Postprint of R-S-N Model Based on High-Cycle Fatigue Experimental Data of Surfacing-Welded Titanium Alloy

Authors: Zhu Liang, Wang Jing, Li Xiaohui, Suo Hongbo, Zhang Yiliang

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Abstract

To address the issue of stress ratio effects on fatigue life, fatigue experiments were conducted on TC18 titanium alloy welded-formed specimens (prepared by multi-layer welding) under three stress ratios (R=0.5, R=0.06, R=-1) to obtain the corresponding fatigue limits, and six S-N curves were derived using the "stress amplitude life model" and the "three-parameter life model". Based on the integral relationship between crack propagation rate and fatigue life, and using the two fatigue life mathematical models as a foundation, the relationship between stress ratio (R) and fatigue life curve (S-N) was systematically investigated, and a fatigue life (R-S-N) mathematical model considering stress ratio was proposed. According to the correction formula proposed in this paper, two R-S-N mathematical models applicable to TC18 titanium alloy welded-formed material were established. The results show that: the stress amplitude life model can accurately predict the medium fatigue life region, while the three-parameter life model is more suitable for predicting the medium-to-long life region. The two proposed R-S-N mathematical models show good agreement with experimental values and can predict fatigue life curves at arbitrary stress ratios in engineering applications.

Full Text

Preamble

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R-S-N Model Based on High-Cycle Fatigue Experimental Data of Build-Up Welded Titanium Alloy

ZHU Liang[1], WANG Jing[1], LI Xiaohui[1], SUO Hongbo[2], ZHANG Yiliang[1]

[1]College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, Beijing 100022, China
[2]Science and Technology on Power Beam Processing Laboratory, Beijing Aeronautical Manufacturing Technology Research Institute, Beijing 100024, China

Abstract: To address the influence of stress ratio on fatigue life, fatigue experiments were conducted on TC18 titanium alloy specimens fabricated by build-up welding (prepared using a multi-layer build-up welding method) under three stress ratios (R=0.5, R=0.06, R=-1). The corresponding fatigue limits were obtained, and six S-N curves were derived using both the "stress amplitude life model" and the "three-parameter life model." Based on the integral relationship between crack growth rate and fatigue life, and building upon two fatigue life mathematical models, the relationship between stress ratio (R) and fatigue life curve (S-N) was systematically investigated, leading to the proposal of a stress ratio-considering fatigue life (R-S-N) mathematical model. Using the modified formulas proposed in this paper, two R-S-N mathematical models applicable to TC18 build-up welded materials were established. The results demonstrate that the stress amplitude life model can accurately predict the medium fatigue life region, while the three-parameter life model is more suitable for predicting the long-life region. Both proposed R-S-N mathematical models show good agreement with experimental values and can predict fatigue life curves under any stress ratio for engineering applications.

Keywords: foundational discipline in materials science, stress ratio, fatigue life, fatigue limit, R-S-N mathematical models

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Introduction

The fundamental method for evaluating the fatigue performance of metallic materials involves experimentally determining the S-N curve (fatigue curve) or fatigue limit under a specific stress ratio R, thereby obtaining the stress-life curve equation. In actual engineering applications, however, the stress ratio R varies between [-1, 1] due to the complexity of loading conditions, particularly for titanium alloy welded structures where changes in R significantly affect material fatigue life and fatigue limit. Therefore, obtaining a stress-life equation with R as a variable holds important theoretical and practical significance for structural design and life assessment.

Numerous studies have investigated the effect of stress ratio on material fatigue performance [1-4]. Xue et al. [5] conducted fatigue tests on titanium-aluminum alloys under different stress ratios and concluded that stress ratio has a significant influence on fatigue life. Sun [6] and Ishihara et al. [7] investigated the effects of stress ratio on fatigue life and crack propagation. Sakai et al. [8] studied the influence of stress ratio on the long-life region of bearing steel, while Tokaji [9] examined its effect on the fatigue properties of aluminum alloys.

This paper focuses on establishing a mathematical expression relating stress ratio R to the S-N fatigue life curve. Based on the integral relationship between crack growth rate and fatigue life, and using two fatigue life mathematical models as a foundation, a stress ratio–considering fatigue life (R-S-N) model is proposed. Using TC18 titanium alloy build-up welded material as the research object, fatigue tests were conducted under three stress ratios (0.5, 0.06, -1). Both the "stress amplitude power function life model" and the "three-parameter life model" were applied for fitting to obtain S-N curves and verify the applicability of the two R-S-N mathematical models for this material.

