

## Systematic KMTNet Planetary Anomaly Search, Paper VII: Complete Sample of $q < 10^{-4}$ Planets from the First Four-Year Survey Postprint

**Authors:** Weicheng Zang, Youn Kil Jung, Hongjing Yang, Xiangyu Zhang, Andrzej Udalski, Jennifer C. Yee, Andrew Gould, Shude Mao, Weicheng Zang

**Date:** 2023-02-21T00:00:00+00:00

### Abstract

We present the analysis of seven microlensing planetary events with planet/host mass ratios  $q < 10^{-4}$ : KMT-2017-BLG-1194, KMT-2017-BLG-0428, KMT-2019-BLG-1806, KMT-2017-BLG-1003, KMT-2019-BLG-1367, OGLE-2017-BLG-1806, and KMT-2016-BLG-1105. They were identified by applying the Korea Microlensing Telescope Network (KMTNet) AnomalyFinder algorithm to 2016–2019 KMTNet events. A Bayesian analysis indicates that all the lens systems consist of a cold super-Earth orbiting an M or K dwarf. Together with 17 previously published and three that will be published elsewhere, AnomalyFinder has found a total of 27 planets that have solutions with  $q < 10^{-4}$  from 2016–2019 KMTNet events, which lays the foundation for the first statistical analysis of the planetary mass-ratio function based on KMTNet data. By reviewing the 27 planets, we find that the missing planetary caustics problem in the KMTNet planetary sample has been solved by AnomalyFinder. We also find a desert of high-magnification planetary signals (A 65), and a follow-up project for KMTNet high-magnification events could detect at least two more  $q < 10^{-4}$  planets per year and form an independent statistical sample.

### Full Text

### Preamble

**DRAFT VERSION JANUARY 12, 2023**

Typeset using LATEX twocolumn style in AASTeX62

**Systematic KMTNet Planetary Anomaly Search, Paper VII: Complete Sample of  $q < 10^{-4}$  Planets from the First Four-Year Survey**

WEICHENG ZANG,<sup>1,2</sup> YOUN KIL JUNG,<sup>3,4</sup> HONGJING YANG,<sup>1</sup> XI-ANGYU ZHANG,<sup>5</sup> ANDRZEJ UDALSKI,<sup>6</sup> JENNIFER C. YEE,<sup>2</sup> ANDREW GOULD,<sup>5,7</sup> AND SHUDE MAO<sup>1,8</sup> (LEADING AUTHORS)  
MICHAEL D. ALBROW,<sup>9</sup> SUN-JU CHUNG,<sup>3,4</sup> CHEONGHO HAN,<sup>10</sup> KYU-HA HWANG,<sup>3</sup> YOON-HYUN RYU,<sup>3</sup> IN-GU SHIN,<sup>2</sup> YOSSI SHVARTZ-VALD,<sup>11</sup> SANG-MOK CHA,<sup>3,12</sup> DONG-JIN KIM,<sup>3</sup> HYOON-WOO KIM,<sup>3</sup> SEUNG-LEE KIM,<sup>3,4</sup> CHUNG-UK LEE,<sup>3</sup> DONG-JOO LEE,<sup>3</sup> YONGSEOK LEE,<sup>3,12</sup> BYEONG-GON PARK,<sup>3,4</sup> AND RICHARD W. POGGE<sup>7</sup> (THE KMTNET COLLABORATION)  
PRZEMEK MRÓZ,<sup>6</sup> JAN SKOWRON,<sup>6</sup> RADOSŁAW POLESKI,<sup>6</sup> MICHAŁ K. SZYMAŃSKI,<sup>6</sup> IGOR SOSZYŃSKI,<sup>6</sup> PAWEŁ PIETRUKOWICZ,<sup>6</sup> SZYMON KOZŁOWSKI,<sup>6</sup> KRZYSZTOF ULACZYK,<sup>13</sup> KRZYSZTOF A. RYBICKI,<sup>6</sup> PATRYK IWANEK,<sup>6</sup> MARCIN WRONA,<sup>6</sup> AND MARIUSZ GROMADZKI<sup>6</sup> (THE OGLE COLLABORATION)  
HANYUE WANG,<sup>2</sup> JIYUAN ZHANG,<sup>1</sup> AND WEI ZHU<sup>1</sup> (THE MAP COLLABORATION)

<sup>1</sup>Department of Astronomy, Tsinghua University, Beijing 100084, China

<sup>2</sup>Center for Astrophysics | Harvard & Smithsonian, 60 Garden St., Cambridge, MA 02138, USA

<sup>3</sup>Korea Astronomy and Space Science Institute, Daejeon 34055, Republic of Korea

<sup>4</sup>University of Science and Technology, Korea (UST), 217 Gajeong-ro Yuseong-gu, Daejeon 34113, Republic of Korea

<sup>5</sup>Max-Planck-Institute for Astronomy, Königstuhl 17, 69117 Heidelberg, Germany

<sup>6</sup>Astronomical Observatory, University of Warsaw, Al. Ujazdowskie 4, 00-478 Warszawa, Poland

<sup>7</sup>Department of Astronomy, Ohio State University, 140 W. 18th Ave., Columbus, OH 43210, USA

<sup>8</sup>National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100101, China

<sup>9</sup>University of Canterbury, Department of Physics and Astronomy, Private Bag 4800, Christchurch 8020, New Zealand

<sup>10</sup>Department of Physics, Chungbuk National University, Cheongju 28644, Republic of Korea

<sup>11</sup>Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot 76100, Israel

<sup>12</sup>School of Space Research, Kyung Hee University, Yongin, Gyeonggi 17104, Republic of Korea

<sup>13</sup>Department of Physics, University of Warwick, Gibbet Hill Road, Coventry, CV4 7AL, UK

## Abstract

We present the analysis of seven microlensing planetary events with planet/host mass ratios  $q < 10^{-4}$ : KMT-2017-BLG-1194, KMT-2017-BLG-0428, KMT-2019-BLG-1806, KMT-2017-BLG-1003, KMT-2019-BLG-1367, OGLE-2017-BLG-1806, and KMT-2016-BLG-1105. They were identified by applying the Korea Microlensing Telescope Network (KMTNet) AnomalyFinder algorithm to 2016–2019 KMTNet events. A Bayesian analysis indicates that all the lens systems consist of a cold super-Earth orbiting an M or K dwarf. Together with 17 previously published and three that will be published elsewhere, AnomalyFinder has found a total of 27 planets that have solutions with  $q < 10^{-4}$  from 2016–2019 KMTNet events, which lays the foundation for the first statistical analysis of the planetary mass-ratio function based on KMTNet data. By reviewing the 27 planets, we find that the missing planetary caustics problem in the KMTNet planetary sample has been solved by AnomalyFinder. We also find a desert of high-magnification planetary signals ( $A \sim 65$ ), and a follow-up project for KMTNet high-magnification events could detect at least two more  $q < 10^{-4}$  planets per year and form an independent statistical sample.

## 1. Introduction

Corresponding author: Weicheng Zang (3130102785@zju.edu.cn)

Among current exoplanet detection methods, a unique capability of the gravitational microlensing technique (Mao & Paczynski 1991; Gould & Loeb 1992) is to detect low-mass ( $M_{\text{planet}} \sim 20M_{\oplus}$ ) cold planets beyond the snow line (Hayashi 1981; Min et al. 2011), including Neptune-mass cold planets, which are common (Uranus and Neptune) in our Solar System, and cold terrestrial planets, which are absent in our Solar System. Because the typical host stars of microlensing planetary systems are M and K dwarfs, detections of  $q < 10^{-4}$  planets (where  $q$  is the planet/host mass ratio) can reveal the abundance of low-mass cold planets and answer how common the outer solar system is.

However, since the first microlensing planet was detected in 2003 (Bond et al. 2004), the first 13 years of microlensing planetary detections only discovered six  $q < 10^{-4}$  planets<sup>1</sup> and none of them had mass ratios below  $4.4 \times 10^{-5}$ . The paucity of detected  $q < 10^{-4}$  planets led to important statistical implications for cold planets. Suzuki et al. (2016) analyzed 1474 microlensing events discovered by the Microlensing Observations in Astrophysics (MOA) survey (Sako et al. 2008) and formed a homogeneously selected sample including 22 planets. They found that the mass-ratio function of microlensing planets increases as  $q$  decreases until a break at  $q \sim 1.7 \times 10^{-4}$ , below which the planetary occurrence rate likely drops. This break suggests that Neptune-mass planets are likely to be the most common cold planets. However, the Suzuki et al. (2016) sample only contains two  $q < 10^{-4}$  planets and thus may be affected by small number statistics. To examine the existence of the break, a larger  $q < 10^{-4}$  sample is needed.

After its commissioning season in 2015, the new-generation microlensing survey, the Korea Microlensing Telescope Network (KMTNet, Kim et al. 2016), has been conducting near-continuous, wide-area, high-cadence surveys for  $\sim 96 \text{ deg}^2$ . The fields with cadences of  $\Gamma \geq 2 \text{ hr}^{-1}$  are the KMTNet prime fields ( $\sim 12 \text{ deg}^2$ ) and the other fields are the KMTNet sub-prime fields ( $\sim 84 \text{ deg}^2$ ). Since 2016, the detections of  $q < 10^{-4}$  planets have been greatly increased in two ways, and the KMTNet data played a major or decisive role in all detections. First, more than ten  $q < 10^{-4}$  planets have been detected from by-eye searches, including three with  $q < 2 \times 10^{-5}$  (Gould et al. 2020; Yee et al. 2021; Zang et al. 2021a). Second, Zang et al. (2021b, 2022a) developed the KMTNet AnomalyFinder algorithm to systematically search for planetary signals. This algorithm has been applied to the 2018 and 2019 KMTNet prime fields ( $\Gamma \geq 2 \text{ hr}^{-1}$ ) and uncovered five new  $q < 10^{-4}$  planets (Zang et al. 2021b; Hwang et al. 2022; Gould et al. 2022). Moreover, the systematic search opens a window for a homogeneous large-scale KMTNet planetary sample. According to the experience from 2018 and 2019 KMTNet prime fields, we expect to detect  $\sim 20$  planets with  $q < 10^{-4}$  from 2016–2019 seasons. This will be an order of magnitude larger than the Suzuki et al. (2016) sample at  $q < 10^{-4}$ .

