Improved Cohen-Sutherland algorithm for TGS transmission imaging

Authors: Yucheng Yan, Mingzhe Liu, Xingyu Li, Yuting Xiong, Yao Tan, Zhuo Zuo, Lan Chang, Cui Li, Mingzhe Liu

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Abstract

Laminar γ-scanning (TGS), an advanced γ-ray nondestructive analysis technique, is capable of locating and analyzing the nuclides in radioactive nuclear waste, and the scanning of TGS falls into two parts (e.g., transmission measurement and emission measurement). To be specific, transmission measurement lays the basis for the accurate measurement of non-uniform radionuclide content in TGS scanning. The scan data were obtained using the Monte Carlo tool Geant4 simulation, and a total of 25 voxels were divided into 5 in length and 5 in width in a square barrel. In this study, the encoding cropping algorithm based on the draped foot vector judgment was adopted to rapidly calculate the voxel trace matrix within the square bucket of nuclear waste, and the transmission images were reconstructed using the Ordered Subsets Expectation-Maximization(OSEM). The results indicated that the cropping speed of the improved coding algorithm was significantly increased compared with the original algorithm, and the relative mean deviation (RMD) and root mean square error(RMSE) between the reconstructed attenuation coefficient and the reference standard value tended to decrease with the increase of the cropped line segments in the voxel, and the Pearson correlation coefficient(PCC) tended to converge to be 1.0. The image quality evaluation parameters of high media density materials were better than those of low media density materials in the above three indexes. The reconstruction effect was relatively poor under the more complex filling material. When there were more than 10 cropped line segments in the voxel, the reconstruction data generally tended to be stable. The graphical trimming algorithm is capable of rapidly calculating the trace matrix of the scanned voxels, it shows the advantages of speed and efficiency, and it can serve as a novel method to solve the trace matrix of TGS nuclear waste transmission scans.

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Preamble

Improved Cohen-Sutherland Algorithm for TGS Transmission Imaging

Yu-Cheng Yan[1,2,3], Ming-Zhe Liu[1*], Xing-Yu Li[2], Yu-Ting Xiong[2], Yao Tan , Zhuo Zuo[1,2,3], Lan Chang[2,3], Cui Li[2,3]

[1]State Key Laboratory of Geohazard Prevention and Geoenvironment Protection, Chengdu University of Technology, Chengdu 610059, China
[2]The Engineering & Technical College of Chengdu University of Technology, Leshan, Sichuan 614000, China
[3]Southwestern Institute of Physics, Chengdu, Sichuan 610225, China
Sichuan University of Science & Engineering, 643000, China

*Corresponding author. E-mail addresses: liumz@cdut.edu.cn (Mingzhe Liu)

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Abstract

Objective: Tomographic Gamma Scanner (TGS) is an advanced -ray nondestructive analysis technique that can locate and analyze nuclides in radioactive nuclear waste. TGS can be categorized into two types: transmission measurement and emission measurement. Specifically, transmission measurements provide the basis for accurate quantification of nonuniform radionuclide content in TGS scanning.

Methods: Scan data were obtained using the Monte Carlo tool Geant4 simulation, with 25 voxels divided into five lengths and five widths in a square barrel. This study adopted an encoding cropping algorithm based on draped foot vector judgment to rapidly calculate the voxel trace matrix within a square bucket of nuclear waste, and transmission images were reconstructed using ordered subset expectation maximization (OSEM).

Results: The results indicated that the cropping speed of the improved coding algorithm was significantly higher than that of the original algorithm. The relative mean deviation (RMD) and root mean square error (RMSE) between the reconstructed attenuation coefficient and the reference standard value tended to decrease with an increase in the cropped line segments in the voxel, while the Pearson correlation coefficient (PCC) tended to converge to 1.0. The image quality evaluation parameters for high media-density materials were better than those for low media-density materials across all three indexes. The reconstruction effect was relatively poor for more complex filling materials. When there were more than 10 cropped line segments in the voxel, the reconstruction data generally tended to be stable.

Limitations: Only the first-layer scan data of the validation sample model were used for verification because of the small number of sample voxels, sparse voxel grid partitioning, simple preset materials employed in the project simulation, and insufficient energy of the transmitted source. Moreover, the results were not supported by relevant experimental data.

