Geomorphic Evolution Characteristics of Loess Small Watersheds Based on Potential Energy Information Entropy (Postprint)

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Abstract

The geomorphological evolution characteristics of watershed geomorphological systems in the Loess Plateau are exceedingly complex, with numerous scientific issues yet to be investigated in greater depth. Previous research has predominantly concentrated on specific aspects such as erosion and development characteristics of watershed geomorphological evolution, while lacking studies that deeply analyze the geomorphological evolution features of field multi-rock-soil layer loess small watersheds from the perspective of watershed geomorphological systems and their potential energy information entropy. Therefore, based on systems theory perspectives and methods, a mathematical model for the multi-rock-soil layer loess small watershed geomorphological system and its potential energy information entropy was established, and the geomorphological evolution characteristics were investigated using the Xindian Gully watershed as a case study. The results demonstrate that: (1) The constructed conceptual model of the field multi-rock-soil layer loess small watershed geomorphological system and the mathematical model of its potential energy information entropy can effectively conduct numerical simulation of the Xindian Gully watershed. (2) The geomorphological evolution process of the Xindian Gully watershed, dominated by loess erosion from 2000 to 2019, represents an entropy reduction process in its potential energy information entropy and a continuous erosion process of loess landforms. (3) The potential energy information entropy of the Xindian Gully watershed can satisfactorily reflect the geomorphological evolution stage and erosion process of this small watershed.

Full Text

Abstract

Research on the evolution characteristics of loess erosion and the development of watershed geomorphic systems in the Loess Plateau of China is a hotspot in the study of loess landforms, and many related scientific issues still need to be studied. Most previous studies have focused on the erosion and development characteristics of watershed geomorphology, but there is a lack of research on in-depth analysis of the geomorphic evolution characteristics of loess small watersheds with multiple rock and soil layers in the field from the perspective of watershed geomorphology system and its potential energy information entropy (PEIE). Therefore, based on the viewpoint and method of system theory, mathematical models of the geomorphic system and its PEIE of a small loess watershed with multiple rock and soil layers were built. Taking the Xindiangou small watershed at Suide County, Shaanxi Province, China as the research sample area, the digital elevation model data of 5 periods from 2000 to 2019 were used to investigate the entropy change law of the PEIE and the characteristics of landform evolution in the watershed. The results show that from 2000 to 2019, the geomorphic evolution process dominated by loess erosion in the Xindiangou small watershed is the process of entropy reduction of its PEIE and continuous erosion of the loess landform. Combined with the entropy change of PEIE in the Xindiangou small watershed and the indoor small watershed, is speculated that the watershed geomorphic system will form a W-shaped PEIE change curve in its complete geomorphic evolution process, and its PEIE can better indicate the watershed geomorphic development demarcation points in its infancy, maturity, and old age. The research results also confirm the effectiveness of the conceptual model of the loess watershed geomorphic system and its mathematical model of PEIE in the numerical simulation of the Xindiangou small watershed. This model is an extension of the existing mathematical model of PEIE of watershed geomorphic systems with homogeneous single loess layers, which can be used in heterogeneous multiple rock and soil layer loess watershed geomorphic systems and has wider applicability. The research results provide ideas for further research on the geomorphic formation mechanism and evolution law of the Xindiangou small watershed in the future and guide soil and water conservation, ecological restoration, and regional sustainable development of the Xindiangou small watershed, which has important theoretical significance and good application prospects.

Keywords: digital elevation model (DEM); digital terrain analysis; potential energy information entropy; watershed geomorphic system; Xindiangou small watershed

Introduction

Since the 1960s, many geomorphologists have fully recognized the importance of applying general system theory to geomorphology research and have conducted further studies. Strahler [?] established the dynamic basis of geomorphology, while Culling [?] and Howard [?] applied system theory to analyze geomorphological systems. Phillips [?, ?] discussed deterministic chaos and emergent equilibrium in geomorphology. Chinese scholars such as Li Qiang et al. [?] and Ma Xinzhong et al. [?] also made significant contributions to watershed geomorphology system research. The concept of entropy in geomorphology was first introduced by Leopold and Langbein [?], but its quantitative calculation remained challenging until the development of digital elevation model (DEM) technology.

Shannon [?] first proposed the concept of information entropy, which was later applied to cartography and geomorphology. Ai Nanshan [?] introduced geomorphic information entropy to erosional drainage systems, while Yue Tianxiang et al. [?] further proposed the concept of geomorphic superentropy. These concepts have been widely applied in gully head activity evaluation [?], glacial debris flow risk assessment [?], and spatial variation of gully density [?]. However, most previous studies focused on homogeneous loess layers, and there remains a lack of research on heterogeneous multi-layer watershed systems.

