Transport Model Study of Conserved Charge Fluctuations and QCD Phase Transition in Heavy-Ion Collisions (Postprint)

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Abstract

The Relativistic Heavy Ion Collider (RHIC)-STAR (Solenoid Tracker at RHIC) experiment measured the cumulants of multiplicity distributions for net-protons (representative of net baryons), net-charge, and net-kaons (representative of net strangeness) in Au+Au collisions at sNN = 7.7~200 GeV, and found that the ratio of the fourth-order to second-order cumulants (σ^2) for net-protons exhibits non-monotonic energy dependence. In relativistic heavy ion collision experiments, only information about final-state particles can be measured. Therefore, based on the A Multi-Phase Transport (AMPT) model, the fluctuation properties of conserved charges (baryon number, charge number, and strangeness) in Au+Au collision systems were studied, and it was found that the AMPT model results can basically describe the RHIC-STAR experimental results. More importantly, using the AMPT model, we have understood the influence of several key effects in the dynamical evolution of relativistic heavy ion collisions (production and diffusion of conserved-charge particles, hadronization, hadronic rescattering, and weak decays) on the evolution of conserved-charge fluctuations and their particle correlation functions. It was found that the correlation between positive and negative charges may originate from the string melting mechanism, the baryon (proton) correlation function is consistent with expectations from baryon number conservation, and the strangeness (net-kaon) correlation function originates from pair production. The correspondence between these representative quantities and conserved charges is qualitatively consistent in behavior, but differs quantitatively. Although the AMPT model currently lacks a critical fluctuation mechanism, our results can provide a baseline for the study of conserved-charge fluctuations, which helps in searching for possible critical behavior near the Quantum Chromodynamics (QCD) Critical End Point (CEP) in relativistic heavy ion collisions. Preliminary inclusion of critical density fluctuations in the model shows that they play a certain role.

Full Text

Preamble

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Transport Model Study of Conserved Charge Fluctuations and QCD Phase Transition in Heavy-Ion Collisions

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Abstract

The RHIC-STAR (Relativistic Heavy Ion Collider-Solenoid Tracker at RHIC) experiments have measured the cumulants of net-proton (a proxy for net-baryon), net-charge, and net-kaon (proxy of net-strangeness) multiplicity distributions in Au+Au collisions at center-of-mass energies ranging from 7.7 GeV to 200 GeV. Recent results have shown that the ratio of the fourth-order net-proton cumulant over the second-order one (σ^2) exhibits a nonmonotonic energy dependence. In relativistic heavy-ion collision experiments, only information about the final state particles can be measured. Therefore, we investigated the fluctuations of the conserved charges (baryon, electric charge, and strangeness) in Au+Au collisions using a multiphase transport (AMPT) model. This model can basically describe the results measured by the RHIC-STAR experiment. More importantly, the AMPT model is used to understand the key impacts of the dynamical evolution of relativistic heavy-ion collisions on fluctuations and correlation functions, including the creation and diffusion of conserved charges, hadronization, hadronic rescatterings, and weak decays. It was discovered that the correlation between positive and negative charges may originate from the string melting mechanism. Baryon (proton) correlation functions are consistent with the expectation of baryon number conservation. Net-strangeness (net-kaon) originates from pair production. We studied the correspondence between representative quantities and their conserved charges and found that their behaviors are qualitatively consistent yet quantitatively different. Although the physics of quantum chromodynamics (QCD) critical fluctuations is not included in the AMPT model, our results are expected to provide a baseline for the search of possible critical behavior at the QCD critical end point in relativistic heavy-ion collisions. We incorporated critical density fluctuations into the model and

found that they play a role.

Keywords: Conserved charge, Fluctuation, Cumulants, Correlation functions, QCD phase transition, Critical end point (CEP)

Classification Codes: O41, O56
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Introduction

Quantum Chromodynamics (QCD) is the fundamental theory describing the strong interaction between quarks and gluons. QCD theory has successfully described numerous experimental phenomena, from hadron spectra to few-body processes in deep inelastic scattering. Studying the properties of hot and dense QCD matter is crucial for understanding natural phenomena such as the early universe state and the formation and evolution of compact celestial objects. Relativistic heavy-ion collision experiments are currently the only experimental means to study the properties of high-temperature and high-density QCD matter. In relativistic heavy-ion collisions, numerous high-energy nucleons participate in the collision, easily creating conditions of high temperature or energy density that can lead to QCD phase transitions.

QCD theoretical studies indicate that at high temperature or high density, quarks and gluons confined within hadrons (confinement phase) undergo deconfinement to produce Quark-Gluon Plasma (QGP) (deconfinement phase). Therefore, research on QCD phase diagram structure has been a key focus of QCD matter strong interaction theory and experimental studies for many years. The thermodynamic properties of a many-body system can be represented by a phase diagram in thermodynamic parameter space. In the case of QCD matter (Figure 1 [Figure 1: see original paper]), there exists a T- B QCD phase diagram as a function of temperature T and baryon chemical potential B. Different regions in the phase diagram correspond to different existence states of QCD matter with different symmetries. The most important location in the phase diagram is the Critical End Point (CEP), which can inform us about the basic structure of the phase diagram.

Lattice QCD first-principles calculations show that the phase transition at zero baryon chemical potential (B) is a smooth crossover, with a critical temperature of Tc 160 MeV. Some effective QCD models predict that a first-order phase transition will occur in the large B region, and the point where the first-order phase transition curve intersects with the smooth crossover region is called the QCD critical point. Experimentally, many measurement methods have been adopted to search for the CEP, such as collective flow, HBT (Hanbury-Brown-Twiss) radii, light nuclei yield ratios, and conserved charge fluctuations. Theoretically, extensive research has been conducted in these areas. The correlation length diverges near the critical point, and since conserved charge fluctuations are very sensitive to the correlation length, lattice QCD calculations indicate that conserved charge fluctuations are related to susceptibilities. Therefore, conserved charge fluctuations are considered a promising observable signal.

For research on QCD phase diagram structure, relativistic heavy-ion collision experiments have conducted extensive work at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) and the Large Hadron Collider (LHC) at CERN. These experiments provide baryon chemical potentials ranging from B 750 MeV to zero baryon chemical potential region at chemical freeze-out. Future domestic experimental facilities will be able to reach larger baryon chemical potential regions. The first phase of the RHIC Beam Energy Scan has measured the cumulants of net-proton (proxy for net-baryon), net-charge, and net-kaon (proxy for net-strangeness) multiplicity distributions in Au+Au collision systems at sNN = 7.7–200 GeV. Recently, RHIC experiments based on net-proton higher-order cumulant measurements predict that critical phenomena may occur at sNN < 20 GeV. This paper will focus on the hot topic of conserved charge fluctuations in high-energy heavy-ion collisions, introducing relevant cutting-edge research progress at home and abroad and summarizing a series of research results we have conducted over the years using transport models.

1.1 Calculation Methods

In statistics, various characteristics of probability distributions can be described by different moments, such as mean (M), variance (σ), skewness (S), and kurtosis ( ). The cumulants of a random variable refer to a series of quantities that provide the same information as moments. Cumulants can typically be calculated using a cumulant generating function. Currently, experimental detection technology can only detect information about final-state particles such as multiplicity, charge, and momentum. For conserved charge fluctuation problems, research is mainly conducted by calculating the cumulants of conserved charge (baryon number, charge number, and strangeness) multiplicity distributions event-by-event.

The nth-order cumulants Cn of conserved charge multiplicity distributions are given by:

$C_1 = \ N \ ,$
$C_2 = \ (\delta N)^2 \ ,$
$C_3 = \ (\delta N)^3 \ ,$
$C_4 = \ (\delta N)^4 \ - 3 \ (\delta N)^2 \ ^2$

where N is the number of particles in each event, $\delta N = N - \ N$ is the deviation, and ... denotes the event average. Based on the definition of cumulants, we can obtain various moments:

$M = \ N \ ,$
$\sigma^2 = \ (\delta N)^2 \ ,$
$S = \ (\delta N)^3 \ /\sigma^3,$
$\ = \ (\delta N)^4 \ /\sigma^4 - 3$

where the mean (M) represents the average value of the distribution, variance (σ^2) represents the dispersion, skewness (S) represents the asymmetry, and kurtosis ( ) describes the degree of peak deviation from a normal distribution.

