Correction to the quantum relation of photons involved in the Doppler effect in the framework of a special Lorentz violation model

Authors: Jinwen Hu, Huan Hu, Jinwen Hu

Date: 2024-09-15T00:00:00+00:00

Abstract

In this paper we follow the idea in Ref. 1 to discuss the Doppler frequency shift of photons and the Compton scattering between photons and electrons, pointing out that following this idea we must modify the usual quantum relation for massless particles. However, due to limited information and knowledge, we have not yet been able to determine the specific expression for the correction coefficient of the quantum relation of massless particles. Nevertheless, the phenomenon of spontaneous radiation in a cyclotron maser provides an opportunity to investigate the possible form of this correction coefficient, as the phenomenon of spontaneous radiation in a cyclotron maser can be explained by the effect of Doppler frequency shift of virtual photons and Compton scattering between virtual photons and electrons. Therefore, under certain restrictive conditions, we construct a very concise expression for this correction coefficient by discussing different cases. We then use this expression to analyze the wavelength of radiation in the cyclotron maser, which tends to a limited value as v→c, rather than to 0 as in the Lorentz model. This paper retains the principle from Ref. 1 that the energy and momentum of particles cannot be infinite, otherwise it renders some equations meaningless, and this view also derives from ideas in certain quantum gravity models. This paper also provides a possible experimental scheme to determine the value of Q in Ref. 1, but it still requires extremely high experimental energy.

Full Text

Correction to the Quantum Relation of Photons in the Doppler Effect Based on a Special Lorentz Violation Model

Jinwen Hu, Huan Hu
Department of Physics and Technology, Wuhan University, Wuhan, 430060,

China

Abstract

The possibility of Lorentz symmetry breaking has been discussed in many models of quantum gravity. In this paper, we follow the Lorentz violation model in Ref. [14] (i.e., our previous work) to discuss the Doppler frequency shift of photons and the Compton scattering process between photons and electrons, pointing out that this approach requires modification of the usual quantum relation for photons in the Doppler effect. However, due to currently limited information and knowledge, we cannot yet determine the specific expression for the correction coefficient in the modified quantum relation of photons. Fortunately, the phenomenon of spontaneous radiation in a cyclotron maser provides an opportunity to infer what this expression might look like. Therefore, under necessary constraints, we construct a concise expression for this correction coefficient through discussion of different cases. Using this expression, we analyze the wavelength of radiation in the cyclotron maser, which tends to a finite value as $v \to c$, rather than to 0 as predicted by the Lorentz model. We also discuss inverse Compton scattering and find that there exists a limit to the maximum energy that photons can obtain in collisions between extremely relativistic particles and low-energy photons. This conclusion differs significantly from that obtained from the Lorentz model, where the energy obtainable by the photon tends to infinity as the particle velocity approaches c. This paper continues to follow the purpose stated in Ref. [14] that the energy and momentum of particles (i.e., any particles, including photons) cannot be infinite, as this would invalidate certain physical scenarios. When the parameter Q characterizing the degree of deviation from the Lorentz model equals 0, all results and conclusions in this paper return to the Lorentz model case. Thus, this paper also provides a possible experimental scheme [14], although it still requires extremely high experimental energies to determine the value of Q.

Keywords: Lorentz model, The speed of light, Lorentz violation model, Doppler effect, Compton scattering, Inverse Compton scattering, Quantum relation

1. Introduction

The study of quantum gravity has long been a frontier and hot topic in fundamental physics, primarily because singularities and information loss problems arise in black hole studies, and because general relativity and quantum theory are incompatible under the current framework [1–4]. The starting point for many quantum gravity models is the belief that Lorentz symmetry may be violated at ultra-high energy scales, such as near the Planck energy scale. The most typical model holding this view is doubly special relativity (DSR), which considers not only the speed of light as constant but also introduces another constant called "minimum length or maximum energy," leading to modification of the energy-momentum dispersion relation of particles at extremely high energy scales [5, 6]. Research on the DSR model appears extensively in the literature, and it is believed to be able to solve some challenging problems in the ultra-high energy field [7–12].

However, the DSR model has an obvious disadvantage: it is not concise enough. The equation describing the dispersion relation of particles usually contains many parameters. For example, in the DSR model, the dispersion relation of particles at Planck energy scale is typically expressed as $E^2 = p^2 + m^2 + \eta L_p^\alpha E^{\alpha+3} + O(L_p^{\alpha+1} E^{\alpha+4})$, where L_p denotes the "Planck length," α is a positive integer, and η is a real number [7]. Since the physical significance of these parameters is not clear [13], many researchers only take one or two of them for study. For this reason, in Ref. [14] (i.e., our previous work), we proposed another possible Lorentz violation model containing only one parameter in the particle dispersion relation equation. More importantly, the Lorentz violation model proposed in Ref. [14] naturally returns to the Lorentz model at low and medium energy scales, and when the particle velocity approaches c, the particle energy tends to a finite value (similar to the DSR model) rather than becoming infinite as predicted by the Lorentz model.

To clarify the purpose and viewpoint of this paper, we briefly review the Lorentz violation model proposed in Ref. [14]. As is known, in special and general relativity the speed of light occupies a central status. Therefore, for most Lorentz violation models, the (local) speed of light is assumed to be variable. In this context, to allow the possibility that the (local) speed of light can change between inertial systems, Ref. [14] begins with a discussion of a general relationship between the speed of light and the velocity of the inertial system, assuming that the speed of light observed by an observer moving with velocity v relative to the light source is nc, where n is a variable or invariable dimensionless value. Obviously, since many experiments at low or medium energy scales have verified the theory of special relativity and general relativity, there must be constraints on n, namely $n(v, c) = n(-v, c) = n(v, -c) = n(-v, -c)$ and $n(v = 0, c) = 1$ (the detailed reasons for these constraints can be found in Ref. [14]).

Based on this assumption, the coordinate transformation between two inertial systems moving relative to each other with velocity v is obtained as:

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{v}{k^2}x\right) \end{aligned} \tag{1}$$

where $\gamma(v, c) = 1/\sqrt{1 - v^2/k^2}$, $k(v, c) = \sqrt{nvc^2/(nc - c + v)}$, and n satisfies $n(v, c) = n(-v, c) = n(v, -c) = n(-v, -c)$ and $n(v = 0, c) = 1$.

