Out-of-Plane Static Compression Properties of Open-Cell Paper Honeycomb Cores

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Abstract

Focusing on the out-of-plane static compression characteristics of paper honeycomb cores with double-layer perforated side walls, this study investigates the compressive deformation process of non-perforated and perforated paper honeycomb cores at 85% out-of-plane compression through experimental analysis, examining the effects of relative humidity and circular hole diameter on their strength properties. Based on the symmetry of honeycomb structures and utilizing two basic unit cell models, the honeycomb cell walls under out-of-plane loading are treated as non-perforated and perforated simply supported elastic thin plates, from which predictive formulas for elastic buckling critical stress and plastic buckling plateau stress are derived. The results demonstrate good agreement between theoretical calculations and experimental results. Combining the orthotropic properties of paper substrates with the out-of-plane compression characteristics of paper honeycomb cores, finite element models for non-perforated and perforated paper honeycomb are established to simulate the bending deformation evolution process of honeycomb cell walls during elastic and plastic buckling stages, which is consistent with the deformation process observed in static compression experiments.

Full Text

Out-of-Plane Static Compression Properties of Open-Cell Paper Honeycomb Cores

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Abstract: To investigate the out-of-plane static compression properties of paper honeycomb cores with holes in the double-layer side walls, experiments were conducted to analyze the compression deformation processes of paper honeycomb cores without holes and with holes when the out-of-plane compression amount was 85%, and the effects of relative humidity and circular-hole diameter on their strength performance were studied. Based on the symmetry of the honeycomb structure, using two basic unit models, the honeycomb cell walls bearing out-of-plane loading were regarded as simply supported elastic thin plates without holes and with holes, and prediction formulas for the elastic buckling critical stress and the plastic buckling plateau stress were derived. The results show that the theoretical calculations agree well with the experimental results. Combining the orthotropic properties of the paper substrate and the out-of-plane compression properties of the paper honeycomb core, finite element models of paper honeycombs without holes and with holes were established to simulate the evolution of the bending and folding deformation of the honeycomb cell walls during the elastic and plastic buckling stages; the deformation process observed in the static compression tests was consistent with the simulation.

Keywords: open-cell paper honeycomb core; static compression; relative humidity; critical stress; plateau stress

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Out-of-plane static compression of paper honeycomb core layer with hole

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Abstract: In view of the out-of-plane static compression characteristics of paper honeycomb core layer with hole on double-layer cell walls, the compression deformation processes of paper honeycomb core layer without and with hole when the compression amount is 85% are investigated by experiments, and the effects of relative humidity and pore diameter on its strength performance are studied. Based on the symmetry of honeycomb structure and using two element models, the honeycomb cell walls loaded along the out-of-plane direction are regarded as simply-supported elastic plate without and with hole. The prediction formulas of the critical stress of elastic buckling and plastic plateau stress are derived. The results show that analytical calculations are in good agreement with the experimental results. Combining the orthotropic properties of the paper sheet and the out-of-plane compressive feature of paper honeycomb core layer, the finite element models of paper honeycomb without and with hole are established. The simulation of the bending and folding deformation evolution processes for honeycomb cell walls during the elastic-plastic buckling stages is

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sion of paper honeycomb core layer with hole[J]. Chinese Journal of Applied Mechanics, 2023, 40(1): 189-198.

consistent with the deformation processes observed in the static compression experiment.

Key words: paper honeycomb core layer with hole; static compression; relative humidity; critical stress; plateau stress

Honeycombs, owing to their excellent light weight, impact resistance, and energy-absorption capability, have extremely important engineering application value[^1-^2]. Therefore, paper honeycomb structures have been widely used in the protective packaging of military and civilian products[^3]. PEHLIVAN et al.[^4] experimentally analyzed the out-of-plane compressive properties of carbon-fiber composite honeycombs and found that regular hexagonal honeycombs have superior compression performance and compression efficiency compared with square and circular honeycombs. The out-of-plane compressive characteristics of regular hexagonal paper honeycomb structures are mainly determined by the properties of the core layer, and the key influencing factors include the grammage of the honeycomb core paper, cell side length and thickness, adhesive, and environmental humidity. With respect to the influence of environmental humidity, studies by GUO et al.[^5], NAVARANJAN et al.[^6], and Wang Zhiwei et al.[^7] show that, under medium- and high-humidity conditions, the strength properties and elastic modulus of corrugated paperboard and paper honeycomb board decrease significantly. LEPPÄNEN et al.[^8] used an elastic-plastic model to analyze the rheological properties of paperboard under different humidity conditions. BOSCO et al.[^9] proposed a progressive homogenization model of the microstructure of paper fibers to predict the mechanical behavior of paper sheets under coupled humidity-force conditions. The above studies indicate that the inherent moisture absorption of paper-based materials makes honeycombs relatively sensitive to the medium and high humidity in logistics transportation and service environments. Moreover, the adhesive used in paper honeycomb boards contains a certain amount of water, which readily causes warping deformation and strength reduction of the honeycomb paperboard, and may even lead to dew-like phenomena. To solve practical engineering problems, drawing on the perforation design of current honeycomb structures and paper packaging, a perforated paper honeycomb core structure is proposed, and its static compressive strength is studied. WANG et al.[^10] proposed perforating the side faces of hexagonal aluminum honeycombs to promote gas flow inside the cells in both in-plane and out-of-plane directions, thereby improving the heat dissipation performance of aluminum honeycombs. Jia Lianguang et al.[^11] effectively avoided brittle fracture at beam-column joints by perforating the webs while ensuring the energy-dissipation capacity of honeycomb steel structures. FADIJI et al.[^12] studied the compressive strength, optimum perforation position, and ventilation characteristics of perforated, ventilated corrugated-paperboard packaging. NAVICKAITE et al.[^13] analyzed the heat-transfer performance of corrugated tubes by taking advantage of the periodic one-way ventilation and communication with the external air in corrugated structures. Zhao Junxia et al.[^14] proposed a design method for perforating the double-layer side walls of paper honeycomb cores, improving the channels for internal and external airflow while ensuring structural strength, so as to increase the drying rate and effectiveness of honeycomb paperboard.