In a thermodynamic equilibrium system (such as the grand canonical ensemble), the system can be characterized by the logarithm of the QCD partition function, i.e., its dimensionless pressure:

$$P/T^4 = (1/VT^3)\ \ln[Z(V,\ T,\ \ B,\ \ Q,\ \ S)]$$

where V and T are the system volume and temperature, and B, Q, and S are the baryon (B), charge (Q), and strangeness (S) chemical potentials, respectively. For thermodynamic equilibrium systems with different degrees of freedom and interactions, the fluctuations of these quantities can be determined. In lattice QCD theory, these fluctuations can be quantified by conserved charge susceptibilities:

$$Q \quad = \quad ^{+\,+} (P/T^4)/\hat{}\quad \hat{}Q\ \ \hat{}S$$

where ˆq = q/T and the subscript q = B, Q, S. The relationship between these conserved charge cumulants and the corresponding susceptibilities can be expressed as:

$$C\ Q \quad = VT^3 \quad Q \quad (T,\ \ B,\ \ Q,\ \ S)$$

Through equations (2) and (5), we can obtain cumulant ratios (moment products), defined as follows:

$$C_2/C_1 = \sigma^2/M,$$
$$C_3/C_2 = S\sigma,$$
$$C_4/C_2 = \ \sigma^2$$

Fluctuations and correlations are inseparable partners. We can also obtain correlation functions from the cumulants of multiplicity distributions. Since cumulants have the disadvantage of mixing different orders of correlation functions, studying multi-particle (integral) correlation functions is often more instructive. Using the following relations, n-particle correlation functions n (also called factorial cumulants) can be obtained from cumulants:

$$_1 = C_1 = \ N\ ,$$
$$_2 = C_2 - C_1,$$
$$_3 = C_3 - 3C_2 + 2C_1,$$
$$_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

Conversely, the relationship between cumulants and correlation functions from n-particle correlation functions n is:

$$C_1 = \ _1,$$
$$C_2 = \ _2 + \ _1,$$
$$C_3 = \ _3 + 3\ _2 + \ _1,$$
$$C_4 = \ _4 + 6\ _3 + 7\ _2 + \ _1$$

Cumulant ratios can also be expressed using normalized correlation functions $\ n/\ _1$ (n>1):

$C_2/C_1 = \ _2/\ _1 + 1,$
$C_3/C_2 = (\ _3/\ _1 + 3\ _2/\ _1 + 1)/(\ _2/\ _1 + 1),$
$C_4/C_2 = (\ _4/\ _1 + 6\ _3/\ _1 + 7\ _2/\ _1 + 1)/(\ _2/\ _1 + 1)$

Therefore, according to equation (9), fluctuations can be further understood using normalized correlation functions. However, it should be noted that equations (7)–(9) only apply to analyzing correlation functions of one type of particle. For calculating net-particle correlation functions, two types of particles need to be considered. For example, net-kaons (ΔNK = N_K$^+$ + N_K$^-$) involve K$^+$ and K$^-$, requiring more complex calculation methods. According to the factorial moments mentioned in reference [68], correlation functions can be calculated, and the relationship between factorial moments and correlation functions is as follows:

For K$^+$ and K$^-$ as an example, where Fi,k = N_K$^+$ N_K$^-$ represents the factorial moment of i K$^+$ mesons and k K$^-$ mesons, and $^{(,)}$ represents the n-particle (n = i + k) correlation function. If i and k are not both zero, then $^{(,)}$ is a mixed correlation function. If i and k are not zero, $^{(,)}$ represents the n-particle correlation function containing i K$^+$ and k K$^-$. If either i or k is zero, it represents a pure correlation function containing only one type of particle. For example, $^{(,0)}$ and $^{(0,)}$ represent pure correlation functions containing only i K$^+$ and k K$^-$, respectively. Through these correlation functions, their relationship with net-kaon cumulants can also be obtained:

C_1 = N_K$^+$ - N_K$^-$,
C_2 = N_K$^+$ + N_K$^-$ + 2 $_2^{(1,1)}$,
C_3 = N_K$^+$ - N_K$^-$ + 3 $_3^{(2,1)}$ - 3 $_3^{(1,2)}$,
C_4 = N_K$^+$ + N_K$^-$ + 6 $_4^{(2,2)}$ + 4 $_4^{(3,1)}$ + 4 $_4^{(1,3)}$

In the following sections, we will use the above formulas to calculate relevant physical quantities for different conserved charge fluctuations.

1.2 The AMPT Model

The A Multi-Phase Transport (AMPT) model has been widely used to study relativistic heavy-ion collisions. The AMPT model with string melting mechanism is a hybrid model containing four sub-packages that simulate the four main stages of relativistic nucleon collisions. It uses the Heavy Ion Jet Interaction Generator (HIJING) to simulate initial conditions, primarily providing the spatial and momentum distributions of partons from both hard QCD processes and soft excitation strings. Many parent hadrons (including strange mesons and strange baryons) are briefly produced from the melting of mini-jet partons and excited strings. To simulate partonic matter, these parent hadrons are split into original quarks and antiquarks according to the flavor and spin structure of their quark constituents. Zhang's Parton Cascade (ZPC) model simulates parton interactions within the quark-gluon plasma. Currently, this model only considers two-body elastic scattering using perturbative QCD cross sections (for example, typically 3 mb). When all partons stop interacting, the hadronization process combines partons into hadrons through a quark coalescence model. The A Relativistic Transport (ART) model simulates interactions between hadrons in the hadronic phase, including meson-meson, meson-baryon, and baryon-baryon scattering processes. Both antiparticle and strangeness production are considered, along with nucleon mean field and isospin effects. To compare with experimental data, our simulations also include resonance decays, including those of unstable strange hadrons such as Λ, Ω, Ξ, Σ, and ϕ.

The above describes the publicly released AMPT model. To better study conserved charge fluctuations, we have also made relevant improvements to the publicly released AMPT model. The main two points are introduced below:

First, we adopted a charge-conservation version of the AMPT model, which ensures conservation of various conserved charges (baryon number, charge number, and strangeness) in all hadronic reaction channels during the hadronic phase evolution. In the old version of the AMPT model, total charge was not conserved mainly for two reasons: 1) In the old version, only K^+ and K^- were explicitly introduced in the hadronic rescattering process, while K^0 and $\bar{K}^0$ were ignored. To effectively include K^0 and $\bar{K}^0$, the old version replaced K^0 with K^+ and $\bar{K}^0$ with K^- before hadronic rescattering, and then replaced half of the K^+ and K^- with K^0 and $\bar{K}^0$ after hadronic rescattering, violating charge conservation and strangeness conservation. 2) In the old version, not all possible isospin modes of hadronic reaction channels or resonance decays were considered. Instead, isospin-averaged cross-section treatment was used, where the charge of final-state particles was randomly selected from all possible charges regardless of the initial-state charge, causing total charge non-conservation. For example, $\pi^+ + \pi^+$ should be allowed to enter $^+ + ^+$ rather than $^+ + ^-$ and $^- + ^-$. To solve these two main problems, in the charge-conservation version of the AMPT model, K^0 and $\bar{K}^0$ are explicitly introduced. On the other hand, all problematic reaction channels have been corrected to ensure that all reaction channels satisfy baryon number, charge number, and strangeness conservation, ensuring that each pair of forward and reverse reaction channels is in equilibrium under the principle of detailed balance.

Second, a new quark coalescence model was introduced, which improves the treatment of the hadronization process. In the new quark coalescence model, quarks determine whether to form mesons or baryons based on the distance to their merging partners, rather than simply merging the nearest quarks into hadrons as in the previous quark coalescence model. Studies have shown that in relativistic heavy-ion collisions, the string melting version of the AMPT model with the new quark coalescence mechanism can better describe the properties of matter, especially the production of strange hadrons. On the other hand, in the new quark coalescence model, net charge, net baryon number, and net strangeness are conserved, rather than requiring unnecessary separate conservation as in the previous quark coalescence model.

This review mainly elaborates on the study of fluctuations using the AMPT model. In the following sections, we will discuss in detail the fluctuations of different conserved charges through the study of different types of particle properties. Relativistic heavy-ion collisions are actually a complex dynamical evolution process, including several important evolution stages. To better understand the dynamical evolution of fluctuation observables, it is necessary to study the cumulants and correlation functions at each evolution stage. We mainly focus on the several evolution stages shown in Table 1 .

