

Optimization method of Hadamard coding plate in γ -ray computational ghost imaging Postprint

Authors: Zhi Zhou, Sangang Li, Qingshan Tan, Li Yang, Mingzhe Liu, Ming Wang, Lei Wang, Yi Cheng, Sangang Li

Date: 2023-06-08T00:00:00+00:00

Abstract

Owing to the constraints on the fabrication of γ -ray coding plates with many pixels, few studies have been carried out on γ -ray computational ghost imaging. Thus, the development of coding plates with fewer pixels is essential to achieve γ -ray computational ghost imaging. Based on the regional similarity between Hadamard subcoding plates, this study presents an optimization method to reduce the number of pixels of Hadamard coding plates. First, a moving distance matrix was obtained to describe the regional similarity quantitatively. Second, based on the matrix, we used two ant colony optimization arrangement algorithms to maximize the reuse of pixels in the regional similarity area and obtain new compressed coding plates. With full sampling, these two algorithms improved the pixel utilization of the coding plate, and the compression ratio values were 54.2% and 58.9%, respectively. In addition, three undersampled sequences (the Harr, Russian dolls, and cake-cutting sequences) with different sampling rates were tested and discussed. With different sampling rates, our method reduced the number of pixels of all three sequences, especially for the Russian dolls and cake-cutting sequences. Therefore, our method can reduce the number of pixels, manufacturing cost, and difficulty of the coding plate, which is beneficial for the implementation and application of γ -ray computational ghost imaging.

Full Text

Optimization Method for Hadamard Coding Plates in γ -Ray Computational Ghost Imaging

Zhi Zhou¹, Sangang Li^{1,2*}, Qingshan Tan¹, Li Yang^{3,4}, Mingzhe Liu¹, Ming Wang¹, Lei Wang¹, Yi Cheng^{1}

¹ College of Nuclear Technology and Automation Engineering, Chengdu University of Technology, Chengdu 610059, China

² Applied Nuclear Technology in Geosciences Key Laboratory of Sichuan Province, Chengdu University of Technology, Chengdu 610059, China

³ Institute of Plasma Physics, Hefei Institutes of Physical Science, Chinese Academy of Sciences, Hefei 230031, China

⁴ University of Science and Technology of China, Hefei 230026, China

Corresponding author: Sangang Li, Chengdu University of Technology, Chengdu, Sichuan 610059, China. Email: lisangang@cdut.edu.cn

Abstract

Owing to fabrication constraints on γ -ray coding plates with many pixels, few studies have been carried out on γ -ray computational ghost imaging. Thus, developing coding plates with fewer pixels is essential to achieve γ -ray computational ghost imaging. Based on the regional similarity between Hadamard subcoding plates, this study presents an optimization method to reduce the number of pixels in Hadamard coding plates. First, a moving distance matrix was obtained to describe regional similarity quantitatively. Second, based on this matrix, we used two ant colony optimization arrangement algorithms to maximize pixel reuse in regions with similarity and obtain new compressed coding plates. Under full sampling, these two algorithms improved pixel utilization of the coding plate, achieving compression ratios of 54.2% and 58.9%, respectively. In addition, three undersampled sequences (the Harr, Russian dolls, and cake-cutting sequences) with different sampling rates were tested and discussed. At various sampling rates, our method reduced the number of pixels for all three sequences, particularly for the Russian dolls and cake-cutting sequences. Therefore, our method can reduce the number of pixels, manufacturing cost, and fabrication difficulty of coding plates, which is beneficial for implementing and applying γ -ray computational ghost imaging.

Keywords: γ -ray computational ghost imaging; regional similarity; Hadamard coding plate

1. Introduction

Computational ghost imaging (CGI) offers a less expensive alternative to traditional γ -ray imaging, thereby reducing scientific costs. By replacing conventional detectors with bucket detectors and spatial modulators, CGI has attracted considerable attention as a novel imaging modality. Moreover, crosstalk noise between detectors and radiation intensity significantly affects image quality. CGI can effectively avoid crosstalk noise between detectors and reduce the radiation intensity required for imaging. In particular, it circumvents the limitation of detector size on image spatial resolution, offering new possibilities for more precise radiation imaging.