With respect to strength calculation for honeycomb structures, Liang Sen et al.[^15] transformed the honeycomb core layer under out-of-plane loading into a rectangular thin plate with four simply supported edges, established a complete theoretical analysis model for regular hexagonal honeycomb sandwich cells, and derived a formula for calculating the critical stress of honeycomb sandwich core cells. Du Yixian et al.[^16] used an energy homogenization method to establish an optimization model based on the macroscopic mechanical properties of a mesoscopic lattice structure, and carried out analytical analysis and experimental verification of the calculation method for the shear modulus of a regular hexagonal honeycomb. Tan et al.[^17] theoretically calculated the equivalent elastic parameters of an aluminum honeycomb sandwich structure and the stress and deformation under uniformly distributed loading. On the basis of the symmetry of honeycomb structures, researchers have used a Y-shaped structure composed of two basic folded elements with 120° angles as a representative unit. WIERZBICKI and CHEN[^18–^19] theoretically analyzed the buckling, folding, collapse, and strength properties of hexagonal metal honeycombs with one set of double-layer cell walls in the cells, and systematically studied the super-folding element method and simplified folding-element method. E et al.[^20] treated the initial shape of paper honeycomb cells as a Y-shaped element and used the super-folding element method to derive an analytical formula for the plateau stress of paper honeycombs under out-of-plane compression. Yin Hanfeng et al.[^21] used the Chen–Wierzbicki simplified basic folding-element method to theoretically analyze the out-of-plane average compressive stress of three common honeycomb structures, and the results of LS-DYNA nonlinear finite-element simulations agreed well with the theoretical results. BAI et al.[^22] regarded the paper honeycomb sandwich layer as an ideal rigid–plastic model and analyzed the plateau stress and energy-absorption characteristics at different strain rates. SOROHAN et al.[^23] studied a homogenization model of honeycomb structures; the results showed that, for an equivalent orthotropic homogeneous solid, finite-element analysis could establish the stress distribution in the nodes and verify analytical relations. Wang Xuan et al.[^24] used the ABAQUS/Explicit solver to simulate and analyze the quasi-static lateral compression performance and progressive damage characteristics of a honeycomb sandwich core structure with perforation damage in a single face sheet. Xia Yuanming et al.[^25], through quasi-static compression tests and ABAQUS finite-element analysis, studied the compressive deformation and strength characteristics of metal honeycombs with perforations in single and double cell walls. Zhang Xinchun et al.[^26] numerically studied the influence of localized defects (cell-wall loss) on the in-plane impact performance of triangular and quadrilateral honeycomb materials;

the results showed that, as the defect rate increased, the plateau stress and energy-absorption capacity of the specimens gradually decreased. Guo Chunxia et al.[^27] studied the fracture toughness and energy-absorption performance of regular-sinusoidal Poisson' s-ratio honeycomb beams under in-plane three-point bending; the results showed that the energy-absorption performance of negative-Poisson' s-ratio beams was superior to that of conventional square-cell honeycomb beams, but as the negative-Poisson' s-ratio effect increased, their energy-absorption performance gradually decreased.

In view of the practical engineering problems of honeycomb paperboard in logistics transportation, service environments, and production processes, and in combination with the drying process of paper sheets, this paper draws on existing perforation methods for honeycomb structures and paper packaging, and proposes a perforated paper honeycomb core structure. Perforations are designed in the double-layer cell walls of the paper honeycomb core layer to improve the communication channels for internal and external airflow, so as to increase the drying rate and effectiveness of honeycomb paperboard while ensuring its out-of-plane static compressive strength. For the proposed perforated paper honeycomb core structure, this paper theoretically analyzes and experimentally studies its out-of-plane static compressive characteristics, and uses tensile tests to measure

under different relative humidity conditions, the elastic modulus, buckling strength, fracture tensile force, and tensile strength of single-layer honeycomb cell walls and double-layer honeycomb cell walls before and after bonding were measured; the critical load and plastic plateau stress of the elastic buckling of open-cell paper honeycomb cores under different humidity conditions were analytically calculated; and the compressive deformation behavior of paper honeycomb cores after perforation was analyzed by simulation, providing a basis for the perforation design of paper honeycomb cores and for the drying process of honeycomb paperboard.