Table 1 Five evolution stages in the AMPT model with a string melting mechanism

Evolution Stage	Description
(a) Initial state	The initial state of partonic matter consisting of quarks and antiquarks created by A+A collisions
(b) After parton cascade	The final state of partonic matter consisting of quarks and antiquarks that have undergone parton cascade
(c) After hadronization	The initial state of hadronic matter transformed from the freeze-out partonic matter through the hadronization of coalescence
(d) After hadronic rescatterings	The freeze-out hadronic matter which has undergone hadronic rescatterings
(e) Final state	The final state of hadronic matter with considering weak decays

To provide an intuitive understanding, Figures 2 [Figure 2: see original paper]–4 [Figure 4: see original paper] respectively show the density distributions of positive/negative charges, positive/negative baryon numbers, and positive/negative strangeness in the transverse plane at different evolution stages for central Au+Au collision events at sNN = 7.7 GeV. Note: For the study of charge number fluctuation properties, we ignore the weak decay stage because Figures 2(d) and 2(e) show that weak decay has little effect on charge number density distribution. For the study of baryon number fluctuation properties, we directly combine hadronic rescattering and weak decay effects into the final state because Figures 3(d) and 3(e) also show that weak decay has little effect on baryon number density distribution. However, for the study of strangeness fluctuation properties, we will consider each evolution stage (Figure 4 [Figure 4: see original paper]), which will be discussed in detail later.

Net-Charge Fluctuation Properties

In 2014, the STAR collaboration first published the results of different order moments of net-charge multiplicity distributions in Au+Au collision systems. Although the results deviated from the expectations of Poisson or negative binomial distributions, both the collision centrality dependence and energy dependence showed monotonic behavior, and no nonmonotonic behavior was observed. We used the AMPT model to calculate the dynamic evolution of net-charge multiplicity distribution moments, focusing on studying the dynamical evolution process connecting lattice QCD calculations and final-state experimental measurements. For comparative analysis, we selected the same selection criteria as the STAR experiment, choosing charged particles within transverse momentum 0.2 GeV < pT < 2.0 GeV and pseudorapidity | | < 0.5 to calculate the moments and moment products of net-charge multiplicity distributions. We used charged particle multiplicity to divide collision centrality. To avoid self-correlation effects (ACE), charged particles in pseudorapidity | | < 0.5 were excluded when defining centrality, and the Δ theoretical formula was applied to calculate statistical errors. To suppress volume fluctuations caused by centrality interval width and initial nuclear volume (geometry), i.e., the Centrality Bin Width Effect (CBWE), we used Centrality Bin Width Correction (CBWC) to calculate moments and cumulants, as both STAR experimental results and AMPT results show that CBWC significantly improves higher-order cumulants or moments, especially for more central collisions.

2.1 Energy Dependence of Net-Charge Moment Products

Based on the AMPT model, we studied the energy dependence of net-charge moment products, selecting the most central and peripheral collisions for analysis, and considering both AMPT model data with and without CBWC, all compared with STAR experimental data. From Figure 5 [Figure 5: see original paper], it can be seen that for a given collision centrality (0%–5% and 70%–80%), the σ^2/M values show an exponential increase trend with energy. Whether CBWC is considered or not, the σ^2/M values for both most central and peripheral collisions are close to experimental data, and the values for peripheral collisions (70%–80%) are greater than those for most central collisions (0%–5%). For Sσ and σ^2 values, Sσ decreases with increasing energy, while σ^2 results are independent of energy changes. In the AMPT model, the CBWC-corrected values of Sσ and σ^2 are smaller than the uncorrected values, consistent with the conclusion that CBWC has a greater impact on higher-order moments. From Figure 5, it can be found that the AMPT model results are basically consistent with experimental measurements, and no nonmonotonic energy dependence of net-charge multiplicity distribution moment products is observed, perhaps because the AMPT model does not include QCD critical fluctuation physics mechanisms, or because charge fluctuations are not sensitive to critical fluctuations.

2.2 Positive-Negative Charge Correlations

As mentioned earlier, heavy-ion collisions are actually a complex dynamical evolution process, including several important evolution stages. Therefore, studying the evolution process of net-charge moment products will help us understand the dynamical information of fluctuations. We calculated the results of four evolution stages in the AMPT model at two energies, sNN = 7.7 GeV and 200 GeV. To elucidate the possible contributions of dynamics to moments or moment products, a reference baseline of Poisson distribution without any correlations is crucial. To obtain the Poisson expectation moments of net-charge distribution, we first assume that the distributions of positively charged particles and negatively charged particles are two independent Poisson distributions, and obtain the Poisson expectations K_Q^{+n} and K_Q^{-n} for positive and negative charge particles, respectively. In the Poisson expectation case, the net-charge distribution is a combination of two Poisson distributions, also known as a Skellam distribution, with Poisson expectation K_Q^{+n} + (-1)^{n}K_Q^{-n}, reflecting the situation where there is no correlation between positively and negatively charged particles.

To illustrate the close relationship between fluctuations and correlations, the correlation strength can be defined as: ΔK = K_{netq}^{n} - K_{Poisson}^{n}, where K_{netq}^{n} and K_{Poisson}^{n} represent the actual cumulants and Poisson expectations of net-charge, respectively. The following relationship can be obtained:

ΔK_2 C(1,1)
ΔK_3 C(2,1) - C(1,2)
ΔK_4 C(3,1) + C(1,3) - 2C(2,2)

where C(n,m) represents the (n+m)-th order correlation function of n positively charged particles and m negatively charged particles. Note: We only focus on the correlation between positively and negatively charged particles, ignoring the correlation between same-sign charged particles. We can observe that the difference between net-charge moments and their Poisson expectations comes from two-particle, three-particle, or four-particle correlations between positively and negatively charged particles. This paper mainly focuses on two-particle correlations, with more detailed descriptions of three-particle and four-particle correlations available in reference [91].

To intuitively illustrate the two-particle correlation between the multiplicities of positively and negatively charged particles, Figure 6 [Figure 6: see original paper] shows the distribution diagrams of Q^+ and Q^- at four different evolution stages in Au+Au collisions at sNN = 7.7 GeV and 200 GeV from the AMPT model. We can observe that the two-particle correlations differ at different evolution stages, and changes in two-particle correlation strength may lead to the dynamical evolution of σ^2/M (equation (11)) for net-charge distribution in relativistic heavy-ion collisions, as shown in Figure 7 [Figure 7: see original paper].

Figure 7 shows the centrality dependence (represented by the number of participating nucleons Npart) of σ^2/M values at four different evolution stages in Au+Au collisions at sNN = 7.7 GeV and 200 GeV, with STAR experimental results and Poisson expectations also shown for comparison. It is worth noting that experimental results are obtained from measured final-state hadrons, so only comparison with AMPT model results after the hadronic rescattering stage is meaningful. In Au+Au collisions, AMPT model data can reasonably describe experimental data at sNN = 7.7 GeV and central collisions at sNN = 200 GeV, but overestimates results for peripheral collisions. From Figure 7, we can observe that σ^2/M values gradually increase from the initial state to the final state. Comparing AMPT model results with their Poisson expectations, we find that AMPT model results are always lower than the corresponding Poisson expectations at four different evolution stages, indicating the existence of positive correlation between positively and negatively charged particles, i.e., the two-particle correlation function on the right side of equation (12) is always positive, meaning two oppositely charged particles always appear or disappear simultaneously. Although this difference changes with evolution, it indicates that positive correlation between positive and negative charges is formed in the early stage of heavy-ion collision dynamical evolution, and the correlation gradually strengthens with evolution.

The positive correlation between positive and negative charges in the AMPT model may originate from: the initial stage correlation may stem from the string melting mechanism, where excited strings are split into quarks and antiquarks with different charges. The parton cascade process changes the kinematic information of partons due to parton collisions, thus changing the two-particle correlation strength. However, the quark coalescence mechanism in the hadronization process changes the degrees of freedom of the system, causing significant changes in two-particle correlation strength. It can be observed that ΔK is almost zero after hadronization, meaning the correlation almost disappears at this stage. The hadronic rescattering stage can change correlation strength not only through hadronic rescattering (similar to parton cascade) but also provide additional two-particle correlation through resonance decay, which transforms a "parent" hadron into two "daughter" hadrons carrying opposite charges. This seems to indicate that the experimentally observed σ^2/M values are mainly produced during the hadronic rescattering process. It is worth mentioning that studies using the Hadron Resonance Gas (HRG) model also show that moment products are significantly affected by resonance decays, enhancing σ^2/M values at high collision energies. The results from the last two evolution stages of the AMPT model are basically consistent with HRG model results. However, the degree of this effect may be different because our model includes not only resonance decays but also elastic and inelastic reactions during the hadronic phase evolution. Therefore, it would be meaningful to separately study the effects of resonance decays and hadronic reaction channels on conserved charge fluctuations in the future.