However, designing and manufacturing a suitable spatial modulator has always been a challenge for achieving CGI. The earliest breakthroughs occurred in the visible light regime, where spatial light modulators (SLM) or digital micromirror devices (DMD) have been used to modulate the light field. However, neither SLM nor DMD can modulate ray particles with certain penetration capabilities. In the field of high-energy radiation, the only viable approach is to exploit the attenuation of rays by coding plates.

The difficulty of fabricating coding plates has significantly delayed the realization of X-ray and neutron ghost imaging. In 2020, He et al. used electroplating to fabricate Hadamard coding plates made of Au and photochemical etching to fabricate Hadamard coding plates made of Cu, with pixel sizes of $10 \times 10 \text{ }\mu\text{m}$ and $150 \times 150 \text{ }\mu\text{m}$, and thicknesses of $10 \text{ }\mu\text{m}$ and $150 \text{ }\mu\text{m}$, respectively. Using these two coding plates as spatial modulators, they achieved X-ray CGI with resolutions of $10 \text{ }\mu\text{m}$ and $150 \text{ }\mu\text{m}$. In a subsequent study on neutron CGI, He et al. used the Bosch process for deep silicon etching and Gd_2O_3 powder filling to produce a thermal neutron coding plate with a thickness of $300 \text{ }\mu\text{m}$, conducting the first thermal neutron CGI experiment on the Dongguan spallation neutron source facility.

However, few studies on γ -ray CGI are currently available, primarily because the penetration capability of γ -rays is generally higher than that of X-rays and thermal neutrons. For example, if the energy of γ -rays exceeds 100 keV , the thickness required for high-Z coding plates must be more than 1 mm , which is currently impossible to achieve using etching processes. Similar devices are also needed in γ cameras to correct γ rays, such as those employing uniformly redundant arrays (URA) and modified uniformly redundant arrays (MURA) coding plates. These coding plates are fabricated by cutting metal into N small blocks, with each metal block placed into a coding plate according to the numerical distribution of the respective correction matrices. However, to achieve the same imaging resolution, N^2 pixels are required in the CGI coding plate, making it difficult to employ this method to create coding plates with many pixels.

Furthermore, to achieve higher imaging resolution, the number of required measurements has increased dramatically, and the number of subcoding plates required has also risen significantly, resulting in a very large number of pixels on the coding plate. This eliminates the cost advantage of a single-pixel imaging system, as the savings from detectors are offset by coding plate manufacturing costs. For a high-quality γ -ray imaging system, it is difficult to create coding plates with a large number of pixels. Therefore, developing coding plates with fewer pixels is essential to achieve γ -ray CGI.

To date, few good solutions to this problem have been reported for two main reasons: (1) relevant research is insufficient due to the short history of radiation CGI; and (2) in the optical field, it is easy to change the distribution of modulated light fields because of the widespread use of SLM and DMD. Consequently, studies of light-field modulation have primarily focused on achieving high-quality imaging under sampling.

If undersampling methods are used in γ -ray CGI, the number of required coding plate pixels can also be reduced. Nevertheless, this reduction remains insufficient for γ -ray CGI. Fabricating coding plates suitable for γ -rays will be easier if the number of pixels can be reduced further.

Compressed sensing (CS) theory provides an excellent theoretical framework for image reconstruction in CGI with undersampling. It often employs special measurement patterns, such as Hadamard masks and random masks, and iterative optimization methods, such as the L1-magic algorithm, orthogonal matching pursuit (OMP), and TVAL3 algorithm, for image reconstruction. Undersampling can reduce the number of pixels required by coding plates, but this also imposes higher standards on coding plate design. Specifically, within a limited number of measurements, the coding plate should maximize the degree of “fluctuation” in the measurement data to retain as much valid information as possible. Random masks have the advantage of being easy to manufacture, and one effective method to further reduce the sampling rate is to use orthogonal basis types of masks, such as Hadamard coding plates, to remove redundancy in random measurements.

