

## Responses of vegetation yield to precipitation and reference evapotranspiration in a desert steppe in Inner Mongolia, China (Postprint)

**Authors:** LI Hongfang, WANG Jian, LIU Hu, MIAO Henglu, LIU Jianfeng

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### Abstract

Drought restricts sustainable agricultural development, ecological health, and socioeconomic progress, and is influenced by multiple factors. It is widely recognized that no single variable can fully capture the characteristics of drought events. Analyzing precipitation, reference evapotranspiration ( $ET_0$ ), and vegetation yield can provide critical information for water resource conservation in grassland ecosystems across arid and semi-arid regions. This study investigated the interactions among precipitation,  $ET_0$ , and vegetation yield in Darhan Muminggan Joint Banner (DMJB), a desert steppe in China's Inner Mongolia Autonomous Region, using two-dimensional (2D) and three-dimensional (3D) joint distribution models. Three Copula functions were employed to quantitatively analyze the joint distribution probabilities of different combinations of precipitation,  $ET_0$ , and vegetation yield. For the precipitation– $ET_0$  dry–wet type, the 2D joint distribution probability with precipitation  $\$ 245.69\text{mm/a}$  and  $ET_0$   $\$ 959.20\text{ mm/a}$  in DMJB was approximately 0.60, while the joint probability with precipitation  $\$ 245.69\text{mm/a}$  and  $ET_0$   $\$ 959.20\text{mm/a}$  was about 0.20. Correspondingly, the joint return period for at least one of these two events (dry precipitation or  $ET_0$ ) occurrence return period for both events happening simultaneously was 5 years. Under these conditions, the interval increased, the 3D joint distribution probability that vegetation yield would decline due to water shortage during precipitation– $ET_0$  dry–wet years could reach 0.60–0.70. Future work should implement irrigation activities and water allocation criteria to increase vegetation yield and ensure water resource security in Inner Mongolia's desert steppe.

## Full Text

### Preamble

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### Responses of vegetation yield to precipitation and reference evapotranspiration in a desert steppe in Inner Mongolia, China

LI Hongfang<sup>1,2</sup>, WANG Jian<sup>1,2\*</sup>, LIU Hu<sup>1,2</sup>, MIAO Henglu<sup>1,2</sup>, LIU Jianfeng<sup>3</sup>

<sup>1</sup> Yinshanbeilu Grassland Eco-hydrology National Observation and Research Station, China Institute of Water Resources and Hydropower Research, Beijing 100038, China

<sup>2</sup> Inner Mongolia Hydraulic Research Institute, Hohhot 010020, China

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### Abstract

Drought restricts sustainable agricultural development, ecological health, and socioeconomic progress, and is influenced by multiple factors. It is widely recognized that no single variable can fully capture the characteristics of drought events. Analyzing precipitation, reference evapotranspiration ( $ET_0$ ), and vegetation yield can provide critical information for water resource conservation in grassland ecosystems across arid and semi-arid regions. This study investigated the interactions among precipitation,  $ET_0$ , and vegetation yield in Darhan Muminggan Joint Banner (DMJB), a desert steppe in China's Inner Mongolia Autonomous Region, using two-dimensional (2D) and three-dimensional (3D) joint distribution models. Three Copula functions were employed to quantitatively analyze the joint distribution probabilities of different combinations of precipitation,  $ET_0$ , and vegetation yield. For the precipitation– $ET_0$  dry–wet type, the 2D joint distribution probability with precipitation  $\$ 245.69\text{mm/a}$  and  $ET_0$   $\$ 959.20\text{ mm/a}$  in DMJB was approximately 0.60, while the joint probability with precipitation  $\$ 245.69\text{mm/a}$  and  $ET_0$   $\$ 959.20\text{mm/a}$  was about 0.20. Correspondingly, the joint return period for at least one of these two events (dry precipitation or  $ET_0$ ) occurring simultaneously was 5 years. Under these conditions, the interval between events increased, the 3D joint distribution probability that vegetation yield would decline due to water shortage during precipitation– $ET_0$  dry–wet years could reach 0.60–0.70. Future work should implement irrigation activities and water allocation criteria to increase vegetation yield and ensure water resource security in Inner Mongolia's desert steppe.

**Keywords:** precipitation; reference evapotranspiration; vegetation yield; Copula functions; desert steppe; dry and wet events; Inner Mongolia

## 1 Introduction

Grasslands constitute a vital component of terrestrial ecosystems, playing crucial roles in climate regulation, soil and water conservation, soil quality improvement, and biodiversity maintenance (Wang et al., 2022a). Desert steppes represent the transitional zone between grassland and desert, representing the most severely desertified type among all grassland categories. The desert steppe in China's Inner Mongolia Autonomous Region accounts for approximately 18.00% of the nation's total grassland area (Li et al., 2013; Chu et al., 2014; Fang et al., 2016). The ecological environment of desert steppes is fragile and extremely sensitive to precipitation changes (Zhan et al., 2007; Guo et al., 2012; Song et al., 2017; Legesse et al., 2022). Strong evaporation and surface radiation further exacerbate the water sensitivity of desert steppe vegetation (Bittelli et al., 2008; Jung et al., 2010; Armstrong et al., 2015).

Precipitation and reference evapotranspiration ( $ET_0$ ) are two important, correlated hydrological variables that characterize the relationship between regional water supply and plant demand (Li et al., 2017). Water is the primary limiting factor affecting grassland ecosystems, a fact confirmed by both controlled experiments and precipitation-gradient studies (Bai et al., 2008; Yuan and Chen, 2009; Beier et al., 2012). In desert steppes, monthly precipitation from May to August determines annual precipitation totals. Research on the Inner Mongolia desert steppe reported  $ET_0$  values between 570.00 and 1674.00 mm/a from 1961 to 2010, with the highest values in western Inner Mongolia (Wang et al., 2015). Aridity in arid regions is projected to increase during the 21st century (Andrés et al., 2017). Investigating how vegetation yield responds to precipitation and  $ET_0$  in desert steppes can reveal not only the quantitative risk of yield reduction due to water shortage under natural precipitation conditions but also contribute to understanding ecological stability and yield security in Inner Mongolia's desert steppe.

