Bayesian inference of the crust-core transition density via the neutron-star radius and neutron-skin thickness data

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Abstract

In this work we perform a Bayesian inference of the crust-core transition density t of neutron stars based upon the neutron-star radius and the neutron-skin thickness data within a thermodynamical method. The uniform and Gaussian distributions for the t prior are adopted in the Bayesian approach. It has a larger probability to have values higher than 0.1 fm 3 for t as the uniform prior and the neutron-star radius data are used. We found that this is controlled by the curvature Ksym of the nuclear symmetry energy. This phenomenon will not happen if Ksym is not extremely negative, i.e., Ksym > -200 MeV. The obtained t is 0.075+0.005-0.01fm^3 at 68% confifidence level when both the neutron-star radius and the neutron-skin thickness data are taken into account. The strongly anti-correlations between t and the slope L, curvature of the nuclear symmetry energy are observed. The dependence of the three L-Ksym correlations predicted in the literature on the crust-core density and pressure is quantitatively investigated. The most probable value of 0.08 fm^3 for t is obtained from the L-Ksym relation raised by Holt et al. and the larger values are preferred by the other two relations.

Full Text

Preamble

Bayesian Inference of the Crust-Core Transition Density via Neutron-Star Radius and Neutron-Skin Thickness Data

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In this work, we perform a Bayesian inference of the crust-core transition density (ρ_t) of neutron stars based on neutron-star radius and neutron-skin thickness data within a thermodynamical method. The Bayesian approach adopts both uniform and Gaussian distributions for the ρ_t prior. When using the uniform prior with neutron-star radius data, there is a higher probability for ρ_t to have values above 0.1 fm^{-3}. We find that this phenomenon is controlled by the curvature K_{sym} of the nuclear symmetry energy. This does not occur if K_{sym} is not extremely negative, i.e., $K_{\mathrm{sym}} > -200$ MeV. The obtained ρ_t is $0.075^{+0.005}_{-0.01}$ fm^{-3} at 68% confidence level when both neutron-star radius and neutron-skin thickness data are taken into account. Strong anti-correlations between ρ_t and the slope L and curvature of the nuclear symmetry energy are observed.

The dependence of three L-K_{sym} correlations predicted in the literature on the crust-core density and pressure is quantitatively investigated. The most probable value of 0.08 fm^{-3} for ρ_t is obtained from the L-K_{sym} relation proposed by Holt et al., while the other two relations prefer larger values.

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The values predicted by the dynamical method are usually smaller than those from the thermodynamical method by about 0.01 fm^{-3} [?].

Introduction

Determining the crust-core transition density ρ_t in neutron stars (NSs) is important not only for predicting bulk NS properties [?] but also for understanding finite nuclei properties [?, ?, ?]. However, due to the intricate structure of the inner crust in NSs, constraining the transition density remains challenging. Various theoretical models have been used to estimate the transition density in recent years, including the dynamical method [?, ?, ?, ?, ?], the Thomas-Fermi approximation method [?], the random phase approximation [?], the thermodynamical method [?, ?, ?], the Vlasov method [?], the compressible liquid-drop model [?], and the meta-modeling approach [?]. These methods yield different predictions: for example, $\rho_t = 0.071 \pm 0.011$ fm^{-3} estimated in the meta-modeling approach [?], $\rho_t = 0.0955 \pm 0.0007$ fm^{-3} obtained by comparing with excitation energies of giant resonances, energy-weighted pygmy dipole strength, and dipole polarizability data using relativistic nuclear energy density functionals [?], $\rho_t = 0.04-0.065$ fm^{-3} limited by using the EOS including momentum-dependent interaction of neutron-rich nuclear matter constrained by isospin diffusion data in heavy-ion reactions [?], $\rho_t = 0.069 - 0.098$ fm^{-3} from the thermodynamical method [?], and $\rho_t = 0.058 - 0.092$ fm^{-3} with the Thomas-Fermi method [?], among others.