Jiang Qiong [?] proposed the concept of potential energy information entropy (PEIE) for watershed geomorphic systems and constructed a mathematical model for homogeneous loess watersheds. Zhao Weidong et al. [?] applied this concept to study valley network evolution in small loess watersheds. However, these conclusions were based on indoor artificial rainfall simulation experiments with homogeneous loess, and their applicability to natural field conditions with multiple rock and soil layers requires further investigation. This study addresses this gap by constructing a mathematical model for multi-layer loess watershed geomorphic systems and applying it to the Xindiangou small watershed.

1. Study Area Overview

This study focuses on the Xindiangou small watershed in the Loess Plateau [Figure 1: see original paper]. Located on the left bank of the Wuding River in Suide County, Yulin City, Shaanxi Province, the watershed spans 110°16′~110°17′E and 37°29′~37°31′N. It belongs to the first subregion of the loess hilly-gully region with severe surface erosion [?]. The climate is semi-arid temperate continental, with precipitation concentrated in the flood season from June to September. The soil is loose, dominated by loessial soil, mainly distributed in areas with gentle terrain or good vegetation cover [?].

To determine the spatial distribution of rock and soil layers and their density differences, field investigations and sampling were conducted in July 2019 [Figure 2: see original paper]. The results show clear stratification in the lower part of the watershed, with sandstone at the bottom, shale in the middle, and loess on top. Sandstone and shale densities were measured at 2.80 $\mathrm{g \cdot cm^{-3}}$ and 2.65 $\mathrm{g \cdot cm^{-3}}$, respectively. Loess density was measured at multiple sampling points

, with an average value of 1.21 $g \cdot cm^{-3}$ used for calculations.

2.1 Data Sources and Processing

To study the geomorphic evolution of the Xindiangou watershed, DEM data from different periods were collected . The data sources include SRTM V3, ASTER GDEM, TanDEM, and high-resolution DEMs from various platforms. All data were uniformly projected to the WGS_{{{1984}}_{{UTM}}} coordinate system using ArcGIS software and resampled to a consistent resolution.

2.2 Conceptual Model Construction

A watershed geomorphic system is a typical open natural system that continuously exchanges matter and energy with the external environment [?]. To construct the conceptual model, system boundaries must first be defined. For the Xindiangou watershed, the upper boundary is the topographic surface, the lateral boundaries are vertical planes along the watershed divide, and the bottom boundary is a horizontal plane at the lowest elevation point near the watershed outlet, which serves as the potential energy reference plane.

Based on field investigations, the watershed has three distinct layers: sandstone at the bottom, shale in the middle, and loess on top. This stratification forms the basis for the three-dimensional discretization of the multi-layer watershed geomorphic system.

2.3 Mathematical Model of Potential Energy Information Entropy

2.3.1 Discretization of Multi-layer Watershed Geomorphic System

The potential energy of a watershed geomorphic system is defined as the total potential energy of all rock and soil masses within the system boundaries, calculated relative to the bottom boundary reference plane. To calculate this, the watershed is discretized using a square grid technique in the horizontal plane, creating m×n grid cells. In the vertical direction, the system is stratified according to rock/soil density differences. For the Xindiangou watershed, three layers are defined corresponding to sandstone, shale, and loess.

Each grid cell may contain different rock/soil layers, forming a three-dimensional discretized body with m×n×k basic units. The total potential energy is obtained by summing the potential energy of all individual units.

2.3.2 PEIE Mathematical Model Construction

Jiang Qiong [?] proposed the PEIE concept for homogeneous loess watersheds. The potential energy of a single unit is calculated as:

$$E_P = \frac{1}{2} g \rho d^2 h^2$$

where g is gravitational acceleration, ρ is density, d is cell size, and h is the height relative to the reference plane.

For multi-layer systems, the model must account for different densities and layer thicknesses. The height of bedrock at grid cell (i, j) is calculated using:

$$H_{ij} = \cos\alpha \times \tan\beta \times \sin(\theta_{ij} - \theta_0) + \cos\beta \times (H_0 - H_{min})$$

where α is bedrock dip direction, β is dip angle, (λ_0, ϕ_0, H_0) are coordinates of the sampling point, $(\lambda_{ij}, \phi_{ij})$ are coordinates of the grid cell, and H_{min} is the minimum elevation at the outlet.

The total potential energy of the multi-layer system is:

$$E_{total} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} g \rho_k d^2 h_{k,ij}$$

where ρ_k is the density of layer k, $h_{k,ij}$ is the height of layer k in cell (i, j), and K is the number of layers.

The PEIE is then calculated based on the probability distribution of potential energy levels. Research shows that the number of energy levels is closely related to PEIE magnitude [?]. The optimal number of levels is determined when the rate of PEIE change becomes negligible. For this study, 16 levels were selected as the optimal classification number, corresponding to a change rate of negative one ten-thousandth.