Currently, Hadamard coding plates are widely used in CGI because the Hadamard matrix possesses interesting features, such as good orthogonality and the ability to remove redundant information in natural scenes. This raises the question: which subcoding plates should be selected to form a new plate as small as possible while maintaining high-quality imaging? Researchers have proposed various undersampling measurement sequences based on the Hadamard matrix, hoping that careful design of multiplexing matrices will yield potential performance improvements in CGI.

Sun et al. proposed a Russian dolls measurement sequence in 2017. This sequence was designed according to both the characteristics of numerical distributions and the total number of subcoding plate blocks, achieving good recovery results with a sampling rate of only 16%. In 2019, Yu et al. found that rearrangement of the measurement sequence of Hadamard coding plates might change the coherent area of each imaging point, which could in turn affect imaging quality. If the measurement sequence of coding plates is arranged in ascending order in terms of the number of blocks, the coherent region of each imaging point will naturally become smaller, resulting in high-quality imaging recovery at lower sampling rates. This measurement sequence, known as the “cake-cutting” sequence, demonstrates better performance than the Russian dolls and random Hadamard measurement sequences in undersampling. In addition, because the image reconstruction effect of compressed sensing algorithms is closely related to the selected sparse matrix, some researchers have proposed measurement sequences based on different sparse matrices, such as the Harr sequence, which can better restore image characteristics at a sampling rate of 25%.

In 2020, Xiao proposed two new measurement sequences from the perspective of graph decomposition: the total variation (TV) method and the total wavelet transformed coefficients (TW) method. Subcoding plates are reordered accord-

ing to their TV and TW values in ascending order to achieve optimal performance. Numerical simulation and experimental results show that the TV, CC, and TW orders can provide almost identical reconstruction quality at deep compressive sampling, and even better performance at sampling ratios above 30%.

Yu proposed a concept called selection history to record the Hadamard spatial folding process and built a model based on it to reveal the formation mechanisms of different orderings and deduce the mutual conversion relationships among them. Based on this, a weight ordering of the Hadamard basis was proposed, with both numerical simulation and experimental results demonstrating better reconstruction quality at lower sampling rates compared to traditional sorting methods.

Despite the emergence of many orderings, their relationships remain unclear, and a unified theory to explain the inherent mathematical and physical mechanisms is lacking. In particular, there is a lack of mathematical explanation for why good imaging quality can be obtained for each undersampling sequence. Furthermore, most researchers have concentrated on how to compose a multiplexing matrix in the undersampling method to reduce the number of pixels in coding plates, while ignoring methods that directly reduce the number of pixels by changing the connection between subcoding plates.

In this study, on the premise that the size of coding plates cannot be reduced on a large scale due to the lack of unified theoretical frameworks, we investigate the internal correlation of subcoding plates from another perspective and study new connection modes between subcoding plates to reduce the number of pixels in coding plates.

The specific approach is as follows: we propose an optimization algorithm that can create “compressed coding plates” by compressing traditional coding plates. The algorithms satisfy the requirements of both full sampling and undersampling. This study consists of three parts: (1) characterization of Hadamard coding plates to obtain a moving distance matrix that quantifies regional similarity between subcoding plates; (2) design of two ant colony optimization arrangement algorithms to maximize pixel reuse in regions with similarity; and (3) testing of the optimization method and discussion of its outcomes for both full sampling and undersampling.

2.1 Computational Ghost Imaging

The schematic of a typical γ -ray CGI device is shown in Fig. 1 [Figure 1: see original paper]. The conceptual drawing illustrates the experimental setup, with the left and right parts showing the overall layout and a partial enlargement, respectively. The collimator (1) ensures that particles produced by the γ -source are incident only on the imaging area. The coding plate (mask) (2) comprises all subcoding plates, with patterns encoded on metal. Particles are modulated on the subcoding plate (3), and the object can be placed between the collimator

and subcoding plate. The single-pixel detector (4) collects ray intensity, while a computer (5) reconstructs the images.