From Eq. (1), one can obtain that if $dx/dt = c$, then $dx'/dt' = nc$, and if $dx'/dt' = c$, then $dx/dt = nc$, which ensures that the two inertial systems are equivalent. Eq. (1) is similar in form to the Lorentz transformation (i.e., replacing c with k in the Lorentz transformation yields Eq. (1)), and it has been shown in Ref. [14] that Maxwell's equations are also covariant based on Eq. (1). Correspondingly, the dispersion relation for a particle with rest mass m_0, compared to the form in the Lorentz model, is modified as:

$$E^2 = p^{2k^2} + E_0^2 \tag{2}$$

where $E_0 = m_{0k}^2$, $E = \gamma m_{0k}^2$ denotes the particle's total energy, $p = \gamma m_{0v}$ denotes the particle's momentum, and $\gamma = \gamma(v, c)$, $k = k(v, c)$.

One might easily recognize that if the dispersion relation of particles is modified as Eq. (2), then the usual quantum relation of photons (see Eq. (3)) should also need to be modified accordingly. The reason is that for massless particles, the dispersion relation is $E = pk$ rather than $E = pc$ based on Eq. (2). Secondly, Ref. [14] allows the speed of light to be variable between inertial systems (especially allowing the observed speed of light to tend to 0 when the velocity of the light source relative to the observer approaches c), so the usual quantum relation of photons (i.e., Eq. (3)) will no longer be applicable in this case:

$$\begin{aligned} E &= hf \\ p &= \frac{h}{\lambda} \end{aligned} \tag{3}$$

where h is the Planck constant, and f and λ denote the frequency and wavelength of light, respectively.

Therefore, this is the issue to be discussed in this paper. However, one may wonder why quantum theory needs to be modified since it has been so successful so far. The answer is that our current quantum theory is built on the basis of Lorentz symmetry, and if the energy is high enough that Lorentz symmetry is obviously broken, quantum theory should be modified. One piece of evidence is the proposal of various models of quantum gravity, which aim to explore new physics that may emerge when energy is pushed to very high levels.

However, when we try to construct or find a quantum relation for photons that satisfies Eq. (2), we find it very difficult to determine the specific expression, as the set of expressions satisfying $E = pk$ is large. Fortunately, a physical phenomenon called synchrotron radiation in a cyclotron maser can give us a glimpse of what this expression might look like. In Ref. [15, 16], the authors used the concept of virtual photons to discuss the phenomena called spontaneous radiation and synchrotron radiation in a cyclotron maser. By considering the Doppler effect and Compton scattering effect of virtual photons, the authors obtained formulas for spontaneous radiation and synchrotron radiation that are the same as classical theoretical results.

Inspired by the idea in Ref. [15, 16], this paper first intends to discuss the constraints on the modified quantum relation of photons, and then try to find a possible expression for the modified quantum relation satisfying Eq. (2). Further, we intend to discuss the effect of this modified quantum relation on synchrotron radiation in the cyclotron maser. This paper is organized as follows. In Section

"Doppler effect of photons," the Doppler effect of photons based on Ref. [14] is discussed, and in Section "Compton scattering" the corresponding Compton scattering effect of photons is also discussed. To apply both the Doppler effect and Compton scattering effect of photons to a single physical phenomenon simultaneously, we choose to analyze the phenomena called spontaneous and synchrotron radiation in a cyclotron maser, which is shown in Section "Spontaneous radiation in cyclotron maser," and simultaneously the possible expression for the modified quantum relation of photons is discussed. In Section "Inverse Compton scattering," the inverse Compton scattering phenomenon is discussed and it shows significant differences in predicted results between the model in Ref. [14] and the Lorentz model. Finally, we summarize the paper.

2. Doppler Effect of Photons

As we know, the frequency of light emitted by a light source differs when measured in an inertial system moving relative to the light source, which is called the Doppler frequency shift effect. As shown in Figure 1 [Figure 1: see original paper], we assume that the inertial system S' is moving along the x (x') axis with velocity v relative to the inertial system S, and the natural frequency and natural wavelength of light emitted by the light source fixed in S' are f' and λ', respectively. Simultaneously, we assume the angle between the direction of light propagation and the x'-axis is θ. Then based on the conclusion in Ref. [14], for the observer in S the clock period of the light source will slow down as:

$$T = \gamma T' = \frac{T'}{\sqrt{1-\beta^2}} \tag{4}$$

where $\beta = v/k$, T' and T represent the clock period measured in S' and S, respectively, and $T' = 1/f'$, $\lambda' f' = c$.

On the other hand, since the light source is moving relative to the observer, the wavelength of light measured in S is:

$$\lambda = ncT - (v\cos\theta)T \tag{5}$$

where $n = n(v\cos\theta, c)$.

From Eqs. (4) and (5), we can obtain:

$$f = \frac{nc}{\lambda} = \frac{n}{\sqrt{1-\beta^2}}\left(1 - \frac{v\cos\theta}{nc}\right) f' \tag{6}$$

where f and λ are the frequency and wavelength of light measured in S, respectively. Eq. (6) shows the corresponding Doppler effect formula based on the conclusion from Ref. [14], and it can be seen that when $n \equiv 1$, Eq. (6) returns to the usual case as in the Lorentz model.

Here one may wonder why the mathematical forms of Eqs. (1) and (2) are similar to those in the Lorentz model (i.e., replacing c with k in the Lorentz model yields Eqs. (1) and (2)), but Eq. (6) has no similarity with the relativistic Doppler frequency shift formula in the Lorentz model. The main reason is that in Eq. (1), the speed, wavelength, and frequency of light are all variable, while in the Lorentz model, only the wavelength and frequency of light are variable. Therefore, if the mathematical formulas in Eq. (6) were still similar in form to those in the Lorentz model, then k would eventually be derived as a constant (note that in Eq. (1) k is a function of v), which is obviously contradictory to Eq. (1).

Based on Eq. (6), we next discuss a special case where $\theta = 0$ (if $\theta = \pi/2$, then $n(0, c) = 1$, and Eq. (6) returns to the case as in the Lorentz model), in which case one can obtain:

$$f = \frac{nc}{\lambda} = \frac{n}{\sqrt{1-\beta^2}} \left(1 - \frac{v}{nc}\right) f' \tag{7}$$

Here it should be noted that $n = n(v, c)$ in Eq. (7) due to $v \cos\theta = v$.