Conclusions: The graphical trimming algorithm can rapidly calculate the trace matrix of scanned voxels, exhibiting advantages of speed and efficiency, and can serve as a novel method to solve the trace matrix of TGS nuclear waste transmission scans.

Key words: TGS; Cohen-Sutherland; transmission scanning; trace matrix

1 Introduction

Tomographic gamma scanning (TGS) is a nondestructive assay technique that represents a passive gamma analysis method. Both TGS and -medium radiographic imaging technologies are popular topics in the field of radiation detection imaging [1][2]. The TGS technique was developed based on segmented gamma scanning (SGS) and was specifically created to address the inability of the SGS technique to determine high- and medium-density non-uniformly distributed nuclear waste [3],[3]. Linear attenuation coefficient reconstruction is vital for TGS transmission measurement reconstruction. Reconstruction algorithms are primarily classified into two types. One type comprises analytical reconstruction algorithms based on Radon transform theory that directly and mathematically invert the image to be reconstructed; this category includes the filtered backprojection algorithm (FBP) and the Radon inverse transform algorithm [3]. The second type refers to iterative reconstruction algorithms that convert the voxel and projection values of the reconstructed image into a series of linear equations and obtain the image to be reconstructed by solving these equations; this category includes algebraic iterative reconstruction algorithms and statistical iterative reconstruction algorithms. Generally, algebraic iterative reconstruction algorithms involve algebraic reconstruction techniques (ART algorithms) and simultaneous iterative class reconstruction algorithms. Synchronous iterative class reconstruction algorithms include the simultaneous algebraic reconstruction technique (SART) and the diagonally relaxed orthogonal projection (DROP) algorithms [6]. Statistical iterative reconstruction algorithms include maximum likelihood expectation maximization (MLEM) and ordered subset expectation maximization (OSEM) algorithms[8]. Currently, common TGS image reconstruction algorithms include the expectation-maximization (EM) algorithm and the ART algorithm[8][9]. The EM algorithm is an iterative algorithm based on mathematical statistics that exhibits good operability and convergence, high noise immunity, and superior reconstructed image quality within a certain number of iterations; however, its convergence speed is too low [8]. Due to its large computation time and long reconstruction time, the ART algorithm spends most of its time calculating projection and backprojection. The reconstruction efficiency of the ART algorithm has been improving for years through algorithm optimization and hardware acceleration, and it exhibits good noise immunity [11][12][13].

The two key factors for TGS transmission imaging are the solutions of the trial and projection matrices. Common methods adopted to calculate the trial matrix include the average method, Monte Carlo simulation, and computer graphics cropping. Specifically, the cropping method processes defined graphics along the window boundary and displays them inside the window based on preset window parameters. To offer clear and distinct objects for image identification and processing, cropping seeks to remove all graphics from the image except for the user-defined window. The cropping of line segments is a critical problem that must be solved in computer graphics. The four classical cropping algorithms that have been extensively used include the You-Dong Liang-Barsky algorithm in parametric form [12], the Nicholl-Lee-Nicholl algorithm based on region partitioning [15], the Cyrus-Beck algorithm for polygonal windows [16], and the Cohen-Sutherland algorithm based on region coding [17]. The You-Dong Liang-Barsky algorithm employs a parametric representation of the line segment to simplify the calculation of intersection point coordinates between the line segment location and the rectangular cropping window border, calculating the parameter value corresponding to the intersection point. Subsequently, the valid intersection point is determined by comparing the parameters of the intersection point with the parameter definition interval of the line segment being cropped to obtain the part of the line segment that should be retained after cropping. The Nicholl-Lee algorithm follows the coding algorithm to decrease the number of intersection-point calculations by adding more regional tests. The Cyrus-Beck algorithm is an early cropping algorithm proposed to deal with convex polygon cropping windows. For concave polygons, there is no general algorithm, whereas the method of splitting concave polygons into convex polygons has typically been adopted (e.g., extension line splitting and rotation splitting). In this study, an optimized Cohen-Sutherland algorithm is proposed based on the judgment of the vertical foot vector, which can effectively avoid defects caused by the original algorithm (e.g., cutting invalid intersection points) and significantly increase the efficiency of solving the trace matrix.