The nuclear symmetry energy plays a dominant role in accurately describing the crust-core interface of NSs. It is known that the crust-core transition density is highly sensitive to the isospin dependence of the nuclear equation of state (EOS)

[?]. In particular, the slope L and curvature K_{sym} of the nuclear symmetry energy, as well as the L-K_{sym} correlation, have been reported to be strongly correlated with the crust-core transition density [?, ?, ?]. In a recent work [?], a Bayesian approach was used to infer the distribution of ρ_t based on low-density constraints for neutron and symmetric nuclear matter from effective field theory. However, the obtained ρ_t distribution and the correlations between ρ_t and EOS parameters depend heavily on the surface energy parameter.

In the present work, we perform a Bayesian inference of the crust-core transition density in NSs based on NS radius and neutron-skin thickness data. We discuss the dependence of the L-K_{sym} correlations predicted in the literature on the posterior distributions of ρ_t.

The rest of the paper is organized as follows. In the next section, we outline the theoretical framework including the thermodynamical method, the EOS meta-modeling method, the nuclear droplet model, and the Bayesian inference approach. In Section III, we probe the crust-core transition density and its correlations with EOS parameters via NS radius and neutron-skin thickness data in the Bayesian framework. We also explore the effect of the L-K_{sym} correlation on the crust-core transition density and pressure. A summary is given at the end.

II. Theoretical Framework

A. Crust-Core Transition Density and Isospin-Dependent Parametric EOS for the Core

The crust-core transition density in this work is estimated by adopting the thermodynamical approach under the condition that the energy per nucleon $E(\rho, \delta)$ in nuclear matter at nucleon density ρ and isospin asymmetry $\delta \equiv (\rho_n - \rho_p)/\rho$ is approximated by the isospin-parabolic expansion:

$$E(\rho, \delta) \approx E_0(\rho) + E_{\text{sym}}(\rho) \cdot \delta^2. \tag{1}$$

The crust-core transition point is obtained when uniform matter begins to separate into a mixture containing single nucleons and clusters. The transition density is specifically calculated by the vanishing effective incompressibility of uniform NS matter under β-equilibrium and charge neutrality conditions:

$$K_\mu = \rho^2 \frac{d_0^{2E}}{d\rho^2} + 2\rho \frac{dE_0}{d\rho} + \rho^2 \frac{d_{\text{sym}}^{2E}}{d\rho^2} + 2\rho \frac{dE_{\text{sym}}}{d\rho} - 2E_{\text{sym}}^{-1} \left(\rho \frac{dE_{\text{sym}}}{d\rho} \right)^2 = 0. \tag{2}$$

Thus, one can obtain the transition density ρ_t by solving the equation $K_\mu = 0$. It is noteworthy that the crust-core transition density can be overestimated when the parabolic approximation (Eq. (1)) is used [?]. As indicated in Fig. 5 of Ref.

[?], this overestimation mainly appears in the region where the parameters L and $K_{\rm sym}$ are large, i.e., $L > 60$ MeV and $K_{\rm sym} > -100$ MeV. However, after filtering by NS radius data, the probability that they fall in the abovementioned intervals is very small according to our earlier calculations [?]. Therefore, employing the parabolic approximation will hardly change the present results. Furthermore, it can also reduce the discrepancy between the thermodynamical method used in this work and other approaches like the dynamical method when these EOS parameters are confronted with NS radius data.

The transition pressure can be approximately written as [?]:

$$P_t \approx K_0 \frac{\delta_t^2}{9} + \rho_t \delta_t^2 \left[-E_{\rm sym}(\rho_t) + \left. \frac{dE_{\rm sym}(\rho)}{d\rho} \right|_{\rho_t} \right] \tag{3}$$

where δ_t is the isospin asymmetry corresponding to ρ_t. E_0 and $E_{\rm sym}$ in Eqs. (1) and (2) are the energy per particle in symmetric nuclear matter (SNM) and the nuclear symmetry energy, respectively. They can be parameterized as:

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3, \tag{4}$$

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\rm sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{\rm sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3, \tag{5}$$

where $E_0(\rho_0) = -15.9$ MeV is the energy per particle for SNM at saturation density ρ_0. $K_0 = 9\rho_0^2[\partial_0^{2E}(\rho)/\partial\rho^2]_{\rho=\rho_0}$ and $J_0 = 27\rho_0^3[\partial_0^{3E}(\rho)/\partial\rho^3]_{\rho=\rho_0}$ represent the incompressibility and skewness parameters of SNM at ρ_0, respectively. $E_{\rm sym}(\rho_0)$, $L = 3\rho_0[\partial E_{\rm sym}(\rho)/\partial\rho]_{\rho=\rho_0}$, $K_{\rm sym} = 9\rho_0^2[\partial_{\rm sym}^{2E}(\rho)/\partial\rho^2]_{\rho=\rho_0}$, and $J_{\rm sym} = 27\rho_0^3[\partial_{\rm sym}^{3E}(\rho)/\partial\rho^3]_{\rho=\rho_0}$ are, respectively, the magnitude, slope, curvature, and skewness of nuclear symmetry energy at ρ_0.