In CGI, it is necessary to move the coding plate after each measurement to change the mask shape in the imaging area and thus the distribution of the ray field. In this study, the mask corresponding to each measurement is called a subcoding plate. The measurement process can be expressed using Eq. 1, where \mathbf{O} represents $N \times N$ pixel values in an object transformed into a column vector, \mathbf{P} represents $N \times N$ modulation values corresponding to the i th subcoding plate (a measured pattern) transformed into a row vector, and I represents the measured value obtained from the i th measurement.

In CGI, the measured value I is mathematically equivalent to the inner product of each pattern \mathbf{P} and object image \mathbf{O} . Spatial information of objects is acquired by a detector and subcoding plates, not by array detectors. Traditional ghost imaging is a well-known technique where an object image O can be reconstructed using \mathbf{P} and I , with \cdot denoting the average of all CGI measurements.

With the development of ghost imaging technology, some researchers have reconstructed images from the perspective of solving equations. For example, Zhang proposed pseudo-inverse ghost imaging, and Xue proposed singular value decomposition ghost imaging. Subsequently, optimization algorithms based on compressed sensing have been employed to reconstruct images under undersampling, such as the orthogonal matching pursuit (OMP) and TVL3 algorithms. Various patterns, including Gaussian, Bernoulli, DCT, and Hadamard matrices, have been used in CGI. The Hadamard matrix can generate better-quality images compared to other measurement matrices, making the Hadamard coding plate widely used in CGI.

2.2 Regional Similarity

Due to the uncorrelated character of any two columns or rows, the Hadamard matrix, as a modulation orthogonal matrix, can effectively avoid correlation noise among pixels and greatly improve reconstructed image quality at the same sampling rate. The Hadamard matrix is generally constructed recursively from low to high order, as shown below. Traditional Hadamard coding plates comprise $K \times K$ -block subcoding plates, with the molding method converting 1D code elements into 2D patterns, as illustrated in Fig. 2 Figure 2: see original paper.

Owing to the recursive property of the Hadamard matrix, the front part of a row in the matrix is identical to the back part of another row (Fig. 2(a)). This rule causes the left area (Fig. 2(b)) of one subcoding plate to be completely consistent (Fig. 2(d)) with the right area (Fig. 2(c)) of another subcoding plate, a phenomenon known as regional similarity between two subcoding plates.

To utilize regional similarity for compressing coding plates, it is necessary to

quantitatively calculate the degree of similarity between subcoding plates. A moving distance matrix \mathbf{M} is employed to describe regional similarity. Table 1 presents the moving distance matrix M_{64} corresponding to the Hadamard matrix with size 64^2 (64 subcoding plates). According to this matrix, regional similarity between subcoding plates exhibits two key characteristics: (1) approximately 90% of elements in the moving distance matrix take the value of 8, indicating no regional similarity between most subcoding plates, yet 320 pairs of subcoding plates do exhibit regional similarity, suggesting that pixel reduction can be achieved by adjusting the order of subcoding plates; and (2) sporadic continuous oblique distributions of low-value elements indicate interrelated characteristics between subcoding plates with regional similarities, enabling the use of continuous subcoding plates with such similarities.

Therefore, based on the moving distance matrix, we reuse identical regions between subcoding plates to reconstruct a new coding plate with fewer pixels, as shown in Fig. 2(d). This new coding plate can compress a traditional coding plate and is called a compressed coding plate.

2.3.1 Ant Colony Algorithms

To maximize utilization of regional similarity between subcoding plates and thus minimize pixels in the compressed coding plate, it is necessary to redesign a measurement sequence of subcoding plates according to the moving distance matrix. The problem is modeled as follows: given a set of points $[1, 2, 3, \dots, N]$ where N represents the number of points, m_{ij} represents the distance from point i to point j (expressed by moving matrix \mathbf{M}), S represents the sum of all distances in a path, and t represents the t -th point in a path ensuring calculation of distance between adjacent points.

