Fourier Demodulation Method Based on Continuous Rotating Waveplate Modulation: Postprint

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Abstract

This paper presents the derivation process of Fourier analysis and demodulation formulas under the continuous rotating waveplate modulation mode, and verifies the accuracy of the formulas using theoretical polarization profiles. Two demodulation methods, Fourier analysis and demodulation matrix, are respectively employed to quantitatively simulate the influence of three factors on measurement accuracy under continuous modulation mode: initial azimuth angle error of the waveplate, rotational positioning error, and the time difference ratio during detector exposure. The main conclusions are: (1) Simple Fourier analysis is no longer applicable for demodulating results from continuous modulation. The new Fourier analysis method presented in this paper and the demodulation matrix method can obtain essentially consistent results when conducting error analysis. (2) Considering the influence caused by initial azimuth angle error, we find that for linear polarization signals, using Fourier analysis and demodulation matrix yields highly consistent results, whereas for circular polarization, the method based on demodulation matrix produces relatively smaller errors. However, both demodulation methods reflect that initial azimuth angle error has consistent effects on circular and linear polarization, and the magnitude of relative error is related to the strength of the polarization signal itself. The initial azimuth angle error must be on the order of ten arcseconds to satisfy the requirement that the relative error of results be below the order of 10-3. (3) When considering the influence of rotational positioning error, we find that the results obtained using Fourier analysis and demodulation matrix are very close. Both simultaneously demonstrate that rotational positioning error has a more pronounced effect on linear polarization signals, and the magnitude of relative error is also related to signal strength. When the waveplate' s repeat positioning accuracy is around 10'' , for weak polarization signals on the order of 10-2, the measurement error can also be around the order of 10-3. From this perspective, we find that the continuous modulation mode has significantly higher requirements for waveplate rotational positioning accuracy than the step-

wise modulation mode. (4) Results from both demodulation methods show that under continuous modulation mode, the time difference (Δ) between the waveplate' s modulation period and detector exposure duration can cause crosstalk between linear polarization signals, meaning the time difference ratio (Δ/T) has a more pronounced effect on linear polarization signals than on circular polarization signals. When Δ/T is less than 1%, the relative error of linear polarization signals reaches the order of 10-3.

Full Text

Preamble

Fourier Demodulation of the Continuously Rotating Waveplate Modulation and the Error Analysis

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Abstract

This paper presents the derivation process and demodulation formulas for Fourier analysis in the context of continuously rotating waveplate modulation mode, and validates the accuracy of these formulas using synthetic polarization profiles. We quantitatively simulate the impact of three factors on measurement precision under continuous modulation mode—initial azimuth angle error of the waveplate, rotational positioning error, and the time difference ratio during detector exposure—using both Fourier analysis and demodulation matrix methods. The main conclusions are: (1) Simple Fourier analysis is no longer applicable for demodulating results from continuous modulation. The new Fourier analysis method presented in this paper yields essentially consistent results with the demodulation matrix method when conducting error analysis. (2) Considering the effects of initial azimuth angle error, we find that for linear polarization signals, Fourier analysis and demodulation matrix methods produce highly consistent results, while for circular polarization, the demodulation matrix method yields relatively smaller errors. However, both methods reveal that initial azimuth angle error has consistent effects on circular and linear polarization, and the magnitude of relative error is related to the strength of the polarization signal itself. The initial azimuth angle error must be controlled at the level of a few tens of arcseconds to achieve relative errors below 10^{-3}. (3) When examining the effects of rotational positioning error, we find that results from Fourier analysis and demodulation matrix methods are very close. Both demonstrate that rotational positioning error has a more pronounced impact on linear polarization signals, and the magnitude of relative error is also related to signal strength. When the waveplate' s repeatability accuracy is around a certain value, measurement errors for weak polarization signals at the 10^{-2} level can be maintained around the 10^{-3} level. From this perspective, we find that continuous modulation mode imposes significantly higher requirements on waveplate rotational positioning accuracy compared to stepwise modulation mode. (4) Results from both demodulation methods show that under continuous modulation mode, the time difference between the waveplate modulation period and detector exposure duration causes crosstalk between linear polarization signals, meaning the time difference ratio has a more significant impact on linear polarization than on circular polarization. When the ratio is less than 1%, the relative error of linear polarization signals reaches the 10^{-3} level.