Net-Baryon Fluctuation Properties

Theoretical studies show that higher-order cumulants of net-baryon (net-proton) multiplicity distributions are most sensitive to QCD phase transitions and the QCD critical point. STAR experimental results show that the ratio of the fourth-order to second-order cumulants (σ^2) of net-protons exhibits nonmonotonic energy dependence, which is considered a possible signal of the QCD critical point. Therefore, based on the AMPT model, we explore QCD matter phase structure-related issues by studying proton and baryon cumulants and their correlation functions, providing a baseline for searching for possible critical behavior at the critical point in relativistic heavy-ion collisions. Recently, we extracted the evolution paths of different energy collision systems in the phase diagram using Boltzmann statistics and quantum statistics. Since critical fluctuation behavior may appear in regions near the CEP, the research results indicate that the beam energy interval that may approach the critical region is 5 GeV < sNN < 20 GeV. This predicted range is close to the recent STAR experimental result based on net-proton higher-order cumulant ratio analysis, which predicts that critical phenomena may occur in the interval 3 GeV < sNN < 20 GeV. Therefore, the following research work mainly selects sNN = 7.7 GeV energy for more comprehensive analysis of Au+Au collisions.

3.1 Centrality Dependence of Proton Cumulants (Ratios)

The STAR collaboration recently published the centrality dependence of cumulants and correlation functions of proton, antiproton, and net-proton multiplicity distributions in Au+Au collisions at sNN = 7.7–200 GeV. Using the charge-conservation version of the AMPT model, we focused on discussing the centrality dependence of cumulants, cumulant ratios, correlation functions, and normalized correlation functions of protons, antiprotons, and net-protons in Au+Au collisions at sNN = 7.7 GeV, and compared them with STAR results. Therefore, we selected the same selection criteria as the STAR experiment, choosing protons and antiprotons within transverse momentum 0.4 GeV < pT < 2.0 GeV and rapidity |y| < 0.5 to calculate the cumulants and correlation functions of their multiplicity distributions. The collision centrality was defined using charged particle multiplicity in pseudorapidity | | < 1. To avoid self-correlation effects, protons and antiprotons were excluded when defining centrality. The Δ theoretical formula was also used to calculate statistical errors, and CBWC correction was applied to eliminate volume fluctuation effects.

Figure 8 [Figure 8: see original paper] shows the Npart (centrality) dependence of cumulants Cn (n=1,2,3,4) of proton, antiproton, and net-proton multiplicity distributions in Au+Au collisions at sNN = 7.7 GeV. With increasing number of participating nucleons Npart, the cumulants Cn of antiprotons are almost zero, reflecting the low antiproton yield at low energies. Therefore, the Cn of net-protons mainly comes from proton contributions. Both proton and net-proton cumulants Cn show Npart dependence, and the AMPT model results can basically describe the experimental data.

Figure 9 [Figure 9: see original paper] shows the Npart (centrality) dependence of cumulant ratios [C_2/C_1 (σ^2/M), C_3/C_2 (Sσ), and C_4/C_2 (σ^2)] of proton, antiproton, and net-proton multiplicity distributions in Au+Au collisions at sNN = 7.7 GeV. These cumulant ratios can eliminate possible volume effects. In Figure 9(a), the σ^2/M results from the AMPT model are consistent with the trend of STAR experiments, but the values are slightly smaller. In Figure 9(b), the Sσ values of protons and net-protons from the AMPT model are consistent with STAR results, but the AMPT model slightly overestimates the Sσ values of antiprotons. It is generally believed that if σ^2 shows nonmonotonic energy dependence, it indicates that critical fluctuations may occur when the system passes through a region near the CEP. However, the σ^2 values of protons and net-protons in the AMPT model are consistent with the Poisson baseline within large error ranges and show no nonmonotonic behavior, as shown in Figure 9(c), possibly because the AMPT model does not contain any critical fluctuation mechanism at the CEP.

According to equation (9), Figure 10 [Figure 10: see original paper] shows the Npart (centrality) dependence of normalized correlation functions (n/ $_1$) of proton and antiproton multiplicity distributions in Au+Au collisions at sNN = 7.7 GeV. We do not show the normalized correlation functions of net-protons here because at sNN = 7.7 GeV, the antiproton yield is much lower than the proton yield, and proton fluctuations are similar to net-proton fluctuations, which has been proven in Figures 8 and 9 where proton and net-proton cumulants and cumulant ratios basically coincide. Net-protons consist of two particle types (protons and antiprotons) and require calculation using the mixed correlation function method. From Figure 10(a), we can observe that the trend of the two-proton normalized correlation function $_2$/ $_1$ in the AMPT model is consistent with STAR experiments, showing significant negative two-proton normalized correlation functions, and the two-proton correlation strength increases with Npart. However, the two-proton correlation strength in the AMPT model is stronger than in experiments. The AMPT model shows a slightly increasing positive normalized correlation function for two antiprotons, which seems inconsistent with experimental measurements showing near-zero or slightly negative correlation. However, we have verified that the positive two-antiproton correlation is caused by the simple quark coalescence model in our model, and this positive two-antiproton correlation phenomenon disappears when using the new quark coalescence model in the AMPT model. In Figure 10(b), the AMPT model shows positive three-proton normalized correlation functions $_3$/ $_1$ that increase with Npart, while three-antiproton normalized correlation functions are close to zero, consistent with experimental measurements within large uncertainties. In Figure 10(c), the $_4$/ $_1$ values of protons and antiprotons in the AMPT model are both zero. Since experimental data points also have large uncertainties, we cannot conclude whether there is inconsistency between the

AMPT model and experimental measurements. In fact, the Npart (centrality) dependence of multi-proton correlation functions in the AMPT model is the result of baryon stopping effect evolution under baryon number conservation, which will be discussed in §3.3.

3.2 Effects of Acceptance on Cumulant Ratios and Correlation Functions

Experimentally, due to the limited acceptance range of the STAR detector's Time Projection Chamber (TPC) and Time-of-Flight (TOF) spectrometer, certain acceptance range selections are made during data processing and analysis to ensure particle identification purity. Results from references [66,96–98] show that when the rapidity acceptance range (Δy) is much smaller than the system's typical correlation length ( ), i.e., Δy , cumulants (Cn) and correlation functions (n) are proportional to the nth power of the average particle multiplicity in the accepted rapidity range, Cn, n ΔN ^{n}. Conversely, when the rapidity acceptance range (Δy) is much larger than the system's typical correlation length ( ), i.e., Δy , cumulants (Cn) and correlation functions (n) are linearly related to the mean particle multiplicity or Δy in the accepted rapidity range, Cn, n Δy. Therefore, this section focuses on discussing the acceptance dependence of cumulant ratios and correlation functions of protons, antiprotons, and net-protons in most central collisions (0%–5%) of Au+Au collisions at sNN = 7.7 GeV. The following mainly shows the effects of rapidity acceptance range variations on cumulant ratios and their normalized correlation functions, while detailed discussions of transverse momentum acceptance range variations can be found in reference [89].

Figure 11 [Figure 11: see original paper] shows the dependence of cumulant ratios [C_2/C_1 (σ^2/M), C_3/C_2 (Sσ), and C_4/C_2 (σ^2)] of proton, antiproton, and net-proton multiplicity distributions on the upper limit of rapidity acceptance ymax in most central collisions (0%–5%) of Au+Au collisions at sNN = 7.7 GeV. The selected particle rapidity y should satisfy |y| < ymax, so the rapidity acceptance range is Δy = 2ymax. Therefore, increasing ymax actually expands the rapidity acceptance range. The selected particle transverse momentum range is 0.4 GeV < pT < 2.0 GeV. Since antiproton yield is very small, we find that proton cumulant ratios are roughly equal to net-proton cumulant ratios. From Figures 11(a) and (b), we can observe that AMPT model results are consistent with the trend of STAR experimental data, but the σ^2/M values for protons and net-protons in the AMPT model are slightly smaller than experimental values. Figure 11(c) shows that the σ^2 values for protons and net-protons in the AMPT model show a decreasing trend. According to equation (9), cumulant ratios are related to interactions between different orders of normalized correlation functions n/ $_1$. We note that proton cumulant ratios σ^2/M, Sσ, and σ^2 in the AMPT model all deviate from the Poisson baseline. In subsequent analysis, we find this is caused by negative two-proton correlation effects.