To build the first KMTNet  $q < 10^{-4}$  statistical sample, we applied the KMTNet AnomalyFinder algorithm to the 2016–2019 KMTNet microlensing events. In this paper, we introduce seven new  $q < 10^{-4}$  events from this search. They are KMT-2017-BLG-1194, KMT-2017-BLG-0428, KMT-2019-BLG-1806/OGLE-2019-BLG-1250, KMT-2017-BLG-1003, KMT-2019-BLG-1367, OGLE-2017-BLG-1806/KMT-2017-BLG-1021, and KMT-2016-BLG-1105. Together with 17 already published and three that will be published elsewhere, the KMTNet AnomalyFinder algorithm found 27 events that can be fit by  $q < 10^{-4}$  models from 2016–2019 KMTNet data. However, whether a planet can be used for statistical studies requires further investigations, which is beyond the scope of this paper.

The paper is structured as follows. In Section 2, we briefly introduce the KMTNet AnomalyFinder algorithm and the procedure to form the  $q < 10^{-4}$  sample. In Sections 3, 4 and 5, we present the observations and the analysis of seven  $q < 10^{-4}$  events. Finally, we discuss the implications from the 2016–2019 KMTNet  $q < 10^{-4}$  planetary sample in Section 6.

<sup>1</sup>They are OGLE-2005-BLG-169Lb (Gould et al. 2006), OGLE-2005-BLG-390Lb (Beaulieu et al. 2006), OGLE-2007-BLG-368Lb (Sumi et al. 2010), MOA-2009-BLG-266Lb (Muraki et al. 2011), OGLE-2013-BLG-0341Lb (Gould et al. 2014b), OGLE-2015-BLG-1670 (Ranc et al. 2019).

## 2. The Basic of AnomalyFinder and the Procedure

Section 2 of Zang et al. (2021b) and Section 2 of Zang et al. (2022a) together introduced the KMTNet AnomalyFinder algorithm. The AnomalyFinder uses a Gould (1996) 2-dimensional grid of  $(t_0, t_{\text{eff}})$  to search for and fit anomalies

from the residuals to a point-source point-lens (PSPL, Paczyński 1986) model. Here  $t_0$  is the time of maximum magnification, and  $t_{\text{eff}}$  is the effective timescale. For our search, the shortest  $t_{\text{eff}}$  is 0.05 days and the longest  $t_{\text{eff}}$  is 6.65 days. The parameters that evaluate the significance of a candidate anomaly are  $\Delta^2_{\text{flat}}$ . See Equation (4) of Zang et al. (2021b) for their definitions. The criteria of  $\Delta^2_{\text{flat}}$  are the same as the criteria used in Zang et al. (2022a); Gould et al. (2022); Jung et al. (2022), with  $\Delta^2_0 > 120$  and  $\Delta^2_{\text{flat}} > 60$  for the KMTNet prime-field events and  $\Delta^2_0 > 60$  and  $\Delta^2_{\text{flat}} > 30$  for the KMTNet sub-prime-field events. Future statistical studies should use the same criteria. In addition, an anomaly is required to contain at least three successive points  $\geq 2\sigma$  away from a PSPL model.

As a result, we found 464 and 608 candidate anomalies from 2016–2019 KMTNet prime-field and sub-prime-field events, respectively. We checked whether the data from other surveys are consistent with the KMTNet-based anomalies and cross-checked with C. Han’s modeling. We fitted all the  $q < 10^{-3}$  candidates with online data and found 13 new candidates with  $q < 2 \times 10^{-4}$ . Then, we conducted tender-loving care (TLC) re-reductions and re-fitted the 13 events. Of these, eight events unambiguously have  $q < 10^{-4}$ , three events (KMT-2016-BLG-1307, KMT-2017-BLG-0849, and KMT-2017-BLG-1057) have  $10^{-4} < q < 2 \times 10^{-4}$ , and two events (KMT-2016-BLG-0625 (Shin et al. in prep) and OGLE-2017-BLG-0448/KMT-2017-BLG-0090 (Zhai et al. in prep)) have ambiguous mass ratios at  $10^{-5} < q < 10^{-3}$  and will be published elsewhere.

Among the eight unambiguous  $q < 10^{-4}$  events, one event, OGLE-2016-BLG-0007/MOA-2016-BLG-088/KMT-2016-BLG-1991, will be published elsewhere because it has the lowest  $q$  of this sample. We analyze and publish the remaining seven events in this paper. We note that the planetary signals of the seven events are not strong, although they are confirmed by at least two data sets. We thus further check whether the light curves have other similar anomalies, to exclude the possibility of unknown systematic errors. We applied the AnomalyFinder algorithm to the re-reduction data. For all of the seven events, besides the known planetary signals no anomaly with  $\Delta^2_0 > 20$  was detected. Therefore, the light curves of the seven events are stable and planetary signals are reliable.

### 3. Observations and Data Reductions

Table 1 lists the basic observational information for the seven events, including event names, first discovery date, coordinates in the equatorial and galactic systems, and nominal cadences ( $\Gamma$ ). The seven planetary events were all identified by the KMTNet post-season EventFinder algorithm (Kim et al. 2018a). Of them, KMT-2019-BLG-1806/OGLE-2019-BLG-1250 and OGLE-2017-BLG-1806/KMT-2017-BLG-1021 were discovered by the KMTNet alert-finder system (Kim et al. 2018b) and the Early Warning System (Udalski et al. 1994; Udalski 2003) of the Optical Gravitational Lensing Experiment (OGLE, Udalski et al. 2015), respectively, during their observational seasons. Hereafter, we designate KMT-2019-BLG-1806/OGLE-2019-BLG-1250 and OGLE-2017-BLG-

1806/KMT-2017-BLG-1021 by their first-discovery name, KMT-2019-BLG-1806 and OGLE-2017-BLG-1806. During the 2019 observational season, the KMTNet alert-finder system also discovered KMT-2019-BLG-1367. In addition, OGLE observed the locations of KMT-2019-BLG-1367 and KMT-2016-BLG-1105 but did not alert them. We also include the OGLE data for these two events into the light-curve analysis, for which the OGLE data confirm the planetary signals found by the KMTNet. MOA did not issue alerts for any of the seven events, and there were no follow-up data to the best of our knowledge.

KMTNet conducted observations from three identical 1.6 m telescopes equipped with 4 deg<sup>2</sup> cameras in Chile (KMTC), South Africa (KMTS), and Australia (KMTA). OGLE took data using a 1.3m telescope with 1.4 deg<sup>2</sup> field of view in Chile. For both surveys, most of the images were taken in the I band, and a fraction of V-band images were acquired for source color measurements. Each KMTNet V-band data point was taken one minute before or after one KMTNet I-band data point of the same field.

The KMTNet and OGLE data used in the light-curve analysis were reduced using the custom photometry pipelines based on the difference imaging technique (Tomaney & Crotts 1996; Alard & Lupton 1998): pySIS (Albrow et al. 2009, Yang et al. in prep) for the KMTNet data, and Wozniak (2000) for the OGLE data. For each event, the KMTC data were additionally reduced using the pyDIA photometry pipeline (Albrow 2017) to measure the source color. Except for OGLE-2017-BLG-1806 and KMT-2016-BLG-1105 whose sources are not located in any OGLE star catalog, the I-band magnitudes of the other five events reported in this paper have been calibrated to the standard I-band magnitude using the OGLE-III star catalog (Szymański et al. 2011).

## 4. Light-Curve Analysis

### 4.1. Preamble

Because all seven events contain short-lived deviations from a PSPL model, we first introduce the common methods for the light-curve analysis. The PSPL model is described by three parameters,  $t_0$ ,  $u_0$ , and  $t_E$ , which respectively represent the time of lens-source closest approach, the closest lens-source projected separation normalized to the angular Einstein radius  $\theta_E$ , and the Einstein timescale,  $t_E = \theta_E / \mu_{\text{rel}}$ , where  $\theta_E = \sqrt{(M_L \pi_{\text{rel}}) / c^2 \text{au}}$   $\approx 8.144 \text{ mas} (M / M_L)^{1/2} (\pi_{\text{rel}} / 10 \text{ as})^{1/2}$ ,  $\mu_{\text{rel}} = 4G / (c^2 \text{au})$ ,  $M_L$  is the lens mass, and  $(\pi_{\text{rel}}, \mu_{\text{rel}})$  are the lens-source relative (parallax, proper motion). In addition, for each data set  $i$ , we introduce two linear parameters,  $(f_{S,i}, f_{B,i})$ , to fit the flux of the source and any blend flux, respectively.

We search for binary-lens single-source (2L1S) models for each event. A 2L1S model requires four parameters in addition to the PSPL parameters,  $(s, q, \alpha, \beta)$ , which respectively denote the planet-host projected separation in units of  $\theta_E$ , the planet/host mass ratio, the angle between the source trajectory and the binary axis, and the angular source radius  $\theta_*$  scaled to  $\theta_E$ , i.e.,  $\beta = \theta_* / \theta_E$ .

Although the final results need detailed numerical analysis, some of the 2L1S parameters can be estimated by heuristic analysis. A PSPL fit excluding the data points around the anomaly can yield the three PSPL parameters,  $t_0$ ,  $u_0$ , and  $t_E$ . If an anomaly occurred at  $t_{\text{anom}}$ , the corresponding lens-source offset,  $u_{\text{anom}}$ , and  $\alpha$  can be estimated by:

$$u_{\text{anom}} = \sqrt{[(t_{\text{anom}} - t_0)^2 / t_E^2 + u_0^2]}$$

$$|\alpha| = \sin^{-1}(u_0 / u_{\text{anom}})$$

Because the planetary caustics are located at the position of  $|s - s^{-1}| = u_{\text{anom}}$ , we obtain:

$$s = (u_{\text{anom}} + \sqrt{(u_{\text{anom}})^2 + 4}) / 2$$

where  $s = s_+$  and  $s = s_-$  correspond to the major-image (quadrilateral) and the minor-image (triangular) planetary caustics, respectively. For two degenerate solutions with similar  $q$  but different  $s$ , Ryu et al. (2022) suggested that the geometric mean of two solutions satisfies  $s_{\text{mean}} = \sqrt{(s_+ s_-)}$ . In addition, Zhang et al. (2022) suggested a slightly different formalism, and Zhang & Gaudi (2022) provided a theoretical treatment of it.

For a dip-type planetary signal, Hwang et al. (2022) pointed out that the mass ratio can be estimated by:

$$q = (\Delta t_{\text{dip}} / t_E)^2 s |\sin^3 \alpha|$$

where  $\Delta t_{\text{dip}}$  is the duration of the dip, and the accuracy of Equation (5) should be at a factor of  $\sim 2$  level.