According to the systematics of terrestrial nuclear experiments and predictions of various nuclear theories, K_0, $E_{\rm sym}(\rho_0)$, and L have the ranges of $220-260$ MeV, $28.5-34.9$ MeV, and $30-90$ MeV [?, ?, ?, ?, ?], respectively. Meanwhile, the parameters J_0, $K_{\rm sym}$, and $J_{\rm sym}$ characterizing the nuclear EOS at high densities have wide ranges of $-800 \le J_0 \le 400$ MeV, $-400 \le K_{\rm sym} \le 100$ MeV, and $-200 \le J_{\rm sym} \le 800$ MeV, based only on theoretical predictions [?, ?].

In the framework of the minimum NS model, the non-rotating NS consists of neutrons, protons, electrons, and muons under β-equilibrium and charge neutrality conditions. The relation between pressure and nucleon density in the core of NSs, $P(\rho,\delta) = \rho^2 d\epsilon(\rho,\delta)/d\rho$, is controlled by the energy density $\epsilon(\rho,\delta) = \rho[E(\rho,\delta) + M_N] + \epsilon_l(\rho,\delta)$, with M_N and $\epsilon_l(\rho,\delta)$ being the average nucleon mass and the energy densities for leptons (calculated by the noninteracting

Fermi gas model) [?], respectively. The isospin asymmetry δ can be obtained through the charge neutrality condition $\rho_p = \rho_e + \rho_\mu$ and the β-equilibrium condition $\mu_n - \mu_p = \mu_e = \mu_\mu \approx 4\delta E_{\text{sym}}(\rho)$, where μ denotes the chemical potential calculated by the expression $\mu_i = \partial\epsilon(\rho, \delta)/\partial\rho_i$ for the ith particle.

One can construct the EOS for the core of NSs using expressions (4), (5), and (6). Below the crust-core transition density, the NV EOS [?] for the inner crust and the BPS EOS [?] for the outer crust are employed.

B. Neutron Skin and Nuclear Droplet Model

The neutron skin thickness of a finite nucleus in the nuclear droplet model (DM) is obtained according to the expression [?, ?]:

$$\Delta R_{np} = \frac{3e^{2Z}}{70E_{\text{sym}}(\rho_0)} + \frac{t}{b_n + b_p}R, \tag{7}$$

where $e^{2Z}/70E_{\text{sym}}(\rho_0)$ is a correction term due to the Coulomb interaction, $R = r_{0A}^{1/3}$ is the nuclear radius, and b_n and b_p denote the surface widths of the neutron and proton density profiles, respectively (typically $b_n = b_p = 1$ fm is used in the standard DM). The quantity t in Eq. (7) is calculated as [?]:

$$t = \frac{3E_{\text{sym}}(\rho_0)}{L(1-x)} x A^{1/3}(\delta - \delta_C), \tag{8}$$

where $x = \rho_A/\rho_0$, and

$$\delta_C = \frac{20E_{\text{sym}}(\rho_0)R}{L(1-x)A^{1/3}}. \tag{9}$$

Here $\rho_A = 0.1\ \text{fm}^{-3}$ for the nucleus ^{208}Pb and $\rho_A = 0.08\ \text{fm}^{-3}$ for the nucleus ^{48}Ca.

C. Bayesian Inference Approach

The Bayesian theorem is expressed as:

$$P(M|D) = C \cdot P(D|M)P(M). \tag{10}$$

Here C is a normalization constant, $P(M|D)$ represents the obtained posterior probability distribution function (PDF) of the model M when the data set D is given, $P(D|M)$ is the likelihood function obtained by confronting the theoretical results from model M with data D, and $P(M)$ is the prior probability of model M representing knowledge about the theoretical parameters before comparison with data.