1 Physical-Property Testing of Honeycomb Core Paper

1.1 Test Method

Honeycomb core paper is a multiphase and heterogeneous paper-fiber composite material. To compare the transverse and longitudinal properties of the honeycomb core paper, and to consider the differences between single-layer honeycomb cell walls and double-layer honeycomb cell walls before and after bonding, the tensile test method was used to determine its elastic modulus, buckling strength, fracture tensile force, and tensile strength, thereby providing material parameters for analytical calculations and finite-element analysis. The selected honeycomb core paper had an average thickness of 0.255 mm and an average grammage of 150 g/m^2. Specimens with a width of (15 ± 0.1) mm and a length

Fig. 1

Figure 1: Fig. 1

of 250 mm were cut along the machine direction (MD) and cross direction (CD) of the honeycomb core paper. Single-layer honeycomb core paper (i.e., single-layer honeycomb cell wall), double-layer honeycomb core paper (i.e., unbonded double-layer honeycomb cell wall), and adhesive-bonded double-layer honeycomb cell walls were prepared and denoted by the symbols S, D, and DA, respectively. Before testing, in accordance with GB/T 10739, "Paper, board and pulps—Standard atmosphere for conditioning and testing," the specimens were conditioned for 6 h at a temperature of 23 °C and relative humidities of 50%, 60%, 70%, and 80%, respectively. With reference to GB/T 12914, "Paper and board—Determination of tensile properties," a D2-5DL electronic high-temperature testing machine was used to stretch the specimens at a quasi-static constant rate of 5 mm/min, and the tensile force–deformation curves were measured.

1.2 Analysis of the Physical Properties of Honeycomb Core Paper

The tensile force–deformation curves of the four types of honeycomb core paper specimens under different relative humidity conditions are shown in Fig. 1. S-CD denotes single-layer honeycomb core paper for which the tensile direction is perpendicular to the fiber direction; S-MD denotes single-layer honeycomb core paper for which the tensile direction is the same as the fiber direction; D-MD denotes double-layer honeycomb core paper for which the tensile direction is the same as the fiber direction; and DA-MD denotes adhesive-bonded double-layer honeycomb cell walls for which the tensile direction is the same as the fiber direction. From the curves, it can be seen that the honeycomb core paper is not a completely linear-elastic material; its in-plane mechanical behavior consists of a linear part and a nonlinear part. Table 1 gives the average values of the elastic modulus, buckling strength, fracture tensile force, and tensile strength of the honeycomb core paper calculated from the tensile force–deformation curves. Here, tensile strength refers to the maximum tensile force that the honeycomb core paper can withstand before fracture per unit width. The experimental analysis results show that, under the four relative humidity conditions, the tensile strength of specimen D-MD in the longitudinal fiber direction is 1.84–2.06 times that of specimen S-MD; the elastic modulus, buckling strength, and tensile strength of the single- and double-layer honeycomb cell walls all decrease as humidity

(a) S-CD

(b) S-MD

(c) D-MD

(d) DA-MD

Fig. 1 Tensile force and deformation curves of core paper at different relative humidity

increases.

increase and then decrease. Under the same humidity conditions, the elastic moduli in the longitudinal fiber direction of the single-layer and double-layer honeycomb walls are relatively close, while the buckling strength and tensile strength increase with the number of honeycomb-wall layers. The elastic modulus of the honeycomb core paper in the longitudinal fiber direction is significantly higher than that in the transverse direction. The bonding effect of the adhesive on the double-layer paper markedly improves the buckling strength of the honeycomb core paper, whereas its enhancement of tensile strength is relatively small.

Table 1 Physical performances of honeycomb core paper at different relative humidity

Tab. 1 Physical performances of honeycomb core paper at different relative humidity

Relative humidity / %	Specimen type	Elastic modulus / MPa	Buckling strength / ($kN \cdot m^{-1}$)	Fracture load / N	Tensile strength / ($kN \cdot m^{-1}$)
50	S-CD	1 127.46	0.51	30.40	2.03
	S-MD	3 088.71	1.48	78.66	5.24
	D-MD	2 808.71	1.85	157.85	10.52
	DA-MD	2 744.57	3.37	179.11	11.94
60	S-CD	1 112.29	0.61	28.14	1.88
	S-MD	2 897.20	1.38	75.21	5.01
	D-MD	2 743.90	1.57	154.70	10.31
	DA-MD	2 697.59	3.06	169.72	11.32
70	S-CD	990.71	0.51	23.65	1.68
	S-MD	2 263.08	1.15	67.98	4.53
	D-MD	2 240.82	1.35	125.20	8.35
	DA-MD	2 231.57	2.24	139.47	9.30
80	S-CD	803.40	0.31	20.68	1.38
	S-MD	1 735.26	0.97	61.38	4.09
	D-MD	2 000.95	1.19	113.84	7.59
	DA-MD	1 834.80	2.04	117.74	7.85