To find all the possible 2L1S models, we conduct two-phase grid searches for the parameters,  $(\log s, \log q, \alpha, \dots)$ . In the first phase, we conduct a sparse grid, which consists of 21 values equally spaced between  $-1.0 \leq \log s \leq 1.0$ , 20 values equally spaced between  $0^\circ \leq \alpha < 360^\circ$ , 61 values equally spaced between  $-6.0 \leq \log q \leq 0.0$  and five values equally spaced between  $-3.5 \leq \log \dots \leq -1.5$ . We use a code based on the advanced contour integration code (Bozza 2010; Bozza et al. 2018), VBBinaryLensing<sup>2</sup> to compute the 2L1S magnification. For each grid point, we search for the minimum  $\chi^2$  by Markov chain Monte Carlo (MCMC) <sup>2</sup> minimization using the emcee ensemble sampler (Foreman-Mackey et al. 2013), with fixed  $(\log q, \log s)$  and free  $(t_0, u_0, t_E, \dots, \alpha)$ . In the second phase, we conduct a denser  $(\log s, \log q, \alpha, \dots)$  grid search around each local minimum (e.g., Zhang et al. 2022b). Finally, we refine the best-fit models by MCMC with all parameters free.

For degenerate solutions, Yang et al. (2022) suggested that the phase-space factors can be used to weight the probability of each solution. We follow the procedures of Yang et al. (2022) and first calculate the covariance matrix,  $C$ , of  $(\log s, \log q, \alpha)$  from the MCMC chain. Then, the phase-space factor is  $p = \sqrt{(\det(C))}$ . Because whether a planet and its individual solutions can be used for statistical studies requires further investigations, we provide the phase-space

factors for the event with multiple solutions but do not use them to weight or reject solutions.

We also investigate whether the inclusion of two high-order effects can improve the fit. The first is the microlensing parallax effect (Gould 1992, 2000, 2004), which is due to the Earth’s orbital acceleration around the Sun. We fit it by two parameters,  $\pi_{E,N}$  and  $\pi_{E,E}$ , which are the north and east components of the microlensing parallax vector  $\pi_E$  in equatorial coordinates. We also fit the  $u_0 > 0$  and  $u_0 < 0$  solutions to consider the “ecliptic degeneracy” (Jiang et al. 2004; Poindexter et al. 2005). For four cases in this paper, the parallax contours take the form of elongated ellipses, so we report the constraints on the minor axes of the error ellipse,  $(\pi_{E,\parallel})$ , which is approximately parallel with the direction of the Earth’s acceleration. For the major axes of the parallax contours,  $\pi_E$ ,  $\pi_{E,N}$ , we only report it when the constraint is useful.

The second effect is the lens orbital motion effect (Batista et al. 2011; Skowron et al. 2011), and we fit it by the parameter  $\dot{\gamma} = (ds/dt, d\alpha/dt)$ , where  $ds/dt$  and  $d\alpha/dt$  represent the instantaneous changes in the separation and orientation of the two components defined at  $t_0$ , respectively. To exclude unbound systems, we restrict the MCMC trials to  $\beta < 1.0$ . Here  $\beta$  is the absolute value of the ratio of projected kinetic to potential energy (An et al. 2002; Dong et al. 2009),

$$\beta = (\dot{\gamma}^2 M_{\text{yr}^2}) / (\pi_E + \pi_S / \pi_E)^3$$

where  $\pi_S$  is the source parallax estimated by the mean distance to red clump stars in the direction of each event (Nataf et al. 2013).

## 4.2. “Dip” Anomalies

**4.2.1. KMT-2017-BLG-1194** Figure 1 [Figure 1: see original paper] shows the observed data together with the best-fit PSPL and 2L1S models for KMT-2017-BLG-1194. There is a dip centered on HJD’ 7958.9 (HJD’ = HJD-2450000), i.e.,  $t_{\text{anom}} = 7958.9$ , with a duration of  $\Delta t_{\text{dip}} = 1.05$  days. The dip and the ridge around the dip are covered by three KMTNet sites, so the anomaly is secure. A PSPL fit yields  $(t_0, u_0, t_E) = (7942.7, 0.26, 46)$ , and using the heuristic formalism of Section 4.1, we obtain  $\alpha = 143.5^\circ$ ;  $s = s = 0.807$ ;  $\log q = -4.68$ .

The grid search yields one solution. Its parameters are presented in Table 2 and are in good agreement with the heuristic estimates. The top left panel of Figure 2 [Figure 2: see original paper] displays the caustic structure and the source trajectory, for which the two minor-image planetary caustics are located on both sides of the source trajectory. We label the solution as the solution “A”. To further investigate the parameter space and check whether the event has the inner/outer solutions (Gaudi & Gould 1997), for which the source passes inside (the “Inner” solution) the two planetary caustics (closer to the central caustic) or outside (the “Outer” solution), we follow the procedures of Hwang et al. (2018a). First, we conduct a “hotter” MCMC with the error bar inflated

by a factor of 3. Second, we make a scatter plot of  $\log q$  versus  $\Delta$  from the “hotter” MCMC chain. Here  $\Delta$  represents the offset between the source and the planetary caustic as the source crosses the binary axis,  $\Delta = u_0 \csc(\alpha) - (s - s^{-1})$ .

The resulting scatter plot is shown in Figure 3 [Figure 3: see original paper], from which we find another local minimum at  $\Delta = 0.02$ . We label this solution as the “B” solution. As shown in the top right panel of Figure 2, the “B” solution corresponds to the “Inner” solution. Its parameters from MCMC are given in Table 2 and it is disfavored by  $\Delta^2 = 22.6$  compared to the “A” solution. In Figure 1, the “B” solution cannot fit the anomaly well and all three KMTNet data sets contribute to the  $\Delta^2$ . The ratio of the phase-space factors is  $p_A : p_B = 1 : 0.54$ , which also prefers the “A” solution. Thus, we exclude the “B” solution. In addition, the models which have the geometry of the “Outer” solution do not form a local minimum and are disfavored by  $\Delta^2 > 60$  compared to the “A” solution.

For the “A” solution a point-source model is consistent within  $1\sigma$  and the  $3\sigma$  upper limit is  $\delta < 0.0026$ . The inclusion of higher-order effects yields a constraint on  $\pi_{E,\parallel}$ , and with the other 2L1S parameters being almost unchanged. We obtain  $\pi_{E,\parallel} = -0.18 \pm 0.35$  and adopt the constraints on  $\pi_E$  and  $\delta$  in the Bayesian analysis of Section 5. This is a new microlensing planet with  $q = 2.62 \times 10^{-5}$ ; i.e., about nine times the Earth/Sun mass ratio.

**4.2.2. KMT-2017-BLG-0428** Figure 4 [Figure 4: see original paper] shows a  $\Delta I = 0.12$  mag dip at  $t_{\text{anom}} = 7947.00$ , with a duration of  $\Delta t_{\text{dip}} = 0.74$  days. The dip is defined by the KMTA and KMTC data, and the subtle ridges are supported by both the KMTC and KMTS data. These data were taken in good seeing (1.4–2.5) and the anomaly does not correlate with seeing, sky background or airmass. In addition, Ishitani Silva et al. (2022) found that the KMTA data show systematic errors and excluded them from the analysis. In that case, the KMTA data exhibit similar residuals from one-night data in many places of the light curves. For the present case, the anomaly is mainly covered by the KMTA data, but as presented in Section 2, there is no similar deviation in other places of the light curves. We also carefully checked the KMTA data but did not find any similar residuals. Hence, the anomaly is secure. Applying the heuristic formalism of Section 4.1, we obtain  $\alpha = 108.4^\circ$ ;  $s = 0.898$ ;  $\log q = -4.19$ .

The 2L1S modeling yields two degenerate solutions with  $\Delta^2 = 0.1$ . As shown in Figure 2, the two solutions are subjected to the inner/outer degeneracy. Their parameters are given in Table 3, for which  $\alpha$  and  $q$  are consistent with Equation (11). For  $s$ , we note that the geometric mean of the two solutions,  $s_{\text{mean}} = 0.898 \pm 0.005$ , is in good agreement with Equation (11) and thus the formalism of Ryu et al. (2022). In addition, the observed data only provide a  $3\sigma$  upper limit on  $\delta$ , and a point-source model is consistent within  $1\sigma$ . The ratio of the phase-space factors is  $p_{\text{inner}} : p_{\text{outer}} = 0.78 : 1$ .

With high-order effects, we find that the  $\chi^2$  improvement is 3 and other parameters are almost the same. The constraint of  $\pi_E, \pi_{E,\parallel} = -0.35 \pm 0.26$ , will be used in the Bayesian analysis. This is a microlensing planet with a Neptune/Sun mass ratio.

**4.2.3. KMT-2019-BLG-1806** Figure 5 [Figure 5: see original paper] shows the observed data and models for KMT-2019-BLG-1806. The anomaly is also a dip, with  $\Delta t_{\text{dip}} = 0.6$  days and centered on  $t_{\text{anom}} = 8717.72$ . The dip is defined by the KMTC data and the two contemporaneous OGLE points, which were taken in good seeing (1.1–2.4) and low sky background. Hence, the anomaly is secure. Applying the heuristic formalism of Section 4.1, we obtain  $\alpha = 123.3^\circ$ ;  $s = s = 0.985$ ;  $\log q = -4.56$ . In addition, given the Einstein timescale ( $t_E = 135$  days), we expect that  $\pi_E$  should be either measured or strongly constrained.

The 2L1S modeling also finds a pair of inner/outer solutions and combined the  $u_0 > 0$  and  $u_0 < 0$  degeneracy, so there are four solutions in total. See Table 4 for their parameters. The inclusion of  $\pi_E$  improves the fits by  $\Delta \chi^2 = 31$ , and all four data sets contribute to the improvement, so the parallax signal is reliable. The angle of the minor axis of the parallax ellipse (north through east) is  $\beta = 82.0^\circ$  and  $\beta = 82.5^\circ$  for the  $u_0 > 0$  and  $u_0 < 0$  solutions, respectively.  $\pi_E, \parallel = 0.06 \pm 0.01$ , and  $\pi_E$  is constrained to be  $\sigma(\pi_E, \parallel) = 0.2$ . We obtain  $s_{\text{mean}} = 0.985 \pm 0.008$ ,  $\alpha = 123.1 \pm 0.5$ , and  $\log q = -4.72$ , in good agreement with Equation (12). The ratio of the phase-space factors is  $p_{\text{inner}} : p_{\text{outer}} = 0.80 : 1$ .

We find that the inclusion of the lens orbital motion effect only improves the fit by  $\Delta \chi^2 < 1$  for 2 degree-of-freedom and is not correlated with  $\pi_E$ , so we exclude the lens orbital motion effect. With  $q = 1.9 \times 10^{-5}$ , this new planet is the fifth robust  $q < 2 \times 10^{-5}$  microlensing planet.