The goal is to find a path that minimizes S , with constraints that each point can be visited only once and all points must be visited. This problem resembles the traveling salesman problem (TSP), a mathematical combinatorial optimization problem that is NP-hard. The ant colony algorithm is a mature and effective method for solving TSP and other combinatorial optimization problems. In this study, each subcoding plate serves as a node in the ant colony path space, with the moving distance matrix representing node spacing. The path of an ant passing through all nodes constitutes a feasible solution, and all paths of the entire ant population form a feasible solution space.

Influenced by ant communication mechanisms, pheromone release, and pheromone evaporation, more pheromones accumulate on short paths (optimal solutions) while pheromones on other solutions evaporate slowly with iterations. Ants prefer paths with high pheromone concentrations. As iterations increase, more pheromones remain on shorter paths and more ants choose these paths. Eventually, through positive feedback, ants converge on the best path, and the corresponding subcoding plate sequence represents the optimal solution.

When ant k is at point t , the probability $P_{kj}(t)$ of choosing the next point j from

point i considers two factors: the heuristic factor $\eta_{ij}(t)$ representing distance between points i and j , and the pheromone concentration $\tau_{ij}(t)$ representing feasibility between points i and j . This probability can be calculated using the formula where α represents the importance of pheromones, β represents the importance of the heuristic factor, and the set of selectable points.

After obtaining the path of a generation of ants, the path is used to update pheromones according to the update formula: $\tau_{ij}(t+1) = (1-\rho) \cdot \tau_{ij}(t) + \Delta\tau_{ij}$, where ρ is the pheromone evaporation coefficient, antN is the number of generation ants, $\Delta\tau_{ij}$ is the sum of pheromones left by ants on the path between points i and j , $\Delta\tau_{ij}^k$ is the pheromone left by the k -th ant on the path between points i and j , antQ is the total pheromone of an ant, and L^k is the total length of the path taken by the k -th ant.

2.3.2 Method for Optimum Coding Plate

According to the algorithm described in Section 2.3.1, the measurement sequence of subcoding plates under maximum regional similarity utilization can be calculated. However, the coding plate manufactured according to this sequence would be very long. Considering practical production and usage requirements, we need to optimize the coding plate further by designing a rectangular compressed coding plate with no significant difference in segment lengths while minimizing total pixel count. Two methods are employed to obtain the optimum coding plate: the one-objective ant colony segmentation algorithm (OACSA) and the multi-objective ant colony reconstruction optimization algorithm (MACRA).

(1) One-Objective Ant Colony Segmentation Algorithm

The core idea of this method is to divide the long measurement sequence obtained by the ant colony algorithm into multiple short measurement sequences by selecting appropriate segmentation points, without changing the original long sequence. These short sequences are reconstructed to obtain corresponding rectangular compressed coding plates (with supplemental pixels and parallel combinations). The reconstruction process may increase the total number of pixels in the coding plate. Analysis indicates that segmentation points should meet three criteria: (1) each segment length should be approximately equal; (2) segmentation points should preserve complete subcoding plates to avoid unnecessary pixel supplementation; and (3) segmentation points should be selected between subcoding plates with low regional similarity to minimize supplemental pixels in the reconstruction process.

Based on these criteria, this study proposes a segmentation-point selection method for single-sequence coding plates, with its flowchart shown in Fig. 3 [Figure 3: see original paper]. According to the segmentation points selected by the algorithm, the single-sequence coding plate is divided into multiple segments, which are then reconstructed to obtain the corresponding compressed coding plate (with supplemental pixels and parallel combination).