Keywords: Polarization measurement; Polarization modulation; Polarization demodulation; Fourier demodulation

0 Introduction

Polarization measurement is a crucial means for obtaining astrophysical conditions. Anisotropy in light sources or propagation paths is the primary mechanism causing polarization, thus polarized radiation can provide important information about vector fields such as magnetic fields or radiation fields in celestial bodies. In solar physics, high-precision polarization measurements using certain atomic spectral lines are currently a primary method for detecting solar vector magnetic fields, with polarization measurement precision typically required at the 10^{-3} level or higher.

We typically use the Stokes polarization vector, denoted as $\mathbf{S} = (I, Q, U, V)^T$ (where T indicates transpose), to describe the polarization characteristics of a light beam. Since detectors can only measure intensity signals, we need to linearly combine the four Stokes parameters through multiple measurements (or polarization modulation) before we can retrieve the complete polarization information (or polarization demodulation) from the measured intensity signals. The optical component assembly that achieves linear combination of polarization vectors is called a polarization analyzer, typically consisting of two parts: a polarization modulator and an analyzer. The mathematical expression of the polarization modulation process can be represented as $\mathbf{I} = \mathbf{M} \cdot \mathbf{S}$. Here, $\mathbf{M}$ is an $n \times 4$ matrix (where n is the number of measurements, $n \geq 4$), known as the modulation matrix. The demodulation process is the inverse of the modulation process, expressed as $\mathbf{S} = \mathbf{D} \cdot \mathbf{I}$. Here, $\mathbf{D}$ is called the demodulation matrix, which is the inverse of the modulation matrix [**?**]. With the correct modulation and demodulation matrices, we can calculate the input polarization signal from observations.

Waveplates or waveplate combinations are common polarization modulator elements. There are various forms of polarization modulation using waveplates, such as rotating the fast axis of an optical waveplate with fixed retardation driven by a motor, or changing the retardation or optical axis direction of a liquid crystal waveplate by varying the voltage. Compared to using liquid crystal waveplates, the advantages of using optical waveplates mainly include the following points [?]: (1) More stable performance, with better surface figure accuracy and retardation uniformity across the effective clear aperture. (2) Retardation can be optimized according to scientific requirements, typically considering that different scientific objectives have different measurement efficiency requirements for various polarization vectors. (3) Applicable to a wide wavelength range, where achromatic waveplates enable simultaneous multi-band observations. Therefore, many solar telescopes have adopted rotatable optical waveplates as polarization modulators, such as ground-based instruments like the Advanced Stokes Polarimeter (ASP) on the Vacuum Tower Telescope [?] and the 1.6-meter Goode Solar Telescope (GST) [?], China' s one-meter New Vacuum Solar Telescope (NVST) [?] and Fiber Arrayed Solar Optical Telescope (FASOT) [?], etc. Space-based instruments include the SP payload on the SOT instrument of Japan' s Hinode satellite [?] and the sounding rocket CLASP2 [?]. India' s Visible Emission Line Coronagraph (VELC) for coronal magnetic field measurements also uses rotating waveplates as polarization modulators [?]. From the polarization modulation methods of these instruments, we find that waveplate rotation can adopt two modes: one is stepwise rotation, where the detector collects data after the waveplate rotates to each position and stops (e.g., NVST and FASOT); the other is continuous rotation, where the detector synchronously collects intensity variations during waveplate rotation (e.g., GST, Hinode/SP, VELC, etc.).