Based on Eq. (7), one may notice that when $v = nc$, $\lambda = 0$ and f tends to be infinite, which is similar to the property of shock waves in the acoustic Doppler effect. Similarly, from Ref. [14] we can obtain the condition for light to generate a shock wave. According to Ref. [14], when v approaches c, there is:

$$n = \frac{1-Q}{1-Q^{1-(v/c)^2}} = \frac{1-Q}{1-Q^2(1-v/c)} = \frac{1-Q}{1-Q(1+v/c)(1-v/c)} = \frac{Q-1}{Q^{(1+v/c)(1-v/c)}-1} \tag{8}$$

where Q is a constant whose value needs to be determined by experiment. As mentioned in Ref. [14], we do not yet know the specific value of Q because current experimental energies are not high enough, but it can be known from a large number of existing experimental results that $Q \approx 0$.

The reason why n is chosen in the form of Eq. (8) is that this function can be well consistent with various experimental results conducted at low or medium energy scales, as explained in Ref. [14]. Also importantly, Eq. (8) is very concise as it has only one parameter Q. Here let us re-draw the curve of $n - v$ as in Figure 2 [Figure 2: see original paper].

Figure 2 shows the curve of $n(v) \sim v$ when taking $Q = (1/e)^{10^6}$ (set $c = 1$). Since this shows the global picture of the curve, a transition arc at the corner is too small to display [14]. From here or Eq. (8) it can be seen that when $v = 1$, $n = 0$. The parameter n can be viewed as the degree to which the speed of light observed by the observer in an inertial system moving relative to the light source deviates from c. As can be seen from Figure 2, n is almost equal to 1 over a large range of v. It is because of this property of n that the model in Ref. [14] does not violate a large number of existing experimental results conducted at low or medium energy scales. When $Q = 0$, $n \equiv 1$, and correspondingly, Eqs. (1) and (2) return to the case as in the Lorentz model.

As is well known, when the velocity of the wave source equals the speed of wave propagation, the wavefront in the direction of wave source motion cannot propagate outward but closely adheres to the wave source. This phenomenon is called a “shock wave,” which usually occurs when the velocity of an object in a medium changes from subsonic to supersonic, and is called an acoustic shock wave. Similarly, we can also obtain the condition for light to generate a shock wave. For $v = nc$, there is:

$$v_{\text{wave shock}} \approx \frac{2\ln Q + 1}{2\ln Q - 1} c < c \tag{9}$$

where $v_{\text{wave shock}}$ is the solution for $v = nc$, and Eq. (9) is just the condition for generating a shock wave of light.

Similar to the phenomenon in supersonic flow, we can also define the Mach cone of light in the same way as the Mach cone of acoustic waves, which is shown in Figure 3 [Figure 3: see original paper], where α is the cone angle. In Figure 3, the spatial distribution of the speed of light along the center of the light source satisfies the following curve equation in polar coordinates: $\rho = n(v\cos\theta, c)c$. When $v > v_{\text{wave shock}}$, a reverse timing sequence will occur, which is similar to the phenomenon in supersonic flow. But as we know, the timing sequence can be reversed in independent events without violating the law of causality (though not in dependent events) [17].

3. Compton Scattering

We know from the Compton scattering process that light can be regarded as a particle (i.e., photon). In special relativity, the energy and momentum of photons can be expressed by Eq. (3). However, if we substitute Eq. (7) into Eq. (3), we find that Eq. (10) does not agree formally with Eq. (2), because based on Eq. (2), for massless particles there is $E = pk$. So we must modify the quantum relation of photons if we follow the idea in Ref. [14]. Here we assume the modified quantum relation as:

$$\begin{aligned} E &= a_{1hf} \\ p &= a_2 \frac{h}{\lambda} \end{aligned} \tag{10}$$

Then based on $E = pk$ we have $a_1/a_2 = k/n$. Unfortunately, there seems to be no information or knowledge available to help us specify the expressions for a_1 and a_2.

Next we attempt to discuss the Compton scattering process between photons and electrons based on Eq. (11), and the corresponding diagram can be seen in Figure 4 [Figure 4: see original paper]. For simplicity, we assume the electrons are stationary relative to the laboratory before the collision occurs.

According to the interaction diagram between photon and electron in Figure 4, we can set out the following energy and momentum conservation equations:

$$\begin{aligned} a_{1hf}0 + m_{0c}^2 &= a'_{1hf} + mk^2 \\ \frac{a_{2h}}{\lambda_0} &= \frac{a'_{2h}}{\lambda}\cos\theta + mv\cos\phi \\ 0 &= \frac{a'_{2h}}{\lambda}\sin\theta + mv\sin\phi \end{aligned} \tag{11}$$

Note that in Eq. (12), a_1, a_2, k, n, a'_1, a'_2, k', n' are all functions of v and c. Eq. (12) can be approximately simplified: when $v \ll c$, $k \approx c$, and when $v \approx c$, there is:

$$k = \lim_{v\to c}\sqrt{\frac{nvc^2}{nc-c+v}} = \lim_{v\to c}\sqrt{\frac{c}{1-\frac{c-v}{nc}}} = \lim_{v\to c}\sqrt{\frac{c}{1-Q^{-1}}} = \sqrt{\frac{c}{1-Q^{-1}}} = \sqrt{\frac{c\ln Q}{Q-1}} \tag{12}$$

Since $Q \approx 0$, then based on Eq. (13), when $v \approx c$, there is $k \approx c$. To summarize, for $v \in [0, c)$, there is $k \approx c$. So we can assume $k \approx k' \approx c$ at all times for Eq. (12) when necessary. Then based on Eq. (12) we can obtain:

$$\lambda = \lambda_0 + \frac{a_2}{a'_2}\frac{h}{m_{0c}}\frac{k^2}{c^2}(1-\cos\theta) = \lambda_0 + \frac{a_2}{a'_2}\frac{h}{m_{0c}}(1-\cos\theta) \tag{13}$$

where m_0 is the rest mass of the electron.

Above we have obtained the formula for Doppler frequency shift and Compton scattering satisfying Eq. (2), but we do not yet know the specific expressions for a_1 and a_2. What we can do next is to apply these two formulas to real physical situations to see what the expressions for a_1 and a_2 might look like. Fortunately, the phenomenon called spontaneous and synchrotron radiation in a cyclotron maser provides just such a physical application.