Figure 12 [Figure 12: see original paper] shows the dependence of normalized correlation functions (n/ $_1$) of proton and antiproton multiplicity distributions on the upper limit of rapidity acceptance ymax in most central collisions (0%–5%) of Au+Au collisions at sNN = 7.7 GeV. From Figure 12(a), we can see that the values of proton normalized correlation function $_2$/ $_1$ are all negative and decrease monotonically with increasing rapidity acceptance. However, we observe that the negative correlation strength of proton normalized correlation function $_2$/ $_1$ in the AMPT model is stronger than in experimental data. Figures 12(b) and (c) show that the $_3$/ $_1$ and $_4$/ $_1$ values of protons and antiprotons in the AMPT model are close to zero, unable to produce obvious rapidity dependence of $_4$/ $_1$ or long-range four-proton correlation functions.

3.3 Relationship Between Baryons and Protons in Fluctuation Calculations

Cumulants and correlation functions may also be affected by nonequilibrium effects during the dynamic expansion of the QGP fireball, diffusion, and hadronic rescatterings. Relativistic heavy-ion collisions are actually a complex dynamical evolution process. To understand the stage-by-stage evolution characteristics of fluctuation observables, it is necessary to compare cumulants and correlation functions at several important evolution stages. On the other hand, total baryon number is absolutely conserved in relativistic heavy-ion collisions, defined by the total number of nucleons from the projectile and target nuclei. Baryon number conservation is also satisfied in the AMPT model. In the initial state, baryons are decelerated and stay in the mid-rapidity region due to the baryon stopping effect. References [6,105] proposed that under the baryon stopping effect, the distribution P(N) of measured baryon number N can be simply given by a binomial distribution:

P(N) = N!/(N!(B-N)!) p (1-p) $^-$

where p is the probability that an initial nucleon finally stays within the selected acceptance range, and B is the total baryon number. The n-baryon correlation function n is derived as:

$_1$ = N = pB,
$_2$ = -p(1-p)B,
$_3$ = 2p(1-p)(1-2p)B,
$_4$ = -6p(1-p)(1-6p+6p^2)B

Equation (14) shows that baryon number conservation leads to baryon correlation functions n whose signs depend on the order (-1)$^{n+1}$, with strength proportional to N ^{n}. Since protons are not globally conserved quantities, to observe the stage evolution of conserved charge cumulants, this section mainly uses the AMPT model to focus on studying the dynamical laws of baryon cumulants and correlation functions at four different evolution stages in Au+Au collisions at sNN = 7.7 GeV. Since hadronic scattering and weak decay have similar effects on baryon fluctuations, for simplicity we directly show the final state, thus containing only four evolution stages.

Figure 13 [Figure 13: see original paper] shows the Npart (centrality) dependence of baryon cumulant ratios at different evolution stages in Au+Au collisions at sNN = 7.7 GeV. Since baryon number is composed of quarks, we calculate the baryon number of quarks in the first two evolution stages as one-third. Observing Figure 13, we find that compared with the initial stage, cumulant ratios decrease after the parton cascade stage, while baryon cumulant ratios increase after the hadronization stage. However, the hadronic rescattering stage has little effect on baryon cumulant ratios. The reason for different fluctuation phenomena at different evolution stages is: baryons diffuse to larger phase space due to system expansion, leading to decreased cumulant ratios after the parton cascade stage. The hadronization stage increases cumulant ratios because quark coalescence brings more baryons into the acceptance range. The baryon diffusion effect is weaker in the hadronic phase. As shown in equation (9), if there are no multi-particle correlations, the Poisson baselines for the three cumulant ratios are consistent. At four different evolution stages, the σ^2/M values of baryons in the AMPT model are always lower than the Poisson baseline, indicating that negative two-baryon correlation always exists during heavy-ion collisions. For Sσ and σ^2, they are more complex than σ^2/M because they include both two-baryon and multi-baryon correlations. More detailed information about different orders of multi-baryon correlation functions will be discussed later. However, it is certain that cumulant ratios change with the evolution of heavy-ion collisions. Therefore, the dynamical evolution effect of cumulant ratios must be considered when searching for possible critical fluctuation behavior at the CEP. In addition, we find that the centrality dependence of proton cumulant ratios at the final state is similar to that of baryon cumulant ratios, with only slight differences in numerical values.

Multi-particle correlation functions are clearer than cumulant ratios for analyzing fluctuations. Figure 14 [Figure 14: see original paper] shows the Npart (centrality) dependence of n-baryon correlation functions n at different evolution stages in Au+Au collisions at sNN = 7.7 GeV. In the initial stage, we observe negative two-baryon correlation, positive three-baryon correlation, and negative four-baryon correlation. These sign changes are consistent with expectations from baryon number conservation. The correlation strength of multi-baryon correlation functions all increases with Npart, indicating that more baryons are decelerated and stay in the mid-rapidity region due to the baryon stopping effect in more central collisions. In addition, we observe that multi-baryon correlation functions gradually weaken with the evolution of heavy-ion collisions. Although multi-baryon correlation strength weakens, it indicates that correlation persists throughout the evolution of heavy-ion collisions. We observe that multi-proton correlation functions are similar in trend to multi-baryon correlation functions, but their correlation strength is weaker, indicating that when measuring multi-baryon correlation functions, protons can be considered as a representative of baryons to a certain extent.

According to references [106–109], there is a relationship between multi-baryon correlation functions and multi-proton correlation functions: $\_B^n = q^{-n}\ \_p^n$, where q is the effective acceptance factor, representing the ratio of measured protons to baryons within limited acceptance and efficiency. If N__B = N__p/q, this relationship is also consistent with baryon number conservation expectations, as shown in equation (14). The effective acceptance factor q can be extracted by calculating the nth power of the ratio of multi-proton correlation functions to multi-baryon correlation functions, i.e., q = (__p^n/ __B^n)^1/^{n}. Figure 15 [Figure 15: see original paper] shows the Npart (centrality) dependence of the effective acceptance factor __p^n/ __B^n in Au+Au collisions at sNN = 7.7 GeV. We find that the effective acceptance factor is almost independent of centrality, fitting to a constant of about 0.475, which is slightly different from the 1/2 proposed in references [107,108]. From Figures 13 and 14, we can see that in the AMPT model, multi-baryon correlation functions and multi-proton correlation functions should come from the same source, i.e., baryon number conservation. The effective acceptance factor can reflect the extent to which protons can represent baryons in baryon number conservation. However, if there are contributions from other sources, especially critical fluctuations, the relationship between baryon number and proton number fluctuations may be more complex and requires further study.

Net-Strangeness Fluctuation Properties

Since projectile and target nuclei in heavy-ion collisions do not carry strangeness, the production of final-state strange hadrons can provide a unique method to understand the characteristics of the produced system. For example, it was predicted that a large number of strange quarks would be produced in QGP, leading to enhancement of strange hadrons, which is one of the significant features of QGP signals. Another famous experimental signal is the observation of a peak in the energy dependence of the K^+/π^+ ratio in central Pb+Pb collisions at 30A GeV energy, which is considered a possible beginning of QCD matter deconfinement. Strange baryons produced in high-energy A+A collisions, when scaled by the number of participating nucleons, are enhanced relative to those measured in p+p reactions because the yield of particles containing multiple strange quarks increases in the presence of QGP. Different types of hadron yields and fluctuations can reveal information related to chemical freeze-out in heavy-ion collisions. Experimental data analysis shows that light hadrons (containing only u,d quarks) seem to have lower chemical freeze-out temperatures than strange hadrons (containing s quark). Other results indicate that the temperature for deconfinement of strange quarks in the smooth crossover phase transition region may occur at positions greater than the critical temperature Tc. Net-strangeness fluctuations have also attracted the attention of many theoretical physicists and are considered a unique probe of QCD phase transitions.