(2) Multi-Objective Ant Colony Reconstruction Optimization Algorithm

Reconstructing a rectangular compressed coding plate is a problem of reconstructing multiple independent segments—an optimization problem with constraints. To solve this, we included the constraint (each segment being essentially the same length) as a penalty term in the objective function, as shown in Eq. 7: $f = \sum d_k + \gamma \cdot \max |s_k - s_{k-1}|$, where segN is the total number of segments (also representing the number of ants), d_k is the distance traveled by the k -th ant in the same group (representing the length of the k -th segment), kN is the number of points the ant needs to cross, and γ is the penalty factor coefficient. MACRA was employed to solve this model, and through continuous iterative optimization, the total distance was gradually reduced while gaps between segment lengths were bridged.

2.4 Evaluation Index

To quantitatively measure the degree of pixel reduction in coding plates using ant colony optimization arrangement algorithms, this study defines a quantitative index—the compression ratio α . It represents the ratio of pixel numbers between compressed and traditional coding plates, where smaller values indicate better compression. The formulas are as follows: $\alpha = \sum D_i / (K^2 \cdot \sum P_i)$, where $D_i = K \cdot (K + \sum d_{ij})$, D_i represents the number of pixels for the i -th segment, K represents the order of a subcoding plate, P_i represents the number of subcoding plates in each segment, d_{ij} denotes the minimum movement distance from the j -th to the $(j+1)$ -th subcoding plate in the i -th segment, i varies from 1 to M , and M is the number of segments.

3. Results and Discussions

We applied the ant colony optimization arrangement algorithms described in Section 2 to reconstruct the full-sampling traditional Hadamard coding plate and obtain a compressed coding plate. Subsequently, to demonstrate under-sampling feasibility, we obtained compression ratios of compressed coding plates corresponding to the Harr, cake-cutting, and Russian dolls sequences at different sampling rates, comparing and analyzing the compression effects.

3.1 Compressed Coding Plate in Full Sampling

For all subcoding plates corresponding to the 1024-order Hadamard matrix, regional similarity between subcoding plates was quantified using the moving distance modeling method to obtain the moving distance matrix. The OACSA and MACRA (with main parameters listed in Table 2) were used to reconstruct rectangular Hadamard compressed coding plates, shown in Fig. 4 [Figure 4: see original paper]. Compression results are presented in Fig. 5 [Figure 5: see original paper].

As shown in Fig. 5(a), both OACSA and MACRA significantly reduce pixel count, with corresponding compression rates α of 54.2% and 58.9%, respectively. Moreover, OACSA demonstrates better compression performance for the following reasons: with constraints, MACRA is the basic solution to this combinatorial optimization problem but is more complex than OACSA, requiring longer computation time and being easily affected by initial conditions. We believe OACSA may provide a better solution to this challenging problem. OACSA operates in two phases: Phase 1 uses the one-way ant colony algorithm without constraints (to make coding plates rectangular) to solve the optimization problem, creating relatively simple single-sequence coding plates with minimal pixels; Phase 2 involves segmenting and combining single-sequence coding plates to create rectangular compressed coding plates when constraints are applied. If appropriate segmentation points are selected through the segmentation point selection method, the entire process results in only a minor increase in pixel count.

The minimum movement distances between adjacent subcoding plates in the single-sequence coding plate generated by OACSA are shown in Fig. 5(b). Points with the maximum value of 32 account for 59.9% of the distribution, which is wide. This phenomenon is favorable for the segmentation point selection method, resulting in only a small number of added pixels. Consequently, there is no significant difference between pixel counts in rectangular compressed coding plates and single-segment arrangement schemes, leading us to conclude that OACSA outperforms MACRA.

3.2 Compressed Coding Plate in Undersampling

The previous section demonstrated that both optimization algorithms achieve good compression effects for Hadamard coding plates under full sampling, but their undersampling compression effects remained unknown. Therefore, this section applies OACSA (which shows better compression performance) to reconstruct compressed coding plates for the Harr, Russian dolls, and cake-cutting sequences at sampling rates of 5%–40%. The α values of compressed coding plates are shown in Fig. 6 Figure 6: see original paper.