2.1 Model Construction and Scanning

The TGS transmission measurement system is primarily adopted to quantitatively measure the content of radioactive material in nonuniform medium solid nuclear waste or nuclear waste drums, while obtaining the attenuation coefficient and activity distribution of radioactive material in nuclear waste drums [18]. To acquire more accurate reconstruction information, the samples measured in the nuclear waste drums were tested in equally spaced layers. Moreover, it is imperative to ensure that the same transmission scanning mode is employed for the respective layers of the material to be measured, and that the projection data, system matrix acquisition, and image reconstruction algorithm are all the same. On this basis, the linear attenuation coefficient and activity of the radioactive source in the drums were calculated. The TGS transmission measurement system comprises a radioactive source collimator, a nuclear waste drum, a detector shield, and an HPGe detector, and the scanning device is illustrated in Fig. 1 [Figure 1: see original paper].

A P-type HPGe coaxial GEM20P4-70 detector was used for the simulation in this study; its internal structure is shown in Fig. 2 [Figure 2: see original paper]. A P-type coaxial GEM20P4-70 detector was designed to detect high-energy charged particles and generate signals. It consists of a P-type silicon chip, a coaxial GEM, and a 70-micrometer thick detection layer. The silicon chip provides support and an interface, the coaxial GEM amplifies electrons to enhance signal intensity and resolution, and the detection layer detects the energy deposition of charged particles. A cylindrical lead material served as a collimator to reduce the background. The centers of the detector probe, voxel center of the sample model, and the center of the transmission source were maintained on the same line during measurement. The physical model of the TGS transmission measurement system was simplified, and an idealized "tensor" model was established, i.e., the transmission source was considered a point source and was located at the center of the sample voxel, and the beam had a certain width. The transmission source was a common ^{13} Cs point source, and the experimental source for simulation was a Co point source, with energies of 0.661 MeV, 1.17 MeV and 1.33 MeV, respectively. Based on the actual levels of low and medium radioactive solid waste container steel boxes used in China [18], a voxel model of the sample nuclear waste square barrel was built with 25 voxels per layer, and the size of the respective voxel was 12 cm $\times$ 12 cm $\times$ 12 cm. To acquire sufficient measurement data, voxel information was collected maximally by examining different positions and angles during the respective layer scans. In addition, TGS transmission measurements were performed using three scanning methods (step, rotational, and vertical scans) in combination with layer-by-layer scanning to obtain the medium-filled material of the sample model. Before each scan, it was confirmed that the transmission source was in the same straight line as the detector and that the transmission source was placed outside the packaging body. The transmission source was also selected using a cylindrical collimator such that it could emit rays at a small solid angle. Fig. 3 [Figure 3: see original paper] shows the scanning test diagrams of the simulated transmission experiments with the medium to be tested rotated by 0°, 45°, 90°, and 135°.

The scanning measurement process is as follows:

1. Five equally spaced measurement points were selected, and the average value was taken as the measurement value by performing three measure-

ments at the respective measurement positions, after which the projection data were calculated.

2. After the horizontal measurement was completed, the sample model of the packaging body was rotated clockwise with its geometric center as a circle, and the rotation angles were 0°, 45°, 90°, and 135°. In the same step as above, five equally spaced measurement points were selected at the respective rotation angles, and the average value was examined three times at the respective measurement position. Next, the projection data were calculated.
3. After all the scans were completed, the scale of one layer was completed, and the above operation was repeated to finish scanning all the sample models to acquire the scanned data. Although each layer was relatively independent, the calculation method for each layer was the same, such that only the measurement results of any one layer would be considered for simulation verification.

2.1.1 Mass Attenuation Coefficient Reconstruction

Based on the attenuation law of rays in matter (i.e., the Lambert–Beer law) [20], the attenuation law in complex mixed non-homogeneous materials can be expressed as

$$\frac{1}{n}\sum_{i=1}^{n}\mu_i$$

where n indicates that there are n materials among them, μ_{mi} represents the linear attenuation coefficient of the ith material, and c_i represents the weight percentage of the ith material in the mixed material.

When the -rays are attenuated after entering the material, the transmittance is expressed as

$$S_i = \frac{C_i}{C_{i0}}$$

where S_i is the transmittance of photons at the ith measurement position [21], C_i represents the count rate of photons after the detector has undergone attenuation at the ith measurement position after the rays have passed through the sample model, and C_{i0} is the count rate of photons measured by the detector at the ith position that have not undergone attenuation.