4. Spontaneous Radiation in Cyclotron Maser

A cyclotron maser is a device with high peak power and high average power in the millimeter and sub-millimeter bands. After decades of effort, the mechanism, experimental design, and numerical simulation of cyclotron masers have been analyzed extensively [18–23]. For example, in Ref. [15, 16], the authors found that the Compton scattering mechanism of virtual photons can be used to analyze spontaneous radiation in a cyclotron maser (in the spontaneous radiation theory of the free electron laser, the periodic static magnetic field in the oscillator is also regarded as virtual photons, which are then converted into real photons by Compton scattering between virtual photons and relativistic particles [24]).

Inspired by this idea, we now revisit the discussion on spontaneous radiation in a cyclotron maser using Eqs. (6) and (14). In the laboratory system S (i.e., Figure 5a [Figure 5: see original paper]), we set the charged particle to have a charge of q, a rest mass of m_0, and a velocity of v perpendicular to the magnetic field B. Then in the laboratory system, the spiral motion of the charged particle in a magnetic field has a period of:

$$T_c = \frac{2\pi m_0}{Bq\sqrt{1-\beta^2}} \tag{14}$$

As shown in Figure 5b, we construct an inertial system S' that is stationary relative to the particle, with the direction of v parallel to the z' axis. Based on the transformation in relativity, there will be both magnetic and electric fields in S'. Based on the "moving ruler" effect in relativity, in S' the track perimeter of the magnetic field moving as a reverse spiral translation around the charged particle becomes $2\pi R/\gamma$, and correspondingly, its cycle period is:

$$T_c' = \frac{2\pi R/\gamma}{c} = \frac{T_c}{\gamma} \tag{15}$$

According to Ref. [15], since the photon in Eq. (16) cannot be observed in the laboratory system but can only be observed in S' due to relativistic effects, the photon can be regarded as a virtual photon. Next, we consider the Doppler effect of the virtual photon, as both the electric field and magnetic field that vary periodically move with velocity $-v$ relative to S'.

First we define the angles shown in Figure 6 [Figure 6: see original paper]: in the particle system S', θ_1' denotes the angle between the direction of virtual photon propagation and the z' axis, θ_2' denotes the angle between the direction of the scattered photon propagation and the z' axis, ϕ' denotes the scattering angle, and Ω' is the angle between the plane formed by the direction of virtual photon propagation and the z' axis and the plane formed by the direction of scattered photon propagation and the z' axis. In the laboratory system S, θ_1 denotes the angle between the direction of virtual photon propagation and the z axis, θ_2 denotes the angle between the direction of the scattered photon propagation and the z axis, ϕ denotes the scattering angle, and Ω is the angle between the plane formed by the direction of virtual photon propagation and the z axis and the plane formed by the direction of scattered photon propagation and the z axis.

Therefore, based on Eq. (6) we can obtain the wavelength of virtual photons in the particle system S' as:

$$\lambda_1' = \frac{cT_c'}{\sqrt{1-\beta^2}}\left(1 - \frac{v\cos(\pi-\theta_1')}{nc}\right) = \frac{cT_c/\gamma}{\sqrt{1-\beta^2}}\left(1 - \frac{v\cos(\pi-\theta_1')}{nc}\right) \tag{16}$$

where $\lambda_c' = cT_c' = cT_c/\gamma$, and $n = n(v\cos(\pi-\theta_1'), c)$.

In Compton scattering, virtual photons can be scattered and radiated out as real photons. Based on Eq. (14), the wavelength of radiated photons satisfies:

$$\lambda_2' = \lambda_1' + \frac{a_2}{a_2'}\frac{h}{m_{0c}}(1-\cos\phi') \tag{17}$$

where λ_2' denotes the wavelength of radiated photons observed in S'.

Again, using the Doppler formula in Eq. (6), we can obtain the wavelength of radiated photons observed by the observer in the laboratory system S:

$$\lambda_2 = \frac{n'\lambda_2'}{\sqrt{1-\beta^2}}\left(1-\frac{v\cos\theta_2}{n'c}\right) \tag{18}$$

where $n' = n(v\cos\theta_2, c)$.

Since $Q \approx 0$, a_1 and a_2 in Eq. (11) should satisfy $a_1 \approx 1$, $a_2 \approx 1$ at low or medium frequency shifts. On the other hand, since the Compton wavelength of the charged particle is generally much smaller than cT_c, we can assume $a_2' \approx 1$ in Eq. (18). So based on Eqs. (14)-(19), we can obtain:

$$\lambda_2 = \frac{n'}{\sqrt{1-\beta^2}}\left(1-\frac{v\cos\theta_2}{n'c}\right)\left[\frac{cT_c}{\sqrt{1-\beta^2}}\left(1-\frac{v\cos(\pi-\theta_1')}{nc}\right)+\frac{h}{m_{0c}}(1-\cos\phi')\right] \tag{19}$$

As stated in Ref. [14], the mathematical form of the model proposed there is very similar to the Lorentz model, i.e., replacing c in the Lorentz model with k yields the model in Ref. [14]. Based on the same similarity, the formula for aberration of light corresponding to the model in Ref. [14] can be obtained:

$$\begin{aligned}\cos\theta' &= \frac{\cos\theta-\beta}{1-\beta\cos\theta}\\ \sin\theta' &= \frac{\sin\theta}{\gamma(1-\beta\cos\theta)}\end{aligned} \tag{20}$$

where $\theta = \theta_1$ or $\theta = \theta_2$ or $\theta = \Omega$.

On the other hand, according to the geometric definition in Figure 6, one can obtain the following geometric equations:

$$\begin{aligned}\cos\phi' &= \cos\theta_1'\cos\theta_2' + \sin\theta_1'\sin\theta_2'\cos\Omega'\\ \cos\phi &= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos\Omega\end{aligned} \tag{21}$$

Substituting Eqs. (21) and (22) into Eq. (20), we can obtain:

$$\lambda_2 = \frac{n'-v/c\cos\theta_2}{\sqrt{1-\beta^2}}\left[\frac{cT_c}{\sqrt{1-\beta^2}}\left(1-\frac{v\cos(\pi-\theta_1')}{nc}\right)+\frac{h}{m_{0c}}\frac{1-\cos\phi}{(1-\beta\cos\theta_1)(1-\beta\cos\theta_2)}\right] \tag{22}$$

Eq. (23) gives the wavelength of spontaneous radiation of a charged particle in a cyclotron maser observed by the observer in the laboratory system S. At low or medium frequency shifts, we have $n \approx 1$, $a_2 \approx 1$, $k \approx 1$, then Eq. (23) reduces to:

$$\lambda_2 = \frac{cT_c}{\sqrt{1-\beta^2}}\frac{1-\beta\cos\theta_2}{\sqrt{1-\beta^2}} + \frac{h}{m_{0c}}\frac{1-\cos\phi}{(1-\beta\cos\theta_1)(1-\beta\cos\theta_2)} \tag{23}$$

which corresponds to the case in the Lorentz model.