As shown in Fig. 6(a), the ant colony optimization arrangement algorithm achieves compression effects on all three sequences at different sampling rates, particularly for the Russian dolls and cake-cutting sequences, where pixel reduction can reach 40%. According to relevant literature, the Russian dolls sequence can achieve good imaging results at a 25% sampling rate, meaning the coding plate requires only 25% of the pixels of a traditional Hadamard encoding plate. If the coding plate is further compressed using the optimization arrangement algorithm, pixel count can be reduced even more while maintaining the same imaging effect without changing the modulation matrix. Therefore, combining undersampling methods with the ant colony optimization arrangement algorithm will significantly reduce coding plate pixel count, helping lower manufacturing difficulty and cost and establishing a foundation for realizing γ -ray

CGI.

The compression effect is greatest for the cake-cutting sequence, followed by the Russian dolls sequence, while the Harr sequence shows minimal improvement. This demonstrates that the ant colony optimization arrangement algorithm has varying compression effects on different sequence types. The primary reasons are as follows: a large area of identical pixel values is likely to appear between subcoding plates with small or similar numbers of blocks, making regional similarities easier to identify. The cake-cutting sequence is arranged in ascending order of block numbers. Russian dolls sequences exploit the fact that higher-order Hadamard matrices (H_2) contain lower-order Hadamard matrices (H_2 , $m < n$), preferentially measuring patterns of subcoding plates with scaled versions of low-order matrices, which also preferentially measures subcoding plates with low block numbers. This creates a cyclic growing trend in block numbers for Russian dolls. The Harr sequence is arranged in ascending order of Harr values. Based on these different sorting rules, the trends of block numbers for different sequences are shown in Fig. 6(b). Both cake-cutting and Russian dolls sequences are sorted according to block numbers, whereas the Harr sequence is not, resulting in much better compression effects for the former two sequences.

4. Conclusion

This study proposes optimum ant colony algorithms based on regional similarity to reduce pixel count and reconstruct rectangular compressed coding plates. Without sacrificing imaging quality, compression rates can reach 60% under full sampling. In undersampling, the proposed algorithm achieves compression effects on all three sequences at different sampling rates, with particularly obvious effects on the Russian dolls and cake-cutting sequences. Combining undersampling with the proposed algorithm significantly reduces coding plate pixel count, helping reduce manufacturing difficulty and cost and building a foundation for realizing γ -ray CGI. Finally, beyond γ -ray CGI, the proposed algorithms can also be applied to other types of CGI requiring modulation by mechanical movement of coding plates.

Acknowledgments

The authors acknowledge support from the Youth Science Foundation of Sichuan Province (grant Nos. 22NSFSC3816 and 2022NSFSC1231), the General Project of the National Natural Science Foundation of China (grant Nos. 12075039 and 41874121), and the Key Project of the National Natural Science Foundation of China (grant No. U19A2086).

Author contributions: All authors contributed to study conception and design. Material preparation, data collection, and analysis were performed by Zhi Zhou, Sangang Li, and Qingshan Tan. The first draft was written by Zhi Zhou, and all authors commented on previous versions. All authors read and approved the final manuscript.