P_i is defined as the projected data at the ith measurement position:

$$P_i = \ln(S_i)$$

The equations for the respective measurement points are derived from the decay law. On this basis, the measurement equation of the ith layer is obtained:

$$\sum_{i=1}^{n} P_i x_i = \mu$$

where x_i denotes the trace matrix of the rays of the ith layer, A_i represents the matrix of the projection data of the ith layer, and B_i represents the matrix of the decay coefficients of the ith layer. Furthermore, if S_i is a square matrix, a unique solution to Eq. (5) is obtained as:

$$B_i = A_i \times S_i$$

The first layer of the established voxel model was scanned to obtain 20 sets of count rates, and the obtained count vector was recorded as I_K. To calculate the transmittance, the sample package was removed and measured three times, and the average count rate was taken as the initial count of the radioactive source I_0. Then, according to Eq. (2), the projection data vector can be obtained as follows:

$$\left(\ln\left(\frac{I_{k1}}{I_0}\right) \quad \ln\left(\frac{I_{k2}}{I_0}\right) \quad \ln\left(\frac{I_{k3}}{I_0}\right) \quad \cdots \quad \ln\left(\frac{I_{k20}}{I_0}\right)\right)^T$$

We then solved the attenuation coefficients according to Eq. (6). Twenty sets of linear equations were established based on the scan results. The attenuation coefficients were calculated using Eq. (6) with 20 sets of linear equations.

2.1.2 Cohen-Sutherland Coding Cropping Algorithm and Its Improvement

The Cohen-Sutherland clipping algorithm was one of the earliest and most widely used clipping algorithms. The basic concept is to first use an area code to identify the location of the end of a line segment. The specific position of the line segment is identified based on the code. Line segments that are not entirely inside or outside the window must find their intersection with the window. The part outside the window is discarded, and the remainder is judged as a new line segment. After two clipping judgments, it was possible to determine whether the line segment should be partially or completely cut. Because the Cohen-Sutherland algorithm cannot effectively judge whether a line segment is outside the window, which greatly reduces computational efficiency, an improved scheme is proposed.

The Cohen-Sutherland algorithm uses the four edges of the rectangular cropping window to divide the two-dimensional plane into nine regions, which are marked with the 4-bit binary code $C_t C_b C_r C_l$, as shown in Fig. 4 [Figure 4: see original paper]. The basic idea of the algorithm is as follows: The codes of the two ends of the straight-line segment to be cropped are recorded as $code1$ and $code2$, respectively, and there are three cases with the window: (1) When $code1 = code2 = 0$ (P_1P_2), it is completely visible and is retained. (2) When $code1 \& code2 \neq 0$ (P_3P_4) (& is a bit AND operation), it is not visible at all and is discarded. (3) When the "retain" or "discard" condition is not met, the line segment is divided into two segments at the intersection with the window boundary, one of which is completely outside the window and discarded; then, the above process is repeated for the other segment.

The advantage of the Cohen-Sutherland trimming algorithm is that the first and second cases can be separated without performing intersection operations. However, the third case is computationally intensive. As depicted in the figure, line segment P_7P_8 does not satisfy the above two cases, and according to the original algorithm, the intersection operation must be performed. In contrast, line segment P_7P_8 is outside the window. Thus, it is meaningless to intersect it. To address this problem, the algorithm was further optimized, as shown in Fig. 5 [Figure 5: see original paper]. The respective endpoint of the window forms a vertical line to the line segment and determines whether foot x is within the window. A line segment is considered to have passed through the window (which allows the intersection operation) if there are one or more intersection points. If there are fewer intersection points, the line segment is considered to be outside the window and is immediately discarded with no further processing.

Let the coordinates of the two endpoints of any line AB be (x_a, y_a) and (x_b, y_b), and the coordinates of the four vertices of the window be (x_1, y_1), (x_2, y_1), (x_1, y_2), and (x_2, y_2). The coordinates of the vertical foot F are to line segment drape.