2 Strength of Paper Honeycomb Core Layers without and with Perforations

2.1 Out-of-plane stress analysis of paper honeycomb core layers without perforations

2.1.1 Critical stress for elastic buckling During out-of-plane compression of a paper honeycomb core layer, the paper honeycomb wall—that is, the honeycomb core paper—is usually approximated as a unidirectionally and uniformly compressed orthotropic simply supported thin plate[15]. In this case, the critical stress for elastic buckling of the thin plate, σ_{cr}, is

$$\sigma_{\text{cr}} = K \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{a}\right)^2 \tag{1}$$

where K is the buckling coefficient or stability coefficient. For an elastic thin plate with the same aspect ratio, when orthotropic properties degenerate into isotropic properties, the critical stress will increase. Considering the properties of the paper substrate and the influence of the plate aspect ratio on the critical stress of a rectangular plate, the stability coefficient should be appropriately reduced[6,20,28], and is taken as 1.80; E is the elastic modulus, corresponding to the tested values of the elastic modulus for specimen D-MD in Table 1 at humidities of 50%, 60%, 70%, and 80%; ν is Poisson' s ratio[29], taken as 0.2; t is the thickness of the simply supported thin plate, i.e., the paper thickness, 0.255 mm; and a is the width of the simply supported thin plate, i.e., the side length of the honeycomb cell, 8.66 mm.

The paper honeycomb core layer is divided into a series of rectangular or triangular elements, as shown in Fig. 2. One of these elements is selected as the Y-shaped basic unit. The total compressive load borne by the Y-shaped basic unit is F. According to Eq. (2), the compressive stress value σ is calculated, which is the critical stress for elastic buckling of the paper honeycomb core layer without perforations ($d_{\text{hole}} = 0$ mm). The calculation results are shown in Table 2.

$$\sigma = \frac{F}{A} = \frac{4 \cdot \frac{a}{2} \cdot t \cdot \sigma_{\text{cr}}}{\frac{3a}{2} \cdot \frac{\sqrt{3}a}{2}} = \frac{8}{3\sqrt{3}} \left(\frac{t}{a}\right) \sigma_{\text{cr}} \tag{2}$$

Fig. 2 Paper honeycomb core layer without perforation and Y-Shape basic unit

Table 2 Critical stress of paper honeycomb core layer

Tab. 2 Critical stress of paper honeycomb core layer

Relative humidity / %	Theoretical value / MPa d_{hole} = 0 mm	Theoretical value / MPa d_{hole} = 2 mm	Theoretical value / MPa d_{hole} = 3 mm	Experimental value / MPa d_{hole} = 0 mm	Experimental value / MPa d_{hole} = 2 mm	Experimental value / MPa d_{hole} = 3 mm
50	0.170	0.162	0.151	0.169	0.161	0.152
60	0.166	0.158	0.148	0.164	0.159	0.146
70	0.136	0.129	0.121	0.140	0.129	0.122
80	0.121	0.118	0.110	0.123	0.114	0.107

2.1.2 Plateau stress for plastic buckling According to the energy-balance relation[21], the work done by the out-of-plane quasi-static average compressive force is equated to the sum of the extended bending energy and the horizontal hinge-line energy of the Y-shaped basic unit during compression and folding. The energy-balance relationship for the paper honeycomb core layer during out-of-plane uniform compression is obtained as

$$p_{\mathrm{m}}\delta_{\mathrm{e}}2h = U_{\mathrm{m,B}} + U_{\mathrm{b,B}} \tag{3}$$

$$U_{\mathrm{m,B}} = 4 \cdot \frac{1}{2}\sigma_{\mathrm{ys}}th^2 = 2\sigma_{\mathrm{ys}}th^2 \tag{4}$$

$$\begin{aligned} U_{\mathrm{b,B}} &= \frac{\pi}{2}\left[8 \cdot \frac{\sigma_{\mathrm{ys}}t^2}{4} \cdot \frac{a}{2} + 4 \cdot \frac{\sigma_{\mathrm{ys}}(2t)^2}{4} \cdot \frac{a}{2}\right] \\ &= 6\pi M_0 a \end{aligned} \tag{5}$$

where: p_{m} is the out-of-plane quasi-static mean compressive force, N; δ_{e} is the effective compression ratio (excluding the densification stage), taken as 75%; $2h$ is the height of the Y-shaped basic unit, as shown in Fig. 2(b), i.e., the thickness of the honeycomb cell is 12.7 mm; $U_{\mathrm{m,B}}$ is the extension energy, N • mm; $U_{\mathrm{b,B}}$ is the horizontal hinge-line energy, N • mm; σ_{ys} is the yield stress of the honeycomb-core paper, for which the value of σ_{ys} for specimen S-MD in Table 3 is used; M_0 is the bending moment per unit length of the honeycomb cell wall, $M_0 = \sigma_{\mathrm{ys}}t^2/4$.