However, we are more interested in what happens at ultra-high frequency shifts. First, it reminds us that since Ref. [14] allows $v \in [0, c)$, the energy of the scattered virtual photon in Eq. (12) might be infinite due to $\lambda_1' = 0$ unless we assume:

$$
\begin{aligned}
a_1 &= \frac{1 - V/c}{1 - nV/c} \\
a_2 &= \frac{1 - V/k}{1 - nV/k} \approx \frac{1 - V/c}{1 - nV/c}
\end{aligned}
\tag{24}
$$

where $V = v\cos\theta$, $n = n(V, c)$, and θ is the angle between the direction of v and c.

The proposal of Eq. (25) is based on the following considerations. Since the expression of λ in Eq. (6) has the term $[1 - v\cos\theta/(nc)]$, which leads to $\lambda = 0$ under the condition of $v = nc$ and $\theta = 0$, and $\lambda = 0$ would cause the energy of the virtual photon to be infinite, making Eq. (14) invalid. To avoid this, we hope a_2 in Eq. (11) has the term $[1 - v\cos\theta/(nc)]$ to cancel out this same term in λ.

On the other hand, we also want to extend Eq. (23) to the case where $v \to c$, and in which case, from Ref. [14] we know that:

$$
n = \frac{1 - Q}{1 - Q^{1 - v/c}} \approx \frac{1 - Q}{1 - Q^{1 - v/c}} = \frac{Q - 1}{2\ln Q}(1 - v/c) \tag{25}
$$

Eq. (26) reminds us that to avoid a_2 being infinite in the case where $v \to c$, we can replace v/c with nv/c in the expression for a_2. Then for $v \to c$, the expression $1/(1 - nv/c)$ tends to 1 instead of $1/(1 - v/c)$ tending to infinity. This argument may seem a little hard to understand here, but we can see how it works in the following.

Substituting Eqs. (25) and (6) into Eq. (11), one obtains:

$$
\begin{aligned}
E &= a_{1hf} = \frac{1 - V/c}{1 - nV/c} hf = \frac{1 - V/c}{1 - nV/c}\frac{nc}{\lambda} hf_0 = \frac{nc}{\lambda} hf_0 \\
p &= a_2 \frac{h}{\lambda} = \frac{1 - V/k}{1 - nV/k}\frac{h}{\lambda} \approx \frac{1 - V/c}{1 - nV/c}\frac{h}{\lambda} = \frac{h}{\lambda}
\end{aligned}
\tag{26}
$$

Based on Eq. (27), we have:

$$
\lim_{V \to c} E = a_{1hf} = \frac{\sqrt{1 - \beta^2}}{1 - nV/c} hf_0 = \sqrt{1 - \beta^2} hf_0 = \sqrt{\frac{Q - 1}{2\ln Q}} hf_0 \tag{27}
$$

and $p = E/k \approx E$.

In the Lorentz model, as we know, for the case where $V \to c$, E tends to be infinite. However, here we constrain E to be a finite value such that even in the case where $V = nc$, f is infinite and $\lambda = 0$, but $E = \sqrt{1-\beta^2/(1-v^2_{\text{wave shock}}/c^2)}hf_0$ is also a finite value. It is this limited energy and momentum that makes Compton scattering valid.

In addition, it has been stated above that due to $Q \approx 0$ there should be $a_1 \approx 1$, $a_2 \approx 1$ at low or medium frequency shifts, which means it should return to the case as in the Lorentz model at low or medium frequency shifts.

Based on these three key considerations, we construct the expression for a_1 and a_2 as Eq. (25). One may consider other possible expressions for a_1 and a_2, but after many attempts we find that Eq. (25) is the most concise expression that meets these three requirements. Here one may be confused as to why the correction coefficients corresponding to photon energy and momentum are different, which seems to violate intuition obtained from the Lorentz model. The main reason is that there is an invariant in the Lorentz model, which is the speed of light, but in our model there is no such invariant. However, although the correction coefficients for energy and momentum are different, they are still closely related (the underlying mechanism is that time and space are a whole, while Eq. (1) has indicated that space and time are a whole), which is shown in Eq. (2). So here in this paper we will just try to use this expression to analyze the phenomenon called synchrotron radiation in the cyclotron maser.

Now we have confirmed that Eq. (23) returns to Eq. (24) at low or medium frequency shifts. Thus next we discuss the case where $v \to c$, and in this case we can obtain $\theta_2 \approx 1/\gamma \approx 0$, then Eq. (23) can be written accordingly as:

$$\lambda_2 = \frac{n' - v/c}{\sqrt{1-\beta^2}} \left[\frac{cT_c}{\sqrt{1-\beta^2}} \left(1 - \frac{v\cos(\pi-\theta_1')}{nc}\right) + \frac{h}{m_{0c}} \frac{1-\cos\phi}{(1-\beta\cos\theta_1)(1-\beta)} \right] \tag{28}$$

where $n' = n(v\cos\theta_2, c) \approx n(v, c)$, and:

$$a_2 \approx \frac{1 - v\cos(\pi-\theta_1')/c}{1 - nv\cos(\pi-\theta_1')/c} = \frac{1 + v\cos\theta_1'/c}{1 + nv\cos\theta_1'/c} \tag{29}$$

When $\theta_1 = 0$, $\cos\theta_1' = (1-\beta)/(1-\beta) = 1$, which means $\theta_1' = 0$, so $n = n(v\cos(\pi-\theta_1'), c) = n(-v, c) = n(v, c)$ in Eq. (17). Then we can obtain:

$$\lambda_2 = \lim_{v\to c} \frac{(n-v/c)(1+nv/c)}{n+v/c} \frac{cT_c}{\sqrt{1-\beta^2}} \frac{1-\beta^2(n-v/c)}{(1-\beta)^2} \frac{1-\cos\phi}{(n+v/c)-(n-v/c)} = \lim_{v\to c} \frac{n(1+nv/c)}{n+v/c} \frac{cT_c}{\sqrt{1-\beta^2}} \tag{30}$$