From the vector-perpendicular relationship, it follows that:

$$\overrightarrow{AF} \cdot \overrightarrow{AB} = 0$$

Thus, it can be obtained that:

$$\overrightarrow{AF} = K\overrightarrow{AB}$$

The vertical foot F is located on line segment AB, which is known from vector collinearity. Substituting the equation gives:

$$K = \frac{(x_f - x_a)(x_b - x_a) + (y_f - y_a)(y_b - y_a)}{(x_b - x_a)^2 + (y_b - y_a)^2}$$

Then, the coordinates of the vertical foot of VTL are:

$$x_{fTL} = x_a + K(x_b - x_a)$$

$$y_{fTL} = y_a + K(y_b - y_a)$$

If the pendant foot is inside the window, then:

$$x_1 \le x_{fTL} \le x_2$$
$$y_1 \le y_{fTL} \le y_2$$

Similarly, the coordinates of the perpendiculars of the remaining vertices of the window to any line segment are: VBL to the perpendicular line of the line segment is (x_{fBL}, y_{fBL}), VTR to the perpendicular line of the line segment is (x_{fTR}, y_{fTR}), and VBR to the line segment vertical foot is (x_{fBR}, y_{fBR}). If the coordinates of the foot are within the window, the line passes through the window. The specific flow chart is shown in Fig. 6 [Figure 6: see original paper].

According to the TGS model, because the "tensor" model is between the source and the detector, the range path of -rays is nearly straight, and the effective line segments in the cropping area are summed up and averaged to approximate the length of the trajectory of -rays through the voxel, i.e.,

$$L_n = \frac{1}{n} \sum_{i=1}^{n} L_i$$

The resulting visuals of cropping 2 and 10 line segments are shown in Fig. 7 [Figure 7: see original paper] [13].

To test the cropping efficiency of the two algorithms, experiments were conducted on a Windows 10 computer configured with an AMD Ryzen 32300U processor at 2.0 GHz with four cores and eight threads. The control variable method was adopted to ensure that both algorithms were performed under the same conditions, and the experimental results showed that both algorithms obtained the same trail matrix when performing voxel segmentation on square nuclear waste buckets; however, the cropping times were different, as shown in Table 1 .

As indicated by the running time data in Table 1, the improved Cohen-Sutherland algorithm runs faster than the original algorithm in cutting voxel split line segments because the original algorithm cannot effectively judge whether the line segments are outside the window in the cutting area and will repeat the judgment of invalid intersection points. By introducing the judgment of the vertical foot vector, the improved algorithm avoids the calculation of invalid intersection points, significantly shortening the cropping time and improving computational efficiency.

2.1.3 OSEM Algorithm

The ordered subset (OS) algorithm is also a common acceleration algorithm used in numerical computation [22]. The OS algorithm was proposed by Hudson et al. to address the problems of low computational efficiency and slow convergence in the statistical iterative reconstruction of CT images. The OS algorithm divides the projection data into n subsets by arranging them, and these subsets of projection data are also called ordered subsets. The level of the subset (OS level) is determined by the number of subset divisions n. The reconstruction of an image using the OS algorithm refers to a process in which the reconstructed image is continuously corrected, and the reconstructed image is updated a total of n times. As the OS algorithm adopts projection data inside each ordered subset, it can realize the alignment of each pixel of the image once, and the reconstructed image is updated once as a result. However, because the projection data comprise n ordered subsets, the OS algorithm should correct each pixel n times, and the reconstructed image is updated n times accordingly.

The OSEM algorithm, also known as the ordered subset maximum expectation algorithm, is an application of the OS method to EM algorithms. Each iteration of the EM algorithm consists of two steps (i.e., the E step and the M step), where the E step determines the expectation and the M step determines the maximum. In the EM algorithm, the correction value of the image is obtained using all the projection values. Additionally, in the OSEM algorithm, the correction value of the image is determined from the projection data within the respective subset. OSEM applies the EM algorithm to each subset of the projection data. In the OSEM algorithm, the projection matrix is divided into n ordered subsets, the standard EM algorithm is adopted to maximize the likelihood function for each subset of the projected data in turn, and the reconstructed subset serves as the initial value for the next subset. Similar to the OS algorithm, the OSEM algorithm reconstructs an image with iterative correction updates, and the image is updated n times. When the OSEM algorithm completes the correction of the pixel points using the nth subset of the projection data, the first iteration is completed, and the reconstruction result serves as the initial value for the next iteration. However, unlike the OS algorithm, the OSEM algorithm adds the maximum likelihood function of the previous subset to the next subset and participates as the initial value of the next subset. Thus, the correlation between the reconstructed images increases.

In the OSEM algorithm, data subsets are typically divided in terms of projection angles, and the OSEM algorithm tends to distribute the projected data into ordered subsets according to the symmetric balance principle to guarantee that the pixels contribute approximately equally to each subset.