In this work, we randomly sample the transition density in the range $0.03\ \mathrm{fm}^{-3} \leq \rho_t \leq 0.2\ \mathrm{fm}^{-3}$, and the matched six-parameter set can be found by solving the equation $K_\mu = 0$ using Eq. (2). We adopt two approaches to generate the posterior PDF of the transition density. The first is based only on observed NS radius data, and the second is based on both NS radius data and neutron-skin thickness data.

For the first approach, we use the matched six parameters to construct the NS EOS within the minimum NS model described above and input them into the TOV equation to compute theoretical NS radii. We then obtain the likelihood of the transition density or the matched parameter set by confronting these theoretical NS radii with observed values using the likelihood function:

$$P(D|M) = \exp\left[-\frac{(R_{\mathrm{th}} - R_{\mathrm{obs}})^2}{2\sigma^2}\right], \tag{11}$$

where R_{th} represents theoretical values, and R_{obs} and σ represent observed values and their 1σ error bars for NS radii. The generated NS EOS must satisfy thermodynamical stability ($dP/d\epsilon \geq 0$, where P denotes pressure inside NSs) and causality ($0 \leq v_s^2 \leq c^2$, where v_s and c denote the speed of sound and light, respectively) at all densities, and must be stiff enough to support the maximum observed NS mass. A sharp cutoff at $1.97M_\odot$ is used in this analysis.

For the second approach, after obtaining the matched six-parameter set by sampling the transition density and solving Eq. (2), we use them as input into the DM to calculate theoretical neutron-skin thickness values for ^{208}Pb and ^{48}Ca. We then discard parameter sets and transition densities when the calculated neutron-skin thickness values are far from experimental values using the likelihood function:

$$P(D|M) = \frac{1}{\sqrt{2\pi}\sigma'} \exp\left[-\frac{(\Delta R_{np}^{\mathrm{th}} - \Delta R_{np}^{\mathrm{exp}})^2}{2\sigma a'^2}\right], \tag{12}$$

where $\Delta R_{np}^{\mathrm{th}}$ and $\Delta R_{np}^{\mathrm{exp}}$ are respectively the theoretical and experimental values, and σ' represents the 1σ error bar for the experimental data. The remaining parameter sets are then used to construct the NS EOS and calculate posterior PDFs of the transition density as described above.

The observed NS radius data and experimental neutron-skin thickness data used in this work are summarized in Table I. These NS data include: (i) $R_{1.4} = 11.9^{+1.4}_{-1.4}$ km at 90% confidence level (CFL) from analyzing the GW170817 source reported by the LIGO/Virgo Collaboration [?]; (ii) $R_{1.4} = 10.8^{+2.1}_{-1.6}$ km at 90% CFL from analyzing the same GW170817 source [?]; (iii) $R_{1.4} = 11.7^{+1.1}_{-1.1}$ km at 90% CFL reported earlier from analyzing quiescent low-mass X-ray binaries (QLMXBs) [?]; (iv) Values reported by the NICER Collaboration [?, ?] at 68% CFL: $R = 12.71^{+1.14}_{-1.19}$ km for PSR J0030+0451 with mass $1.34^{+0.15}_{-0.16}M_\odot$ [?], $R =$

$13.7^{+2.6}_{-1.5}$ km for PSR J0740+6620 with mass $2.08^{+0.07}_{-0.07} M_\odot$ [?]. The neutron-skin thickness data are $\Delta R_{np} = 0.121^{+0.026}_{-0.026}$ fm for ^{48}Ca and $\Delta R_{np} = 0.283^{+0.071}_{-0.071}$ fm for ^{208}Pb, taken from recent reports by the CREX and PREX-2 Collaborations [?, ?].

The Metropolis-Hastings algorithm within a Markov-Chain Monte Carlo (MCMC) approach is adopted to generate posterior PDFs of the model parameters. Both PDFs of individual parameters and PDFs for two-parameter correlations can be calculated by integrating over all other parameters using the marginal estimation approach. Initial samples in the so-called burn-in period are discarded [?] so that the MCMC process can start from an equilibrium distribution. We use 40,000 steps for the burn-in progress and the remaining one million steps for calculating the PDF of the transition density.