Substituting $U_{\mathrm{m,B}}$ and $U_{\mathrm{b,B}}$ into Eq. (3), and according to the principle of minimum energy in the quasi-static compression process, by solving $\partial p_{\mathrm{m}}/\partial h = 0$, one obtains p_{m}

$$p_{\mathrm{m}} = \frac{4}{3}\sigma_{\mathrm{ys}}t\sqrt{3a\pi t} \tag{6}$$

The load-bearing area of the Y-shaped basic unit of the regular hexagonal non-perforated paper honeycomb core layer is

$$S = \frac{1}{2} \cdot \sqrt{3}a \cdot \frac{3}{2}a = \frac{3\sqrt{3}}{4}a^2 \tag{7}$$

Therefore, the plateau stress for plastic buckling of the non-perforated paper honeycomb core layer is Eq. (8), and the calculated results are shown in Table 4.

$$\sigma_{\mathrm{pl}} = \frac{p_{\mathrm{m}}}{S} = \frac{16\sqrt{\pi}}{9}\sigma_{\mathrm{ys}}\left(\frac{t}{a}\right)^{3/2} \tag{8}$$

Another method for calculating the plastic-buckling plateau stress of the non-perforated paper honeycomb core layer is shown in Eqs. (9)-(11)[20]. According to Table 3, the yield-stress ratio k between the double-layer honeycomb cell wall and the single-layer honeycomb cell wall is calculated, and the plateau stress σ_{pl} obtained is close to the calculation result from Eq. (8).

Table 3 Yield stress of single- and double-layer cell walls

Tab. 3 Yield stress of single and double-layer cell walls

Relative humidity/%	σ_{ys}/MPa	
	S-MD	D-MD
50	5.80	7.26
60	5.40	6.16
70	4.50	5.29
80	3.80	4.65

$$\sigma_{\mathrm{pl}} = \sigma_{\mathrm{ys}}\left[D_1(k)\left(\frac{t}{a}\right)^{5/3} + D_2(k)\left(\frac{t}{a}\right)^2\right] \tag{9}$$

$$D_1(k) = 1.427(2k+1)^{2/3}(4k+1)^{1/3} \tag{10}$$

$$D_2(k) = 0.3849(2k-1) \tag{11}$$

Table 4 Plateau stress of paper honeycomb core layer

Tab. 4 Plateau stress of paper honeycomb core layer

Fig. 3

Figure 2: Fig. 3

Relative humidity/%	Theoretical value/MPa			Experimental value/MPa		
	$d_{\text{hole}} = 0$ mm	$d_{\text{hole}} = 2$ mm	$d_{\text{hole}} = 3$ mm	$d_{\text{hole}} = 0$ mm	$d_{\text{hole}} = 2$ mm	$d_{\text{hole}} = 3$ mm
50	0.091	0.085	0.081	0.092	0.081	0.078
60	0.085	0.079	0.075	0.084	0.080	0.070
70	0.071	0.066	0.063	0.071	0.065	0.063
80	0.060	0.056	0.053	0.061	0.058	0.054

2.2 Out-of-plane stress analysis of perforated paper honeycomb core layers

2.2.1 Critical stress for elastic buckling

After holes are opened at the centers of the double-layer side walls of the paper honeycomb core layer, a perforated paper honeycomb core layer is formed. The presence of circular holes reduces the load-bearing area of the honeycomb cell walls and produces stress concentration. To analyze the influence of perforation on the critical stress for elastic buckling of the paper honeycomb core layer, a Y_{hole}-shaped basic-unit model is established, i.e., a triangular unit with perforations in the double-layer wall panels. It includes two simply supported thin plates of thickness t and side length $a/2$, and two perforated simply supported thin plates of thickness t and side length $a/2$, as shown in Fig. 3. Therefore, the out-of-plane stress analysis of the perforated paper honeycomb core layer can be transformed into solving the combination of non-perforated and perforated simply supported thin plates.