3.1 Geomorphic Erosion Characteristics of PEIE

PEIE represents the spatial disorder of potential energy distribution in a watershed geomorphic system, analogous to how thermodynamic entropy represents molecular disorder [?]. The change in PEIE reflects internal energy changes, similar to thermodynamic entropy changes [?]. Jiang Qiong [?] demonstrated that PEIE reduction effectively reflects erosion processes in homogeneous loess watersheds under indoor rainfall simulation.

To verify this relationship for natural multi-layer watersheds, the correlation between PEIE and gully density in the Xindiangou watershed was analyzed. The PEIE values for 2000–2019 were calculated, and gully networks were extracted using a flow accumulation threshold determined by the second derivative method [?]. The results show that PEIE decreased continuously from 0.312 to 0.285 (a reduction of 8.30%), while gully density increased slightly from 1.654 to 1.771 $\mathrm{km} \cdot \mathrm{km}^{-2}$ (an increase of 7.06%) [Figure 5: see original paper].

Polynomial fitting reveals a strong cubic relationship between gully density and PEIE, allowing estimation of gully density from PEIE calculations. This confirms that the entropy reduction process in Xindiangou watershed corresponds to a slight enhancement of erosion, consistent with Jiang's conclusions and validating the applicability of PEIE to natural multi-layer watersheds.

3.2 Geomorphic Development Characteristics of PEIE

Jiang Qiong [**?**] showed that PEIE can indicate development stages in homogeneous loess watersheds under indoor conditions. To test this for natural heterogeneous watersheds, the relationship between PEIE and geomorphic development stages in Xindiangou watershed was investigated.

The hypsometric integral (HI) was calculated using the relief ratio method [**?**, **?**]:

$$HI = \frac{H_{mean} - H_{min}}{H_{max} - H_{min}}$$

The HI values for five periods (2000, 2005, 2010, 2015, 2019) range from 0.406 to 0.448 , indicating that the watershed has been in the maturity stage throughout the study period.

However, unlike indoor homogeneous watersheds where PEIE increases during the maturity stage, Xindiangou watershed shows continuous PEIE decrease during maturity [Figure 6: see original paper]. This discrepancy can be explained by Davis's geographical cycle theory [**?**]. In the initial stage (infancy), a perfectly horizontal surface yields maximum PEIE ($\log_2$N). As erosion creates relief, PEIE decreases. In the final stage (old age), when the surface approaches a peneplain, PEIE returns to $\log_2$N. This complete cycle produces a W-shaped standard PEIE curve [Figure 7: see original paper].

The standard PEIE curve suggests that: - During infancy: PEIE decreases from maximum to minimum - During maturity: PEIE increases from minimum toward maximum - During old age: PEIE decreases again toward maximum

Both indoor homogeneous and Xindiangou heterogeneous watersheds follow this W-shaped pattern, with turning points at stage boundaries. The difference lies in their position along this curve: the indoor watershed was studied during its infancy-maturity transition (showing PEIE increase), while Xindiangou is in the maturity-old age transition (showing PEIE decrease).

4. Discussion

The geomorphic evolution of loess watersheds is complex, influenced by stratigraphy, vegetation, rainfall, and human activities. This study constructed a conceptual and mathematical model for the Xindiangou multi-layer watershed system, demonstrating that PEIE can effectively simulate natural watershed evolution. The entropy reduction process observed in Xindiangou (2000–2019) aligns with Jiang's [?] indoor experimental results, confirming that PEIE reduction reflects erosion enhancement.

The proposed standard W-shaped PEIE curve provides a new approach for fine-scale division of geomorphic development stages, though further validation is needed. Future research should examine more watersheds at different developmental stages to refine the curve morphology and validate the geomorphic significance of its inflection points.

5. Conclusions

1) Based on previous research and field data from Xindiangou watershed, this study successfully constructed a conceptual model and mathematical model of PEIE for natural multi-layer loess watersheds. The model effectively simulates the Xindiangou watershed, extending the existing homogeneous loess model to heterogeneous multi-layer systems with broader applicability.
2) The geomorphic evolution of Xindiangou watershed (2000–2019) is characterized by continuous PEIE reduction (entropy decrease) and loess erosion, confirming that PEIE can effectively reflect erosion processes in natural multi-layer watersheds.
3) PEIE may indicate the boundary between maturity and old age in Xindiangou watershed, while indoor homogeneous watershed PEIE better indicates the infancy-maturity boundary. It is speculated that loess watershed systems develop a W-shaped standard PEIE curve during complete evolution, which can effectively indicate stage boundaries and provide a new approach for watershed geomorphic development stage division.

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Note: Figure translations are in progress. See original paper for figures.