In both modes, equal-interval acquisition is a common approach, where the waveplate rotation angle is $2\pi k/n$, with n being the number of steps required for one waveplate rotation or the number of detector acquisitions. Taking the stepwise case as an example, the relationship between the intensity signal I_k collected by the detector, the waveplate rotation angle θ_k, the waveplate retardation δ, and the incident polarization signal $\mathbf{S}$ is [?]:

$$I_k = \frac{1}{2}\left[I + Q\cos^2(2\theta_k) + U\sin(4\theta_k) + V\sin(2\theta_k)\right]$$

We find that Q and U are modulated into I_k at the same frequency (four times the waveplate rotation frequency), but with a phase difference of $\pi/2$. V is modulated into I_k at twice the waveplate rotation frequency. The modulation amplitudes of Q, U, and V (i.e., coefficients a, b, and c) depend only on the waveplate retardation δ. Therefore, besides using the demodulation matrix (i.e., calculating the incident $\mathbf{S}$ using the aforementioned formula $\mathbf{S} = \mathbf{D} \cdot \mathbf{I}$), we can also demodulate $\mathbf{S}$ through Fourier analysis:

$$Q = \mathrm{Re}\{FFT[I_k](f_4)\} \tag{1}$$
$$U = \mathrm{Im}\{FFT[I_k](f_4)\} \tag{2}$$
$$V = \mathrm{Im}\{FFT[I_k](f_2)\} \tag{3}$$

Here, the symbols Re and Im represent the real and imaginary parts of the Fourier analysis frequency, while the numbers '4'or '2'represent the corresponding Fourier analysis frequencies, corresponding to the value of f.

However, in continuous rotation mode, formula (1) needs to be adjusted through angular integration (see formula derivation below), and the simple Fourier demodulation formula mentioned above is no longer applicable. Therefore, for continuous modulation, people still widely adopt the demodulation method based on the demodulation matrix $\mathbf{D}$ [?].

Compared to stepwise rotation, continuous waveplate rotation has obvious advantages in modulation frequency and is widely used internationally. Therefore, this research focuses on the continuous waveplate rotation mode. We first derive the precise Fourier analysis demodulation formula for this modulation mode and verify its correctness through theoretical simulations. Subsequently, using two different demodulation methods (demodulation matrix and Fourier analysis), we analyze the impact of waveplate initial azimuth angle error, waveplate rotational positioning error, and the time difference ratio during detector acquisition on demodulation results. Finally, we present discussion, analysis, and conclusions.

1 Formula Derivation and Verification

In this section, we first present the derivation process of the Fourier demodulation formula for continuously rotating waveplates, then theoretically simulate continuous modulation observations with known polarization vectors, and finally compare the demodulation results obtained using the newly derived formula and the stepwise formula with the known input signals to verify the formula' s accuracy.

1.1 Formula Derivation

In continuous mode, the intensity collected by the detector in a single exposure is:

$$I_j = \int_{\theta_j}^{\theta_j+\Delta\theta_j} \mathbf{M}(\theta) \cdot \mathbf{S} \, d\theta$$

where $\mathbf{M}$ is the modulation matrix, specifically expressed as:

$$\mathbf{M}(\theta) = \frac{1}{2} \begin{pmatrix} 1 & a + b\cos(4\theta) & b\sin(4\theta) & c\sin(2\theta) \\ 1 & a + b\cos(4\theta) & b\sin(4\theta) & c\sin(2\theta) \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

where a, b, and c are defined the same as in formula (2). $\theta_j = \alpha + (j-1)\frac{2\pi}{n}$ (where n represents the total number of acquisitions by the detector during one waveplate rotation). During the j-th detector exposure, the waveplate fast axis sweeps through an angle $\Delta\theta_j$. When the detector exposure acquisition times are equal, i.e., uniform acquisition, $\Delta\theta_j = \frac{2\pi}{n}$. If $\Delta t_j \neq \Delta t_{exp}$, then $\Delta\theta_j = \frac{2\pi}{n} + (\Delta t_j - \Delta t_{exp})\frac{2\pi}{T}$.