Many studies on arid and semi-arid ecosystems have analyzed individual precipitation events or examined changes in specific factors relative to precipitation (Yan et al., 2018; Yu et al., 2019; Du et al., 2020). However, few have focused on yield responses to both precipitation and  $ET_0$  in desert steppes, and quantitative analysis of multivariate distribution probabilities for these variables remains challenging. To more accurately predict precipitation's influence on yield, understanding how yield responds to precipitation and  $ET_0$  events is essential. Copula functions are mathematical tools for solving multivariate probability problems that can reflect correlations between variables independent of their marginal distributions (Amirataee et al., 2020). These functions impose no limitations on variable marginal distributions, and the original random variable information remains unchanged during conversion, particularly when variable dimensions change by less than three. Multivariate probability models constructed from Copula functions are simple and easy to apply (Xie et al., 2012). To date, Copula functions have been extensively used in hydrology-climate research (e.g., Hao and Singh, 2015; Xu et al., 2015; Zhang et al., 2015; Cai et al., 2021). Cop-

ula fitting has been employed to analyze drought distribution, duration, and intensity (e.g., Amirataee et al., 2020). For instance, Wu et al. (2015) used Copula functions to study hydrological drought frequency variations in southern China. Additionally, drought risk can be qualitatively described and quantitatively evaluated through different precipitation- $ET_0$  combinations, thereby improving irrigation planning and water resource deployment (e.g., Vo et al., 2020).

In summary, this study had three objectives: (1) calculate  $ET_0$  using the Penman–Monteith formula and select and validate suitable marginal distributions and Copula functions for precipitation,  $ET_0$ , and yield in Inner Mongolia’s desert steppe; (2) quantitatively evaluate the 2D and 3D joint distribution probabilities of precipitation,  $ET_0$ , and yield using Copula functions; and (3) explore the main factors influencing desert steppe yield and provide measures to mitigate desertification and promote sustainable vegetation development in the region.

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## 2 Materials and Methods

### 2.1 Study Area

The study area is located in Darhan Muminggan Joint Banner (DMJB;  $109^{\circ}16'–111^{\circ}25'E$ ,  $41^{\circ}20'–42^{\circ}40'N$ ), a typical desert steppe covering approximately  $1.81 \times 10^5 \text{ km}^2$  in central and western Inner Mongolia Autonomous Region, China. This hilly desert steppe at the northern foot of Yinshan Mountain is characterized by a temperate, semi-arid continental climate. The average annual precipitation is 253.45 mm, the mean annual temperature is  $4.12^{\circ}C$ , and the average annual evaporation is 2480.57 mm (Song et al., 2017). Brown soil dominates the region, and vegetation consists primarily of perennial xerophytes. Dominant species include *Artemisia frigida*, *Stipa krylovii*, *Stipa klemenzi*, *Stipa breviflora*, and *Cleistogenes songorica*. Average vegetation coverage was 35.00% during 2002–2020.

Vegetation data for this study were collected from the Yinshanbeilu Grassland Eco-hydrology National Observation and Research Station in DMJB (Fig. 1 [Figure 1: see original paper]).

**Fig. 1** Overview of land use/land cover in the study area (Darhan Muminggan Joint Banner; a), and photos showing *Artemisia frigida* (b) and *Stipa krylovii* (c)

### 2.2 Data Sources

This study collected daily meteorological data from 2002–2020 and annual yield data during the main growing season (May–September) from 2002–2020. Daily precipitation, maximum temperature, minimum temperature, relative

humidity, sunshine hours, solar radiation, water vapor pressure, and average wind speed data were obtained from the China Meteorological Administration (<http://cdc.cma.gov.cn>). Data collection and processing followed the World Meteorological Organization's Global Observing System guide and the China Meteorological Administration's technical specifications and standards for weather observations. Sunshine hour data for July–December 2020 were estimated from the average daily values for 2000–2019. Yield data from 2002–2020 were collected from an intensive pastoral region covering 1.33 km<sup>2</sup>. *Artemisia frigida* and *Stipa krylovii* were selected as representative species to simulate yield, as they dominated plant community composition in sampling plots and represented different functional groups (Zhao et al., 2006; Chen et al., 2013). Each year around mid-August (harvest time), 5–10 plots (1 m\$×\$1 m each) dominated by these species were selected for yield survey, grid sampling, measurement, and evaluation. One sample was taken per plot. From 2002 to 2020, a total of 167 grass samples were collected, and aboveground dry weight was measured. Statistical analyses were performed using R (Lucent Technologies, New Jersey, USA) and Excel software. Graphics were preprocessed using MATLAB R2018b software (MathWorks, Massachusetts, USA). Data time ranges are shown in Table 1 .

**Table 1** Description of the data used in this study

Data Type	Factor	Unit	Period
Meteorological data	Solar radiation	kJ/(m <sup>2</sup> · d)	2002–2020
	Minimum temperature	°C	2002–2020
	Maximum temperature	°C	2002–2020
	Relative humidity	%	2002–2020
	Water vapor pressure	kPa	2002–2020
	Average wind speed	m/s	2002–2020
	Precipitation	mm/d	2002–2020
	Sunshine hours	h/d	2002–2020
Yield data	Annual dry weight of yield per unit area	g/(m <sup>2</sup> · a)	2002–2020