References

1. Y. Bromberg, O. Katz, Y. Silberberg, “Ghost imaging with a single detector,” *Physical Review A* 79, 053840 (2009). doi:10.1103/PHYSREVA.79.053840
2. G.M. Gibson, S.D. Johnson, M.J. Padgett, “Single-pixel imaging 12 years on: a review,” *Optics Express* 28, 28190-28208 (2020). doi:10.1364/OE.403195
3. J.H. Shapiro, “Computational ghost imaging,” *Physical Review A* 78, 6 (2008). doi:10.1364/IQEC.2009.IThK7
4. D. Takhar, J.N. Laska, M.B. Wakin et al., “A new compressive imaging camera architecture using optical-domain compression,” *SPIE* 6065, 43 (2006). doi:10.1117/12.659602
5. M. Chen, E. Li, S. Han, “Application of multi-correlation-scale measurement matrices in ghost imaging via sparsity constraints,” *Applied Optics* 53, 13 (2014). doi:10.1364/AO.53.002924
6. Y.H. He, A.X. Zhang, W.K. Yu et al., “Energy-selective x-ray ghost imaging,” *Chinese Physics Letters* 37, 4 (2020). doi:10.1088/0256-307X/37/4/044208
7. Y.H. He, A.X. Zhang, M.F. Li et al., “High-resolution sub-sampling incoherent x-ray imaging with single-pixel detector,” *APL Photonics* 5, 5 (2020). doi:10.1063/1.5140322
8. Y.H. He, Y.Y. Huang, Z.R. Zeng et al., “Single-pixel imaging with neutrons,” *Science Bulletin* 66, 2 (2021). doi:10.1364/CLEOPR.2020.C1G_1
9. T. Zhang, L. Wang, J. Ning et al., “Simulation of an imaging system for internal contamination of lungs using MPA-MURA coded-aperture collimator,” *Nuclear Science and Techniques* 32, 2 (2021). doi:10.1007/S41365-021-00849-3
10. Y. Ren, P. He, H.L. Wang, “Compressed sensing and Otsu’s method based binary CT image reconstruction technique in non-destructive detection,” *Nuclear Science and Techniques* 26, 5 (2015). doi:10.13538/j.1001-8042/nst.26.050403
11. M.F. Duarte, M.A. Davenport, D. Takhar et al., “Single-pixel imaging via compressive sampling,” *IEEE Signal Processing Magazine* 25, 2 (2008). doi:10.1109/MSP.2007.914730
12. S.J. Olivas, Y. Rachlin, L. Gu et al., “Characterization of a compressive imaging system using laboratory and natural light scenes,” *Applied Optics* 52, 19 (2013). doi:10.1364/AO.52.004515
13. P.G. Vaz, D. Amaral, L.R. Ferreira et al., “Image quality of compressive single-pixel imaging using different Hadamard orderings,” *Optics Express* 28, 8 (2020). doi:10.1364/OE.387612

14. W.K. Pratt, J. Kane, H.C. Andrews, "Hadamard transform image coding," *Proceedings of the IEEE* 57, 1 (1969). doi:10.1109/PROC.1969.6869
15. N. Radwell, K.J. Mitchell, G.M. Gibson et al., "Single-pixel infrared and visible microscope," *Optica* 1, 1 (2014). doi:10.1364/OPTICA.1.000285
16. M.J. Sun, L.T. Meng, M.P. Edgar et al., "A Russian Dolls ordering of the Hadamard basis for compressive single-pixel imaging," *Scientific Reports* 7, 1 (2017). doi:10.1038/S41598-017-03725-6
17. W.K. Yu, C. Cao, Y. Yang et al., "Single-pixel imaging based on weight sort of the Hadamard basis," arXiv preprint arXiv:2203.04659.
18. W.K. Yu, "Super sub-Nyquist single-pixel imaging by means of Cake-Cutting Hadamard basis sort," *Sensors* 19, 19 (2019). doi:10.3390/s19194122
19. M.F. Li, X.F. Mo, L.J. Zhao et al., "Single-pixel remote imaging based on Walsh-Hadamard transform," *Acta Physica Sinica* 2665 (2016). doi:10.7498/aps.65.064201
20. M.F. Li, L. Yan, R. Yang, "Fast single-pixel imaging based on optimized reordering Hadamard basis," *Acta Physica Sinica* 68, 6 (2019). doi:10.7498/aps.68.20181886
21. C. Zhang, S. Guo, J. Cao et al., "Object reconstitution using pseudo-inverse for ghost imaging," *Optics Express* 22, 24 (2014). doi:10.1364/OE.22.030063
22. Z. Xue, X. Meng, X. Yang et al., "Singular value decomposition ghost imaging," *Optics Express* 26, 10 (2018). doi:10.1364/oe.26.012948
23. M. Dorigo, G.D. Caro, L.M. Gambardella, "Ant Algorithms for Discrete Optimization," *Artificial Life* 5, 2 (1999). doi:10.1162/106454699568728

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv — Machine translation. Verify with original.