1.1 R-Sa-N Model Based on Power Function

To quantitatively evaluate the influence of stress ratio R on S-N curves, this paper introduces the effect of stress ratio on life based on the crack growth rate mathematical model from literature [10]. The Paris law [11] for crack growth rate is given by:

$$\frac{da}{dN} = C(\Delta K)^m$$

where da/dN is the crack growth rate, ΔK is the stress intensity factor range, and C and m are parameters related to material crack propagation properties. Research indicates that parameters C and m in the formula are correlated with stress ratio R, with m increasing and C decreasing as R increases [12-14]. Therefore, this paper introduces an influence function $D(R)$ to characterize the effect of stress ratio on crack growth rate:

$$\frac{da}{dN} = D(R)(\Delta K)^m$$

For a single-edge cracked fatigue specimen, the stress intensity factor range can be expressed as:

$$\Delta K = Y \Delta S_a \sqrt{\pi a}$$

where Y is a geometric factor depending on the ratio of crack length a to specimen width.

Substituting equation (3) into equation (2) and integrating from initial crack length $[a_0 - a_f]$ and life $[0 - N]$ yields:

$$\int_{a_0}^{a_f} \frac{da}{(Ya)^{m/2}} = D(R)(YS_a\sqrt{\pi})^m N$$

After rearrangement:

$$D(R) = \frac{\int_{a_0}^{a_f} a^{-m/2} da}{(YS_a\sqrt{\pi})^m N}$$

Taking logarithms of both sides, equation (5) can be expressed as:

$$\lg D(R) = -m \lg S_a + \text{constant}$$

Comparing this with the commonly used S-N curve fitting formula $\lg N = a + b \lg S_a$, it is evident that parameter a in the S-N curve equation has a functional relationship with stress ratio R, i.e., $a = a(R)$. Rearranging equation (8) gives:

$$\lg N = a(R) + b \lg S_a$$

Based on fatigue experimental results, several sets of (a_i, b_i) can be obtained from S-N curves at different stress ratios R. Regression analysis of these values yields the functions $a(R)$ and $b(R)$, thereby establishing the R-Sa-N curve.

1.2 R-Smax-N Model Based on Three-Parameter Model

Another common expression for fatigue life equations is the three-parameter equation:

$$(S_{\max} - S_0)N^b = a$$

where the three parameters are a, b, and S_0, which are constants related to material properties, specimen geometry, and loading conditions. S_0 represents the material fatigue limit.

Similar to the power function expression, the relationship between parameters in the three-parameter model and stress ratio was considered. Through integral transformation, we obtain:

$$\lg N = a(R) + b \lg(S_{\max} - S_0)$$

Rearranging equation (11) yields:

$$\lg(S_{\max} - S_0) = \frac{1}{b}[\lg N - a(R)]$$

Based on fatigue experimental results at different stress ratios R, regression analysis yields parameters a_i and b_i in the three-parameter equation, establishing the R-Smax-N model.

Experimental Details

The experiments utilized TC18 titanium alloy build-up welded material. This material is a near-β high-strength titanium alloy with a microstructure consisting of strip-shaped α phase, β transformed structure, and fine secondary precipitated α phase after heat treatment. Its mechanical properties include: tensile strength $R_m = 1050$ MPa, proof strength $R_{p0.2} = 982$ MPa, elongation after fracture $A = 4\%$, and reduction of area $Z = 12\%$, characterized by high yield-to-tensile ratio and poor ductility. The fatigue specimen dimensions are shown in [Figure 1: see original paper].

Considering that TC18 titanium alloy build-up welded components in actual engineering applications experience tension-tension and tension-compression alternating stresses, fatigue tests were conducted on a QBR-100 high-frequency fatigue testing machine under three stress ratios (R=0.5, R=0.06, R=-1). The loading method was axial with a sinusoidal waveform. The staircase method was employed at the fatigue limit, while the group method was used in the medium-to-long life region.

2.2.1 Fatigue Limit Results from Staircase Method

The fatigue limits were determined using the paired staircase method with a specified cycle base of $N_0 = 10^7$ cycles. The stress step was 27.5 MPa for stress ratios R=0.5 and R=0.06, and 10 MPa for R=-1, all meeting staircase method requirements. The staircase results for different stress ratios are presented in [Figure 2: see original paper].

For stress ratio R=j, the fatigue limit is calculated as:

$$\hat{S}_{50}^{j} = \frac{1}{n}\sum_{i=1}^{n} S_{ri}$$

where $\hat{S}_{50}^{j}$ represents the fatigue limit at 50% reliability for stress ratio j, n is the total number of pairs, S_{ri} is the mean stress level, and n_i is the number of pairs at stress level S_{ri}.

The standard deviation of the fatigue limit sample is:

$$s = \sqrt{\frac{\sum_{i=1}^{n} n_i (S_{ri} - \hat{S}_{50})^2}{n-1}}$$

The coefficient of variation [15] is:

$$C_v = \frac{s}{\hat{S}_{50}}$$

where s is the standard deviation and $\hat{S}_{50}$ is the fatigue limit at 50% reliability. Using the specified coefficient of variation for 95% confidence and 5% error limit as the evaluation criterion, the results are considered valid when the actual coefficient of variation is less than the specified value.

