Ultra-low-noise transimpedance amplifier with a single HEMT in pre-amplifier for measuring shot noise in cryogenic STM

Authors: Ying-Xin Liang, Ying-Xin Liang

Date: 2024-05-06T00:00:00+00:00

Abstract

This paper proposes a design scheme for a transimpedance amplifier (TIA) for cryogenic scanning tunneling microscopy (CryoSTM). The TIA with tip-sample components in CryoSTM is referred to as CryoSTM-TIA. The transimpedance gain of this CryoSTM-TIA is 1 GΩ, and its bandwidth exceeds 300 kHz. A unique feature of the proposed CryoSTM-TIA is that its preamplifier is fabricated from a single cryogenic high-electron-mobility transistor (HEMT), resulting in an instrument equivalent input noise current power spectral density below 4 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz. Furthermore, the application of the "bias-cooling method" can be used to in situ control the density of frozen DX-centers in the HEMT doping region, altering its structure to reduce device noise. Utilizing this instrument, fast scanning tunneling spectroscopy measurements with high energy resolution can be performed. Moreover, it is capable of measuring scanning tunneling shot noise spectroscopy (STSNS) at the atomic scale for various quantum systems, even when the shot noise is extremely low. It provides a powerful tool for investigating novel quantum states through STSNS measurements, such as detecting the presence of Majorana bound states in topological quantum systems.

Full Text

Preamble

Ultra-Low-Noise Transimpedance Amplifier with a Single HEMT in the Pre-Amplifier for Measuring Shot Noise in Cryogenic Scanning Tunneling Microscopy

Ying-Xin Liang1,2∗
1Beijing Academy of Quantum Information Sciences, Haidian 100193, Beijing, China

2College of Physics and Electrical Engineering, Anyang Normal University, Anyang 455000, Henan, China

May 3, 2024

Abstract

This work proposes a transimpedance amplifier (TIA) design for cryogenic scanning tunneling microscopy (CryoSTM). The TIA integrated with the tip-sample component in CryoSTM is referred to as CryoSTM-TIA. With a transimpedance gain of 1 GΩ, the CryoSTM-TIA achieves a bandwidth exceeding 200 kHz. Its distinctive feature is a pre-amplifier consisting of a single cryogenic high electron mobility transistor (HEMT), which yields an equivalent input noise current power spectral density below 6 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz. Additionally, a "bias-cooling method" can be employed to in-situ control the density of frozen DX^- centers in the HEMT doping region, modifying its structure to reduce device noise. This apparatus enables fast scanning tunneling spectroscopy measurements with high energy resolution and allows measurement of scanning tunneling shot noise spectra (STSNS) at the atomic scale for various quantum systems, even when the shot noise is extremely low. It provides a powerful tool for investigating novel quantum states through STSNS measurements, such as detecting Majorana bound states in topological quantum systems.

Introduction

High-performance transimpedance amplifiers for cryogenic scanning tunneling microscopy represent a critical component [1, 2]. Reference [3] proposed a TIA design for CryoSTM, where the TIA integrated with the tip-sample component is called CryoSTM-TIA. That design achieved a transimpedance gain of 1 GΩ, bandwidth exceeding 300 kHz, and an equivalent input noise current power spectral density (PSD) of 21 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz. The CryoSTM-TIA was capable of measuring scanning tunneling shot noise spectra (STSNS) of various quantum systems at atomic resolution, even for noise current PSD as small as a few $(\mathrm{fA})^2/\mathrm{Hz}$. Reference [3] demonstrated how CryoSTM-TIA measures STSNS to investigate novel quantum states, such as detecting Majorana bound states (MBSs) in iron-based superconductors [4]. In that design, a pre-amplifier (Pre-Amp) and post-amplifier (Post-Amp) were cascaded to form an operational amplifier (OPA) termed Macro-OPA. The Pre-Amp in Macro-OPA was a differential amplifier using a pair of high electron mobility transistors (HEMTs), where the HEMT is a type of cryogenic GaAs MESFET [5, 6]. However, selecting cryogenic GaAs MESFETs with identical performance for pairing is quite challenging [2].