Assuming the initial position of the waveplate fast axis has a certain angle α with the transmission axis of the analyzer, then $\theta_j = \alpha + (j-1)\frac{2\pi}{n}$. Substituting the integral formula into equation (4), we obtain the observed intensity profile as:

$$I_j = \frac{1}{2}\left[I + Q\left(a + \frac{b\sin(4\Delta\theta_j)}{4\Delta\theta_j}\cos(4\theta_j + 2\Delta\theta_j)\right) + U\left(\frac{b\sin(4\Delta\theta_j)}{4\Delta\theta_j}\sin(4\theta_j + 2\Delta\theta_j)\right) + V\left(\frac{c\sin(2\Delta\theta_j)}{2\Delta\theta_j}\sin(2\theta\right.\right.$$

Analyzing the above formula, we find that the Q and U signals no longer have a simple correspondence with a specific modulation frequency as shown in formulas (1)-(3). The coefficients of $\cos(4\theta_j)$ contain both Q and U signals. The V signal still only relates to the coefficient of $\sin(2\theta_j)$. Therefore, after a series of algebraic operations, we derive new Fourier analysis demodulation formulas as follows:

$$Q = \frac{1}{b}\left[\mathrm{Re}_4\cos(2\Delta\theta) + \mathrm{Im}_4\sin(2\Delta\theta)\right] \tag{4}$$

$$U = \frac{1}{b}\left[\mathrm{Im}_4\cos(2\Delta\theta) - \mathrm{Re}_4\sin(2\Delta\theta)\right] \tag{5}$$

$$V = \frac{1}{c}\mathrm{Im}_2 \tag{6}$$

where Re_4, Im_4, and Im_2 have the same meanings as in formula (3).

1.2 Simulation Verification of Formula Accuracy

Next, we verify the accuracy of the demodulation method in formula (7) using theoretically simulated signals. The approach is as follows: (1) Use the Rybicki-Hummer (RH) polarized radiative transfer program [**?**] to generate theoretical Fe I atomic line polarization profiles. The calculation adopts a one-dimensional plane-parallel atmosphere assumption, the quiet-Sun FALC atmospheric model, and a typical magnetic field configuration (magnetic field strength of 1000 Gauss, inclination angle of 45°, azimuth angle of 45°). The working spectral line is Fe I with a central wavelength of 630.15 nm. The theoretically calculated polarization profiles are shown as solid lines in Figure 1 [Figure 1: see original paper]. It should be noted that we use polarization profiles $\mathbf{S}(\lambda)$ rather than a specific set of polarization vectors $\mathbf{S}$, because different wavelength points have varying strong and weak polarization signals, which better verifies the universality of the demodulation formula. (2) Simulate an ideal 8-step continuous waveplate modulation mode, i.e., the detector uniformly collects 8 times during half a rotation of the waveplate (one modulation period). Here, $\Delta\theta = \pi/8$, and the waveplate retardation is selected as $\delta = 127^\circ$. Calculate the intensity profile I_j variation during one waveplate rotation according to formula (6). Demodulate the theoretical intensity profile I_j using Fourier analysis methods from formulas (3) and (7) respectively. (3) Compare the known polarization profiles with the two demodulation results.

Figure 1 shows the comparison between the known polarization profiles and the results obtained using simple Fourier analysis (i.e., formula (3)). Each subplot displays I/I_c, Q/I, U/I, and V/I respectively. The solid lines represent the input theoretical polarization profiles, while the dots represent the Fourier demodulation results. The comparison clearly shows that even under ideal conditions, the demodulation results from simple Fourier analysis differ significantly from the input theoretical profiles, particularly in the line core and near wings ($\Delta\lambda \approx 0.05$ nm) where polarization signals are strong. The most obvious discrepancy occurs in the weakest Q/I profile, where even sign reversal happens. The maximum relative differences for U/I and V/I profiles reach 25%. The I/I_c profile appears similar, but the residual shows a maximum relative difference of nearly 2%.

To analyze the relationship between the above errors and the number of acquisitions per period, we increase the acquisition number from 8 to 32 and 128. The results are shown in Figure 2 [Figure 2: see original paper]. The left and right panels display Q/I and V/I signals respectively. We find that as the number of acquisition frames gradually increases, the residual between input and output signals decreases. When $N = 128$, the maximum residual for Q/I signals still reaches 3×10^{-3} (10% of the polarization signal itself), while the residual for V/I signals is very small, with a maximum of about 7.5×10^{-5} (0.04% of the polarization signal itself). However, it is evident that the residual profiles are very similar to the polarization signal profiles themselves, rather than random errors.