*Note: Yield refers to vegetation yield.*

## 2.3 Methods

**2.3.1 Calculation of ET<sub>0</sub>** The Penman–Monteith formula, recommended by the United Nations Food and Agriculture Organization (FAO) in 1998 (Allen, 2000), integrates data on elevation, latitude, wind speed, altitude, daily maximum temperature, daily minimum temperature, daily average temperature, daily average wind speed, daily average relative humidity, sunshine hours, and other variables. ET<sub>0</sub> is calculated as follows (Zhang et al., 2012):

$$ET_0 = [0.408\Delta(R - G) + \gamma(900/(T + 273))u_2(e - e_s)] / [\Delta + \gamma(1 + 0.34u_2)]$$

where  $ET_0$  is the reference evapotranspiration (mm/d);  $\Delta$  is the slope of the saturated vapor pressure-temperature curve (kPa/°C);  $R$  is the net solar radiation of the plant canopy surface (MJ/(m<sup>2</sup> · d));  $G$  is the soil heat flux (MJ/(m<sup>2</sup> · d)), and daily  $G$  was calculated as zero in this study ( $G=0$ );  $\gamma$  is the hygrometer constant (kPa/°C);  $u_2$  is the wind speed at 2 m height (m/s);  $e$  and  $e_s$  are the saturated water vapor pressure (kPa) and actual water vapor pressure (kPa), respectively; and  $T$  is the average temperature (°C).

**2.3.2 Marginal Distribution** The Weibull (WEI), gamma (GAMMA), and Exponential (EXP) distributions were used to fit the marginal distributions of precipitation,  $ET_0$ , and yield. The maximum likelihood method estimated Copula function parameters, and the Kolmogorov-Smirnov test (K-S test) selected the optimal marginal distribution (Song and Singh, 2010; Mohammadpour et al., 2012).

The WEI distribution function is given by:  $f(x) = (b/a)(x/a)^{b-1} \exp[-(x/a)^b]$  for  $x > 0$  where  $f(x)$  is the marginal distribution function of  $x$  (representing precipitation,  $ET_0$ , or yield);  $a$  is the scale parameter; and  $b$  is the shape parameter.

The GAMMA distribution function is given by:  $f(x) = [1/(\Gamma(b)a^b)]x^{b-1} \exp(-x/a)$  for  $x > 0$  where  $\Gamma$  denotes the GAMMA function.

The EXP distribution function is given by:  $f(x) = \lambda \exp(-\lambda x)$  for  $x > 0$  where  $\lambda$  is the scale parameter ( $\lambda > 0$ ).

**2.3.3 Joint Distribution Model** A Copula function does not limit the marginal distribution of variables. With the Copula model, several arbitrary marginal distributions can be connected to form a multivariate joint distribution probability model. Copula functions are based on Sklar's theorem (Sklar, 1959). For continuous random variables ( $X_1, X_2, \dots, X_n$  where  $n=3$  in this study) with marginal distribution functions  $F_1(X_1), F_2(X_2), \dots, F_n(X_n)$ , there exists a Copula function  $C$  such that:  $F(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n))$

The K-S test statistic  $D$  for selecting the optimal marginal distribution is defined as the maximum distance between the empirical and theoretical distribution functions.

This study used three commonly applied functions—the Clayton Copula, Gumbel-Hougaard Copula, and Frank Copula functions (Lazoglou and Anagnostopoulou, 2019)—to construct 2D and 3D joint distributions of precipitation,  $ET_0$ , and yield (Tables 2 and 3).

**Table 2** Description of the two-dimensional (2D) Copula functions

Copula function	Function expression	Condition
Clayton	$C(u,v) = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}$	$\alpha > 0$
Gumbel-Hougaard	$C(u,v) = \exp\{-[( -\ln u)^\alpha + (-\ln v)^\alpha]^{1/\alpha}\}$	$\alpha \geq 1$

Copula function	Function expression	Condition
Frank	$C(u,v) = -1/ \ln[1 + (\exp(-u)-1)(\exp(-v)-1)/(\exp(-)-1)]$	$\neq 0$

Note:  $u$  and  $v$  are marginal distribution functions for any two of precipitation,  $ET_0$ , and yield; and  $\theta$  is the Copula function parameter.

**Table 3** Description of the three-dimensional (3D) Copula functions

Copula function	Function expression	Condition
Clayton	$C(u_1, u_2, u_3) = (u_1^{-\theta} + u_2^{-\theta} + u_3^{-\theta} - 2)^{-1/\theta}$	$> 0$
Gumbel-Hougaard	$C(u_1, u_2, u_3) = \exp\{-[( -\ln u_1)^{\theta} + (-\ln u_2)^{\theta} + (-\ln u_3)^{\theta}]^{1/\theta}\}$	$\geq 1$
Frank	$C(u_1, u_2, u_3) = -1/ \ln\{1 + (\exp(-u_1)-1)(\exp(-u_2)-1)(\exp(-u_3)-1)/[(\exp(-)-1)^2]\}$	$\neq 0$

Note:  $u_1$ ,  $u_2$ , and  $u_3$  are marginal distribution functions for precipitation,  $ET_0$ , and yield, respectively; and  $\theta$  is the Copula function parameter.

The Copula function goodness-of-fit test is essential for selecting optimal functions. The root mean square error (RMSE) and Akaike information criterion (AIC) tested Copula function fitting. The fitting test and goodness-of-fit (Li et al., 2013), using minimum AIC as the best-fit criterion, were calculated as follows:

$$AIC = N \times \ln(MSE) + 2t \text{ RMSE} = \sqrt{(MSE) \text{ MSE}} = (1/N)\Sigma(P - P)^2$$

where MSE is the mean square error; N is the total number of joint observations;  $P$  is the empirical frequency;  $P$  is the theoretical frequency; RMSE is the root mean square error; and t is the number of parameters.

For 2D joint distributions, the empirical frequency is calculated as:  $P(x, y) = (1/N) \times \text{Num}(x \leq x, y \leq y)$

where  $\text{Num}(x \leq x, y \leq y)$  indicates the number of joint observations less than or equal to  $(x, y)$ .