The specific steps of the OSEM algorithm are as follows:

1. Assign an initial value to the unknown quantity $\{x_j\}_{j=1}^{J}$.
2. For the nth subset S_n:

- Estimate all projections within the subset as $p_i = \sum_{j \in J} a_{ij} x_j$, where $i \in S_n$.
- Calculate the error as $\Delta p_i = \frac{p_i}{p_i^0}$.
- Calculate the correction value of the jth unknown quantity as $C_j = \frac{\sum_{i \in S_n} a_{ij}}{\sum_{i \in S_n} a_{ij} \Delta p_i}$.
- Correct the values of x_j; here, we correct them with all rays that pass through the voxel in the subset: $x_j^{(k+1)} = x_j^{(k)} \times C_j$.

3. Repeat operation (2) until n subsets of operations are completed, and one round of iteration is completed. Set $k = k + 1$.
4. Repeat operations (2) and (3) with the result of the previous iteration as the initial value and perform a new round of iterations until a result that meets the convergence requirement is obtained.

The subset level of the OSEM algorithm has a vital effect on the quality of the reconstructed image. When a high subset level of the OSEM algorithm is selected, the reconstructed images converge rapidly. However, as the number of iterations tends to increase, it will lead to undesirable noise levels occurring in image regions with low activity, and the reconstructed image appears divergent. When a low subset level is selected, the reconstructed image converges to a low value, and it preferentially recovers low-frequency element information and loses high-frequency information [23]. In the absence of noise, the convergence speed of the image is proportional to the subset segmentation size. Accordingly, if the OSEM algorithm is employed for reconstruction, choosing a smaller number of segmentation subsets not only requires more time for iteration but may also lack some high-frequency information about the image in the process; however, if a larger number of segmentation subsets is chosen, the image will be scattered in the iterative process. Therefore, different subset levels can significantly affect the convergence speed of the reconstructed images and the quality of the reconstructed images.

The length of the respective voxel in the square barrel is rapidly cropped by the code-cropping algorithm based on the vertical foot vector judgment proposed above, thus providing the path length matrix of the rays in the respective voxel; then, the attenuation coefficients of each layer in the barrel at the transmitted energy are reconstructed using the OSEM algorithm. To verify the accuracy of the OSEM algorithm, two experimental models were selected: a single medium model of mixed soil and polyethylene and the concrete, polyethylene, and aluminum mixture model. The first layer of the square barrel sample model was selected as the object of study, and voxels 7, 8, 12, and 13, shown in Fig. 8 Figure 8: see original paper, were filled with concrete and polyethylene successively; voxels 2, 8, 9, 13, 14, 17, and 22, as shown in Fig. 8(b), were filled with polyethylene, concrete, and aluminum successively, while the other voxels were filled with air. The reference values of the attenuation coefficients of the three media at different transmission energies are listed in Table 2 .

3 Results

To objectively and accurately verify the effectiveness of the computer-based graphic cropping algorithm, three image quality evaluation parameters—relative mean deviation (RMD), RMSE, and PCC—were introduced to evaluate the accuracy of the reconstructed values of all 25 voxels in the square barrel based on the known true distribution of the samples to be tested and their material attenuation coefficient reference values.

RMSE is the square root of the ratio of the square of the deviation of the predicted value from the true value to the number of observations N. It was adopted to measure the degree of deviation between the observed and true values; the smaller the RMSE, the closer the predicted value was to the reference value, that is, the higher the accuracy. The RMSE is written as:

$$X_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(X_i)^2}$$

where X_i represents the difference between the corresponding OSEM algorithm-reconstructed value and the reference value in the ith case, and N denotes the number of cases of that unique identical variable.

The PCC is the quotient of the covariance and standard deviation between X and Y. This parameter was adopted to measure the magnitude of the correlation between two variables. Its value range is $[-1, 1]$; a value of 1 indicates that the two random variables have a positive correlation, a value of -1 reveals a completely negative correlation between the two random variables, and a value of 0 represents no linear correlation between the two random variables. The calculation is as follows:

$$\gamma = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

where n denotes the number of preset voxels, X_i represents the computed value obtained in the ith case using the encoding algorithm, and Y_i expresses the reconstructed value obtained in the ith case using the OSEM algorithm. $\bar{X}$ and $\bar{Y}$ represent the corresponding reference values. The Pearson relationship quantifies the numerical correlation between the reconstructed image and the real image as well as the reconstruction similarity; the closer its value is to 1, the closer the reconstructed image will be to the real image, consistent with its higher reconstruction quality.