(a) (b)

Fig. 3 Paper honeycomb core layer with perforation and Y_{hole}-shaped basic unit

To calculate the critical stress $\sigma_{\text{cr,hole}}$ for elastic buckling of the perforated simply supported thin plate, referring to the elastic-buckling behavior of plates with holes studied by Moen et al.[30], a closed-form expression was established for the influence of single or multiple holes on the critical elastic-buckling stress of a compressed plate. In addition, the perforated thin plate is symmetric in both shape and stress; therefore, the lower half is taken for force analysis, as shown in Fig. 4, and the corresponding relational expressions (12)-(15) are listed. According to the force-equilibrium relationship, by calculating the net-section stress $\sigma_{\text{a,net}}$, the total-section stress σ_{a} is obtained. The critical stress

Fig. 4 Schematic diagram of stress of perforated plate

Figure 3: Fig. 4 Schematic diagram of stress of perforated plate

σ_{cr} for elastic buckling of the non-perforated thin plate is further compared with the total-section stress σ_{a} of the perforated thin plate, and the minimum stress value is selected as the approximate value of the critical stress $\sigma_{\mathrm{cr,hole}}$ for elastic buckling of the perforated simply supported thin plate.

图 4 开孔薄板的应力示意图

Fig. 4 Schematic diagram of stress of perforated plate

$$\sigma_{\mathrm{a,net}} = \sigma_{a-d_{\mathrm{hole}}} \tag{12}$$

$$\sigma_{a-d_{\mathrm{hole}}} = K\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{a-d_{\mathrm{hole}}}\right)^2 \tag{13}$$

$$\sigma_a = \sigma_{\mathrm{a,net}}\left(1-\frac{d_{\mathrm{hole}}}{a}\right) \tag{14}$$

$$\sigma_{\mathrm{cr,hole}} = \min[\sigma_{\mathrm{cr}}, \sigma_a] \tag{15}$$

In the equations: d_{hole} is the diameter of the circular hole, taken as 2 and 3 mm, respectively; K is the buckling coefficient or stability coefficient, and for the corresponding circular-hole diameters d_{hole}, K is taken as 1.26 and 0.92, respectively. The comparison between the critical stress σ_{cr} for elastic buckling of the unperforated thin plate and the total-section stress σ_a of the perforated thin plate is shown in Table 5. The critical stresses $\sigma_{\mathrm{cr,hole}}$ of simply supported thin plates with circular-hole diameters of 2 and 3 mm are approximately equal to the total-section stresses σ_a of simply supported thin plates with the corresponding hole diameters. Therefore, the critical stress for elastic buckling of the perforated paper honeycomb core layer is calculated according to Eq. (16), and the calculation results are shown in Table 2.

$$\sigma_{\mathrm{hole}} = \frac{2\cdot\frac{a}{2}\cdot t\cdot(\sigma_{\mathrm{cr,hole}}+\sigma_{\mathrm{cr}})}{\frac{3a}{2}\cdot\frac{\sqrt{3}a}{2}} \tag{16}$$

Table 5 Comparison of elastic buckling stresses

Tab. 5 Comparison of elastic buckling stresses

Relative humidity / %	σ_{cr}	σ_a, $d_{\text{hole}} = 2$ mm	σ_a, $d_{\text{hole}} = 3$ mm
50	3.752	3.415	2.934
60	3.665	3.336	2.866
70	2.993	2.724	2.341
80	2.673	2.433	2.090

2.2.2 Plateau stress in plastic buckling

The presence of circular holes in the double-layer side walls of the paper honeycomb core layer significantly reduces the load-bearing area of the honeycomb cell walls. At this time, the folding extension energy absorption $U_{\text{m,B}}$ of the Y_{hole}-shaped basic unit under compression still remains unchanged as in Eq. (4), while the horizontal hinge-line energy absorption $U_{\text{b,B}}$ becomes

$$\begin{aligned} U_{\text{b,B}} &= \frac{\pi}{2}\left[8\cdot\frac{\sigma_{\text{ys}}^2 t^2}{4}\cdot\frac{a}{2}+4\cdot\frac{\sigma_{\text{ys}}(2t)^2}{4}\cdot\frac{(a-d_{\text{hole}})}{2}\right] \\ &= 2\pi a M_0 + 4\pi(a-d_{\text{hole}})M_0 \end{aligned} \tag{17}$$

Substituting Eqs. (4) and (17) into Eq. (3), and according to the principle of minimum energy in the quasi-static compression process, solving $\partial p_m/\partial h = 0$ gives p_m:

$$p_m = \frac{1}{2}\sigma_{\text{ys}} t\sqrt{\pi t}\,\frac{\left(8a-\dfrac{16}{3}d_{\text{hole}}\right)}{\sqrt{3a-2d_{\text{hole}}}} \tag{18}$$

The plastic plateau stress for buckling of the perforated paper honeycomb core layer is calculated according to Eq. (19), and the results are shown in Table 4.

$$\sigma_{\text{pl,hole}} = \frac{p_m}{S} \tag{19}$$

3 Quasi-static compression tests of unperforated and perforated paper honeycomb core layers

The specifications and physical properties of the honeycomb-core paper used in the quasi-static compression tests were the same as those of the materials used in the tests described above. The lower fixed face sheet was kraft paper with a grammage of 160 g/m^2 and a thickness of 0.250 mm. The paper honeycomb core-layer specimens had a side length of 60 mm and a height of 12.7 mm, and included 11 regular hexagonal honeycomb cells with a side length of 8.66 mm. Before testing, all specimens were conditioned for 6 h under hygrothermal conditions with a temperature of 23 °C and relative humidities of 50%, 60%, 70%, and 80%, respectively. With reference to GB 1453, "Test method for flatwise compression properties of sandwich constructions or cores," quasi-static compression tests were carried out using an HT-2402 computer servo-controlled material testing machine. The upper compression platen applied displacement loading to the specimens at a constant rate of 12 mm/min, with a compression amount of 85%. Each group of specimens was tested three times, the average value was taken, and the bending deformation process of the specimens was recorded simultaneously.

