The Source of Cosmological Constant—Wave Mechanics of Particles in Curved Space-Time (Part II)

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Abstract

In this article, we will see that the existence of cosmological constant is a natural possible result of the new wave equation, which has been established in the first part of this series. After analyzing various possible sources of cosmological constant, the most likely conclusion is drawn that the cosmological constant is not from the vacuum energy of matter, but from space itself, it is an inherent essential feature of space itself.

Full Text

The Source of the Cosmological Constant: Wave Mechanics of Particles in Curved Spacetime (Part II)

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Abstract

This article demonstrates that the existence of a cosmological constant emerges naturally as a consequence of the new wave equation established in Part I of this series. After analyzing various potential sources of the cosmological constant, we conclude that it most likely originates not from the vacuum energy of matter, but from space itself—it is an inherent, essential feature of spacetime. Contemporary astronomical observations support the existence of a small, positive cosmological constant. While conventional wisdom attributes this to vacuum energy, the enormous discrepancy between the quantum field-theoretic prediction and the observed value has long constituted the cosmological constant problem [1]. Resolving this issue fundamentally requires advances in basic physics [2].

Recent theoretical progress establishing a universal wave equation for particles in curved spacetime [3] provides a new perspective, revealing the cosmological constant as a natural outcome of this framework.

Wave Equation in Curved Spacetime

The wave function of microscopic particles in curved spacetime satisfies equation [3]:

= - l2 (cid:3)2(-g) - 1 @z2 + @2

where lP is the Planck length, the four-dimensional operator (cid:3) = ^{ex} @t2, g = |g(cid:22)(cid:23)| is the determinant of the metric tensor, τ is proper time with $d\tau^2 = -ds^2 = -$g(cid:22)(cid:23)dx(cid:22)dx(cid:23), and (cid:3)^2 = @2 @x2 + @2 @y2 + @2 @x + ^{ey} @y + ^{ez} @z + ^{et}.

Separating the amplitude and phase of the wave function, we let = Re^{iS} where R and S are real functions. Substituting into equation (1) and equating real and imaginary parts yields [3]:

= - 1 R(cid:3)2S + 2(cid:3)R · (cid:3)S + -g · (cid:3)S

and

(cid:3) = - 1 ((cid:3)S)2 + (cid:3)2R (cid:3)2 (cid:3) -g · (cid:3)R.

Equations (3) and (4) are completely equivalent to wave equation (1).

Classical Limit

Under the classical limit condition lP $\to 0$, equation (4) becomes:

((cid:3)S)2 =

Because the four-dimensional velocity U = (cid:3)S, equation (5) can be rewritten as [3]:

((cid:3)S)2 =

Taking the four-dimensional gradient on both sides of equation (6), we get:

+ (U · (cid:3)) U = (cid:3) 1

Because of the relationship dU/dτ = U/ τ + (U ·(cid:3))U, equation (7) can be written as [3]:

dU/dτ = (cid:3) 1

Multiplying both sides of equation (8) by the static mass of the particle m, we obtain:

= (cid:3) 1 = m(cid:3) 1 = -m(cid:3)(- 1 = -m(cid:3)(- 1 where P = mU is the four-dimensional momentum. Equation (10) shows that under the classical limit condition (lP $\rightarrow$ 0), the wave equation (1) returns to the dynamic equation of special relativity, and at low speeds it further reduces to the dynamic law of classical mechanics. Supposing the particle velocity v is far less than the speed of light c, we have dt = dτ, and equation (10) further returns to:

`= -m (- 1`

where pi (i = 1, 2, 3) and Ek are the momentum and kinetic energy in classical mechanics, respectively.

Gravitational Potential

The gravitational force acting on a particle with mass m in a gravitational potential ϕ is:

`F = -m $\phi$`

Comparing formula (11) with formula (12), we obtain:

`$\phi$ = - 1`

where C is a constant. Therefore, formula (13) shows that a natural result of wave equation (1) is that there must be a constant gravitational potential in space.

We now discuss the two possibilities for the gravitational potential value in formula (13):

(1) The first possibility: $\phi = -$ 1 2 + C = 0. The gravitational potential in space is zero everywhere, which is a mediocre situation and produces no new results.