For 3D joint distributions, the empirical frequency is calculated as:  $P(x, y, z) = (1/N) \times \text{Num}(x \leq x, y \leq y, z \leq z)$

where  $\text{Num}(x \leq x, y \leq y, z \leq z)$  indicates the number of joint observations less than or equal to  $(x, y, z)$ .

**2.3.4 Joint Distribution Probability and Return Period** Using  $pf(wet)$  and  $pk(dry)$  values of 37.50% and 62.50%, respectively, as frequency thresholds for classifying annual precipitation or  $ET_0$  into wet, normal, and dry categories, we analyzed interannual differences using the 2D Copula joint distribution model (Yan et al., 2007). We further determined the occurrences of wet, normal, and dry seasons, which can be divided into nine situations (Yan et al., 2007), as shown in Table 4.

**Table 4** Division of occurrences of wet, normal, and dry situations for precipitation- $ET_0$

Situation	Frequency	Precipitation- $ET_0$ type
$p_1$	0.125	Wet-wet
$p_2$	0.125	Wet-normal
$p_3$	0.125	Wet-dry
$p_4$	0.125	Normal-wet
$p_5$	0.125	Normal-normal
$p_6$	0.125	Normal-dry
$p_7$	0.125	Dry-wet
$p_8$	0.125	Dry-normal
$p_9$	0.125	Dry-dry

*Note:  $ET_0$  denotes reference evapotranspiration;  $p_1-p_9$  represent the frequencies of nine situations;  $X$  is the precipitation series;  $Y$  is the  $ET_0$  series;  $pf$  and  $pk$  are frequency values for dividing precipitation or  $ET_0$  into wet and dry, respectively;  $xpf$  and  $xpk$  are threshold values for dividing precipitation into wet and dry, respectively;  $ypf$  and  $ypk$  are threshold values for dividing  $ET_0$  into wet and dry, respectively.*

Given that this study's main aim was to evaluate the risk of decreased yield under wet, normal, or dry conditions in Inner Mongolia's desert steppe, we focused on two joint distribution probabilities of the precipitation- $ET_0$  dry-wet type and their return periods:

$$P(X \leq x \text{ or } Y \geq y) = F(x) + F(y) - F(x,y) \quad P(X \leq x, Y \geq y) = F(x) - F(x,y)$$

where  $G(x, y)$  and  $P(X \leq x \text{ or } Y \geq y)$  refer to the joint distribution probability when at least one of two events occurs ( $X$  not exceeding a certain value or  $Y$  exceeding a certain value);  $G'(x, y)$  and  $P(X \leq x, Y \geq y)$  refer to the joint distribution probability when both events occur simultaneously ( $X$  not exceeding a certain value and  $Y$  exceeding a certain value);  $F(x)$  and  $F(y)$  are marginal distributions for precipitation and  $ET_0$ ;  $F(x, y)$  is the joint probability distribution function for  $X \leq x$  and  $Y \leq y$ ;  $T$  is the joint return period; and  $T'$  indicates the co-occurrence return period when both events occur simultaneously.

### 3 Results

#### 3.1 Marginal Distributions of Precipitation, $ET_0$ , and Yield, and Division of Precipitation and $ET_0$

The time series of precipitation,  $ET_0$ , and yield in DMJB from 2002 to 2020 are shown in Figure 2 [Figure 2: see original paper].  $ET_0$  was generally negatively correlated with precipitation and yield, while precipitation and yield were positively correlated. The difference between precipitation and  $ET_0$  fluctuated between approximately 400.00–900.00 mm/a during the study period, reaching its maximum of 863.66 mm/a in 2009.

**Fig. 2** Relationships between precipitation and reference evapotranspiration ( $ET_0$ ; a), precipitation and yield (vegetation yield; b), and  $ET_0$  and yield (c) from 2002 to 2020

At a significance level of  $\alpha=0.05$ , precipitation,  $ET_0$ , and yield in DMJB conformed to the GAMMA distribution (Table 5), a special form of Pearson type III distribution. When the K-S test significance level was 0.05 ( $n=19$ ), the corresponding quantile was 0.30. The K-S test statistics for precipitation,  $ET_0$ , and yield under the GAMMA distribution were 0.16, 0.11, and 0.17, respectively—all below the corresponding quantile values, with P-values all exceeding 0.05. This indicates that the GAMMA distribution was appropriate and could provide threshold values for classifying precipitation and  $ET_0$  into wet, normal, and dry conditions in DMJB (Table 6).

**Table 5** Parameters of the univariate marginal distributions

Characteristic variable	Marginal distribution function	Parameter	K-S test
		Shape	Scale
Precipitation	GAMMA	$0.04 \times 10^{-2}$	$8.59 \times 10^{-7}$
$ET_0$	GAMMA	$2.15 \times 10^{-2}$	$1.25 \times 10^{-6}$
Yield	GAMMA	$1.87 \times 10^{-2}$	$9.34 \times 10^{-7}$

*Note: WEI, Weibull; GAMMA, gamma; EXP, Exponential; K-S test, Kolmogorov-Smirnov test. The K-S test statistic represents the maximum distance between distributions; smaller values indicate smaller gaps and greater consistency. Larger P-values indicate failure to reject the null hypothesis ( $P>0.05$ ) and greater similarity between distributions.*

**Table 6** Division values for classifying precipitation and  $ET_0$  into wet and dry

Category	Wet (frequency of 37.50%)	Dry (frequency of 62.50%)
Precipitation (mm/a)	312.45	245.69
$ET_0$ (mm/a)	875.30	959.20
Division value	Upper threshold	Lower threshold

### 3.2 Joint Distributions of Precipitation, $ET_0$ , and Yield

The maximum likelihood method calculated Copula function parameters. RMSE between empirical and theoretical frequencies was computed, with minimum AIC as the criterion for selecting optimal fitting functions. Results are shown in Table 7. Since precipitation and yield were negatively correlated with  $ET_0$ , only the Frank Copula function was suitable as their joint function. Precipitation and yield were positively correlated. For the precipitation–yield relationship, the Gumbel-Hougaard Copula function showed the smallest AIC and RMSE values, indicating the best goodness-of-fit. For  $ET_0$ –yield, the Frank Copula function performed best, with AIC and RMSE values of  $-75.53$  and  $0.13$ , respectively.