In this study, two transmission sources, ^{13}Cs and Co, were adopted: the first model was preset with two materials (including concrete and polyethylene) in voxels 7, 8, 12, and 13 in the square barrel, and the second model was filled with a mixture of polyethylene, cement, and aluminum in voxels 2, 8, 9, 13, 14, 17, and 22. Subsequently, a coding cropping algorithm based on the draped foot vector judgment was adopted after calculating the trial matrix of the respective voxel. The OSEM algorithm was used to determine the reconstruction values of the attenuation coefficients when the cropped line segments in the voxels were 2, 12, 100, and 1000. The resulting transmission-reconstructed images are shown in Fig. 9 [Figure 9: see original paper].

The relationships of the PCC, RMSE, and RMD with an increasing number of cut lines for the high-density medium concrete material, low-density medium polyethylene material, and mixed materials at transmission energies of 0.661 MeV, 1.17 MeV, and 1.33 MeV, respectively, are shown in Table 3 .

As shown in Table 3, when the preset material in the square barrel is high-density concrete with a single medium material, the RMD and RMSE between the reconstructed attenuation coefficient and the standard reference value tend to decrease with an increase in the cut line segments in the respective voxels, and the PCC tends to converge to 1. When the preset material in the square barrel is polyethylene with a low single medium density, with an increase in the cut line segment, the reconstructed attenuation coefficient and the reference standard value have the same variation trend for the three evaluation parameters as the high-density medium, whereas the image evaluation quality index is worse than that of the high-density medium. When there are multiple preset materials in the square barrel, with an increase in the number of cut strips, the interaction between different media materials, owing to their different densities and attenuation coefficients, will lead to poorer quality indicators for all three image evaluations. As revealed by this result, the accuracy of the image reconstruction was poor when the preset materials were more complex. A possible reason for this is the mutual scattering effect caused by the rays passing through each material. Furthermore, the attenuation coefficients of both preset models show a stable trend after 100 line segments are cut in the voxel, indicating that the average value can be approximated instead of the path length of -rays through the voxel when more line segments are cut in the voxel.

Conclusion

Using TGS technology, this study proposes an encoding and clipping method based on perpendicular vector judgment that can quickly calculate the voxel trajectory length of nuclear waste packaging. The results demonstrate that the improved coding clipping technique may significantly increase the clipping speed by clipping multiple line segments within sample voxels. The RMD, RMSE, and PCC were used as the three parameters for image evaluation. As the number of line segments clipped within each voxel increased, the accuracy of the attenuation coefficient in image reconstruction gradually improved. High-density materials exhibited better performances than low-density materials for all image reconstruction metrics. The reconstructed attenuation coefficients tended to stabilize when more than 100 line segments were cut within a voxel. The path length of the gamma rays passing through the voxel can be roughly estimated using the average value. Only the first-layer scan data of the validation sample model were used for verification because of the small number of sample voxels, sparse voxel grid partitioning, simple preset materials employed in the project simulation, and insufficient energy of the transmitted source. Moreover, the results were not supported by relevant experimental data. Therefore, by addressing these constraints in future studies, we can further improve and refine our simulations.

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Supporting Information

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Figure Legends

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Fig. 4 Schematic diagram of Cohen-Sutherland cropping algorithm region segmentation
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Figures

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Tables

Table 1 Comparison of the running time of the original algorithm and the improved Cohen-Sutherland algorithm

Number of cropping codes/(entries)	Original algorithm/(s)	Improved algorithm/(s)

Table 2 Reference values of the attenuation coefficient of dielectric materials under different transmission characteristic energy values

Media	Density/(g/cm^3)	Materials	Attenuation coefficient reference value/(cm ^{1})
			0.661MeV

Table 3 Evaluation parameter performance of three different dielectric materials at different energies

Transmission energy/(MeV)	Material	Number of cuts
	Concrete	
	Polyethylene	
	Mixture	
	Concrete	

Transmission energy/(MeV)	Material	Number of cuts
	Polyethylene	
	Mixture	
	Concrete	
	Polyethylene	
	Mixture	

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