p / N_d^2$ at specific collision energies (see Eq. (22)) [42].

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3. Results and Discussion

Coalescence model calculations reveal that light nuclei yields depend on final-state nucleon density fluctuations and correlations. Specifically, nucleon density fluctuations and long-range correlations increase the $N_t N_p / N_d^2$ ratio. To explore the dynamical evolution of density inhomogeneities from first-order QCD phase transitions, we developed a relativistic transport model based on the NJL model. Our calculations show that due to rapid system expansion, density fluctuations from first-order phase transitions survive to late stages, increasing the coalescence-produced light nuclei yield ratio by approximately 25%. These results provide theoretical justification and quantitative predictions for using light nuclei production to search for QCD phase transition signals.

We have further developed an extended NJL transport model where scalar-vector interaction coupling strength can be tuned to move the critical point location, with critical temperatures adjustable from 40 MeV to 150 MeV. By scanning collision energy, centrality, and critical point position, we find that first-order QCD phase transitions in $\sqrt{s_{NN}} \approx 3 - 5$ GeV Au+Au collisions substantially enhance $N_t N_p / N_d^2$, consistent with preliminary STAR measurements.

Due to mean-field approximation and test-particle methods, our current transport model can only describe single-particle distribution evolution and cannot accurately treat two-particle correlation function evolution, limiting our study to first-order transition effects. Incorporating critical fluctuations into transport models remains a significant challenge. Additionally, the present NJL-based transport model includes only quark degrees of freedom without gluons; self-consistently introducing gluon dynamics is another important and challenging task [141].

When non-smooth QCD phase transitions occur in heavy-ion collisions, altered system dynamics affect many observables. We have carefully discussed impacts on light nuclei yields, but QCD phase transitions also influence collective flow [52], HBT correlations [50], dilepton production [51], ρ meson yields [142], and spin alignment [143]. Work on these aspects is ongoing.

Model calculations demonstrate that non-smooth QCD phase transitions in heavy-ion collisions affect light nuclei production, particularly enhancing the $N_t N_p / N_d^2$ yield ratio. Exploiting this effect to search for QCD phase transition signatures is a promising direction. Future theoretical work should clarify light nuclei formation mechanisms and dynamical effects of QCD phase transitions.

Author Contributions

All authors contributed equally to this work.

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Note: Figure translations are in progress. See original paper for figures.