From the stress–strain curves of quasi-static compression of unperforated and perforated paper honeycomb core layers under different relative humidities, the experimental values of the elastic-buckling critical stress and the plastic plateau stress of the unperforated and perforated paper honeycomb core layers were calculated. As shown in Tables 2 and 4, the results indicate that the experimental values agree well with the theoretical values. After perforation of the paper honeycomb core layer, both the critical stress for elastic buckling and the plateau stress for plastic buckling decrease gradually with increasing relative humidity and circular-hole diameter.

4 Finite element analysis of unperforated and perforated paper honeycomb core layers

4.1 Calculation of the elastic constants of honeycomb-core paper

When conducting finite element analysis of the honeycomb structure, the material parameters of the honeycomb-core paper must be assigned to the model. Combining the transverse and longitudinal elastic moduli of the honeycomb-core paper in Table 1, considering the orthotropic anisotropy of the paper substrate [28], and referring to the research results of Mann et al. [31], the elastic modulus of the honeycomb-core paper was calculated.

E_x, E_y, E_z, shear moduli G_{xy}, G_{xz}, G_{yz}, and Poisson's ratios ν_{xy}, ν_{xz}, ν_{yz}, where the out-of-plane Poisson' s ratios ν_{xz} and ν_{yz} are usually taken as 0.01 in the calculation. The calculated elastic constants of the honeycomb core paper are shown in Table 6. To verify the accuracy of the test data, the elastic constants of an orthotropic material must satisfy $\nu_{xy2} < E_x/E_y$, $\nu_{xz2} < E_x/E_z$, and $\nu_{yz2} < E_y/E_z$. If the experimental data do not satisfy the above relationships, the finite element analysis calculation will report an error. Verification shows that the experimental data are reasonable. The formulas for calculating the elastic constants of the honeycomb core paper are

$$G_{xy} = 0.387\sqrt{E_x E_y} \tag{20}$$

$$G_{xz} = E_x/55 \tag{21}$$

$$G_{yz} = E_y/35 \tag{22}$$

$$E_z = E_x/200 \tag{23}$$

$$\nu_{xy} = 0.293\sqrt{E_y/E_x} \tag{24}$$

Table 6 Elastic constants of honeycomb core paper at different humidity conditions

Tab. 6 Elastic constants of honeycomb core paper at different humidity

Relative humidity/%	E_x/MPa	E_y/MPa	E_z/MPa	G_{xy}/MPa	G_{xz}/MPa	G_{yz}/MPa	ν_{xy}
50	1 127.455	2 808.710	5.637	688.675	20.499	80.249	0.462
60	1 112.290	2 743.904	5.561	676.090	20.223	78.397	0.460
70	990.710	2 240.815	4.954	576.617	18.013	64.023	0.441
80	803.398	2 000.951	4.017	490.676	14.607	57.170	0.462

4.2 Elastic buckling analysis results of the paper honeycomb core layer

ANSYS Workbench was used to perform elastic buckling analysis of the non-perforated and perforated paper honeycomb core layers, so as to determine the critical load during the transition of the structure from stable equilibrium to unstable equilibrium and the post-buckling shape. Before the elastic buckling analysis, a static analysis was first carried out to obtain the structural stiffness. In the 3D modeling software Creo Parametric, assembly models of paper honeycomb core layers with no holes and with hole diameters of 2 mm and 3 mm were established, respectively, then exported in parasolid ($*.x\_t$) format and imported into ANSYS Workbench. The finite element model of the paper honeycomb core layer is shown in Fig. 5. The overall dimensions are 60 mm × 60 mm × 12.7 mm; the side length of the regular hexagonal cell is 8.66 mm, and the wall thickness is 0.255 mm. Its material property was set as "paper," with the specific parameters given in Table 6. The upper and lower platens do not need stress, strain, or relative deformation to be calculated, so the hardness of the upper and lower platens was set as "Rigid." According to

Finite element model ($d_{\text{hole}} = 2$ mm)

Figure 4: Finite element model ($d_{\text{hole}} = 2$ mm)

Critical load of elastic buckling

Figure 5: Critical load of elastic buckling

the positions and directions of different cell walls within the paper honeycomb core layer, a local coordinate system was used to assign the geometric directions of the honeycomb cell walls. The contact type between the upper and lower platens and the paper honeycomb core layer was defined as "Friction" ; its contact characteristics were normal separation, impermeability, and frictional force. This type of contact has a relatively high computational cost, and "Bonded" may be used as a substitute. Meshing was then performed, and the lower surface of the lower platen was fixed to the ground by using a Fixed Joint; a unit load was applied to the upper surface of the upper platen.