**Table 7** Parameter values and goodness-of-fit tests of the Copula functions

Relationship of variables	Copula function		AIC	RMSE
Precipitation– $ET_0$	Frank	12.45	$-68.23$	0.15
Precipitation–yield	Gumbel-Hougaard	2.18	$-82.15$	0.11
$ET_0$ –yield	Frank	8.76	$-75.53$	0.13
Precipitation– $ET_0$ –yield	Frank	7.34	$-71.28$	0.18

*Note:* , Copula function parameter; AIC, Akaike information criterion; RMSE, root mean square error. The Copula function with the smallest AIC and RMSE values was selected as the most appropriate probability distribution.

As shown in Figure 3 [Figure 3: see original paper], the empirical and theoretical frequencies of joint distributions for precipitation,  $ET_0$ , and yield in DMJB based on Copula functions were uniformly distributed around the  $45^\circ$  diagonal line, indicating good fit. The best goodness-of-fit occurred for precipitation–yield (Fig. 3c) and  $ET_0$ –yield (Fig. 3d), both with  $R^2$  values of  $0.96$  between empirical and theoretical frequencies, followed by precipitation– $ET_0$  (Fig. 3b) with  $R^2=0.88$ . The precipitation– $ET_0$ –yield relationship showed the lowest  $R^2$  value at  $0.76$  (Fig. 3a). These results demonstrate that the selected Copula functions were reasonable.

**Fig. 3** Empirical frequency and theoretical frequency of joint distributions of precipitation– $ET_0$ –yield (a), precipitation– $ET_0$  (b), precipitation–yield (c), and  $ET_0$ –yield (d)

### 3.3 Joint Distribution Probability of Precipitation, $ET_0$ , and Yield

**3.3.1 2D Joint Distribution Probability** The 2D joint distribution probability contours for precipitation– $ET_0$ , precipitation–yield, and  $ET_0$ –yield, along with actual value distributions during 2002–2020, are plotted in Figure 4 [Figure 4: see original paper]. The patterns in 2D probability values were consistent across all variable pairs. Actual years with precipitation– $ET_0$

joint distribution showing precipitation  $\$ 300.00\text{mm/a}$  and  $ET_0$   $\$ 1000.00\text{mm/a}$  totaled 12 years in DMJB during 2002–2020, accounting for 63.16% of the study period, while the 2D joint distribution probability for precipitation  $\$ 300.00\text{mm/a}$  and  $ET_0$   $\$ 1000.00\text{mm/a}$  was 0.53 (Fig. 4a). For precipitation–yield joint distribution with precipitation  $\$ 300.00\text{mm/a}$  and yield  $\$ 80.00\text{g}/(\text{m}^2\cdot\text{a})$ , actual years totaled 13, accounting for 68.42%–yield joint distribution with  $ET_0$   $\$ 1000.00\text{mm/a}$  and yield  $\$ 80.00\text{g}/(\text{m}^2\cdot\text{a})$ , actual years totaled 13, mainly concentrated in 2002–2016, with a yield in DMJB during 2002–2020. The probability of yield being  $\$ 80.00\text{g}/(\text{m}^2\cdot\text{a})$  was approximately 0.50.

**Fig. 4** Two-dimensional (2D) joint distribution contours of precipitation– $ET_0$  (a), precipitation–yield (b), and  $ET_0$ –yield (c) in DMJB

**3.3.2 Frequency Analysis of 2D Joint Distributions for Wet, Normal, and Dry Conditions of Precipitation– $ET_0$**  Among the nine combinations of wet, normal, and dry conditions, the dry–wet type for precipitation– $ET_0$  was most unfavorable for irrigation scheduling. Figure 5 [Figure 5: see original paper] shows 2D joint distribution contours of precipitation– $ET_0$  and actual precipitation and  $ET_0$  value distributions in DMJB during 2002–2020. Figure 5a indicates the joint distribution probability when at least one of two events occurred (precipitation was dry or  $ET_0$  was wet), while Figure 5b shows the joint probability when both events occurred simultaneously (precipitation was dry and  $ET_0$  was wet). Overall, joint probability patterns were consistent, increasing with higher precipitation and lower  $ET_0$ . The joint distribution probability for the precipitation– $ET_0$  dry–wet type with actual precipitation  $\$ 245.69\text{mm/a}$  or  $ET_0$   $\$ 959.20\text{mm/a}$  was approximately 0.60 during 2002–2020, occurring in 12 years (63.16% of the study period). However, the joint distribution probability for precipitation  $\$ 245.69\text{mm/a}$  and  $ET_0$   $\$ 959.20\text{mm/a}$  was approximately 0.20, occurring in 3 years (15.79% of the study period).

The joint return period for at least one of the two events (dry precipitation or wet  $ET_0$ ) occurring was 2 years, and the co-occurrence return period for both events happening simultaneously was 5 years. These values were calculated from Equation 11 based on the precipitation– $ET_0$  dry–wet type joint distribution probability in DMJB. Water supply–demand imbalance in DMJB is more likely under natural precipitation. Under these conditions, the interval between dry and wet events would be short, and the risk of vegetation water demand not being met would be greater.

**Fig. 5** 2D joint distribution probability contours of precipitation– $ET_0$  with at least one event (precipitation dry or  $ET_0$  wet) occurring (a) and with both events (precipitation dry and  $ET_0$  wet) occurring simultaneously (b)

**3.3.3 3D Joint Distribution Probability** Analysis in Table 7 indicated that the Frank Copula function could be applied to the 3D joint distribution

of precipitation,  $ET_0$ , and yield when precipitation  $\leq u_1$  (where  $u_1$  is precipitation's marginal distribution function),  $ET_0 \leq u_2$  (where  $u_2$  is  $ET_0$ 's marginal distribution function), and yield  $\leq u_3$  (where  $u_3$  is yield's marginal distribution function) all occurred in DMJB.

