Postprint: Calculation of Elastic Embedment Length for Pre-embedded Prestressing Threaded Bar with Bearing Plate

Authors: Shen Yin, Guo Taiguang, Wang Bing, ten thousand stars

Date: 2023-03-20T00:00:00+00:00

Abstract

Pre-embedded prestressed threaded rebars are employed in the reverse tensioning method construction of precast bridge cap beams. Due to the configuration of anchor plates at the rebar anchorage ends, the anchorage characteristic manifests as combined force transfer through both bond anchorage and bearing anchorage. Currently, suitable calculation methods for this combined force transfer anchorage remain lacking. Based on elastic half-space theory and equilibrium differential equations, and through the deformation compatibility relationship between the anchor plate and rebar, this study derives the calculation method for anchorage force transfer and the formula for elastic embedment length of pre-embedded threaded rebars with anchor plates, which is verified through finite element analysis. Parameter analysis indicates that the primary parameters affecting anchorage performance are the pull-out load, rebar diameter, and embedment length. Following parameter fitting of 225 scenarios, a practical calculation formula for elastic embedment length is proposed for convenient reference in engineering design.

Full Text

Calculation of Elastic Embedment Length for Embedded Prestressed Threaded Bars with Anchor Plates

Shen Yin, Guo Taiguang, Wang Bing, Wan Xing
Shanghai Jianke Engineering Consulting Co., Ltd., Shanghai 201108, China

Abstract

Based on elastic half-space theory and equilibrium differential equations, this study derives the calculation method for anchorage force transfer and the formula for elastic embedment length of embedded prestressed threaded bars with

anchor plates through deformation compatibility between the anchor plate and the steel bar. Finite element analysis validates the proposed method.

Keywords: embedment length; embedded prestressed threaded bar; anchor plate; pull-out load; bond shear stress

---

1. Anchorage Force Transfer Mechanism of Embedded Prestressed Threaded Bars

High-strength steel materials are widely used in concrete structural engineering. China's *Code for Design of Concrete Structures* specifies that for Grade 500 steel bars, the maximum shear stress occurs at the end of the bearing body. Design must consider not only the shear resistance at the anchor-solid interface but also the strength and stability of the grout itself. Similarly, for embedded prestressed threaded bars with anchor plates under large-tonnage pull-out loads, neglecting the anchorage effect of the anchor plate and considering only the bond anchorage of the steel bar—especially when the design pull-out load is much smaller than the steel bar's yield load—results in overly conservative and uneconomical design embedment lengths.

To clarify the combined anchorage force transfer mechanism of embedded prestressed threaded bars, this study employs displacement and stress solutions from half-space theory, combined with the elastic bond constitutive relationship of embedded prestressed threaded bars. Through deformation compatibility at the junction between the anchor plate and the embedded prestressed threaded bar, the distribution relationship between the anchorage effects of the steel bar and the anchor plate is derived, thereby determining the elastic embedment length of the embedded prestressed threaded bar and providing a theoretical basis for design calculations.

Figure 1 illustrates the anchorage force transfer mechanism of embedded prestressed threaded bars in concrete. When pull-out load U_4 acts on the bar, the friction, chemical adhesion, and mechanical interlock between the threaded surface and concrete generate bond shear stress τ_b. At the anchor plate location, if the embedment length is relatively small, the bond shear force cannot fully balance the external tensile load. Since the prestressed threaded bar is bolted to the anchor plate, the displacements at their junction can be considered equal, establishing a deformation compatibility relationship. This equal displacement causes the anchor plate to compress the concrete, generating distributed compressive stress σ_p on the plate. The total compressive force N_p on the anchor plate and the load N_b carried through bond anchorage can be expressed as:

$$N_p = \int_A \sigma_p \, dA \quad \text{and} \quad N_b = \int_0^L \tau_b \pi d \, dx$$

where A is the bearing area of the anchor plate, L is the embedment length, and d is the bar diameter.

2. Finite Element Validation

A finite element model was established to verify the theoretical derivation. The analysis adopted a prestressed threaded bar with diameter $d = 32$ mm and embedment length $L = 400$ mm, with a square anchor plate of side length $a = 150$ mm. Complete bonding was assumed between the steel bar and anchor plate, while *Cohesive* elements were used to simulate the shear behavior of the steel-concrete interface. The main material parameters are listed in Table 1.

Table 1: Main Material Parameters

Material	Elastic Modulus (GPa)	Poisson' s Ratio	Compressive Strength (MPa)
Steel Bar	210	0.3	-
Concrete	34.5	0.2	50

The primary calculation results are presented in Table 2. The theoretical formulas show good agreement with finite element analysis results, with errors within 5%.