4.1 Centrality Dependence

In 2018, the STAR collaboration first measured the moments of net-kaon multiplicity distributions in Au+Au collisions, but no nonmonotonic behavior was observed within experimental uncertainties. AMPT model studies can provide reasonable chemical freeze-out temperatures consistent with experimental measurements. Here, we use the AMPT model with the new quark coalescence mechanism and charge conservation law to analyze strangeness fluctuation problems. This section focuses on discussing the cumulants, cumulant ratios, and correlation functions of strange particles in Au+Au collisions at sNN = 7.7 GeV. For comparative analysis, we selected the same selection criteria as the STAR experiment. We chose K^+ and K^- within transverse momentum 0.2 GeV < pT < 1.6 GeV and rapidity |y| < 0.5 for physical analysis, while using charged particle multiplicity in pseudorapidity | | < 1 to define collision centrality. To avoid self-correlation effects, K^+ and K^- mesons were excluded when defining centrality, and CBWC was used to analyze data to eliminate volume fluctuation effects. The bootstrap method was used to calculate statistical errors.

Figure 16 [Figure 16: see original paper] shows the Npart (centrality) dependence of cumulants Cn of net-kaon (ΔNK) multiplicity distributions in Au+Au collisions at sNN = 7.7 GeV. The cumulants Cn of net-kaons (ΔNK) increase monotonically from peripheral to central collisions. We can find that AMPT model results are slightly smaller than experimental data. Assuming that the multiplicity distributions of K^+ and K^- are independent Poisson distributions, the corresponding Poisson expectations for net-kaons (ΔNK) can be obtained: C_n^{Poisson} = C_n^K^+ + (-1)^{n}C_n^K^-, as shown by the dashed lines in Figure 16. Within statistical uncertainties, all cumulant Cn values seem to be consistent with the Poisson expectations of the AMPT model. However, upon closer inspection of Figure 16(b), we can find that the C_2 results in the AMPT model are slightly lower than their Poisson baseline, indicating correlation between K^+ and K^-, which will be discussed later.

Figure 17 [Figure 17: see original paper] shows the Npart (centrality) dependence of cumulant ratios [C_1/C_2 (M/σ^2), C_3/C_2 ($S\sigma$), and C_4/C_2 (σ^2)] of net-kaon (ΔNK) multiplicity distributions in Au+Au collisions at sNN = 7.7 GeV. The Poisson expectations of the AMPT model are shown by dashed lines. From Figure 17(a), we can observe that the M/σ^2 values in the AMPT model can describe STAR experimental data for several centralities near central collisions, but overestimate STAR values in peripheral collisions, so the overall trend is inconsistent with experimental data. This is caused by the new quark coalescence mechanism, as found by comparing results from charge-conservation AMPT models with new and old quark coalescence mechanisms. In Figure 17(b), within allowed errors, the $S\sigma$ values of net-kaons in the AMPT model are consistent with experimental data for non-peripheral collisions and are greater than their Poisson baseline. The σ^2 values of net-kaons in the AMPT model are consistent with experimental data and their Poisson baseline within large error ranges, as shown in Figure 17(c). These differences between net-kaon moment products and the Poisson baseline indicate that the net-kaon multiplicity distribution is not a simple random or independent distribution, and there must be some correlation between net-kaons.

According to equation (10), Figure 18 shows the Npart (centrality) dependence of correlation functions of K^+ and K^- multiplicity distributions in Au+Au collisions at sNN = 7.7 GeV. In Figure 18(a), we observe that all three types of two-particle correlation functions for K^+ and K^- are positive. The two-particle correlation strength between opposite charges (K^+ and K^-) ($_2{}^{(1,1)}$) is greater than that between same charges ($_2{}^{(2,0)}$ and $_2{}^{(0,2)}$). After checking data from new and old quark coalescence models, we find that the opposite trend between the Npart dependence of M/σ^2 in the AMPT model and experimental results may be caused by too large slope (growth rate) of $_2{}^{(1,1)}$ in peripheral collisions. A possible solution is to introduce a centrality-dependent relationship for $_2{}^{(1,1)}$ in the AMPT model with the new quark coalescence mechanism. To achieve this, we can refer to the method in reference [132], where parameters γ_Λ,Ξ,Ω were introduced in the new quark coalescence mechanism to control the yields of these particles. Similarly, parameters γ_K^+ and γ_K^- can be introduced here to weaken the correlation strength between K^+ and K^-, thereby obtaining the desired centrality-dependent relationship for $_2{}^{(1,1)}$. We hope to make this one of our future research objects. This indicates that the main source of strangeness is likely the pair production mechanism, which will be discussed in detail below. However, the three-particle and four-particle correlation functions of K^+ and K^- are consistent with zero within current statistical uncertainties.

4.2 Relationship Between Strange Hadrons and Kaons in Fluctuation Calculations

Since the conserved quantity in relativistic heavy-ion collisions is not net-kaons but net-strange hadrons, this section will focus on discussing the dynamical evolution of cumulants and correlation functions of net-strange hadrons. For strangeness calculations, N_s and N_s̄ represent the number of s and s̄ quarks event-by-event, with strangeness numbers of +1 and -1 carried by s and s̄ quarks, respectively. Different strange hadrons carry different numbers of (anti-)strange quarks. For example, Σ, Ξ, and Ω are composed of 1, 2, and 3 strange quarks, respectively. The number of strange quarks in an event can be expressed as: N_s = Σ_i i·n_s , where i represents the number of s quarks carried by each strange hadron, and n_s represents the number of strange hadrons. Conversely, the number of antistrange quarks in an event is: N_s̄ = Σ_i i·n_s̄ . To calculate the cumulants of net-strangeness multiplicity distributions in the last three hadron evolution stages, we calculated the numbers of s and s̄ constituent quarks inside strange hadrons (K^+, K^0, Λ, Ξ, Ω) and their antiparticles, similar to case 4 in reference [133]. During evolution, strange quarks are continuously produced and diffused and fluctuate event-by-event. Additionally, we attempt to understand the extent to which net-kaons can represent net-strangeness fluctuations in relativistic heavy-ion collisions.

Figure 19 [Figure 19: see original paper] shows the Npart (centrality) dependence of cumulants Cn of net-strangeness or net-kaon multiplicity distributions at five different evolution stages (from left to right) in Au+Au collisions at sNN

= 7.7 GeV. First, we focus on C_1 here. As shown in equation (11), for Cn (n=2,3,4), they contain complex contributions from multi-particle correlation functions, which will be discussed in detail later using Figure 20 [Figure 20: see original paper]. Next, we temporarily focus only on the first four columns and ignore the last column, because the first four evolution stages strictly follow strangeness conservation law, while the last evolution stage includes weak decays that break strangeness conservation law. From the "initial state" to the "hadronic rescattering" stage, we can observe that the first-order cumulant C_1 of net-strangeness approaches zero, because the nucleons participating in the collision in the initial state have no strange hadrons, and the total net strangeness should be zero. However, since we only focus on the mid-rapidity window and make transverse momentum selections, this causes some deviation of the first-order cumulant C_1 from zero. On the other hand, the first-order cumulant C_1 of net-kaons is obviously positive, because strange hadrons include not only kaons but also other strange hadrons, such as many strange baryons. For the final state in the last column, we find that the mean value C_1 of net-strangeness increases significantly compared with the result after the hadronic rescattering stage. This can be understood through the following example: for the reaction channel $\Lambda \rightarrow \pi^- + p$, Λ carries an s quark before weak decay, but after weak decay there are no particles containing s quarks. In addition, at low energies, the yield of strange baryons is greater than that of antistrange baryons. Therefore, the reduction of s quarks should be greater than that of $\bar{s}$ quarks, so net-strangeness increases after weak decay. However, we find that the cumulants Cn of net-kaons remain unchanged in the last three evolution stages, indicating that hadronic rescattering and resonance decay have little effect on net-kaon fluctuations. Reference [69] also observed that the cumulant ratios of net-kaon multiplicity distributions remain basically unchanged in the last three stages. At the final state, the results of net-kaons show similar trends to those of net-strangeness, but with slightly different numerical values. Therefore, we can conclude that net-kaon fluctuations can represent net-strangeness fluctuations to a certain extent, although the magnitude is slightly different. Lattice QCD theory and effective QCD models have also predicted that using net-kaon fluctuations to search for possible net-strangeness QCD critical fluctuations is a good method.