The 3D joint distribution probability of precipitation,  $ET_0$ , and yield is shown in Figure 6 [Figure 6: see original paper]. The average yield per unit area was  $73.05 \text{ g}/(\text{m}^2 \cdot \text{a})$  in DMJB during 2002–2020. When precipitation was dry and  $ET_0$  was wet (as in 2004, 2005, and 2009), average yield per unit area was low ( $48.74 \text{ g}/(\text{m}^2 \cdot \text{a})$ ), and the 3D joint distribution probability for precipitation  $\$ 245.69\text{mm}/\text{a}$ ,  $ET\{0\}\$ \$ 959.20 \text{ mm}/\text{a}$ , and yield  $\$ 48.74\text{g}/(\text{m}^2 \cdot \text{a})$  was 0.07. When precipitation was dry or  $ET\{0\}\$$  was wet, average yield per unit area was  $68.24 \text{ g}/(\text{m}^2 \cdot \text{a})$ , and the 3D joint distribution probability for precipitation  $\$ 245.69\text{mm}/\text{a}$ ,  $ET\{0\}\$ \$ 959.20 \text{ mm}/\text{a}$ , and yield  $\$ 68.24\text{g}/(\text{m}^2 \cdot \text{a})$  was 0.12. Notably, when precipitation remained stable and  $ET\{0\}\$$  increased, the 3D joint distribution probability that yield would decrease due to water shortage in precipitation– $ET_0$  dry–wet years could reach 0.60–0.70. Additionally, as shown in Figure 6, when  $ET_0$  was below  $900.00 \text{ mm}/\text{a}$ , the 3D joint distribution probability of yield decrease was less than 0.30, indicating that desert steppe yield was sensitive to precipitation and other factors such as temperature, wind speed, and sunshine hours.

**Fig. 6** Three-dimensional (3D) joint distribution probability of precipitation,  $ET_0$ , and yield in DMJB

## 4 Discussion

During crop growth periods, precipitation and  $ET_0$  are primary factors causing agricultural drought. Batsukh et al. (2021) used  $ET_0$  to predict evapotranspiration of specific biological communities in Inner Mongolia's desert steppe when meteorological data were scarce. Han et al. (2015) studied climate and vegetation phenology dynamics in Inner Mongolia's desert steppe, finding that primary productivity decreased from southeast to northwest along precipitation gradients. Precipitation affects vegetation growth when transformed into soil water through canopy interception (Sugita et al., 2015). However, previous studies have been relatively one-dimensional and lack multivariate research on desert steppes (Zhu et al., 2022). In our study, the joint distribution probability for the precipitation– $ET_0$  dry–wet type with actual precipitation  $\$ 245.69\text{mm}/\text{a}$  or  $ET\{0\}\$ \$ 959.20 \text{ mm}/\text{a}$  was approximately 0.60 during 2002–2020.

Building on previous research, we quantitatively analyzed and predicted the 2D and 3D joint risk probabilities of precipitation,  $ET_0$ , and yield in Inner Mongolia's desert steppe. Results showed that for the precipitation– $ET_0$  dry–wet type, the 2D joint distribution probability with actual precipitation  $\$ 245.69\text{mm}/\text{a}$  or  $ET\{0\}\$ \$ 959.20 \text{ mm}/\text{a}$  in DMJB was approximately

0.60, while the probability with precipitation  $\$ 245.69\text{mm}/a$  and  $ET\{0\}\$ 959.20\text{mm}/a$  was about 0.20. The joint return period and co-occurrence return period were recalculated as 2 and 5 years, which could increase grassland degradation in Inner Mongolia (Hu et al., 2019). This result aligns with findings that evapotranspiration would increase further under global warming (Li et al., 2016). The desert steppe was also affected by  $ET_0$  changes (Figs. 3c and 6), consistent with the fact that  $ET_0$  in Inner Mongolia's temperate desert steppe can be well estimated using only meteorological variables and the Penman-Monteith equation (Yang and Zhou, 2011). Previous studies also demonstrated that small precipitation events (0.00–5.00 mm) had limited effects on plant assimilation (e.g., Song et al., 2017). Our results showed that when  $ET_0$  was below 900.00 mm/a, the 3D joint distribution probability of decreased yield was less than 0.30 (Fig. 6), suggesting that desert steppe yield was sensitive to precipitation and other factors such as temperature, wind speed, and sunshine hours (Wang et al., 2022b). Therefore, artificial precipitation, reduced grazing, and other measures are needed in precipitation- $ET_0$  dry-wet years to ensure sustainable desert steppe development, and drought adaptability is essential for grassland species growth.

This study plays an important role in analyzing dry and wet event probabilities during crop growth periods in Inner Mongolia's desert steppe and guiding agricultural drought control. However, our work had some data processing limitations. Vegetation data were limited. Since *Artemisia frigida* and *Stipa krylovii* are dominant species in DMJB, rare varieties were ignored during vegetation collection. Additionally, although suitable Copula functions selected by precipitation,  $ET_0$ , and yield marginal distributions showed good fit (Fig. 3; Table 7), some software precision deviations existed. Future desert steppe studies should consider other factors such as biodiversity and explore feedback mechanisms between soil organisms and the hydrological cycle in arid and semi-arid regions.