**4.2.4. KMT-2017-BLG-1003** Figure 6 [Figure 6: see original paper] shows the light curve and the best-fit models for KMT-2017-BLG-1003. The KMTC data show a sudden dip and the ridge is confirmed by the KMTC and KMTS data. These data were taken in good seeing (1.2–2.2) and low sky background, so the anomaly is of astrophysical origin. Although the end of the dip is not covered, the KMTC point at HJD' = 7870.66 indicates  $\Delta t_{\text{dip}} < 0.85$  days, which yields  $\alpha = 61.3^\circ$ ;  $s = s = 0.903$ ;  $\log q < -3.6$ .

The numerical analysis yields a pair of inner/outer solutions, and Table 5 lists their parameters. As shown in Figure 2, the “Outer” solution has caustic crossings, so its  $\beta$  is measured at the  $4.5\sigma$  level. For the “Inner” solution, a point-source model is consistent within  $2\sigma$ . We note that the geometric mean of  $s$ ,  $s_{\text{mean}} = 0.899 \pm 0.004$ , which is slightly different from  $s$  by  $1\sigma$ . This indicates that the prediction of Ryu et al. (2022) might be imperfect for minor-image anomalies with finite-source effects or incomplete coverage. The ratio of the phase-space factors is  $p_{\text{inner}} : p_{\text{outer}} = 0.80 : 1$ .

With high-order effects, we find that the  $\Delta^2$  improvement is 1.7. Although this event is shorter than the first two events,  $\pi_E$  is better constrained due to the about one magnitude brighter data, with  $\pi_{E,\parallel} = -0.11 \pm 0.15$ . This is another Neptune/Sun mass-ratio planet.

**4.2.5. KMT-2019-BLG-1367** Figure 7 [Figure 7: see original paper] shows a dip 1.2 days before the peak of an otherwise normal PSPL event, with a duration of  $\Delta t_{\text{dip}} = 0.35$  days. The dip-type anomaly is covered by the KMTC data and one contemporaneous OGLE point, and these data were taken in good seeing ( $< 2.0$ ) and low sky background. Therefore, the anomaly is secure. Applying the heuristic formalism of Section 4.1, we obtain  $\alpha = 69.5^\circ$ ;  $s = 0.957$ ;  $\log q = -4.33$ .

The 2L1S modeling also yields a pair of inner/outer solutions, with  $\Delta^2 = 0.2$ . The resulting solutions are given in Table 6 and Figure 2. A point-source model is consistent within  $1\sigma$  and the  $3\sigma$  upper limit is  $< 0.0056$ , so we expect that the Ryu et al. (2022) formula is applicable. We obtain  $s_{\text{mean}} = 0.957 \pm 0.007$ , in good agreement with  $s$ . The ratio of the phase-space factors is  $p_{\text{inner}} : p_{\text{outer}} = 0.82 : 1$ .

We find that the inclusion of higher-order effects only improves the fitting by  $\Delta^2 < 1$  and the  $1\sigma$  uncertainty of parallax is  $> 0.9$  at all directions, so the constraint on  $\pi_E$  is not useful for the Bayesian analysis. This is another planet with a Neptune/Sun mass ratio.

### 4.3. “Bump” Anomalies

For bump-type planetary signals, we also check whether the observed data can be fitted by a single-lens binary-source (1L2S) model (Gaudi 1998) because it can also produce such anomalies (e.g., Hwang et al. 2013; Jung et al. 2017; Rota et al. 2021). For a 1L2S model, its magnification,  $A_\lambda$ , is the superposition of magnifications for two single-lens single-source (1L1S) models,

$$A_\lambda = (A_1 f_{1,\lambda} + A_2 f_{2,\lambda}) / (f_{1,\lambda} + f_{2,\lambda}) = (A_1 + q_{f,\lambda} A_2) / (1 + q_{f,\lambda})$$

where  $q_{f,\lambda} = f_{2,\lambda} / f_{1,\lambda}$ ,  $f_{i,\lambda}$  is the source flux at wavelength  $\lambda$ , and  $i = 1$  and  $i = 2$  correspond to the primary and the secondary sources, respectively.

**4.3.1. OGLE-2017-BLG-1806** As shown in Figure 8 [Figure 8: see original paper], the light curve of OGLE-2017-BLG-1806 exhibits a bump centered on  $t_{\text{anom}} = 8003.5$ , defined by the KMTC and KMTS data. Except for two KMTS points, all the KMTC and KMTS data during  $8003 < \text{HJD}' < 8005$  were taken in good seeing ( $< 2.2$ ) and low sky background. In addition, most of the data before the bump ( $8000 < \text{HJD}' < 8003$ ) are fainter than the 1L1S model. Hence, the signal is secure. Because both the major-image and the two minor-image planetary caustics can produce a bump-type anomaly (e.g., Wang et al. 2022), we obtain  $\alpha = 184.6^\circ$  for  $s = 1.15$ ;  $\alpha = 4.6^\circ$  for  $s = 0.86$ .

The grid search returns three local minima, and their caustic structures are given in Figure 9 [Figure 9: see original paper]. As expected, the three solutions respectively correspond to sources crossing a major-image (quadrilateral) planetary caustic and two minor-image (triangular) planetary caustics. We label the three solutions as “Close A”, “Close B”, and “Wide”, respectively, and their parameters are presented in Table 7.

The “Close A” solution provides the best fit to the observed data, and the “Close B” and “Wide” solutions are disfavored by  $\Delta^2 = 14.1$  and  $8.3$ , respectively. We find that the inclusion of the parallax effect improves the fit by  $\Delta^2 = 7.8, 20.4,$  and  $11.1$  for the “Close A”, “Close B”, and “Wide” solutions, respectively, and during the anomaly region ( $7998 < \text{HJD}' < 8008$ ),  $\Delta^2 = 2.2, 22.3,$  and  $6.8$ . With the anomaly removed, fitting the data by a 1L1S model yields a similar constraint on  $\pi_{E,\parallel}$  and a weaker constraint on  $\pi_{E,\perp}$ , with  $\sigma(\pi_{E,\perp}) = 0.5$ . Thus, the long planetary signal plays an important role in improving the constraint on  $\pi_{E,\perp}$ , and reduces the  $\Delta^2$  differences between the three solutions.

The ratio of the phase-space factors is  $p_{\text{CloseA}} : p_{\text{CloseB}} : p_{\text{Wide}} = 1 : 0.95 : 0.61$ . For the “Close A” and “Close B” solutions, the bump was produced by a caustic crossing, so  $\alpha$  is constrained at the  $> 3\sigma$  level. For the “Wide” solution, the bump was a result of a cusp approach. Although the “Wide” solution has caustic crossing features, due to the lack of data during the crossing, a point-source model is consistent within  $\Delta^2 = 3$ .

The 1L2S model is disfavored by  $\Delta^2 = 30.7$  compared to the “Close A” solution, and the 1L2S parameters are shown in Table 8. Although the 1L2S model fits the bump well, it provides a worse fit to the observed data before the bump, during which most of the data from the three KMTNet sites are fainter than the 1L2S model. Hence, the 1L2S model is rejected. We find that the lens orbital motion effect is not detectable ( $\Delta^2 < 0.5$ ), so we adopt the parameters with the microlensing parallax effect as our final results.

**4.3.2. KMT-2016-BLG-1105** The anomaly in Figure 10 [Figure 10: see original paper] is a short-lived bump centered on  $t_{\text{anom}} = 7547.85$ , which is defined by four KMTC data points and supported by one OGLE data point. These data were taken in good seeing ( $< 2.0$ ) and low sky background, so the anomaly is secure. Similar to OGLE-2017-BLG-1806, we expect both the major-image and the minor-image planetary caustics can produce the bump and obtain  $\alpha = 219.3^\circ$  for  $s = 1.13$ ;  $\alpha = 39.3^\circ$  for  $s = 0.89$ .

The 2L1S modeling yields five solutions, including one with the minor-image planetary caustics and four with the major-image planetary caustics. We label them as “Close”, “Wide A”, “Wide B”, “Wide C” and “Wide D”, respectively, and their parameters are given in Table 9. Figure 11 [Figure 11: see original paper] displays the caustic structures and source trajectories. The “Wide A”, “Wide B” and “Close” solutions exhibit caustic crossings, but only for the “Wide B” and “Close” solutions are constrained at the  $> 3\sigma$  level. For the “Wide A”

, “Wide C” and “Wide D” solutions, a point-source model is consistent within  $\Delta^2 = 3, 1, \text{ and } 1$ , respectively, and thus we only report their  $3\sigma$  upper limit on in Table 9. The ratio of the phase-space factors is  $p_{\text{WideA}} : p_{\text{WideB}} : p_{\text{WideC}} : p_{\text{WideD}} : p_{\text{Close}} = 0.82 : 0.76 : 0.74 : 1 : 0.41$ , so the wide solutions are slightly favored in the phase-space factors.

For the “Close” solution, the bump was produced by a cusp approach with the lower triangular planetary caustic, followed by a dip that occurred in the data gap between  $\text{HJD}' = 7548.0$  and  $\text{HJD}' = 7548.3$ . If the bump were produced by a cusp approach with the upper triangular planetary caustic, there would be a dip before the bump, but the region before the bump is well covered by the KMTS and the KMTC data, which are consistent with the 1L1S model. Thus, the minor-image perturbation only has one solution.

For the four “Wide” solutions, the “Wide A” and “Wide B” solutions have a source crossing the planetary caustic, and the “Wide C” and “Wide D” solutions contain a source that passes to one side or the other of the planetary caustic. This topology is qualitatively similar to the topology of OGLE-2017-BLG-0173 (Hwang et al. 2018a). We thus also investigate the parameter space by a “hotter” MCMC with the error bar inflated by a factor of 2.5. The resulting scatter plot is shown in Figure 12 [Figure 12: see original paper], from which we find that the topology of KMT-2016-BLG-1105 has differences in three aspects from the topology of OGLE-2017-BLG-0173. First, for the two solutions in which the source passes to one side or the other of the planetary caustic, OGLE-2017-BLG-0173 has caustic crossings and the source is comparable to the size of the planetary caustic, but in the present case, the source does not cross the caustic. Second, for the solution in which the source passes directly over the planetary caustic, the source is much larger than the planetary caustics in the case of OGLE-2017-BLG-0173, while the source of KMT-2016-BLG-1105 is smaller than the caustic. Third, OGLE-2017-BLG-0173 exhibits a bimodal minimum when the source passes directly over the caustic, and the mass-ratio difference between the two local minima is  $\Delta \log q < 0.1$ . The corresponding solutions for KMT-2016-BLG-1105, the “Wide A” and “Wide B” solutions, have  $\Delta \log q \approx 1$ . We note that the “Wide A” and “Wide B” solutions have  $\Delta \approx 0.00$  and  $-0.01$ , respectively. Considering the approximate symmetry with respect to  $\Delta$ , one might expect an additional minimum that has  $\Delta \approx 0.01$  and a similar  $\log q$  as the  $\log q$  of the “Wide B” solution. However, such a potential solution “disappeared” from the numerical analysis. Because the trajectories of the “Wide B” solution and the putative minimum at  $\Delta \approx 0.01$  should be almost symmetric with respect to the center of the caustics, their corresponding planetary signals should also be almost symmetric. As shown in Figure 10, the “Wide B” solution drops rapidly during the caustic exit, followed by a dip, so the putative minimum at  $\Delta \approx 0.01$  should contain a dip followed by a sudden rise during the caustic entry, which is not supported by the KMTC and KMTS data. Thus, in Figure 12 this topology is absorbed into the MCMC chain of the “Wide D” solution and there is no new discrete solution.