Next, we demodulate using the new Fourier analysis method (formula (7)), with results shown in Figure 3 [Figure 3: see original paper]. The difference between the demodulation results and theoretical input values forms a stark contrast to Figure 1. To highlight the differences, we present residuals in each subplot. From the residuals (dashed lines), the demodulated profiles show almost no noticeable difference from the input profiles, with residuals maintained at an extremely low error range (10^{-16} to 10^{-17}) across the entire profile.

The comparison between the results in Figures 1 and 2 with those in Figure 3 fully demonstrates that the simple stepwise Fourier analysis formula is no longer suitable for demodulating continuously rotating waveplate modulation, while the new Fourier demodulation formula derived in this paper can provide very accurate demodulation results. Therefore, all Fourier analysis or Fourier demodulation mentioned hereafter refers to the method given in formula (7).

2 Error Analysis of Different Demodulation Methods and Continuous Modulation Process

From formula (6), we can see that errors in several variables affect the modulation results of continuously rotating waveplates, including the initial azimuth angle α of the waveplate (the angle between the waveplate' s fast axis and the analyzer' s transmission axis), rotational positioning errors of the waveplate, and mismatches between waveplate rotation speed and detector exposure time. In this section, we quantitatively analyze the impact of these variable errors on demodulation results using both demodulation matrix and Fourier analysis methods.

2.1 Initial Azimuth Angle Error

In the optical path, there exists a certain angle α between the waveplate' s fast axis and the analyzer' s transmission axis, referred to as the initial azimuth angle error of the waveplate. In practice, we cannot know the precise value of α, thus introducing calculation errors. When simulating the impact of this error, our analysis approach is as follows: (1) Input known polarization signals $\mathbf{S} = (1, 0.02, 0.06, 0.15)^T$, representing weak, medium, and strong polarization signals (hereafter, Q, U, and V signals are uniformly referred to as linear and circular polarization signals). Simulate an 8-step continuous modulation process, set the waveplate initial azimuth angle as α, with the waveplate rotating through an angle of $\pi/8$ during a single detector exposure, and waveplate retardation $\delta = 127^\circ$. Generate the observed intensity profile I_j according to formula (6). (2) Demodulate the observed intensity profile using both demodulation matrix and Fourier analysis methods, and compare the relative error between demodulation results and input results, defined as $|\mathbf{S}_{out} - \mathbf{S}_{input}|/|\mathbf{S}_{input}|$. Since the α value is unknown, both demodulation matrix and Fourier analysis calculations assume the ideal case of $\alpha = 0$. The ideal modulation matrix $\mathbf{M}$ for 8-step continuous modulation is as follows, with its demodulation matrix calculated using $\mathbf{D} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T$ [?], yielding demodulation results $\mathbf{S}_{out} = \mathbf{D} \cdot \mathbf{I}$. When using Fourier demodulation, we employ formula (7) (with α set to 0). (3) Vary the initial azimuth angle α within the range of 0° to 0.1°, and quantitatively calculate the relationship between α and the relative error of demodulation results, as shown in Figure 4 [Figure 4: see original paper].

Figure 4 shows the relationship between the relative error of demodulation results and the initial azimuth angle obtained using the two methods. The dashed line represents results from the demodulation matrix method, while the ' ' symbols represent Fourier analysis results. First, regardless of the method used, the relative errors of Q, U, and V all increase significantly with α. For linear polarization signals, the results from both methods are essentially consistent (the residual is very small, below the 10^{-6} level). However, for circular polarization signals, the Fourier analysis results show larger relative errors, approximately twice those obtained using the demodulation matrix. We conducted further analysis on this situation and found that the error can be reduced by increasing the detector acquisition frequency within one modulation period (e.g., from 8 to 16 acquisitions), though this approach is not very practical in real applications, which we will discuss in the results section. Second, the relative errors of Q and U in the figure show some differences, indicating that the magnitude of relative error is related to the signal strength itself. When we further set the input Q, U, and V signals to the same magnitude, their relative errors become essentially identical, demonstrating that initial azimuth angle error has consistent effects on all three. Therefore, for relatively weak polarization signals at the 10^{-2} level, such as the Q signal here ($Q = 0.02$), both demodulation methods consistently require the initial azimuth angle error to be less than 0.03° (approximately $108''$) to keep the relative error within the 10^{-3} level.