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## 5 Conclusions

This study examined yield responses to precipitation and  $ET_0$  in wet and dry years in a typical desert steppe (DMJB) in Inner Mongolia and investigated interactions among precipitation,  $ET_0$ , and yield during 2002–2020. All three variables followed the GAMMA distribution, and joint distribution models for precipitation- $ET_0$ -yield, precipitation- $ET_0$ , and  $ET_0$ -yield established by the Frank Copula function fitted well. The Gumbel-Hougaard Copula function showed the best fit for the precipitation-yield joint distribution model.  $R^2$  values between empirical and theoretical frequencies exceeded 0.70 for all relationships. The 3D joint distribution probability for precipitation  $\$ 245.69\text{mm}/a$ ,  $ET\{0\}\$ 959.20\text{mm}/a$ , and yield  $\$ 48.74\text{g}/(m^2\cdot a)$  was 0.07. Joint distribution probability values for precipitation and  $ET\{0\}\$ both increased with higher precipitation and lower  $ET_0$  in DMJB. Under natural precipitation, DMJB faced relatively high risk of water supply failing to meet$

vegetation water demand, with short intervals between dry and wet events. For the precipitation- $ET_0$  dry-wet type, the 2D joint distribution probability with precipitation  $\$ 245.69mm/a$  or  $ET\{0\}\$ \$ 959.20 mm/a$  in DMJB was approximately 0.60, while the probability with precipitation  $\$ 245.69mm/a$  and  $ET\{0\}\$ \$ 959.20mm/a$  was about 0.20. The joint return period and co-occurrence return period were 2 and 5 years, respectively, for dry-wet years.

Future research should expand to cover all Inner Mongolia grasslands, deduce location-specific patterns through spatial variations, and analyze joint distributions of precipitation,  $ET_0$ , and yield at different locations.

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## References

- Allen R G. 2000. Using the FAO-56 dual crop coefficient method over an irrigated region as part of an evapotranspiration intercomparison study. *Journal of Hydrology*, 229(1-2): 27-41.
- Amirataee B, Montaseri M, Rezaie H. 2020. An advanced data collection procedure in bivariate drought frequency analysis. *Hydrological Processes*, 34(21): 4067-4082.
- Andrés P, Moore J C, Cotrufo F, et al. 2017. Grazing and edaphic properties mediate soil biotic response to altered precipitation patterns in a semiarid prairie. *Soil Biology and Biochemistry*, 113: 263-274.
- Armstrong R N, Pomeroy J W, Martz L W. 2015. Variability in evaporation across the Canadian Prairie region during drought and non-drought periods. *Journal of Hydrology*, 521: 182-195.
- Bai Y F, Wu J G, Xing Q, et al. 2008. Primary production and rain use efficiency across a precipitation gradient on the Mongolia Plateau. *Ecology*, 89(8): 2140-2153.
- Batsukh K, Zlotnik V A, Suyker A, et al. 2021. Prediction of biome-specific potential evapotranspiration in Mongolia under a scarcity of weather data. *Water*, 13(18): 2470, doi: 10.3390/w13182470.
- Beier C, Beierkuhnlein C, Wohlgemuth T, et al. 2012. Precipitation manipulation experiments—challenges and recommendations for the future. *Ecology Letter*, 15(8): 899-911.

- Bittelli M, Ventura F, Campbell G S, et al. 2008. Coupling of heat, water vapor, and liquid water fluxes to compute evaporation in bare soils. *Journal of Hydrology*, 362(3–4): 191–205.
- Cai S H, Song X N, Hu R H, et al. 2021. Spatiotemporal characteristics of agricultural droughts based on soil moisture data in Inner Mongolia from 1981 to 2019. *Journal of Hydrology*, 603: 127104, doi: 10.1016/j.jhydrol.2021.127104.
- Chen L P, Zhao N X, Zhang L H, et al. 2013. Responses of two dominant plant species to drought stress and defoliation in the Inner Mongolia Steppe of China. *Plant Ecology*, 214(2): 221–229.
- Chu C, Havstad K M, Kaplan N, et al. 2014. Life form influences survivorship patterns for 109 herbaceous perennials from six semi-arid ecosystems. *Journal of Vegetation Science*, 25(4): 947–954.
- Du N, Li W, Qiu L, et al. 2020. Mass loss and nutrient release during the decomposition of sixteen types of plant litter with contrasting quality under three precipitation regimes. *Ecology and Evolution*, 10(7): 3367–3382.
- Fang J Y, Bai Y F, Li L H, et al. 2016. Scientific basis and practical ways for sustainable development of China's pasture regions. *Chinese Science Bulletin*, 61(2): 155–164.
- Guo Q, Hu Z M, Li S G, et al. 2012. Spatial variations in aboveground net primary productivity along a climate gradient in Eurasian temperate grassland: Effects of mean annual precipitation and its seasonal distribution. *Global Change Biology*, 18(12): 3624–3631.
- Han F, Zhang Q, Buyantuev A, et al. 2015. Effects of climate change on phenology and primary productivity in the desert steppe of Inner Mongolia. *Journal of Arid Land*, 7(2): 251–263.
- Hao Z C, Singh V P. 2015. Drought characterization from a multivariate perspective: A review. *Journal of Hydrology*, 527: 668–678.
- Hu X X, Mitsuru H, Wu Y N, et al. 2019. Responses in gross primary production of *Stipa krylovii* and *Allium polyrhizum* to a temporal rainfall in a temperate grassland of Inner Mongolia, China. *Journal of Arid Land*, 11(6): 824–836.
- Jung M, Reichstein M, Ciais P, et al. 2010. Recent decline in the global land evapotranspiration trend due to limited moisture supply. *Nature*, 467: 951–954.
- Lazoglou G, Anagnostopoulou C. 2019. Joint distribution of temperature and precipitation in the Mediterranean, using the Copula method. *Theoretical and Applied Climatology*, 135(3–4): 1399–1411.
- Legesse T G, Qu L P, Dong G, et al. 2022. Extreme wet precipitation and mowing stimulate soil respiration in the Eurasian meadow steppe. *Science of the Total Environment*, 851(1): 158130, doi: 10.1016/j.scitotenv.2022.158130.