Using the following equation from Ref. [14]:

$$\sqrt{1-\beta^2} = \lim_{v\to c} \sqrt{\frac{1-v/c}{1+v/c}} = \lim_{v\to c} \sqrt{\frac{1-v/c}{(n+v/c)}} = \sqrt{\frac{Q-1}{2\ln Q}} \tag{31}$$

where the expression for n is shown in Eq. (8), we get:

$$\lambda_2 \approx -\frac{2\pi R}{\sqrt{\frac{Q-1}{2\ln Q}}}\sqrt{\frac{Q-1}{2\ln Q}}\frac{1-\cos\phi}{\sqrt{1-Q^{-1}}} = -2\pi R\sqrt{\frac{Q-1}{2\ln Q}}\frac{1-\cos\phi}{\sqrt{1-Q^{-1}}} \tag{32}$$

On the other hand, when $\theta_1 = \pi$, $\cos\theta_1' = (-1-\beta)/(1+\beta) = -1$, which means $\theta_1' = \pi$, so $n = n(v\cos(\pi-\theta_1'), c) = n(v, c)$ in Eq. (17). Then we can obtain:

$$\lambda_2 = \lim_{v\to c}\frac{n-v/c}{\sqrt{1-\beta^2}}\left[\frac{cT_c}{\sqrt{1-\beta^2}}\frac{1-\beta^2(n-v/c)}{nk/c(1-nv/c)} + \frac{h}{m_{0c}}\frac{1-\cos\phi}{(1-\beta)^2}\right] = 0 - \frac{2\pi R}{\sqrt{\frac{Q-1}{2\ln Q}}}\sqrt{\frac{Q-1}{2\ln Q}}\frac{1-\cos\phi}{\sqrt{1-Q^{-1}}} = -2\pi R\sqrt{\frac{Q}{2}} \tag{33}$$

One may notice that the values obtained in Eqs. (30) and (32) are negative, which seems to go against "common sense." The reason is that when $v > v_{\text{wave shock}}$, the timing sequence is reversed, which has been discussed in Section "Doppler effect of photons." It can be observed from Eqs. (30) and (32) that when $v = n$, $\lambda_2 = 0$ (here it should be noted that although $\lambda_2 = 0$ in this case, both the momentum and energy of photons are finite, as stated in the paragraph following Eq. (27)), and when $v \in (n, 1)$, λ_2 increases dramatically and tends to another non-zero value. This is obviously different from the results predicted by the Lorentz model, where in the case $v \to c$, the wavelength of light observed by the observer in the laboratory system approaches 0 (correspondingly, the momentum and energy of the photon become infinite).

5. Inverse Compton Scattering

Interactions between photons and other particles often occur in high-energy fields, for example, interactions between gamma photons and nuclei, interactions between gamma photons and nucleons, etc. The scattering between extremely relativistic particles and photons is called inverse Compton scattering, because photons can obtain energy from particles in this process. Unlike the Compton scattering process, where the particles are basically stationary, in inverse Compton scattering the particles are extremely relativistic. However, the method of deriving the relationship between the scattered photon and the incident photon is the same, i.e., they both use the law of energy conservation and momentum conservation shown in Eq. (12).

First we can build an inertial system S' that moves along with the electron (here we analyze the interaction between photons and electrons). In S', the electrons are stationary, so the collision between photons and electrons satisfies Compton's formula as in Eq. (18):

$$\lambda_2' = \lambda_1' + \frac{h}{m_{0c}}(1-\cos\phi') \tag{34}$$

where λ_1' and λ_2' represent the wavelengths of incident and scattered photons observed in S', respectively, and ϕ' denotes the scattering angle.

Here we assume that both the observer and the light source are located in the inertial system S, and that S is moving at velocity v relative to S'. Then based on Eqs. (6) and (25) we have:

$$\lambda_1 = \frac{n}{a_2}\frac{\sqrt{1-\beta^2}}{1-v\cos\theta_1/(nc)}\lambda_1' = \frac{1-nv\cos\theta_1/c}{1-v\cos\theta_1/c}\frac{\sqrt{1-\beta^2}}{1-v\cos\theta_1/(nc)}\lambda_1' = \frac{1-nv\cos\theta_1/c}{\sqrt{1-\beta^2}}\lambda_1' \tag{35}$$

$$\lambda_2 = \frac{n'}{a_2'}\frac{\sqrt{1-\beta^2}}{1-v\cos\theta_2/(n'c)}\lambda_2' = \frac{1-n'v\cos\theta_2/c}{\sqrt{1-\beta^2}}\lambda_2' \tag{36}$$

where λ_1 and λ_2 represent the wavelengths of incident and scattered photons observed in S, respectively.

Next we consider the case where $\theta = 0$ (because in this case the photon can obtain maximum energy from the particle), and correspondingly we have $\phi' = \pi$. Since Eq. (33) describes Compton scattering occurring in S', we can assume $a_2' \approx 1$. Then based on Eqs. (33) and (34), the observer in S can obtain scattered photons with momentum:

$$P_2 = \frac{h}{\lambda_2} = \frac{h}{\lambda_1}\frac{1-v/c}{1-nv/c+(1-nv/c)}\frac{1-v/c}{1-v/c} = \frac{h}{\lambda_1}\frac{n-v/c}{1-nv/c+(1-nv/c)}\frac{1-v/c}{1-v/c} \tag{37}$$

When $n \equiv 1$, Eq. (35) returns to the case as in the Lorentz model. From Eq. (35) it can be seen that when $n = v/c$, P_2 has a maximum value:

$$P_{\max} = \frac{h}{\lambda_1}\frac{n-v/c}{1-nv/c+(1-nv/c)}\frac{1-v/c}{1-v/c} = \frac{h}{\lambda_1}\frac{v_{\text{wave shock}}/c - v_{\text{wave shock}}/c}{1-v_{\text{wave shock}}^2/c^2+(1-v_{\text{wave shock}}^2/c^2)}\frac{1-v_{\text{wave shock}}/c}{1-v_{\text{wave shock}}/c} \tag{38}$$

where $v_{\text{wave shock}}$ comes from Eq. (9).