2.2 Waveplate Positioning Error

Waveplate rotational positioning error is mainly caused by unstable rotation speed of the motor driving the waveplate, which results in the rotation angle of the waveplate' s fast axis during each detector exposure no longer being a stable value (i.e., each $\Delta\theta_j$ differs). Based on our investigation, currently available high-precision motors can achieve repeatability accuracy of about 0.01° after reaching constant speed. Therefore, we conducted Monte Carlo simulations for positioning errors of this magnitude. Our analysis approach is as follows: (1) Input known polarization signals $\mathbf{S} = (1, 0.02, 0.06, 0.15)^T$. Simulate 8-step continuous modulation, where the waveplate rotates through an angle of $\pi/8 \pm 0.01^\circ$ during a single detector exposure, with α as a random number within 0° to 0.01°. Waveplate retardation $\delta = 127^\circ$. Generate the observed intensity profile I_j according to formula (6). (2) Demodulate the observed intensity profile using ideal demodulation matrix and Fourier analysis methods respectively, and calculate the relative error between demodulation results and input values.

Figure 5 [Figure 5: see original paper] shows the impact of waveplate positioning error on the relative error of demodulation results. After simulating 100 independent modulation processes, we find: the relative errors of linear and circular polarization signals obtained by the two demodulation methods are very close. When the input signal is $\mathbf{S} = (1, 0.02, 0.06, 0.15)^T$, the RMS values of relative errors for Q, U, and V are 3.7×10^{-3}, 1.1×10^{-3}, and 1.8×10^{-4} respectively. We further calculated that when the input signal is $\mathbf{S} = (1, 0.15, 0.15, 0.15)^T$, the RMS values of relative errors for the three become 3.7×10^{-3}, 3.5×10^{-3}, and 2.9×10^{-3} respectively. This not only shows that the magnitude of relative error is related to signal strength, but also indicates that positioning error has a slightly smaller impact on circular polarization signals than on linear polarization signals. Finally, for Q signals at the 10^{-2} level, a waveplate positioning accuracy of 0.01° produces relative errors also at the 10^{-3} level.

2.3 Measurement Error Caused by Time Difference Ratio

Figure 6 [Figure 6: see original paper] illustrates the difference between the time required for the waveplate to complete one rotation slot during a modulation cycle and the time taken by the detector to collect photons. Assuming the waveplate has no positioning error and rotates uniformly through a specific angle ($\pi/8$) within time T, but the detector' s actual exposure duration is only t_{exp}, there exists a time difference $\Delta t = \Delta t_j - t_{exp}$ during which the detector does not collect light intensity, thus creating an error. We define the ratio of this time difference Δt to time T as the time difference ratio (i.e., $\Delta t/T \times 100\%$). In Figure 7 [Figure 7: see original paper], we simulate the relative error between demodulation results and input values as $\Delta t/T$ increases from 0% to 2%. In this section, the input polarization signal is $\mathbf{S} = (1, 0.15, 0.15, 0.15)^T$, meaning linear and circular polarization signals have equal strength.

Figure 7 shows the effect of time difference ratio on demodulation results. We find that results from both demodulation methods are essentially consistent: as $\Delta t/T$ gradually increases, the relative error of demodulation results increases accordingly. The obvious difference between the two methods still lies in circular polarization demodulation, where the demodulation matrix yields relatively smaller errors. We also observe that compared to circular polarization signals, changes in time difference ratio have a more pronounced effect on linear polarization signals. Analyzing the Q signal reveals that to control relative error within the 10^{-3} level, $\Delta t/T$ must be less than 1%. We also analyzed the case of $\mathbf{S} = (1, 0.02, 0.02, 0.02)^T$, where the Q signal is 10 times weaker, and still found that time difference ratio changes have very weak effects on circular polarization signals, similar to the results in Figure 7, and therefore not shown. This indicates that time difference mainly causes crosstalk between linear polarization signals. Finally, it should be noted that our simulation only addresses one modulation period; if multiple periods are modulated, the small Δt will accumulate, and errors will gradually amplify.