To further understand the physical mechanisms of net-strangeness cumulants at different evolution stages, Figure 20 shows the Npart (centrality) dependence of n-particle (j s constituent quarks and i $\bar{s}$ constituent quarks) correlation functions (n) at five different evolution stages in Au+Au collisions at sNN = 7.7 GeV. The top row of figures actually shows the mean values of s and $\bar{s}$ constituent quarks at different evolution stages, because $_1{}^{(1,0)}$ = N_s and $_1{}^{(0,1)}$ = N_$\bar{s}$. Within our limited statistics, we can observe that three-particle and four-particle correlation functions are basically equal to zero throughout the evolution of heavy-ion collisions. Next, we will only focus on single-particle yields and two-particle correlation functions. For the initial state, we observe that the numbers of s and $\bar{s}$ constituent quarks are very close. The two-particle correlation functions $_2^{(2,0)}$ and $_2^{(0,2)}$ in the figure are both positive, but the two-particle mixed correlation function between s and $\bar{s}$ constituent quarks ($_2^{(1,1)}$) dominates, indicating that s and $\bar{s}$ constituent quarks are produced in pairs due to strangeness conservation law. In the AMPT model, the weak signal of two-particle correlation functions of the same strange quark [$_2^{(2,0)}$ and $_2^{(0,2)}$] can be considered as the result of melting multi-strange baryon parent particles under the string melting mechanism. It is worth noting that the sign of strangeness two-particle correlation functions is different from that of baryons discussed above, because strangeness is newly produced while baryon number is due to the baryon stopping effect. After the parton cascade stage, we can find that the mean values of s and $\bar{s}$ quarks decrease and two-particle correlation strength weakens, which is caused by strangeness diffusion during parton evolution. This is similar to the situation we previously studied in net-baryon fluctuations, where baryon number diffusion also weakens two-baryon correlation during fireball expansion. Since the hadronization stage transforms quarks into hadrons, and the dynamics of forming strange hadrons are different from those of constituent strange quarks, both strangeness yield and two-particle correlation functions are enhanced to some extent within the acceptance range. After the hadronic rescattering stage, strangeness yield and two-particle correlation functions also increase slightly, because some reactions can produce not only two strange mesons carrying s and $\bar{s}$ quarks but also strange baryons carrying two s quarks. For example: i) In the reaction channel $\pi^0 + \pi^0 \rightarrow K^+ + K^-$, K^+ and K^- carry s and $\bar{s}$ quarks respectively, and their production and annihilation will inevitably change $_2^{(1,1)}$; ii) In the reaction channel $\Lambda^0 + \Lambda^0 \rightarrow \Xi^- + \pi^0$, since Ξ^- contains two s quarks, the production and annihilation of Ξ^- will change $_2^{(2,0)}$. For the final state, the number of s quarks decreases because strange baryon decays break strangeness, and two-particle correlation functions weaken, especially $_2^{(2,0)}$. Therefore, these different multi-particle correlation function evolution stage results jointly contribute to the evolution of strangeness cumulants shown in Figure 19, as shown in equation (11). By comparing Figures 18 and 20, we can see that at the final state, the centrality dependence of kaon correlation functions has a similar trend to that of strangeness correlation functions, which also shows that net-kaon fluctuations can well represent net-strangeness fluctuations.

Introducing Density Fluctuations for QCD Phase Transition Studies

As mentioned above, when heavy-ion collision systems pass through the QCD critical point, the correlation length diverges. In actual systems, the value of correlation length is limited by system size. In the fireball produced by heavy-ion collisions, the parton phase space may produce parton clusters or vacancy phenomena. We have conducted some preliminary explorations of density fluctuation phenomena caused by critical fluctuations. To explore the effects of this phenomenon, we extended the AMPT model (version v1.26t5/v2.26t5) by introducing Enhanced Local parton Density Fluctuation (ELDF), which considers baryon number, charge number, and strangeness conservation. The specific method is as follows: assume that the partonic matter before hadronization is composed of many clusters in the transverse plane, with each cluster's central position determined by the local maximum of frozen parton density in each event. The number and size of clusters are free parameters. In the study, the transverse radius of each cluster is set to 1 fm, and the number of clusters is randomly selected from 2, 3, 4, and 5. To generate clusters, each parton outside the cluster region is moved to a random transverse position inside its nearest cluster, while the longitudinal coordinates and momenta of partons remain unchanged.

5.1 Effects of Density Fluctuations on Higher-Order Moments

Figure 21 [Figure 21: see original paper] shows the spatial distribution of hadrons in the transverse plane after parton hadronization when parton density fluctuations are introduced in the AMPT model. It can be found that after the hadronization stage, the introduced parton phase space density fluctuations are still retained in the initial hadrons. Although each event has a specific number of clusters, a large event sample consists of events with different numbers of clusters, i.e., clusters or vacancies are formed at certain positions in the coordinate space of hadrons. We used equation (9) to calculate cumulant ratios and analyzed the effects of introduced local parton density fluctuations on cumulant ratios in Au+Au collisions at sNN = 7.7 GeV, 11.5 GeV, and 19.6 GeV.

For net-baryon number fluctuation calculations, protons (baryons) within transverse momentum 0.2 GeV < pT < 0.8 GeV were selected to calculate cumulant ratios of net-proton (baryon) multiplicity distributions, using the same selection criteria as the experimental results, with statistical errors estimated using the Δ theorem. Figure 22 [Figure 22: see original paper] shows the energy dependence of σ^2 for protons, net-protons, net-nucleons, and net-baryons in most central Au+Au collisions (0%–5%), comparing AMPT, ELDF AMPT model results with STAR experimental data. For proton and net-proton results (Figure 22(a)), AMPT model calculations show weak energy dependence. We find that although ELDF AMPT model calculations include the effects of local parton density fluctuations, they differ little from original AMPT model calculations and can both describe experimental data within error ranges. However, when neutrons and all other baryons are also considered (Figure 22(b)), we can observe that these other baryons have a larger effect on σ^2 values, especially at lower energies.

We also further studied the effect of hadronic interactions on fluctuations by turning off the hadronic interaction stage in the AMPT model, as shown in Figure 23 [Figure 23: see original paper]. Comparing Figures 22 and 23(a), we can observe that at sNN = 7.7 GeV, hadronic interactions have a larger effect on higher-order moments of protons and net-protons, while having smaller effects on σ^2 values at sNN = 11.5 GeV and 19.6 GeV. Similar features are found in the analysis of higher-order moments of net-baryons and net-nucleons in Figure 23(b), where hadronic interactions have important effects, especially at lower collision energies. Our results are consistent with other models, such as QvdWHRG (Quantum van der Waals-HRG) model calculations.

5.2 Effects of Density Fluctuations on Strange Hadron Yield Ratios

Near the QCD phase transition critical point, theoretical predictions indicate that baryon density fluctuations will be enhanced due to the sharp increase in correlation length. In recent years, hydrodynamic methods and microscopic transport model studies have shown that the phase transition from QGP to hadronic matter, caused by Spinodal instability or first-order phase transition, will lead to increased baryon density fluctuations. These density fluctuations will persist to the final freeze-out stage, causing nucleon density fluctuations. Therefore, baryon density fluctuations in relativistic heavy-ion collisions can be used as a probe to study QCD phase transitions. Reference [22] shows that by studying neutron density fluctuations in relativistic heavy-ion collisions ($\Delta n = (\delta n)^2$), the physical quantity can be obtained by constructing nucleon yield ratios through the yields of protons, deuterons, and tritons: $O_{p\text{-}d\text{-}t} = N_d^2/(N_p N_t)$. Results in the SPS energy region show that nucleon yield ratios have nonmonotonic energy dependence. The RHIC STAR experiment also calculated nucleon yield ratios within its energy scan range, showing signs of non-monotonic energy dependence. However, JAM (Jet AA Microscopic transport model) simulation results without critical physics mechanisms show no energy dependence and cannot describe experimental data. This difference suggests that there should be changes in neutron density fluctuations at different collision energies. Since neutron yields are determined after kinetic freeze-out and are greatly affected by the hadronic rescattering stage, using neutron density fluctuations to study QCD phase transitions may not directly reflect the phase transition.

Inspired by reference [146], we proposed to study s quark fluctuations by calculating strange hadron yields in heavy-ion collisions, i.e., calculating the physical quantity through the yields of K^+, Ξ^-, ϕ, and Λ: $O_{K\text{-}\Xi\text{-}\phi\text{-}\Lambda} = N_\phi N_\Lambda/(N_{K^+} N_{\Xi^-})$, using the coalescence model COAL-SH (Coalescence-ShangHai) from reference [147] for the study. In general quark coalescence models, it is assumed that quarks are uniformly distributed in phase space, so quark density is constant, and quark yield is $N_q = V_{cn} q$. In this work, we assume that quark density will be affected by QGP phase transition and fluctuate, mainly considering fluctuations in quark phase space positions in the quark density term (n_q):

$$n_q(r) = \bar{n}_q + \delta n_q(r)$$

where $\bar{n}_q$ represents the mean density of quark q relative to coordinate space, and $\delta n_q(r)$ represents the deviation of quark q density from the mean at position r, with $\delta n_q = 0$. The quark relative density fluctuation term is defined as: $\Delta_q = \delta n_q^2 / \bar{n}_q^2$. Then the correlation term between quark density fluctuations can be obtained: $\alpha_{q_1 q_2} = \delta n_{q_1} \delta n_{q_2} / (\bar{n}_{q_1} \bar{n}_{q_2})$. Higher-order correlation terms are not considered here.