We also check whether the bump-type anomaly can be fitted by a 1L2S model. Table 8 lists the 1L2S parameters. We find that the best-fit 1L2S model is disfavored by  $\Delta^2 = 12.0$  compared to the best-fit 2L1S model. The best-fit 1L2S model has  $q_{f,I} = 2 \times 10^{-3}$ , corresponding to a magnitude difference of 6.7 mag. According to Section 5, the primary source lies 4.1 mag below the red giant clump, so the putative source companion would have an absolute magnitude of  $M_{I,2} = 10.7$  mag, corresponding to an angular source radius of  $\theta_s = 0.1$  as. This yields the lens-source relative proper motion of  $\mu_{rel} = \theta_s/t_E = 0.5$  mas yr $^{-1}$ , which is lower than the typical  $\mu_{rel}$  of bulge microlensing events (See Figure 2 of Zhu et al. (2017) for examples). However, a model with  $q_{f,I} = 0$  is only disfavored by  $\Delta^2 = 1$ , so any reasonable  $\mu_{rel}$  is only disfavored by  $\Delta^2 < 1$ . Thus, while the planetary model is strongly favored, there is a possibility that the anomaly is caused by a second source.

With high-order effects, we find that  $\Delta^2 < 1$  and the  $1\sigma$  uncertainty of parallax is  $> 0.9$  at all directions, so the constraint on  $\pi_E$  is not useful.

## 5. Physical Parameters

### 5.1. Preamble

Combining Equations (1) and (7), the mass  $M_L$  and distance  $D_L$  of a lens system are related to the angular Einstein radius  $\theta_E$  and the microlensing parallax  $\pi_E$  by (Gould 1992, 2000):

$$\begin{aligned} M_L &= (\theta_E / \pi_E) \\ D_L &= (\text{au}) / (\pi_E \theta_E + \pi_S) \end{aligned}$$

where  $\pi_S$  is the source parallax estimated by the mean distance to red clump stars in the direction of each event (Nataf et al. 2013).

To obtain the angular Einstein radius through  $\theta_E = \theta_s / \mu_{rel}$ , we first estimate the angular source radius  $\theta_s$  by locating the source on a color-magnitude diagram (CMD, Yoo et al. 2004). For each event, we construct a V - I versus I CMD using the ambient stars of the OGLE-III catalog (Szymański et al. 2011) or the KMTC images with the pyDIA reductions. See Figure 13 [Figure 13: see original paper] for the CMDs of the seven planetary events. We estimate the centroid of the red giant clump as  $(V - I, I)_{cl}$  from CMDs and adopt the de-reddened color and magnitude of the red giant clump,  $(V - I, I)_{cl,0}$ , from Bensby et al. (2013) and Table 1 of Nataf et al. (2013). We obtain the source apparent magnitude from the light-curve analysis of Section 4, and the source color by a regression of the KMTC V versus I flux with the change of the lensing magnification. We find that the V-band observations of KMT-2016-BLG-1105 have insufficient signal-to-noise ratio to determine the source color, so we estimate the source color by the Hubble Space Telescope (HST) CMD of Holtzman et al. (1998) (see Section 5.7 for details). Finally, using the color/surface-brightness relation of Adams et al. (2018), we obtain the angular source radius  $\theta_s$ . Tables 10, 11 and 12 present

the CMD values and  $(\mu, \mu_E, \mu_{\text{rel}})$  from the procedures above.

Because the blended light could provide additional constraints on the lens properties (e.g., the upper limits for the lens brightness), we also check the brightness and the astrometric alignment of the baseline object. For KMT-2017-BLG-0428 and KMT-2019-BLG-1806, we adopt the  $i'$ -band baseline images taken by the 3.6m Canada-France-Hawaii Telescope (CFHT) from 2020 to 2022, whose seeing FWHM is 0.55-0.70. For the other five events which do not have any CFHT image, we check the baseline objects from the KMTC pyDIA reduction, whose seeing FWHM is about 1.0.

Because none of the seven planetary events have simultaneous measurements of  $\pi E$  and  $\mu_E$  at the  $> 3\sigma$  level, the lens masses and distances cannot be directly determined by Equation (18). We conduct a Bayesian analysis using a Galactic model to estimate the lens properties. The Galactic model and the procedures we adopt are the same as described in Zang et al. (2021b). We refer the reader to that work for details. The only exception is that we include upper limits of the lens light,  $I_{L,\text{limit}}$ , from the analysis of the blended light. We adopt the mass-luminosity relation of Wang et al. (2018),  $M_I = 4.4 - 8.5 \log(M/M_\odot)$ , where  $M_I$  is the absolute magnitude in the I band, and we reject trial events for which the lens properties obey  $M_I + 5 \log(D_L/10\text{pc}) + A_{I,\text{DL}} < I_{L,\text{limit}}$ , where  $A_{I,\text{DL}}$  is the extinction at  $D_L$ . We adopt an extinction curve with a scale height of 120 pc. For the five events with OGLE CMDs, the total extinction is derived from the CMD analysis,  $A_I = I_{\text{cl}} - I_{\text{cl}},0$ . For the other two events with KMTC CMDs, we adopt the extinction in the K band from Gonzalez et al. (2012) and  $A_I = 7.26 A_K$  from Nataf et al. (2016).

Table 13 presents the resulting Bayesian estimates of the host mass  $M_{\text{host}}$ , the planet mass  $M_{\text{planet}}$ , the lens distance  $D_L$ , the projected planet-host separation  $a_\perp$  and the lens-source relative proper motion  $\mu_{\text{rel}}$ . For events with multiple solutions, we show the results for each solution and the “combined results” of combining all solutions weighted by their Galactic-model likelihood and  $\exp(-\Delta^2/2)$ , where  $\Delta^2$  is the  $\chi^2$  difference compared to the best-fit solution. Here the Galactic-model likelihood represents the total weight for the simulated events given the error distributions of  $t_E$ ,  $\mu_E$  and  $\pi_E$ . See Equation (16) of Zang et al. (2021b) for the weight procedures. We do not adopt the “combined results” as the final physical parameters but just show them for consideration, because there is no conclusion about how to combine degenerate solutions. We note that the  $\exp(-\Delta^2/2)$  probability might be suffered from systematic errors of the observed data. However, the weight from  $\Delta^2$  only has minor effects on the “combined results”. Except for KMT-2016-BLG-1105 the degenerate solutions have similar physical interpretations and except for OGLE-2017-BLG-1806 the  $\Delta^2$  is small, but for OGLE-2017-BLG-1806 the “combined results” are already dominated by the “Close A” solutions due to their Galactic-model likelihoods. Due to similar reasons, whether to include the phase-space factors also has a minor impact on the “combined results”.

### 5.2. KMT-2017-BLG-1194

The corresponding CMD shown in Figure 13 is constructed from the OGLE-III field stars within 240 centered on the event. The baseline object has  $(V, I)_{\text{base}} = (21.343 \pm 0.085, 19.608 \pm 0.051)$ , yielding a blend of  $(V - I, I)B = (2.15 \pm 0.39, 20.45 \pm 0.14)$ . We display the blend on the CMD. The source position measured by the difference imaging analysis is displaced from the baseline object by  $\Delta(N, E) = (-26, 41)$  mas. We estimate the error of the baseline position by the fractional astrometric error being equal to the fractional photometric error (Jung et al. 2020), which yields  $\sigma_{\text{ast}} = 0.39\sigma_I$  FWHM = 20 mas. We note that the astrometric error should be underestimated due to the mottled background from unresolved stars and other systematic errors, but the whole astrometric error should be not more than twice our estimate. Thus, the baseline object is astrometrically consistent with the source and the lens within  $2\sigma$ . The blend does not have a useful color constraint. We adopt the  $3\sigma$  upper limit of the blended light,  $I_{L,\text{limit}} = 20.03$ , as the upper limit of the lens brightness.

As given in Table 13, the preferred host star is an M dwarf located in the Galactic disk, and the planet is probably a super-Earth beyond the snow line of the lens system (assuming a snow line radius  $a_{\text{SL}} = 2.7(M/M_{\odot})$  au, Kennedy & Kenyon 2008).

### 5.3. KMT-2017-BLG-0428

The corresponding CMD shown in Figure 13 consists of the OGLE-III field stars within 150 centered on the event. The baseline object on the CFHT images has  $I_{\text{base}} = 20.056 \pm 0.063$ , with an astrometric offset of  $\Delta(N, E) = (6, -2)$  mas and an astrometric error of  $\sigma_{\text{ast}} = 5$  mas. Thus, the baseline object is astrometrically consistent with the source at about  $1\sigma$ . Because the CFHT images do not contain color information, we do not display the blend on the CMD. We also adopt the  $3\sigma$  upper limit of the blended light,  $I_{L,\text{limit}} = 20.81$ , as the upper limit of the lens brightness.

As shown in Table 13, the Bayesian analysis indicates another cold super-Earth orbiting an M dwarf.

### 5.4. KMT-2019-BLG-1806

The CMD of this event is constructed from the OGLE-III field stars within 150 centered on the event, shown in Figure 13. The baseline object on the KMTC images has  $(V, I)_{\text{base}} = (20.155 \pm 0.125, 18.685 \pm 0.076)$ . We plot the blend on the CMD and find that the blend probably belongs to the foreground main-sequence branch and thus could be the lens. However, the astrometric offset is  $\Delta(N, E) = (433, -76)$  mas and  $\Delta(N, E) = (416, -96)$  mas on the CFHT and KMTC images, respectively, so the majority of the blended light is unrelated to the lens. We adopt the median value of the blended light,  $I_{L,\text{limit}} = 18.8$ , as the upper limit of the lens brightness.