3 Discussion and Conclusions

This paper presents the derivation process and demodulation formula (formula (7)) for Fourier analysis in the continuously rotating waveplate modulation mode, and verifies the formula' s accuracy using synthetic polarization profiles. We must emphasize that in formula (7), using the imaginary part rather than the real part for V signal demodulation yields more accurate results. Subsequently, we quantitatively calculate the errors in demodulation results caused by three factors—waveplate initial azimuth angle error, rotational positioning error, and time difference ratio during detector exposure—using both Fourier analysis and the classical demodulation matrix methods, thereby assessing their impact on polarization measurement precision. We discuss and summarize the main conclusions as follows:

(1) **Impact of waveplate initial azimuth angle error**: For linear polarization signals, Fourier analysis and demodulation matrix methods can produce relatively consistent results. When measuring polarization signals at the 10^{-2} level (common in solar physics observations), if the relative error of results must be within 10^{-3}, the initial azimuth angle error must

be less than 0.03° (approximately 108″). However, we find that for circular polarization signals, Fourier analysis yields relatively larger errors. From this perspective, the demodulation matrix method can better overcome the impact of initial azimuth angle error. Additionally, we find that initial azimuth angle error has consistent effects on circular and linear polarization, and the magnitude of relative error is related to the strength of the polarization signal itself. Here we compare with the results of Liang et al. (2019) [**?**]. Their error analysis was based on stepwise rotating waveplate modulation and simple Fourier analysis. Their results (which we confirmed through similar simulations) showed that initial azimuth angle error has almost no effect on the relative error of circular polarization signals. Their analysis based on linear polarization signals is consistent with this paper. Therefore, we conclude that continuous modulation imposes higher requirements on initial azimuth angle positioning accuracy compared to stepwise modulation.

(2) **Impact of waveplate rotational positioning error**: For both linear and circular polarization signals, the results from the two demodulation methods are essentially consistent. Both show that rotational positioning error has a more pronounced effect on linear polarization signals, and the magnitude of relative error is related to signal strength. Quantitative analysis reveals that when the waveplate' s repeatability accuracy is around 0.01° (consistent with the performance specifications of common high-precision motors), measurement errors for weak polarization signals at the 10^{-2} level can be maintained around the 10^{-3} level. Liang et al. (2019) [**?**] showed that for stepwise modulation, the waveplate positioning accuracy can be relaxed to 0.1° when requiring relative measurement errors at the 10^{-3} level. Therefore, we conclude that under the same measurement error requirements, continuous modulation mode imposes higher requirements on waveplate rotational positioning accuracy than stepwise modulation mode.

(3) **Impact of time difference ratio during detector acquisition**: This effect exists only in continuous modulation mode. Under ideal conditions, we require $\Delta t_j = t_{exp}$, but in actual measurements, image storage or data transfer requires certain time Δt, causing the modulation mode to deviate from ideal conditions and resulting in demodulation errors. Δt is generally at the microsecond level, which is very small compared to the tens of milliseconds exposure time typically used in solar observations. Moreover, we find that the effect of time difference mainly causes crosstalk between linear polarization signals. When $\Delta t/T$ is around 1%, its impact on the relative error of circular polarization signals is near the 10^{-4} level. Additionally, in continuous acquisition mode, besides time difference, we should also pay attention to the matching between detector frame rate and waveplate rotation speed, using the detector' s frame rate to set the lower limit for waveplate rotation speed.

(4) **Finally, it is worth noting** that although not quantitatively calculated, we find that increasing the detector acquisition frequency within one modulation period improves polarization measurement precision for both demodulation methods. However, increasing acquisition frequency leads to increased data volume and reduced modulation frequency or time resolution [**?**]. Therefore, after comprehensive consideration of various factors, we adopt the common mode of 8 frames per period in this work.

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Note: Figure translations are in progress. See original paper for figures.