(2) The second possibility: $\phi = -$ 1 2 + C $\neq$ 0. If there exists a non-zero constant gravitational potential $\phi = -$ 1 2 + C everywhere in space, the Schrödinger equation for free particles should be modified to:

`i~   = i~    2  + V0`

where V_0 = mϕ. As long as the gravitational potential ϕ is small enough to satisfy mϕ $p^2/2m$, there will be no obvious contradiction between equation (14) and equation (15), and the Schrödinger equation (14) for free particles remains valid. Therefore, the second possibility is that the gravitational potential ϕ in space is a non-zero, very small constant.

Physical Origin of the Constant Potential

We discuss the physical meaning of the second possibility. If a very small gravitational potential is constant throughout space, what generates it?

(1) First, a particle moving in space experiences this ubiquitous gravitational potential, which cannot be generated by the particle itself. In other words, a particle cannot feel its own gravitational potential.

(2) Could it be the vacuum energy of matter? The vacuum state is a special state of matter particles. In the vacuum state, all particles occupy the lowest energy state 1 $k^2 + m^2$, with energy density [1, 4]:

```
(cid:2)  vac = g (2$\pi$)$^{3}$ k$^{2}$ + m$^{2}$
```

where g = 2s + 1 accounts for the degrees of freedom of the matter particle and kmax is the momentum cutoff.

Consequently, vacuum energy is part of the energy of matter. The total matter energy includes two components: the energy of particles in excited states e_m and the energy of particles in the ground state (lowest energy state 1 $k^2 + m^2$) vac, expressed as:

```
m =  e_m +  vac
```

If a region of space contains no matter particles, then it contains no particles in their lowest energy state either. We call this "pure space" —space where the vacuum energy of matter is zero. For a single particle moving in pure space, wave equation (1) and formula (13) remain valid, and the particle still experiences a non-zero, very small constant gravitational potential. Therefore, this constant gravitational potential cannot be generated by the vacuum energy of matter particles.

(3) Excluding the first two possibilities, this non-zero, very small constant gravitational potential can only be generated by space itself. Space must possess a non-zero, very small energy density s that generates the gravitational potential, and this energy density is constant throughout time and space, i.e., s = const., representing an essential attribute of space.

In other words, space itself has mass, with a constant mass density that is an inherent fundamental attribute of space. The inherent energy density or mass density of space is a fundamental constant of both space and physics. Like other physical constants, its value can only be determined through experimental observation.

Implications for General Relativity

That space itself has mass deepens our understanding of general relativity. Einstein' s theory tells us that massive matter curves spacetime, with the curvature determined by Einstein' s field equations, but it does not explain why matter can curve space. We can now answer: because space itself possesses mass, it is curved by the gravitational attraction of massive objects. Simultaneously, we understand that since charge does not interact with mass, charge cannot curve space. Therefore, we can never geometrize electromagnetism in the same way as gravity.

Modified Einstein Field Equations

Because space itself has constant energy density s, Einstein’ s original field equation should be modified to:

```
R(cid:22)(cid:23) - 1 g(cid:22)(cid:23)R = 8$\pi$GT(cid:22)(cid:23)
R(cid:22)(cid:23) - 1 g(cid:22)(cid:23)R = 8$\pi$G(T(cid:22)(cid:23) -  sg(cid:22)(cid:23))
```

where (T(cid:22)(cid:23) − sg(cid:22)(cid:23)) represents the total energy-momentum tensor generated by both pure matter and space itself. Since vacuum energy (vac in formula (17)) is part of matter energy, its contribution is included in the matter energy-momentum tensor T(cid:22)(cid:23).

Equation (19) can be rewritten in the familiar form:

```
R(cid:22)(cid:23) - 1 g(cid:22)(cid:23)R + (cid:3)sg(cid:22)(cid:23) = 8$\pi$GT(cid:22)(cid:
```

where (cid:3)s is the cosmological constant with (cid:3)s = 8πG s. The subscript s indicates that it originates not from vacuum energy, but from space itself.

Conclusion and Outlook

If the cosmological constant indeed originates from space itself as an essential characteristic, independent of the vacuum energy of matter fields (vac in formula (17)), then to maintain theoretical self-consistency we must answer: why does the vacuum energy of matter fields not contribute to the cosmological constant, and why is the observed vacuum energy density so small? These questions will be addressed in the third part of this series.

References

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Note: Figure translations are in progress. See original paper for figures.