TABLE I: Data for NS radius and neutron-skin thickness used in this work.

Mass ($M_\odot$)	Radius R (km)	Source and Reference
$1.34^{+0.15}_{-0.16}$	$11.9^{+1.4}_{-1.4}$ (90% CFL)	GW170817 [?]
$1.44^{+0.15}_{-0.14}$	$10.8^{+2.1}_{-1.6}$ (90% CFL)	GW170817 [?]
$2.08^{+0.07}_{-0.07}$	$11.7^{+1.1}_{-1.1}$ (90% CFL)	QLMXBs [?]
	$12.71^{+1.14}_{-1.19}$ (68% CFL)	PSR J0030+0451 [?]
	$13.0^{+1.2}_{-1.0}$ (68% CFL)	PSR J0030+0451 [?]
	$13.7^{+2.6}_{-1.5}$ (68% CFL)	PSR J0740+6620 [?]

Nucleus	ΔR_{np} (fm)	Source and Reference
^{48}Ca	$0.121^{+0.026}_{-0.026}$ (68% CFL)	CREX [?]
^{208}Pb	$0.283^{+0.071}_{-0.071}$ (68% CFL)	PREX-2 [?]

III. Results and Discussions

A. Exploring the Crust-Core Transition Density via NS Observations

The posterior PDFs of the crust-core transition density ρ_t and corresponding transition pressure P_t, as well as their correlations with EOS coefficients, are plotted in Fig. 1 [Figure 1: see original paper]. In the calculations, two types of priors for ρ_t are adopted. The first is a uniform distribution, which is a better choice when we have no prior knowledge about ρ_t, and this case is displayed by black curves in Fig. 1. The second is a Gaussian distribution with a mean value of 0.078 fm^{-3} and standard deviation of 0.04, as used in Ref. [?], indicated by purple curves in Fig. 1. The panels in the upper two rows show posterior PDFs of the correlations using uniform priors for ρ_t, while those in the bottom two rows show results using Gaussian priors. These results are based only on observed NS radius data summarized in Table I.

A two-humped posterior distribution for ρ_t is observed for both uniform and Gaussian priors. The first peak is located at $\rho_t = 0.08$ fm^{-3}, often used as its fiducial value in the literature, and the second peak is at $\rho_t = 0.1$ fm^{-3}. The calculated 68% and 90% credible intervals using the highest posterior density interval approach for ρ_t and P_t are listed in Table II. We see that, relative to the prior distributions, the posterior PDFs of ρ_t are narrowed to small intervals, reflecting the fact that the crust-core transition density is sensitive to NS radius. There is a larger probability for ρ_t to fall in the region where values exceed 0.1 fm^{-3} when an uninformative prior is used. This can be attributed to correlations between ρ_t and some EOS parameters, which will be discussed later.

Better constraints on ρ_t are found when using the Gaussian prior compared to the uniform prior. This is understandable because more information is available for the Gaussian prior before confronting the data with NS radius data. The generated ranges of P_t—$0.05^{+1.25}_{-0.04}$ MeV/fm^3 using the uniform prior and $0.1^{+0.28}_{-0.1}$ MeV/fm^3 using the Gaussian prior at 68% confidence level, as listed in Table II—can cover values calculated using meta-modeling, dynamical, and thermodynamical models (see Table I in Ref. [**?**]), though the most probable values are smaller. Our results for ρ_t are consistent with those in the literature.

Turning to explore correlations among ρ_t, P_t, and EOS parameters, we note that we do not focus on correlations among EOS parameters themselves, which are consistent with those reported in our earlier publications [**?**, **?**]. Low-order parameters such as $E_{\mathrm{sym}}(\rho_0)$ and K_0 are hardly correlated with the transition. However, as mentioned in Ref. [**?**], the condition $K_\mu = 0$ requires larger ρ_t as K_0 increases. This phenomenon is displayed in Fig. 1, namely a weak positive correlation between ρ_t and K_0 for cases using both uniform and Gaussian priors.