- Li C, Singh V P, Mishra A K. 2013. A bivariate mixed distribution with a heavy-tailed component and its application to single-site daily rainfall simulation. *Water Resources Research*, 49(2): 767–789.
- Li F, Zhao W Z, Liu H. 2013. The response of aboveground net primary productivity of desert vegetation to rainfall pulse in the temperate desert region of northwest China. *PLoS ONE*, 8(9): e73003, doi: 10.1371/journal.pone.0073003.
- Li H D, Wang A Z, Yuan F H, et al. 2016. Evapotranspiration dynamics over a temperate meadow ecosystem in eastern Inner Mongolia, China. *Environmental Earth Sciences*, 75: 978, doi: 10.1007/s12665-016-5786-z.
- Li M, Fu Q, Singh V P, et al. 2017. An intuitionistic fuzzy multi-objective non-linear programming model for sustainable irrigation water allocation under the combination of dry and wet conditions. *Journal of Hydrology*, 555: 80–84.
- Mohammadpour O, Hassanzadeh Y, Khodadadi A, et al. 2012. Probabilistic risk analysis of flood events using trivariate copulas. *Theoretical and Applied Climatology*, 111: 341–360.
- Sala O E, Lauenroth W K. 1982. Small rainfall events: An ecological role in semiarid regions. *Oecologia*, 53(3): 301–304.
- Sklar A. 1959. Dimension distribution functions and their margins. *Statistical Institute of the University of Paris*, 8: 229–231. (in French)
- Song S, Singh V P. 2010. Frequency analysis of droughts using the Plackett copula and parameter estimation by genetic algorithm. *Stochastic Environmental Research & Risk Assessment*, 24(5): 783–805.
- Song Y F, Guo Z X, Lu Y J, et al. 2017. Pixel-level spatiotemporal analyses of vegetation fractional coverage variation and its influential factors in a desert steppe: A case study in Inner Mongolia, China. *Water*, 9(7): 478, doi: 10.3390/w9070478.
- Sugita M, Yoshizawa S, Byambakhuu I. 2015. Limiting factors for nomadic pastoralism in Mongolian steppe: A hydrologic perspective. *Journal of Hydrology*, 524: 455–467.
- Vo Q T, So J M, Bae D H. 2020. An integrated framework for extreme drought assessments using the natural drought index, Copula and  $G_i^*$  Statistic. *Water Resources Management*, 34(3): 1353–1368.
- Wang S N, Li R P, Wu Y J, et al. 2022a. Vegetation dynamics and their response to hydrothermal conditions in Inner Mongolia, China. *Global Ecology and Conservation*, 34: e02034, doi: 10.1016/j.gecco.2022.e02034.
- Wang S N, Li R P, Wu Y J, et al. 2022b. Effects of multi-temporal scale drought on vegetation dynamics in Inner Mongolia from 1982 to 2015, China. *Ecological Indicators*, 136: 108666, doi: 10.1016/j.ecolind.2022.108666.

- Wang X X, Pan X B, Gu S H, et al. 2015. Trend in reference crop evapotranspiration and meteorological factors affecting trends in Inner Mongolia. *Transactions of the Chinese Society of Agricultural Engineering*, 31(1): 142–152. (in Chinese)
- Wu Z, Lin Q, Lu G, et al. 2015. Analysis of hydrological drought frequency for the Xijiang River Basin in South China using observed streamflow data. *Natural Hazards*, 77(3): 1655–1677.
- Xie H, Luo Q, Huang J. 2012. Synchronous asynchronous encounter analysis of multiple hydrologic regions based on 3D copula function. *Advances in Water Science*, 23(2): 186–193. (in Chinese)
- Xu K, Yang D, Xu X, et al. 2015. Copula based drought frequency analysis considering the spatio-temporal variability in Southwest China. *Journal of Hydrology*, 527: 630–640.
- Yan B W, Guo S L, Xiao Y. 2007. Synchronous–asynchronous encounter probability of rich–poor precipitation between water source area and water receiving areas in the Middle Route of South–to–North Water Transfer Project. *Journal of Hydraulic Engineering*, 38(10): 1178–1185. (in Chinese)
- Yan Z Q, Qi Y C, Dong Y S, et al. 2018. Precipitation and nitrogen deposition alter litter decomposition dynamics in semiarid temperate steppe in Inner Mongolia, China. *Rangeland Ecology and Management*, 71(2): 220–227.
- Yang F L, Zhou G S. 2011. Characteristics and modeling of evapotranspiration over a temperate desert steppe in Inner Mongolia, China. *Journal of Hydrology*, 396(1–2): 139–147.
- Yu S Q, Mo Q F, Li Y W, et al. 2019. Changes in seasonal precipitation distribution but not annual amount affect litter decomposition in a secondary tropical forest. *Ecology and Evolution*, 9(19): 11344–11352.
- Yuan Z Y, Chen H. 2009. Global scale patterns of nutrient resorption associated with latitude, temperature and precipitation. *Global Ecology and Biogeography*, 18(1): 11–18.
- Zhan X, Li L, Cheng W. 2007. Restoration of *Stipa krylovii* steppes in Inner Mongolia of China: Assessment of seed banks and vegetation composition. *Journal of Arid Environments*, 68(2): 298–307.
- Zhang F, Zhou G, Wang Y, et al. 2012. Evapotranspiration and crop coefficient for a temperate desert steppe ecosystem using eddy covariance in Inner Mongolia, China. *Hydrological Processes*, 26(3): 379–386.
- Zhang Q, Xiao M, Singh V P. 2015. Uncertainty evaluation of copula analysis of hydrological droughts in the East River basin, China. *Global & Planetary Change*, 129(7): 1–9.
- Zhao N X, Gao Y B, Wang J L, et al. 2006. Genetic diversity and population differentiation of the dominant species *Stipa krylovii* in the Inner Mongolia steppe. *Biochemical Genetics*, 44(11–12): 513–526.

Zhu Y, Yu K L, Wu Q, et al. 2022. Seasonal precipitation and soil microbial community influence plant growth response to warming and N addition in a desert steppe. *Plant and Soil*, 258: 1–15.

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