2.2.2 Group Method Experimental Results

The group method was applied to obtain S-N curves under different stress ratios. With limited specimens, the stress levels were determined as follows: for R=0.5, four stress levels were selected as equally spaced as possible between the fatigue limit and ultimate strength, using 22 valid specimens; for R=0.06, three stress levels were selected using 14 valid specimens; for R=-1, due to the lower fatigue limit, three stress levels were selected in the medium-to-long life region using 19 valid specimens. The experimental results are listed in .

Based on the power function expression for fatigue curves and considering the actual experimental process where stress amplitude S_a is typically specified and fatigue life N is measured experimentally, a linear relationship exists between $\lg S_a$ and $\lg N$ in double logarithmic coordinates, expressed as:

$$\lg N = a + b \lg S_a$$

or equivalently:

$$S_a^b N = A$$

where a and b are parameters determined by least squares fitting. Using experimental data points (S_{ai}, N_i), the parameters a, b, and A can be obtained from equations (16) and (17), establishing the fatigue curve equation. The curve equations are listed in and plotted in [Figure 3: see original paper].

Similarly, based on equation (10), a linear relationship exists between $\lg(S_{\max} - S_0)$ and $\lg N$ in double logarithmic coordinates, expressed as:

$$\lg N = a + b \lg(S_{\max} - S_0)$$

Using the least squares principle to fit data points $(S_{\max i}, N_i)$ yields parameters a and b, establishing the three-parameter fatigue curve. When fitting the S-N curve, S_0 is taken as the median fatigue limit and N as the average fatigue life at each stress level. The curve equations are listed in and shown in [Figure 4: see original paper].

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3 R-S-N Curve Equations for TC18 Build-Up Welded Material

The coefficients a_i and b_i from the S-N curves for the three R values are listed in . Regression analysis of R versus a and R versus b was performed separately. The results indicate that parameters a and b follow a quadratic relationship with stress ratio R. The calculated functions are:

$$a(R) = 47.72R^2 + 77.78R + 57.53 \quad (R^2 = 0.984)$$

$$b(R) = 23.93R^2 + 35.28R + 19.60 \quad (R^2 = 0.981)$$

Substituting functions $a(R)$ and $b(R)$ into equation (9) yields the R-S-N mathematical model at 50% reliability:

$$\lg N = (47.72R^2 + 77.78R + 57.53) + (23.93R^2 + 35.28R + 19.60)\lg S_a$$

3.2 R-Smax-N Model for TC18 Build-Up Welded Material

Similarly, based on the three-parameter fatigue life curve equations for the three different stress ratios, three sets of coefficients a_i and b_i were obtained, as detailed in .

Separate regression analysis of R versus a and R versus b yielded:

$$a(R) = -5.66R^2 - 0.40R + 10.93$$

$$b(R) = -1.98R^2 + 0.04R + 2.61$$

Substituting functions $a(R)$ and $b(R)$ into equation (12) gives the R-Smax-N mathematical model at 50% reliability:

$$\lg N = (-5.66R^2 - 0.40R + 10.93) + (-1.98R^2 + 0.04R + 2.61)\lg(S_{\max} - S_0)$$

4.1.1 Properties of Sa-N Curves

As shown in [Figure 3: see original paper], two key characteristics are observed: (1) In the Sa-N model, life N is inversely proportional to stress ratio R. Mathematically, stress amplitude S_a relates to maximum stress $S_{\max}$ as $S_a = (1-R)S_{\max}/2$. For the same $S_{\max}$, R is inversely proportional to S_a; a smaller stress ratio R results in larger stress amplitude S_a. (2) The slope of the Sa-N curve is inversely proportional to stress ratio R. Under the same stress difference (ΔS_a), a lower R produces greater change in N. For example, when S_a decreases from 500 MPa to 450 MPa, the fatigue life increases by 2.35×10^5 cycles at $R = -1$, but only by 3.05×10^4 cycles at $R = 0.06$, representing nearly a tenfold difference.

4.1.2 Properties of Smax-N Curves

As shown in [Figure 4: see original paper], three characteristics are evident: (1) At the same maximum stress $S_{\max}$, stress ratio R is directly proportional to life N; higher R yields longer specimen life. (2) The slope of the Smax-N curve is directly proportional to stress ratio R. Under the same stress difference ($\Delta S_{\max}$), larger R produces greater life variation. For instance, when $S_{\max}$ decreases from 900 MPa to 800 MPa, the fatigue life increases by only 2.86×10^3 cycles at $R = -1$, but by 2.31×10^6 cycles at $R = 0.06$. (3) The fatigue limit is directly proportional to stress ratio R. From a mechanical perspective, as R approaches 1, the loading condition approaches static stress, minimizing the effect of alternating stress.