Consistent with results reported in Ref. [**?**], strong correlations among ρ_t and the isovector compressibility K_{sym} and skewness J_{sym} are discovered. A negative correlation between ρ_t and K_{sym} is inconsistent with results in Ref. [**?**], where EOS parameters were filtered by predictions from effective field theory and surface coefficients were determined by nuclear masses in the extended Thomas-Fermi approximation method. The positive correlations between ρ_t and J_{sym}, and between ρ_t and P_t shown in Fig. 1, agree with those in Refs. [**?**, **?**]. The transition is not affected by the skewness of symmetric nuclear matter. L shows a negative correlation with ρ_t. Interestingly, a positive correlation appears in the region at $\rho_t > 0.1$ fm^{-3} for the case using the uniform prior. This does not happen for the case using the Gaussian prior. Does this relate to the shoulder indicated in the posterior PDF of ρ_t using the uniform prior in Fig. 1? To answer this question, we plot the L-K_{sym} correlations from three types of calculations as indicated in Fig. 2 [Figure 2: see original paper]: using uniform and Gaussian priors based on NS radius data, and using uniform priors based on both NS radius and neutron-skin thickness data.

Two phenomena are visible in Fig. 2. One is the anti-correlations shown in left and right panels and the very weak correlation shown in the middle panel between L and K_{sym}. The other, as shown in the left panel of Fig. 2, is that

K_{sym} has a high probability to stay in the region where it is extremely negative, i.e., $K_{\text{sym}} < -200$ MeV. The latter is obviously responsible for the shoulder, because the shoulder in the posterior PDF of ρ_t disappears as shown in Fig. 3 [Figure 3: see original paper] even though the L-K_{sym} anti-correlations are the same in their calculations in the left and right panels. Therefore, K_{sym} plays a more important role in constraining the crust-core transition density of NSs than the L-K_{sym} correlation, as studied in Ref. [?]. The transition pressure is weakly correlated with EOS parameters.

B. Effect of Neutron-Skin Thickness and Comparison with Other Calculations

It is known that neutron-skin thickness is an effective probe of nuclear symmetry energy, particularly its slope parameter [?, ?], which plays an important role in determining the crust-core transition density of NSs. Figure 3 shows results identical to those in Fig. 1 but based on both NS radius and neutron-skin thickness data. In the calculations, we first perform a Bayesian inference of the coefficients $E_{\text{sym}}(\rho_0)$, L, and K_{sym} in the framework of the nuclear droplet model as described in Section II.B, based on neutron-skin thickness data reported by the CREX [?] and PREX-2 [?] Collaborations as listed in Table I. After that, we regard the obtained distributions of these parameters and the matched transition density as their priors to infer the posterior PDF of the transition density based on NS radius data within the minimum NS model. It is worth noting that significant controversy exists about constraining the slope of symmetry energy through neutron-skin thickness data of ^{48}Ca and ^{208}Pb reported by the CREX and PREX-2 Collaborations. A recent study demonstrates that the ranges for slope L obtained from CREX and PREX-2 are completely inconsistent [?]. More accurate measurements are needed. Fortunately, many methods exist to determine neutron-skin thickness, such as configurational information entropy analysis [?, ?].

As seen in Fig. 3, the correlations are roughly the same as those in Fig. 1. A weak anti-correlation between the symmetry energy magnitude $E_{\text{sym}}(\rho_0)$ and ρ_t is observed due to constraints from neutron-skin thickness data on $E_{\text{sym}}(\rho_0)$. The range for K_{sym} is smaller than when using only NS radius data, also because of the effect of neutron-skin thickness data. The constraint on ρ_t is improved compared to that based only on NS data, as listed in Table II. This is because parameters L and K_{sym} are better constrained when neutron-skin thickness data are taken into consideration.

In Fig. 4 [Figure 4: see original paper], we compare our results with those inferred using a compressible liquid-drop model within a Bayesian framework [?], which employed two filters: (i) the low-density (LD) behavior of energy functionals should be rigorously limited to uncertainty intervals from effective field theory calculations for symmetric and pure neutron matter [?], and (ii) the high-density (HD) behavior should obey conditions such as positive symmetry energy at all densities, causality, etc. [?]. Our results overlap with theirs. There is a long tail for P_t in the case using the uniform prior, because there is a large probability for ρ_t to exist in the region $\rho_t > 0.1$ fm^{-3}. Our most probable values for P_t are smaller than those from Ref. [**?**].