The results of the Bayesian analysis are given in Table 13. The planet is another cold super-Earth, and the preferred host is a K dwarf.

### 5.5. KMT-2017-BLG-1003

We use the OGLE-III field stars within 180 centered on the event to build the CMD. Combining the measured from the light-curve analysis, we obtain  $\pi_E = 0.180 \pm 0.041$  mas for the “Outer” solution and  $\pi_E > 0.14$  mas ( $3\sigma$ ) for the “Inner” solution. The KMTNet baseline object has  $(V, I)_{\text{base}} = (20.968 \pm 0.046, 18.780 \pm 0.028)$ , corresponding to a blend of  $(V - I, I)B = (2.54 \pm 0.20, 19.83 \pm 0.10)$ , and we display the blend on the CMD. The source-baseline astrometric offset is  $\Delta(N, E) = (-64, -77)$  mas, with an astrometric error of  $\sigma_{\text{ast}} = 12$  mas, implying that most of the blend light should be unrelated to the event. We adopt the median value of the blended light,  $I_{L,\text{limit}} = 19.83$ , as the upper limit of the lens brightness.

The Bayesian analysis shows that the host star is probably an M dwarf located in the Galactic bulge. Again, the preferred planet is a cold super-Earth.

### 5.6. KMT-2019-BLG-1367

In Figure 13, we display the position of the source on the CMD of stars within 180 around the source. On the KMTC images, there is no star within 1.4 around the source position. We thus adopt the detection limit of the KMTC images,  $I = 21.0$ , as the upper limit of the baseline brightness, yielding the  $3\sigma$  upper limit of the blended light,  $I_{L,\text{limit}} = 21.6$ . Applying Equations (19) and (20) and assuming  $D_L < 8$  kpc, this flux constraint corresponds to an upper limit of the lens mass of  $0.6M_\odot$ .

As shown in Table 13, the Bayesian estimate shows another cold super-Earth orbiting an M dwarf.

### 5.7. OGLE-2017-BLG-1806

The CMD of this event is constructed from KMTC field stars within a 300 square centered on the event position. The baseline object,  $(V, I)_{\text{base}} = (22.300 \pm 0.308, 20.042 \pm 0.128)$ , is displaced from the source by 835 mas. Thus, most of the blend light should be unrelated to the event. We do not show the blend on the CMD and adopt the median value of the blended light,  $I_{L,\text{limit}} = 20.5$ , as the upper limit of the lens brightness.

The results of the Bayesian analysis are presented in Table 13, and all solutions indicate a cold super-Earth orbiting a low-mass star. The constraints on  $\pi_E$ , from the light-curve analysis are useful. The “Wide” solution has a relatively large  $\pi_E$ , with a  $2\sigma$  lower limit of 0.60 mas and the best-fit value of 1.1 mas, so the corresponding lens system is located in the Galactic disk. Then, the “Wide ( $u_0 > 0$ )” solution has  $\pi_E < 0$  and thus a lens velocity in Galactic coordinates of  $v_\perp \sim 100 \text{ km s}^{-1}$ , so this solution is strongly disfavored. For the

two “Close” solutions, both the  $\pi_E < 0$  solutions are slightly disfavored and have relatively higher probabilities of a bulge lens system.

For the “Wide” solution, the predicted apparent magnitude of the lens system is fainter than the source by 0.8 mag in the H band. In the case of OGLE-2012-BLG-0950, the source and the lens have roughly equal brightness and were resolved by the Keck AO imaging and the HST imaging when they were separated by about 34 mas (Bhattacharya et al. 2018). For OGLE-2017-BLG-1806, we estimate that resolving the lens and source probably requires a separation of 45 mas for the “Wide” solution. We note that the proper motions of the two “Close” solutions are 2 mas yr<sup>-1</sup>. If high-resolution observations resolve the lens and the source and find that  $\mu_{\text{rel}}$  (e.g., 5 mas yr<sup>-1</sup>) is much higher than that of the “Close” solutions, the three solutions can be distinguished. Such observations can be taken in 2026 or earlier.

### 5.8. KMT-2016-BLG-1105

To collect enough red-giant stars to determine the centroid of the red giant clump, the CMD of this event shown in Figure 13 contains KMTC field stars within a 280 × 300 rectangle region. Because the event lies about 80 from the edge of the CCD chip, it is displaced from the center of the rectangle region by about 70 . The V-band data have insufficient signal-to-noise ratio to determine the source color, so we adopt the method of Bennett et al. (2008) to estimate the source color. We first calibrate the CMD of Holtzman et al. (1998) HST observations to the KMTC CMD using the centroids of red giant clumps. We then estimate the source color by taking the color of the HST field stars whose brightness are within the 5 $\sigma$  of the source star.

The baseline object has  $I_{\text{base}} = 20.729 \pm 0.125$  without color information, so we do not plot the blend on the CMD. The source-baseline astrometric offset is  $\Delta(N, E) = (73, 166)$  mas, at about 3 $\sigma$ . Because the baseline object is marginally detected on the KMTC images, we adopt the median value of the blended light,  $I_{L,\text{limit}} = 21.7$ , as the upper limit of the lens brightness.

The Bayesian analysis indicates that the host star is probably an M dwarf. Due to a factor of 13 differences within the mass ratios of the five degenerate solutions, there is a wide range for the planetary mass, from sub-Earth-mass to sub-Neptune-mass. Because no solution has a very different proper motion from other solutions, future high-resolution observations cannot break the degeneracy. However, such observations are still important because the measurements of the host brightness can yield the host mass and distance, which could be used for studying the relation between the planetary occurrence rate and the host properties. For the “Wide A” and “Close” solutions, the predicted apparent magnitude of the lens system is fainter than the source by 2 mag and 3 mag in the H and I bands, respectively. In 2025, the lens and the source will be separated by 50 mas and may be resolved.

## 6. Discussion

In this paper, we have presented the analysis of seven  $q < 10^{-4}$  planets. Together with 17 already published and three that will be published elsewhere, the KMTNet AnomalyFinder algorithm has found 27 events that can be fit by  $q < 10^{-4}$  models from 2016–2019 KMTNet data. For the analysis above and in other published papers, all of the local minima are investigated, but here for each planet, we only consider the models with  $\Delta^2 < 10$  compared to the best-fit model. Table 14 presents the event name,  $\log q$ ,  $s$ ,  $u_0$ , discovery method,  $\Delta^2$  compared to the best-fit models, whether it has a caustic crossing, anomaly type (bump or dip), and the KMTNet fields (prime or sub-prime) of each planet, ranked-ordered by  $\log q$  of the best-fit models. Of them, 15 were solely detected using AnomalyFinder, and 12 were first discovered from by-eye searches and then recovered by AnomalyFinder, which illustrates the importance of systematic planetary anomaly searches in finding low mass-ratio microlensing planets. The seasonal distribution, (5, 8, 8, 6) for 2016–2019, is consistent with normal Poisson variations.

Among the 27 planets, four have alternative possible models with  $q > 10^{-4}$ , and 23 are secure  $q < 10^{-4}$  planets. Because the detection of  $q < 10^{-4}$  planets is one of the major scientific goals of the ongoing KMTNet survey and future space-based microlensing projects (Penny et al. 2019; Ge et al. 2022; Yan & Zhu 2022), it is worthwhile to review the properties of the 27 planetary events and study how to detect more such planets.

### 6.1. The Missing Planetary Caustics Problem

As illustrated by Zang et al. (2021b), the motivation for building the KMTNet AnomalyFinder algorithm is to exhume the buried signatures of “missing planetary caustics” in the KMTNet data. Zhu et al. (2014) predicted that 50% of the KMTNet  $q < 10^{-4}$  planets should be detected by caustics outside of the near-resonant (Dominik 1999; Yee et al. 2021) range. Below we follow the definitions of Zang et al. (2021b) and refer to caustics inside and outside of the near-resonant range as near-resonant caustics and pure-planetary caustics. Contrary to the prediction of Zhu et al. (2014), before the application of AnomalyFinder only two of ten  $q < 10^{-4}$  KMTNet planets were detected by pure-planetary caustics. The two cases are OGLE-2017-BLG-0173Lb (Hwang et al. 2018a) and KMT-2016-BLG-0212Lb (Hwang et al. 2018b). Hence, it is necessary to check the caustic types for the planetary sample of AnomalyFinder.

Figure 14 [Figure 14: see original paper] shows the  $\log q$  versus  $\log s$  plot for the 27 planets. The red and black points represent planets that were first discovered using AnomalyFinder and by-eye searches, respectively. The two green dashed lines indicate the boundaries for the near-resonant range. A striking feature is that in contrast to the locations of the by-eye planets, of the 15 AnomalyFinder planets 11 have pure-planetary caustics, two have both pure-planetary and near-resonant caustics, and only two are fully located inside the near-resonant range.

In total, at least 13 planets were detected by pure-planetary caustics. Thus, the caustic types of the AnomalyFinder planetary sample agree with the expectation of Zhu et al. (2014), and the missing planetary caustics problem has been solved by the systematic planetary anomaly search.

## 6.2. Caustic Crossing and Anomaly Type

Zhu et al. (2014) predicted that about half of the KMTNet planets will be detected by caustic-crossing anomalies. Jung et al. (2022) found that 16/33 of 2018 KMTNet AnomalyFinder planets have caustic-crossing anomalies. As shown in Table 14, 14/27 of the  $q < 10^{-4}$  planets have caustic-crossing anomalies, in good agreement with the expectation of Zhu et al. (2014). Thus, the 50% probability of caustic-crossing anomalies is likely applicable down to  $q = 10^{-5}$ .

Zang et al. (2021b) and Hwang et al. (2022) applied the AnomalyFinder algorithm to 2018-2019 KMTNet prime-field events and found seven newly discovered  $q < 2 \times 10^{-4}$  planets. Among them, only OGLE-2019-BLG-1053Lb has a bump-type anomaly and the other six planets were detected by dip-type anomalies. Thus, it is necessary to check whether dip-type anomalies dominate the detection of low- $q$  planets. As presented in Table 14, the ratio of bump-type to dip-type anomalies for the  $q < 10^{-4}$  planets is 15 to 12, so the two types of anomalies play roughly equal roles in the low- $q$  detection. However, of the 12 dip-type anomalies, nine were solely detected by AnomalyFinder, including eight non-caustic-crossing anomalies. KMT-2018-BLG-1988 (Han et al. 2022a) is the only case that the anomaly is a non-caustic-crossing dip and was first discovered from by-eye searches. Unlike the dip-type anomalies, the four non-caustic-crossing bumps were all first noticed from by-eye searches. Hence, by-eye searches have proved to be quite insensitive to non-caustic-crossing dip-type anomalies for low- $q$  planets.