4.2 Discussion and Prediction of Stress Ratio Effect on Fatigue Limit

The Goodman theory was employed to correct for stress ratio effects on fatigue limit. The Goodman formula is:

$$\frac{S_a}{S_{-1}} + \frac{S_m}{\sigma_b} = 1$$

where S_a is stress amplitude, S_m is mean stress, S_{-1} is the fatigue limit under symmetric cycling (R=-1), and σ_b is the material tensile strength. Their relationship is:

$$S_m = \frac{1+R}{1-R} S_a$$

$$S_{\max} = S_a + S_m$$

Substituting equations (22) and (23) into (21) yields:

$$S_{\max} = \frac{2S_{-1}\sigma_b}{(1-R)\sigma_b + (1+R)S_{-1}}$$

Rearranging equation (24) gives:

$$S_{\max} = \frac{S_{-1}}{1 - \left(\frac{1+R}{1-R}\right)\frac{S_{-1}}{\sigma_b}}$$

where $S_{\max}$ represents the fatigue limit at stress ratio R. According to equation (25), the fatigue limit at any R can be predicted. At 50% reliability, the predicted and experimental values are compared in . Equation (25) shows that fatigue limit is directly proportional to stress ratio, consistent with experimental results. Furthermore, demonstrates that Goodman theory accurately predicts fatigue limits at various stress ratios with errors less than 6%.

4.3.1 Characteristics of Stress Amplitude Life Model

Based on [Figure 3: see original paper], the stress amplitude life curve continues to decline when fatigue life exceeds 10^7 cycles, showing no horizontal asymptote and failing to reflect the fatigue limit. This indicates that the stress amplitude model is more suitable for predicting the medium life region ($N < 10^6$). After introducing variable R, the life remains directly proportional to stress ratio.

4.3.2 Characteristics of Three-Parameter Life Model

As shown in [Figure 4: see original paper], the three-parameter life curve not only reflects the relationship between maximum stress and fatigue life but also captures fatigue limit effects. When fatigue life exceeds 10^7 cycles, the curves for all three stress ratios remain horizontal, accurately reflecting the fatigue limit at each stress ratio and intuitively explaining the engineering significance of fatigue limit. That is, when stress level is below the fatigue limit, specimen life tends toward infinity, demonstrating applicability in the long-life region (greater than 10^7 cycles). After introducing the stress ratio variable, fatigue life does not exhibit a simple direct or inverse proportional relationship with stress ratio.

4.3.3 Comparison of Theoretical and Experimental Curves

The theoretical curves established using the R-S-N mathematical models proposed in this paper were compared with experimentally obtained S-N curves, as shown in [Figure 5: see original paper].

[Figure 5: see original paper] demonstrates that both the Sa-N and Smax-N theoretical model curves nearly coincide with the measured curves, indicating that the theoretical correction model can accurately reflect fatigue life under different stress ratios and possesses good engineering practicality.

4.4 Generalization of R-S-N Curves for TC18 Build-Up Welded Material

For convenient engineering application, different stress ratios R were substituted into equations (19) and (20) to generate generalized R-S-N curves for TC18 titanium alloy build-up welded material. The typical theoretical curves established from the mathematical models are presented in [Figure 6: see original paper]. Based on [Figure 6: see original paper], the stress-life relationship for any R can be readily determined in engineering practice.

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5 Conclusions

1. Based on experimental results from three stress ratios (R=0.5, R=0.06, R=-1), two R-S-N mathematical models considering stress ratio effects were established:

   - Stress amplitude life model: $\lg N = (47.72R^2 + 77.78R + 57.53) + (23.93R^2 + 35.28R + 19.60)\lg S_a$

   - Three-parameter life model: $\lg N = (-5.66R^2 - 0.40R + 10.93) + (-1.98R^2 + 0.04R + 2.61)\lg(S_{\max} - S_0)$
     Practical analysis demonstrates that both R-Sa-N and R-Smax-N models can accurately predict fatigue life at different stress ratios with good engineering applicability.

2. The R-Sa-N model accurately predicts the medium fatigue life region but cannot reflect the fatigue limit due to its continuously decreasing trend. The R-Smax-N model is more suitable for predicting the medium-to-long life region, as the stress level stabilizes after reaching 10^7 cycles, accurately reflecting the influence of fatigue limit on the life curve.

3. For TC18 titanium alloy build-up welded material, stress ratio is directly proportional to fatigue limit. Increasing the stress ratio from 0.06 to 0.5 (a 7.3-fold increase in R) raises the fatigue limit by 20%. Goodman theory can accurately predict fatigue limits under different stress ratios with errors less than 6%.

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Note: Figure translations are in progress. See original paper for figures.