After the above theoretical considerations, we used the COAL-SH model to calculate the yields of strange hadrons K^+, Ξ^-, ϕ, and Λ by considering both cases without quark density fluctuations and with quark density fluctuation terms. Through rigorous derivation, we obtained that without considering quark density fluctuations, the strange hadron yield ratio O_K-Ξ-ϕ-Λ = 1.1. When introducing quark density fluctuation terms, the quark density fluctuation correlation factor α_q_1q_2 needs to be extracted to obtain the relative density fluctuation value Δ_s of strange quarks. Currently, the value of α_q_1q_2 cannot be directly obtained. Here, we mainly consider two limiting cases: no correlation between quarks or strong correlation between quarks. If there is no correlation between quarks, then $\delta q_1 \delta q_2 = \delta q_1 \;\; \delta q_2 = 0$, and the strange hadron yield ratio O_K-Ξ-ϕ-Λ = g(1 + Δ_s), where g is a constant. It can be seen that the yield ratio O_K-Ξ-ϕ-Λ has a linear relationship with the relative density fluctuation value Δ_s of strange quarks. If there is strong correlation between quarks, then $\delta q_1 \delta q_2 = \delta {q_1}^2 = \delta {q_2}^2$, and α_q_1q_2 = Δ_q_1Δ_q_2. At this time, the strange hadron yield ratio is O_K-Ξ-ϕ-Λ = (1 + Δ_s)(1 + Δ_s + Δ_u)(1 + Δ_s + Δ_d)/(1 + Δ_u + Δ_d). It can be found that if Δ_s = Δ_d = Δ_u, then the yield ratio is a constant g, independent of collision energy. However, due to SU(3) symmetry breaking, the mass of s quark is much greater than that of u,d quarks, and the interactions between s quark and u,d quarks in hot and dense matter are different. Therefore, Δ_s should be different from Δ_u and Δ_d, so the strange hadron yield ratio will definitely be affected by the relative density fluctuation value Δ_s of strange quarks. More detailed derivation can be found in reference [24].

Based on the above analysis, to obtain the strange hadron yield ratio O_K-Ξ-ϕ-Λ, we first need to extract the yields of strange hadrons K^+, Ξ^-, ϕ, and Λ from different heavy-ion collision experiments. At the same time, to illustrate that strange quark relative density fluctuations in heavy-ion collisions may depend on collision energy, Table 2 shows the values of Δ_s extracted with g = 1.1 in the COAL-SH model under the condition α_q_1q_2 = 0.

Table 2 Yields of strange hadrons in full rapidity space from central (0%–7.2% centrality) Pb+Pb collisions at SPS energies measured by the NA49 Collaboration

sNN (GeV)	N_K^+	N_Ξ^-	N_ϕ	N_Λ	O_K-Ξ-ϕ-Λ	Δ_s
6.3	27.1$±0.2	1.50±0.13	1.89±0.31	40.7±0.7	1.19±0.22	0.08±0.20
7.7	36.9±0.3	2.42±0.19	1.84±0.2			

From Table 2, we can see that the dependence of Δ_s on collision energy shows nonmonotonic behavior, with more detailed statistics available in reference [24]. Figure 24 [Figure 24: see original paper] more intuitively shows the relationship between the strange hadron yield ratio O_K-Ξ-ϕ-Λ and collision energy sNN.

From Figure 24, we can clearly see that the strange hadron yield ratio O_K-Ξ-ϕ-Λ shows a nonmonotonic behavior with energy changes. This phenomenon is extremely similar to the nonmonotonic energy dependence of the nucleon yield ratio O_p-d-t proposed in reference [22], and the nonmonotonic behavior changes occur around a center-of-mass energy of 8 GeV. The rightmost line in the figure represents the result of the COAL-SH model without considering quark density fluctuations, which is a constant independent of energy. However, its value is higher than the experimental results at sNN = 200 GeV. At energies above 200 GeV, the phase transition must be in the smooth crossover region and far from the critical point, so fluctuations in physical quantities such as quark density should be very small. Therefore, experimental results at energies above sNN = 200 GeV can be described by the COAL-SH model without considering quark density fluctuations. In the lower energy region, the nonmonotonic energy dependence of the yield ratio can be observed, which can be explained by the COAL-SH model considering quark density fluctuations. According to the above discussion, the relative density fluctuation Δ_s of strange quarks will affect the strange hadron yield ratio. Therefore, the nonmonotonic energy dependence of the strange hadron yield ratio can reflect that the relative density fluctuation Δ_s of strange quarks also has nonmonotonic energy dependence, which is also shown in the above table. The following physical process can explain the phenomena presented in the figure: In central collisions at higher energies, the produced QGP has smaller baryon chemical potential, and the phase transition from QGP to hadronic matter is closer to a smooth crossover second-order phase transition. Therefore, density fluctuations of the produced matter are negligible at these collision energies. As collision energy gradually decreases, the evolution trajectory of the QGP phase transition in the temperature and baryon chemical potential plane may approach or pass through the critical point of the QCD phase diagram, and the matter at this time will produce larger density fluctuations. As collision energy further decreases, its evolution trajectory will move away from the critical point and enter the first-order phase transition region. The hot and dense matter at this time will also produce larger density fluctuations because first-order phase transitions are usually accompanied by Spinodal instability. If collision energy continues to decrease, density fluctuations of matter will decrease because the QGP matter produced at this time has shorter lifetime and smaller size. This physical process is consistent with the nonmonotonic variation behavior shown in Figure 24, indicating that the evolution trajectory of QGP produced in these collisions may have reached or approached the critical point or experienced a first-order phase transition. To verify the above conclusions, we also used the statistical model [150,151] to calculate the strange hadron yield ratio, with the short dashed line in the figure representing the result calculated by the THERMUS program package [151]. It can be found that both the statistical model and the COAL-SH model without considering quark density fluctuations cannot describe the experimental results showing nonmonotonic energy dependence of the strange hadron yield ratio O_K-Ξ-ϕ-Λ. Therefore, we believe that the strange hadron yield ratio O_K-Ξ-ϕ-Λ can be used as a probe in experiments to detect the QCD phase transition critical point.

Conclusion

Using the string melting version of the A Multi-Phase Transport (AMPT) model with charge conservation properties, we studied fluctuation observables such as moments, moment products, cumulants, cumulant ratios, and correlation functions of conserved charge (baryon number, charge number, and strangeness) multiplicity distributions. AMPT model calculations can basically describe STAR beam energy scan experimental data. Our work reveals the effects of several key effects in the dynamical evolution process of relativistic heavy-ion collisions on fluctuations. In the study of baryon number fluctuations, we found that AMPT model results are consistent with expectations from baryon number conservation, caused by multi-baryon (proton) correlations due to baryon number conservation. In the studies of strangeness and charge number fluctuations, we found they are caused by pair production effects. Since the original AMPT model does not contain QCD critical fluctuation physics mechanisms, our results can only provide a baseline for searching for possible critical behavior at the CEP in relativistic heavy-ion collisions. However, we preliminarily introduced critical density fluctuations into the AMPT model and found that they have small effects on higher-order moments of net-proton number, not achieving the expected results. We also considered strange quark density fluctuations in the coalescence model COAL-SH and found that strange hadron yield ratios are affected by relative density fluctuations of strange quarks. These research works provide important theoretical references for understanding critical fluctuations of conserved charges in relativistic heavy-ion collision experiments. In the future, further development of the AMPT model combined with QCD critical fluctuation theory will be more helpful for exploring QCD phase transitions and searching for the QCD critical point in future series of lower-energy fixed-target heavy-ion experiments.

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Author Contributions

Chen Qian was responsible for model development, program design, data analysis, and manuscript writing. Ma Guoliang was responsible for guiding model development, program design, and data analysis, and manuscript revision. Chen Jinhui was responsible for guiding model development, program design, and data analysis, and manuscript review.

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