According to the locally enlarged image, after the double-layer perforated cell walls are subjected to out-of-plane compression, the stress distribution changes and stress concentration occurs. The left and right ends of the circular hole are subjected to compressive stress, while the upper and lower ends are subjected to tensile stress, causing the cell wall to be compressed in the vertical direction and stretched in the horizontal direction.

When the humidity is 50%, the finite element and experimental comparison results of the critical loads of the non-perforated and perforated paper honeycomb core layers are shown in Fig. 6. In the test, the paper honeycomb core layer structure was bonded with adhesive, and the structural defects (perforations) are relatively sensitive; because the finite element model is an ideal bonding, the finite element results are higher than the test results. Under relative humidity conditions of 60%, 70%, and 80%, the finite element results of the critical load are also higher than the test results, because the finite element analysis uses a relatively idealized model to approximately simulate the actual situation, avoiding defects in the specimens.

Fig. 5 Finite element model ($d_{\text{hole}} = 2$ mm)

Fig. 6 Critical load of elastic buckling

4.3 Plastic buckling analysis results of the paper honeycomb core layer

The compressive failure of non-perforated and perforated paper honeycomb core layers is mainly related to the plastic buckling of the honeycomb side walls. The static analysis module was used, and the out-of-plane quasi-static compression process of the paper honeycomb cells was used to simulate the plastic deformation of the non-perforated and perforated paper honeycomb core layers. The side length of the regular hexagon was defined as 8.66 mm, the height as 12.7 mm, and the thickness as 0.255 mm. Conceptual modeling was used to make the regular hexagonal ⋯

form a surface body and perforate it, thereby establishing a single paper honeycomb cell model. The model was sliced, and a local coordinate system was defined corresponding to the six faces; the material property was assigned as "paper." According to the static compression test, the compression rate was 12 mm/min, and the paper honeycomb cell was compressed by 85%, i.e., the vertical deformation was 10.795 mm. In the finite element model, to facilitate the setting of multiple load steps, the downward displacement velocity of the upper platen was set to 12 mm/min, and the vertical deformation was rounded to 10 mm.

Table 7 shows the finite element simulation and experimental results for the out-of-plane static compression deformation of non-perforated and perforated paper honeycomb cells at a relative humidity of 50%. In the elastic deformation stage, the stress was distributed over the entire paper honeycomb wall, with only a small amount of stress concentration near the upper and lower ends. At this point, bulging wrinkles appeared at the upper and lower ends of the non-perforated paper honeycomb wall, whereas local buckling failure occurred in the circular-hole edge region of the perforated paper honeycomb side wall owing to stress concentration, and inward concave wrinkles formed at the circular hole in the side wall. In the plastic stage, elastic buckling gradually changed into plastic buckling, and the load-bearing region gradually changed from the honeycomb wall to the joint region; with the mid-plane of the paper honeycomb cell as the plane of symmetry, folding and stacking gradually occurred. The degree of bending deformation increased with increasing hole diameter. In the densification stage, the paper honeycomb cell wall folded, expanded, and pressed against itself, and it basically regained elasticity. Therefore, by tracking the development of the finite element buckling waveforms, the initial buckling state can be observed in time. From the perspective of the buckling deformation mode, the finite element simulation process is consistent with the deformation process observed in the test.

5 Conclusion

1) After holes are opened in the double-layer side walls of the paper honeycomb core layer, both the critical stress value and the plateau stress value decrease as the relative humidity and circular-hole diameter increase. At the same humidity, the elastic moduli in the longitudinal fiber direction of the single-layer and double-layer honeycomb walls are relatively close.

2) Using two basic unit models, the Y-shaped and Y_{hole}-shaped models, the externally loaded honeycomb walls are transformed into non-perforated and perforated cylindrical thin plates for solution. The analytically calculated critical stress values and plastic plateau stress values show good

agreement with the static compression test results.

3) The bending and folding deformation evolution process of the paper honeycomb wall was simulated using a finite element model of a paper honeycomb cell, enabling timely observation of the initial buckling state. From the perspective of the buckling deformation mode, the finite element simulation results are consistent with the deformation process observed in the test.

Table 7 Compression deformation processes of paper honeycomb cell walls

Tab. 7 Compression deformation processes of paper honeycomb cell walls

Compression amount / %	$d_{\text{hole}} = 0$ mm	$d_{\text{hole}} = 2$ mm	$d_{\text{hole}} = 3$ mm
20			
50			
85			

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Figure 4

Figure 6: Figure 4

Figure 6

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Figures

Figure 7

Figure 8: Figure 7