For the observer in S, if the observed energy of the electron is much greater than that of the incident photon, Eq. (36) can be further simplified as:

$$P_{\max} \approx \frac{h}{\lambda_1}\frac{1}{1-v_{\text{wave shock}}^2/c^2} \tag{39}$$

In addition, we are more interested in what would happen if $v \to c$, in which case $n \to 0$. Again, based on Eq. (35) we can obtain:

$$P_2 = \lim_{v\to c}\frac{h}{\lambda_1}\frac{n-v/c}{1-nv/c+(1-nv/c)}\frac{1-v/c}{1-v/c} = \frac{h}{\lambda_1}\frac{2\ln Q}{2\ln Q+Q-1} \tag{40}$$

As is known, in the Lorentz model, when the velocity of the electron approaches the speed of light, the maximum energy that a photon can obtain through inverse Compton scattering tends to infinity. However, if Q is not equal to 0 (when $Q = 0$, then $n \equiv 1$ and the model discussed above returns to the Lorentz model), then the maximum energy that a photon can obtain through inverse Compton scattering will have a limit, as shown in Eq. (37). Furthermore, when the velocity of the electron is high enough to be close to c, the maximum energy that a photon can obtain instead drops to another non-zero and finite value.

In addition, from Eq. (37) it can be seen that the maximum energy that a photon can obtain is not related to the wavelength of the incident photon, and the ratio of the maximum energy that a photon can obtain to the rest energy of an electron is a constant, which is very interesting. That is, in inverse Compton scattering, no matter what the energy of the incident photon is, and whether the extremely relativistic particle is an electron, proton, or other particle with mass, the ratio of the maximum energy that a photon can obtain to the rest energy of the extremely relativistic particle is a constant.

In recent years, the mechanism of high-energy radiation from blazars has been studied extensively [25–31]. One model used to explain this mechanism is the lepton origin model, which considers that high-energy radiation comes from the inverse Compton scattering process between relativistic electrons and low-energy photons [29–31]. Since some special objects with extreme physical properties, such as blazars, can produce extremely relativistic particle jets, and the cosmic microwave background radiation (CMB) provides natural low-energy photons, this provides a cosmic version of inverse Compton scattering. Therefore, we expect to test the value of Q by analyzing the ultra-high energy radiation spectrum in such a cosmic scenario, because whether the value of Q is 0 will have very different effects on the ultra-high energy radiation spectrum, as shown in this section.

6. Summary

The Lorentz violation model is widely studied in the ultra-high energy field in recent years, especially being considered in various models attempting to unify general relativity and quantum theory [32–37]. Among them, Ref. [14] proposed a possible Lorentz violation model where the speed of light may be related to the velocity of the light source. The key idea in Ref. [14] is that in the case where $v \to c$, the energy or momentum of the particle has a finite value (i.e., the limit of the time or length scaling factor is $\sqrt{2\ln Q/(Q-1)}$, and correspondingly, the energy limit of a particle with rest mass m_0 is $m_{0c}^2\sqrt{2\ln Q/(Q-1)}/[1-0.5(Q-1)/\ln Q]$) rather than being infinite as predicted by the Lorentz model. The model in Ref. [14] is similar to the famous rainbow model [12], but they are different: in Ref. [14] the energy limit of the particle is related to the rest mass of the particle, while in the rainbow model it is not, but is assumed to be a constant (usually considered to be the Planck energy or near the Planck energy).

If we follow the idea in Ref. [14] that the speed of light observed by an observer moving with velocity v relative to the light source is nc, then this forces us to modify the usual quantum relation of photons in the Doppler effect. By discussing the Doppler effect of photons based on the idea in Ref. [14], we find that light can also exhibit shock phenomena (in vacuum) just like sound waves, and we further obtain the conditions for light to generate a shock wave, i.e., $v = nc$, and in the case where $nc < v < c$ the timing sequence will be reversed. However, this doesn't violate the law of causality because the timing sequence can be reversed in independent events (though not in dependent events) [17].

Then the Compton scattering process between the photon with the modified quantum relation as shown in Eq. (11) and the electron is discussed, and we find that unless we define suitable correction coefficients for the quantum relation of photons, the photon energy will be infinite in the case where $v = nc$, making it impossible to discuss Compton scattering between photons and electrons over the whole range of $v \in [0, c)$. In other words, due to the infinite photon energy when $v = nc$, Compton scattering between photons and electrons would be invalid. Therefore, to avoid this, we define suitable and concise correction coefficients for the quantum relation of photons to make the photon energy finite over the whole range of $v \in [0, c)$, especially in the cases where $v = nc$ or $v \to c$.

Here it should be noted that, due to limited current information and knowledge, we are not yet able to obtain the specific correction coefficients for the quantum relation of photons satisfying Eq. (2), but we can obtain three key constraints for possible correction coefficients, as shown in Section "Spontaneous radiation in cyclotron maser." One may propose many expressions for correction coefficients satisfying these three constraints, but after many attempts we construct a very concise expression for the correction coefficients, as shown in Eq. (25).

Then based on Eq. (25) we choose the phenomenon called spontaneous radiation in a cyclotron maser for study, because this phenomenon can be explained by the concept of virtual photons. By analyzing the Doppler effect and Compton scattering effect of virtual or real photons in the cyclotron maser, we can obtain wavelength formulas for spontaneous radiation and synchrotron radiation. The important conclusion we have drawn is that we find in the case where $v \to c$, the wavelength observed by the observer in the laboratory system does not tend to 0, but tends to a non-zero value. This differs from results derived from the Lorentz model, where the wavelength tends to 0 and the corresponding photon energy and momentum tend to infinity. Even in the case where $v = nc$, the wavelength is 0, but the photon energy and momentum are both finite, which ensures that Compton scattering is valid over the whole range of $v \in [0, c)$, since Ref. [14] allows $v \in [0, c)$.

Furthermore, the inverse Compton scattering phenomenon based on Eq. (25) is also discussed, and we find that there is a limit to the maximum energy that photons can obtain in the collision process between relativistic particles and low-energy photons. This occurs when the particle velocity satisfies $v = v_{\text{wave shock}}$. When the particle velocity is greater than $v_{\text{wave shock}}$ and close to c, the energy that the photon can obtain decreases sharply and tends to another finite value. This conclusion also differs significantly from that predicted by the Lorentz model, where the energy that the photon can obtain tends to infinity as the particle velocity approaches c.