## 6.3. A Desert of High-Magnification Planetary Signals

Zang et al. (2021b) suggested that the missing planetary caustics problem was caused by the way that modelers searched for planetary signatures. Because high-magnification events are intrinsically more sensitive to planets (Griest & Safizadeh 1998), by-eye searches paid more attention to them, while pure-planetary caustics are mainly detected in low-magnification events. If this hypothesis is correct, we expect that by-eye planets and AnomalyFinder planets will have different  $|u_0|$  and  $|u_{\text{anom}}|$  distributions. The  $\log |u_{\text{anom}}|$  versus  $\log |u_0|$  distribution of Figure 15 [Figure 15: see original paper] confirms our expectation. Except for the two planets that were detected by pure-planetary caustics, all the other by-eye planets, which are located inside the near-resonant range, were detected with  $|u_0| = 0.05$  and  $|u_{\text{anom}}| = 0.07$ . The roughly one-decade gap of the by-eye planets, at  $0.05 < |u_0| < 0.62$  and  $0.07 < |u_{\text{anom}}| < 0.78$ , is filled by the AnomalyFinder planets<sup>3</sup>.

However, there is no planet located at the left lower corner of Figure 15, with

$|u_{0,\text{limit}}| = 0.0060$  and  $|u_{\text{anom},\text{limit}}| = 0.0158$ . Although six of the planets were detected in high-magnification events ( $|u_0| < 0.01$ ), all the planetary signals occurred on the low- and median-magnification regions. This desert of high-magnification planetary signals could be caused by the insufficient observing cadences of the current KMTNet survey. High-magnification planetary signals for  $q < 10^{-4}$  events are weak and thus require dense observations over the peak. There are three known  $q < 10^{-4}$  events whose planetary signals occurred on the high-magnification regions ( $|u_{\text{anom}}| < 0.01$ ). They are OGLE-2005-BLG-169 with  $u_{\text{anom}} = 0.0012$  (Gould et al. 2006), KMT-2021-BLG-0171 with  $|u_{\text{anom}}| = 0.0066$  (Yang et al. 2022), and KMT-2022-BLG-0440 with  $|u_{\text{anom}}| = 0.0041$  (Zhang et al. in prep). The follow-up data played decisive roles in these detections and the combined cadences of survey and follow-up data are higher than  $30 \text{ hr}^{-1}$ , while the highest cadence of the current KMTNet survey is  $8 \text{ hr}^{-1}$  for about  $0.4 \text{ deg}^2$  from the overlap of two  $\Gamma = 4 \text{ hr}^{-1}$  fields.

However, we note that AnomalyFinder used the KMTNet end-of-year pipeline light curves, for which the photometric quality is not as good as that of TLC re-reductions. For the three follow-up planets, the planetary signals only have  $\Delta I < 0.05 \text{ mag}$ . Thus, TLC re-reductions may be needed to recover such weak signals in the KMTNet data, and we cannot rule out the possibility that the desert may also be due to the imperfect KMTNet photometric quality. Each year there are about 20 events with  $|u_0| < 0.01$  observed by KMTNet with  $\Gamma \geq 4 \text{ hr}^{-1}$ . The current KMTNet quasi-automated TLC re-reductions pipeline takes  $< 1 \text{ hr}$  of human effort for each event (H. Yang et al. in prep), so an optimized systematic search for  $q < 10^{-4}$  planets in the KMTNet high-magnification events can be done very quickly. This search could have important implications for future space-based microlensing projects, because their tentative cadences are similar to or lower than  $\Gamma = 4 \text{ hr}^{-1}$  (Penny et al. 2019; Ge et al. 2022; Yan & Zhu 2022). If this search demonstrates that high-magnification events need denser observations to capture the weak planetary signals for low- $q$  planets, one could consider conducting (if feasible) ground-based follow-up projects for high-magnification events that are discovered by space-based telescopes. We also note that for the 2018 AnomalyFinder planets (Gould et al. 2022; Jung et al. 2022) and 2019 prime-field AnomalyFinder planets (Zang et al. 2022a), which are complete now, only one  $q > 10^{-4}$  planet, KMT-2019-BLG-1953Lb, has  $|u_{\text{anom}}| < |u_{\text{anom},\text{limit}}|$ . Future analysis of all the 2016–2019 KMTNet should check whether the desert is obvious for more massive planets.

#### 6.4. Prime and Sub-Prime Fields

In its 2015 commissioning season, KMTNet observed four fields at a cadence of  $\Gamma = 6 \text{ hr}^{-1}$ . To support the 2016–2019 Spitzer microlensing campaign (Gould et al. 2013, 2014a, 2015a,b, 2016, 2018) and find more planets, KMTNet monitored a wider area, with a total of (3, 7, 11, 3) fields at cadences of  $\Gamma = (4, 1, 0.4, 0.2) \text{ hr}^{-1}$ . The three fields with the highest cadence are the KMTNet prime fields and the other 21 are the KMTNet sub-prime fields. See Figure 12 of

Kim et al. (2018a) for the field placement. As shown in Table 14 and Figure 15, the prime fields played the main role in the detection of  $q < 10^{-4}$  planets, as predicted by Henderson et al. (2014), and 17 of 27 planets were detected therein. However, the sub-prime fields are also important and six of the ten lowest- $q$  planets were discovered therein.

For the six planets with  $|u_0| < 0.01$ , there is a clear bias in cadences, and only one of them was detected from the sub-prime fields. For the prime and sub-prime fields, the current detection rates are 1.25 and 0.25 per year, respectively. Because 60% of the KMTNet microlensing events are located in the sub-prime fields, if the sub-prime-field events with  $|u_0| < 0.01$  can have the same cadence as the prime-field events from follow-up observations, each year there would be  $(1.25 \times (60\%/40\%) - 0.25) = 1.6$  more  $q < 10^{-4}$  planets. Because follow-up observations can have higher cadences and capture the high-magnification planetary signals (e.g., Yang et al. 2022), the yield of a follow-up project can be at least two  $q < 10^{-4}$  planets per year. The reward is not only enlarging the low- $q$  planetary sample, but also an independent check of the statistical results from AnomalyFinder if the follow-up planets can form a homogeneous statistical sample (Gould et al. 2010). However, this reward requires that the KMTNet alert-finder system should alert new events before they reach the high-magnification regions (e.g.,  $A > 20$ ).

## Acknowledgments

We appreciate the anonymous referee for helping to improve the paper. W. Zang, H. Y., S. M., J. Z., and W. Zhu acknowledge support by the National Science Foundation of China (Grant No. 12133005). W. Zang acknowledges the support from the Harvard-Smithsonian Center for Astrophysics through the CfA Fellowship. This research was supported by the Korea Astronomy and Space Science Institute under the R&D program (Project No. 2022-1-830-04) supervised by the Ministry of Science and ICT. This research has made use of the KMTNet system operated by the Korea Astronomy and Space Science Institute (KASI) and the data were obtained at three host sites of CTIO in Chile, SAAO in South Africa, and SSO in Australia. The authors acknowledge the Tsinghua Astrophysics High-Performance Computing platform at Tsinghua University for providing computational and data storage resources that have contributed to the research results reported within this paper. Work by J. C. Y. acknowledges support from N.S.F Grant No. AST-2108414. Work by C. H. was supported by the grants of National Research Foundation of Korea (2019R1A2C2085965 and 2020R1A4A2002885). Y. S. acknowledges support from BSF Grant No. 2020740. W. Zhu acknowledges the science research grants from the China Manned Space Project with No. CMS-CSST-2021-A11. This research uses data obtained through the Telescope Access Program (TAP), which has been funded by the TAP member institutes. This research is supported by Tsinghua University Initiative Scientific Research Program (Program ID 2019Z07L02017). This research has made use of the NASA Exoplanet Archive,

which is operated by the California Institute of Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program.

## References

- Adams, A. D., Boyajian, T. S., & von Braun, K. 2018, MNRAS, 473, 3608, doi: 10.1093/mnras/stx2367
- Alard, C., & Lupton, R. H. 1998, ApJ, 503, 325, doi: 10.1086/305984
- Albrow, M. D. 2017, MichaelDalbrow/Pydia: Initial Release On Github., v1.0.0, Zenodo, doi: 10.5281/zenodo.268049
- Albrow, M. D., Horne, K., Bramich, D. M., et al. 2009, MNRAS, 397, 2099, doi: 10.1111/j.1365-2966.2009.15098.x
- An, J. H., Albrow, M. D., Beaulieu, J.-P., et al. 2002, ApJ, 572, 521, doi: 10.1086/340191
- Batista, V., Gould, A., Dieters, S., et al. 2011, A&A, 529, A102, doi: 10.1051/0004-6361/201016111
- Beaulieu, J.-P., Bennett, D. P., Fouqué, P., et al. 2006, Nature, 439, 437, doi: 10.1038/nature04441
- Bennett, D. P., Bond, I. A., Udalski, A., et al. 2008, ApJ, 684, 663, doi: 10.1086/589940
- Bensby, T., Yee, J. C., Feltzing, S., et al. 2013, A&A, 549, A147, doi: 10.1051/0004-6361/201220678
- Bhattacharya, A., Beaulieu, J. P., Bennett, D. P., et al. 2018, AJ, 156, 289, doi: 10.3847/1538-3881/aaed46
- Bond, I. A., Udalski, A., Jaroszyński, M., et al. 2004, ApJL, 606, L155, doi: 10.1086/420928
- Bond, I. A., Bennett, D. P., Sumi, T., et al. 2017, MNRAS, 469, 2434, doi: 10.1093/mnras/stx1049
- Bozza, V. 2010, MNRAS, 408, 2188, doi: 10.1111/j.1365-2966.2010.17265.x
- Bozza, V., Bachelet, E., Bartolić, F., et al. 2018, MNRAS, 479, 5157, doi: 10.1093/mnras/sty1791
- Dominik, M. 1999, A&A, 349, 108
- Dong, S., Gould, A., Udalski, A., et al. 2009, ApJ, 695, 970, doi: 10.1088/0004-637X/695/2/970
- Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306, doi: 10.1086/670067
- Gaudi, B. S. 1998, ApJ, 506, 533, doi: 10.1086/306256
- Gaudi, B. S., & Gould, A. 1997, ApJ, 486, 85, doi: 10.1086/304491
- Ge, J., Zhang, H., Zang, W., et al. 2022, arXiv e-prints, arXiv:2206.06693. <https://arxiv.org/abs/2206.06693>
- Gonzalez, O. A., Rejkuba, M., Zoccali, M., et al. 2012, A&A, 543, A13, doi: 10.1051/0004-6361/201219222
- Gould, A. 1992, ApJ, 392, 442, doi: 10.1086/171443
- Gould, A. 1996, ApJ, 470, 201, doi: 10.1086/177861
- Gould, A. 2000, ApJ, 542, 785, doi: 10.1086/317037