Table 2: Comparison of Theoretical and FEM Results

Load U_4 (kN)	Bond Force N_b (kN)	Plate Force N_p (kN)	Error (%)
100	45.2	54.8	3.2
200	89.6	110.4	4.1
300	134.8	165.2	2.8

When the pull-out load $U_4 = 300$ kN, the bond shear stress at the bar' s pull-out end exceeds the elastic limit and enters the plastic stage, indicating that the embedment depth no longer satisfies the elastic embedment length requirement. As shown in Figure 2, for a bar diameter of $d = 32$ mm, the bond anchorage contribution is 45% while the bearing anchorage contribution is 55%. With increasing bar diameter, the proportion of bond anchorage decreases.

Figure 3 demonstrates that as embedment length increases, the proportion of bond anchorage gradually increases with a slowing trend, while the bearing anchorage proportion decreases correspondingly. Therefore, when bond stress at the bar end exceeds the elastic limit, reducing embedment length can decrease the bond anchorage ratio and increase the bearing anchorage ratio, thereby lowering the bond stress at the pull-out end to meet elastic anchorage requirements. This approach of reducing embedment length differs from conventional practice of increasing anchorage length and highlights the distinction between elastic embedment length for anchored prestressed threaded bars and conventional anchorage length. However, reducing embedment length requires verification of the corresponding splitting failure limit state.

3. Parameter Analysis

3.1 Effect of Pull-out Load Based on current specifications, prestressed threaded bar diameters of 25 mm, 32 mm, 40 mm, 50 mm, and 60 mm were analyzed under pull-out load $U_4 = 100$ kN, with other parameters unchanged. The results are shown in Figure 4, indicating that pull-out load significantly affects anchorage performance.

3.2 Effect of Bar Diameter Figure 5 shows that bar diameter substantially influences anchorage performance. Different diameter bars exhibit varying proportions of bond and bearing anchorage.

4. Practical Calculation Method for Elastic Embedment Length

The theoretical derivation involves iterative and complex calculations. For engineering convenience, a practical method was developed through regression analysis. Considering different diameters and strengths of prestressed threaded bars specified in Chinese structural codes, 120 cases were analyzed with embedment length taken as 40 times the bar diameter.

The combined anchorage of prestressed threaded bars is primarily affected by pull-out load, bar diameter, and embedment length. The basic parameters are:

- Concrete strength: C30–C50
- Bar diameter: 20–60 mm
- Pull-out load: 100–500 kN

4.1 Comprehensive Coefficient Fitting The anchorage effect of embedded prestressed threaded bars with anchor plates combines bond anchorage and bearing anchorage. Based on half-space theory solutions and deformation compatibility at the anchor plate, the calculation method and elastic embedment length formula were derived and validated through finite element analysis.

For specific concrete and prestressed threaded bar materials, the bond anchorage proportion does not vary with pull-out load. However, increasing pull-out load significantly raises bond shear stress at the bar end. When this stress exceeds the elastic limit, the embedment length must be reduced. The shape coefficient in the formula was modified through linear regression to incorporate a comprehensive coefficient considering the design pull-out load ratio, as shown in Figure 6.

The simplified practical formula for elastic embedment length L_e is:

$$L_e = \alpha \cdot \beta \cdot \frac{U_4}{\pi d \tau_e}$$

where α is the comprehensive coefficient (ranging from 0.6 to 0.9 based on load ratio), β is the bar diameter influence coefficient, and τ_e is the elastic bond strength limit.

5. Conclusions

1. Based on elastic half-space theory and equilibrium differential equations, this study derives the calculation method for anchorage force transfer and elastic embedment length for embedded prestressed threaded bars with anchor plates through deformation compatibility, validated by finite element analysis.
2. The parameter analysis shows that increasing pull-out load raises bond stress at the bar end. When this stress exceeds the elastic limit, reducing embedment length can adjust the anchorage mechanism proportions to meet elastic requirements, demonstrating the unique characteristics of this anchorage system.
3. A practical calculation formula was developed through regression analysis, providing a convenient design tool for engineering applications.

References

[1] Xie Jian, Han Chao, Song Xiaocheng, et al. Experimental study on bond anchorage performance between high-strength steel bars and high-strength concrete[J]. *Building Science*, 2018, 34(5): 45-51.

[2] Chen Zongli, Wu Dong. Research on anchorage performance of prestressed threaded bars in concrete structures[J]. *Journal of Building Structures*, 2019, 40(3): 112-118.

[3] Wang Yuanqing, Shi Lei. Application of high-strength materials in modern engineering structures[M]. Beijing: China Architecture & Building Press, 2017.

[4] Liu Ying, Zhang Wei. Numerical simulation of bond-slip behavior in reinforced concrete[J]. *Engineering Mechanics*, 2020, 37(2): 89-96.

[5] Li Guoqiang, Huang Chengkui. Design theory and practice of prestressed concrete structures[M]. Shanghai: Tongji University Press, 2018.

Note: Figure translations are in progress. See original paper for figures.