Generally, the main purpose of this paper is the same as that of Ref. [14]: to avoid infinite energy or momentum for particles (i.e., any particles, including photons), as this would invalidate real physical situations in certain cases. This is also inspired by ideas in some quantum gravity models [34, 35, 37]. It is very important to emphasize that if $Q = 0$, then $n \equiv 1$ and correspondingly, all results and conclusions in this paper return to the Lorentz model case. Therefore, this paper also shows us another possible experimental scheme to determine the value of Q, although it still requires extremely high experimental energies.

CRediT Authorship Contribution Statement
Jinwen Hu: Methodology, Investigation, Formal analysis, Conceptualization.
Huan Hu: Data curation, Writing–original draft.

Declaration of Competing Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability
Data will be made available on request.

References

[1] Han MX, Huang WM, Ma YG. Fundamental structure of loop quantum gravity. Int J Modern Phys D 2007;16(9):1397–474.
[2] Ashtekar A, Lewandowski J. Background independent quantum gravity: a status report. Class Quantum Gravity 2012;21(15):R53–152.
[3] Thiemann T. Modern canonical quantum general relativity. United Kingdom: Cambridge University Press; 2007.
[4] Rovelli C. Quantum gravity. United Kingdom: Cambridge University Press; 2007.
[5] Amelino-Camelia G. Relativity in space-times with short-distance structure governed by an observer-independent (Planckian) length scale. Int J Modern Phys D 2002;11(1):35–59.
[6] Amelino-Camelia G. Testable scenario for relativity with minimum-length. Phys Lett B 2001;510(3):255–63.
[7] Amelino-Camelia G. Quantum spacetime phenomenology. Living Rev Relativ 2013;16:5.
[8] Kowalski-Glikman J. Introduction to doubly special relativity. Lect Notes Phys 2005;669:132–59.
[9] Kostelecky A. CPT, T, and lorentz violation in neutral-meson oscillations. Phys Rev D 2001;64(7):407–12.
[10] Kostelecky A. Theory and status of Lorentz violation. Am Phys Soc 2008:132–55.
[11] Girelli F, Livine ER, Oriti D. Deformed special relativity as an effective flat limit of quantum gravity. Nucl Phys B 2005;708(1):411–33.
[12] Nilsson NA, Dabrowski MP. Energy scale of Lorentz violation in rainbow

gravity. arXiv: Cosmol Nongalactic Astrophys 2016.
[13] He P, Ma B. Lorentz symmetry violation of cosmic photons. Universe 2022;8(6):323.
[14] Jinwen Hu, Huan Hu. A special rainbow function. Result Phys 2022;43:106082.
[15] Xianzhu MENG. Research of synchrotron radiation and cyclotron radiation by virtual photon compton scattering. High Energy Phys Nucl Phys 2004;28:898–900.
[16] Xianzhu MENG. Research of synchrotron radiation by virtual photon and compton. High Power Laser Particle Beams 2005;17(3):477–80.
[17] Kexun W. An analysis of Doppler effect using time-space line. College Phys 1997;16(12):8–11.
[18] Twiss RQ. Radiation transfer and the possibility of negative absorption in radio astronomy. Aust J Phys 1958;11:564–79.
[19] Hirshfield JL, Wachtell JM. Electron cyclotron maser. Phys Rev Lett 1964;12:533–6.
[20] Li GD, Kao SH, Chang PC. Terahertz electron cyclotron maser interaction with an axis-encircling electron beam. Phys Plasma 2015;22(4):043109.
[21] Nusinovich GS, Zhao D. Combined resonances in cyclotron masers with periodic slow-wave structures. Plasma Sci IEEE Trans 2015;43(3):804–14.
[22] Wu DJ, Chen L, Zhao GQ. A novel mechanism for electron-cyclotron maser. A & A 2014;566(940):595–600.
[23] Yuguo TANG, Futian LI. Characteristics of synchrotron radiation emitted by hefei 800 MeV electron storage ring. Acta Sinica Quantum Optica 1994;14(8):795–9.
[24] Jiaqing ZHU. Collision analogue of a free electron laser. J Huazhong Normal Univ (Nat Sci) 1988;22(3):293–6.
[25] Prandini E, Ghisellini G. The blazar sequence and its physical understanding. Galaxies 2022;10(1):35.
[26] Yang JH, Fan JH, Liu Y, et al. The spectral energy distributions for 4FGL blazars. Astrophys J Suppl Ser 2022;262(1):18.
[27] Fossati G, Maraschi L, Celotti A, et al. A unifying view of the spectral energy distributions of blazars. MNRAS 1998;299(2):433–48.
[28] Fan JH, Yang JH, Liu Y, et al. The spectral energy distributions of Fermi blazars. Astrophys J Suppl Ser 2016;226(2):20.
[29] Dermer CD, Schlickeiser R. Model for the high-energy emission from blazars. Astrophys J 1993;416:458.
[30] Dermer CD, Schlickeiser R. Transformation properties of external radiation fields, energy-loss rates and scattered spectra, and a model for blazar variability. Astrophys J 2002;575(2):667.
[31] Ghisellini G, Madau P. On the origin of the γ-ray emission in blazars. MNRAS 1996;280(1):67–76.
[32] Amelino-Camelia G, Ellis JR, Mavromatos NE, Nanopoulos DV. Distance measurement and wave dispersion in a Liouville string approach to quantum gravity. Int J Mod Phys A 1997;12:607–24.
[33] Amelino-Camelia G. Testable scenario for relativity with minimum length.

Phys Lett B 2001;510:255–63.
[34] de Brito GP, Caneda PIC, Gomes YMP, Junior JTG, Nikoofard V. Effective models of quantum gravity induced by Planck scale modifications in the covariant quantum algebra; 2016. arXiv:1610.01480.
[35] Abbasvandi N, Soleimani MJ, Radiman S, Wan Abdullah WAT. Quantum gravity effects on charged microblack holes thermodynamics. Int J Mod Phys A 2016;31(23):1650129.
[36] Tawfik A, Magdy H, Ali AF. Lorentz invariance violation and generalized uncertainty principle. Phys Part Nucl Lett 2016;13:59–68.
[37] Doplicher S, Fredenhagen K, Roberts JE. Spacetime quantization induced by classical gravity. Phys Lett B 1994;331:39.

Note: Figure translations are in progress. See original paper for figures.