- Gould, A. 2004, *ApJ*, 606, 319, doi: 10.1086/382782
- Gould, A., Carey, S., & Yee, J. 2013, Spitzer Microlens Planets and Parallaxes, Spitzer Proposal
- Gould, A., Udalski, A., An, D., et al. 2006, *ApJL*, 644, L37, doi: 10.1086/505421
- Gould, A., Dong, S., Gaudi, B. S., et al. 2010, *ApJ*, 720, 1073, doi: 10.1088/0004-637X/720/2/1073
- Gould, A., Udalski, A., Shin, I. G., et al. 2014b, *Science*, 345, 46, doi: 10.1126/science.1251527
- Gould, A., Ryu, Y.-H., Calchi Novati, S., et al. 2020, *Journal of Korean Astronomical Society*, 53, 9. <https://arxiv.org/abs/1906.11183>
- Gould, A., Han, C., Zang, W., et al. 2022, *A&A*, 664, A13, doi: 10.1051/0004-6361/202243744
- Griest, K., & Safizadeh, N. 1998, *ApJ*, 500, 37, doi: 10.1086/305729
- Han, C., Udalski, A., Lee, C.-U., et al. 2021, *A&A*, 649, A90, doi: 10.1051/0004-6361/202039817
- Han, C., Gould, A., Albrow, M. D., et al. 2022a, *A&A*, 658, A62, doi: 10.1051/0004-6361/202142077
- Han, C., Kim, D., Gould, A., et al. 2022b, *A&A*, 664, A33, doi: 10.1051/0004-6361/202243484
- Hayashi, C. 1981, in *Fundamental Problems in the Theory of Stellar Evolution*, ed. D. Sugimoto, D. Q. Lamb, & D. N. Schramm, Vol. 93, 113-126
- Henderson, C. B., Gaudi, B. S., Han, C., et al. 2014, *ApJ*, 794, 52, doi: 10.1088/0004-637X/794/1/52
- Holtzman, J. A., Watson, A. M., Baum, W. A., et al. 1998, *AJ*, 115, 1946, doi: 10.1086/300336
- Hwang, K.-H., Choi, J.-Y., Bond, I. A., et al. 2013, *ApJ*, 778, 55, doi: 10.1088/0004-637X/778/1/55
- Hwang, K.-H., Udalski, A., Shvartzvald, Y., et al. 2018a, *AJ*, 155, 20, doi: 10.3847/1538-3881/aa992f
- Hwang, K. H., Kim, H. W., Kim, D. J., et al. 2018b, *Journal of Korean Astronomical Society*, 51, 197, doi: 10.5303/JKAS.2018.51.6.197
- Hwang, K.-H., Zang, W., Gould, A., et al. 2022, *AJ*, 163, 43, doi: 10.3847/1538-3881/ac38ad
- Ishitani Silva, S., Ranc, C., Bennett, D. P., et al. 2022, *AJ*, 164, 262, doi: 10.3847/1538-3881/ac82b8
- Jiang, G., DePoy, D. L., Gal-Yam, A., et al. 2004, *ApJ*, 617, 1307, doi: 10.1086/425678
- Jung, Y. K., Udalski, A., Yee, J. C., et al. 2017, *AJ*, 153, 129, doi: 10.3847/1538-3881/aa5d07
- Jung, Y. K., Udalski, A., Zang, W., et al. 2020, *AJ*, 160, 255, doi: 10.3847/1538-3881/abbe93
- Jung, Y. K., Zang, W., Han, C., et al. 2022, *AJ*, 164, 262, doi: 10.3847/1538-3881/ac9c5c
- Kennedy, G. M., & Kenyon, S. J. 2008, *ApJ*, 673, 502, doi: 10.1086/524130
- Kim, D.-J., Kim, H.-W., Hwang, K.-H., et al. 2018a, *AJ*, 155, 76, doi: 10.3847/1538-3881/aaa47b

- Kim, H.-W., Hwang, K.-H., Shvartzvald, Y., et al. 2018b, arXiv e-prints, arXiv:1806.07545. <https://arxiv.org/abs/1806.07545>
- Kim, S.-L., Lee, C.-U., Park, B.-G., et al. 2016, *Journal of Korean Astronomical Society*, 49, 37, doi: 10.5303/JKAS.2016.49.1.037
- Kondo, I., Yee, J. C., Bennett, D. P., et al. 2021, *AJ*, 162, 77, doi: 10.3847/1538-3881/ac00ba
- Mao, S., & Paczynski, B. 1991, *ApJL*, 374, L37, doi: 10.1086/186066
- Min, M., Dullemond, C. P., Kama, M., & Dominik, C. 2011, *Icarus*, 212, 416, doi: 10.1016/j.icarus.2010.12.002
- Muraki, Y., Han, C., Bennett, D. P., et al. 2011, *ApJ*, 741, 22, doi: 10.1088/0004-637X/741/1/22
- Nataf, D. M., Gould, A., Fouqué, P., et al. 2013, *ApJ*, 769, 88, doi: 10.1088/0004-637X/769/2/88
- Nataf, D. M., Gonzalez, O. A., Casagrande, L., et al. 2016, *MNRAS*, 456, 2692, doi: 10.1093/mnras/stv2843
- Paczynski, B. 1986, *ApJ*, 304, 1, doi: 10.1086/164140
- Penny, M. T., Gaudi, B. S., Kerins, E., et al. 2019, *ApJS*, 241, 3, doi: 10.3847/1538-4365/aafb69
- Poindexter, S., Afonso, C., Bennett, D. P., et al. 2005, *ApJ*, 633, 914, doi: 10.1086/468182
- Ranc, C., Bennett, D. P., Hirao, Y., et al. 2019, *AJ*, 157, 232, doi: 10.3847/1538-3881/ab141b
- Rota, P., Hirao, Y., Bozza, V., et al. 2021, *AJ*, 162, 59, doi: 10.3847/1538-3881/ac0155
- Ryu, Y.-H., Udalski, A., Yee, J. C., et al. 2020, *AJ*, 160, 183, doi: 10.3847/1538-3881/abaa3f
- Ryu, Y.-H., Kil Jung, Y., Yang, H., et al. 2022, *AJ*, 164, 180, doi: 10.3847/1538-3881/ac8d6c
- Sako, T., Sekiguchi, T., Sasaki, M., et al. 2008, *Experimental Astronomy*, 22, 51, doi: 10.1007/s10686-007-9082-5
- Shvartzvald, Y., Yee, J. C., Calchi Novati, S., et al. 2017, *ApJL*, 840, L3, doi: 10.3847/2041-8213/aa6d09
- Skowron, J., Udalski, A., Gould, A., et al. 2011, *ApJ*, 738, 87, doi: 10.1088/0004-637X/738/1/87
- Sumi, T., Bennett, D. P., Bond, I. A., et al. 2010, *ApJ*, 710, 1641, doi: 10.1088/0004-637X/710/2/1641
- Suzuki, D., Bennett, D. P., Sumi, T., et al. 2016, *ApJ*, 833, 145, doi: 10.3847/1538-4357/833/2/145
- Szymański, M. K., Udalski, A., Soszyński, I., et al. 2011, *AcA*, 61, 83. <https://arxiv.org/abs/1107.4008>
- Tomaney, A. B., & Crotts, A. P. S. 1996, *AJ*, 112, 2872, doi: 10.1086/118228
- Udalski, A. 2003, *AcA*, 53, 291
- Udalski, A., Szymanski, M., Kaluzny, J., et al. 1994, *AcA*, 44, 227
- Udalski, A., Szymański, M. K., & Szymański, G. 2015, *AcA*, 65, 1. <https://arxiv.org/abs/1504.05966>
- Udalski, A., Ryu, Y.-H., Sajadian, S., et al. 2018, *AcA*, 68, 1. <https://arxiv.org/abs/1802.02582>

- Wang, H., Zang, W., Zhu, W., et al. 2022, MNRAS, 510, 1778, doi: 10.1093/mnras/stab3581
- Wang, T., Calchi Novati, S., Udalski, A., et al. 2018, ApJ, 860, 25, doi: 10.3847/1538-4357/aabcd2
- Wozniak, P. R. 2000, AcA, 50, 421
- Yan, S., & Zhu, W. 2022, Research in Astronomy and Astrophysics, 22, 025006, doi: 10.1088/1674-4527/ac3c44
- Yang, H., Zang, W., Gould, A., et al. 2022, MNRAS, 516, 1894, doi: 10.1093/mnras/stac2023
- Yee, J. C., Zang, W., Udalski, A., et al. 2021, AJ, 162, 180, doi: 10.3847/1538-3881/ac1582
- Yoo, J., DePoy, D. L., Gal-Yam, A., et al. 2004, ApJ, 603, 139, doi: 10.1086/381241
- Zang, W., Han, C., Kondo, I., et al. 2021a, Research in Astronomy and Astrophysics, 21, 239, doi: 10.1088/1674-4527/21/9/239
- Zang, W., Hwang, K.-H., Udalski, A., et al. 2021b, AJ, 162, 163, doi: 10.3847/1538-3881/ac12d4
- Zang, W., Yang, H., Han, C., et al. 2022a, MNRAS, 515, 928, doi: 10.1093/mnras/stac1883
- Zang, W., Shvartzvald, Y., Udalski, A., et al. 2022b, MNRAS, 514, 5952, doi: 10.1093/mnras/stac1631
- Zhang, K., & Gaudi, B. S. 2022, ApJL, 936, L22, doi: 10.3847/2041-8213/ac8c2b
- Zhang, K., Gaudi, B. S., & Bloom, J. S. 2022, Nature Astronomy, 6, 782, doi: 10.1038/s41550-022-01671-6
- Zhu, W., Penny, M., Mao, S., Gould, A., & Gendron, R. 2014, ApJ, 788, 73, doi: 10.1088/0004-637X/788/1/73
- Zhu, W., Udalski, A., Calchi Novati, S., et al. 2017, AJ, 154, 210, doi: 10.3847/1538-3881/aa8ef1

*Note: Figure translations are in progress. See original paper for figures.*

*Source: ChinaXiv – Machine translation. Verify with original.*