C. Effect of L-K_{sym} Correlations

It has been reported that L-K_{sym} correlations have a significant impact on both crust-core transition density and pressure [**?**]. To further explore this effect, we adopt three typical L-K_{sym} correlations predicted in the literature as priors to infer the posterior PDF of ρ_t.

For completeness and ease of discussion, we briefly describe the three correlations. The first is based on theoretical predictions from 240 Skyrme Hartree-Fock and 263 relativistic mean-field calculations [**?**] by Mondal et al.:

$$K_{\mathrm{sym}} = (-4.97 \pm 0.07)(3E_{\mathrm{sym}}(\rho_0) - L) + 66.80 \pm 2.14 \text{ MeV}. \tag{13}$$

The second is from Ref. [**?**] by Tews et al.:

$$K_{\mathrm{sym}} = 3.50L - 305.67 \pm 24.26 \text{ MeV}. \tag{14}$$

In the framework of Fermi liquid theory, Holt and Lim [**?**] derived the expressions $L = 6.70E_{\mathrm{sym}}(\rho_0) - 148.60 \pm 4.37$ MeV and $K_{\mathrm{sym}} = 18.50E_{\mathrm{sym}}(\rho_0) - 613.18 \pm 9.62$ MeV. Thus, one can formulate the last relation between L and K_{sym} as [**?**]:

$$K_{\mathrm{sym}} = 2.76L - 203.07 \pm 21.69 \text{ MeV}. \tag{15}$$

These correlations are shown in Fig. 5 [Figure 5: see original paper]. The discrepancy between correlations by Mondal et al. and Tews et al. is not too large because they are from the same sets of theoretical predictions. However, those depicted in Figs. 2 and 5 are highly different, illustrating that L-K_{sym} correlations are strongly model-dependent and that constraining them remains a significant challenge.

In the Bayesian inference approach, after considering the above correlations, L and K_{sym} are no longer independent when randomly sampled between their specific ranges. In calculations performed in this subsection, the uniform prior for ρ_t and only NS radius data are employed. The generated posterior PDFs for ρ_t and P_t are shown in Fig. 6 [Figure 6: see original paper], and corresponding confidence intervals are summarized in Table II. As stated in Ref. [**?**], the L-K_{sym} correlations play a significant role in constraining both transition density and pressure. At least two observations can be made: (i) Results from relations by Mondal et al. and Tews et al. are completely consistent, mainly because these two correlations largely overlap within allowed error limits; (ii) Most probable values from the relation by Holt et al. are highly different from the other two cases, with larger transition density and pressure values. This is because K_{sym} is not extremely negative in the relation by Holt et al., i.e., K_{sym} has values higher than about -115 MeV as shown by green curves in Fig. 5. These results are consistent with those in Ref. [?], where authors studied effects of L-K_{sym} correlations on crust-core transition density and pressure by adopting fixed values for other parameters in Eqs. (4) and (5).

IV. Summary

In summary, using the thermodynamical approach to calculate the crust-core transition density and an explicitly isospin-dependent parametric EOS for the core of NSs within the minimum NS model, we perform a Bayesian inference of the crust-core transition density based on NS radius and neutron-skin thickness data. Uniform and Gaussian forms for prior distributions of transition density are employed. We find that the transition density has a larger probability to take values larger than 0.1 fm^{-3} when the uniform prior is used, which does not occur with the Gaussian prior. This phenomenon is responsible for K_{sym} taking values smaller than -200 MeV.

Negative (positive) correlations are observed between ρ_t and L and between ρ_t and K_{sym} (between ρ_t and K_0, between ρ_t and J_{sym}, and between ρ_t and P_t). Based on NS radius data reported thus far, the generated 68% confidence intervals for ρ_t are $0.08^{+0.06}_{-0.005}$ fm^{-3} and $0.08^{+0.01}_{-0.03}$ fm^{-3} when uniform and Gaussian priors are adopted, respectively. When EOS parameters are first filtered by neutron-skin thickness data, the obtained value is $0.075^{+0.005}_{-0.01}$ fm^{-3}. We also examine the impact of L-K_{sym} correlations on posterior PDFs of ρ_t and P_t. Results from L-K_{sym} relations reported by Tews et al. [?] and Mondal et al. [?] completely overlap, while results from Holt et al. [?] are smaller than those from the other two relations.

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Note: Figure translations are in progress. See original paper for figures.