This work proposes a CryoSTM-TIA design featuring a single HEMT in the Pre-Amp. In this CryoSTM-TIA, the Pre-Amp and Post-Amp are cascaded to form an inverting amplifier (Inv-Amp) rather than an OPA. The CryoSTM-TIA maintains a transimpedance gain of 1 GΩ and bandwidth exceeding 200 kHz, but its equivalent input noise current PSD is only 5.3 $(fA)^2/Hz$ at 100 kHz—one quarter of that in Ref. [3]—due to the elimination of one noisy HEMT. Consequently, lower tunneling shot noise can be measured at the atomic scale. Furthermore, using only a single HEMT in the Pre-Amp avoids the difficulty of matching two identical HEMTs. The "bias-cooling method" [5] can also be applied to in-situ reduce the inherent noise of the HEMT. These advantages make this apparatus a more powerful tool for investigating novel quantum states across a broader range of applications than Ref. [3], including high-Tc superconductors [7, 8], topological superconductors [4], MBSs [4, 9, 10, 11, 12], Andreev reflection [13, 14], and Kondo effect [15].

2 Circuit of the Proposed CryoSTM-TIA

The circuit of the proposed CryoSTM-TIA is shown in [Figure 1: see original paper]. It comprises several components: the single HEMT amplifier portion of the Pre-Amp shown in dashed box (a1), the Pre-Amp power supply in dashed box (a2), the Post-Amp in dashed box (b), the compensated feedback network in dashed box (c), and the signal source circuit in dashed box (d). The two-stage amplifier formed by Pre-Amp and Post-Amp is called Inv-Amp. The Inv-Amp connects with the feedback network to form the TIA, whose input G connects to the signal source circuit to complete the CryoSTM-TIA. Components placed in the cryogenic zone are indicated by the dotted box. All component parameters are listed in .

2.1 Design of Pre-Amp

As shown in [Figure 1: see original paper], only a single cryogenic HEMT is used in the Pre-Amp. The HEMT employed in this work is the CNRS-HEMT [5, 6] developed by CNRS/LPN in France, which exhibits excellent cryogenic and noise characteristics. It can operate at 0.5 K with only 0.1 mW power consumption at its ideal operating point (Vds = 100 mV, Ids = 1 mA). Its parameters are listed in , where e^2_H represents its equivalent input noise voltage PSD and i^2_H its equivalent input noise current PSD [6].

The single HEMT amplifier portion of the Pre-Amp appears in dashed box (a1) of [Figure 1: see original paper]. The HEMT source connects to ground through resistor Rs and a small variable resistor Rs1, with RS = Rs + Rs1, and capacitor CS (11 µF) in parallel with RS. The HEMT drain O1 connects to load resistor R1. Resistor Rh connects to ground through variable resistor Rh1, with RH = Rh + Rh1. The other end of Rh, denoted O2, connects to resistor R2, and node L (the other end of R2) connects to node P (the other end of R1) through small variable resistor Rp. R1 = R2 = RL. Node P connects to the Pre-Amp power supply. Nodes P and L are grounded through large capacitors C1 and C2, respectively. In the power supply, RT = Rt + Rt1, where Rt1 is a variable resistor. CA represents the input capacitance of the Pre-Amp. The HEMT, Rs, Rh, R1, and R2 are placed in the cryogenic zone (dotted box). CI is the capacitance of the cable connecting the HEMT gate G to the CryoSTM tip. The

HEMT is positioned as close as possible to the tip to minimize CI, which can be reduced below 0.5 pF [16]. Pre-Amp operating point adjustment is described in Section 4, and Pre-Amp parameters are listed in .

For the Pre-Amp, when AC voltage $\dot{V}$ is applied to input G, the AC voltage difference between O1 and O2 is denoted $\dot{V}_{op}$. The voltage gain is $A_{vP} = \dot{V}_{op}/\dot{V}$. Since the transition frequency for the CNRS-HEMT is $g_m/(2\pi C_{gs}) \approx$ 1 GHz, the Pre-Amp bandwidth exceeds 30 MHz. In the frequency range $\max\{g_m/(2\pi C_S), 1/(2\pi R_S C_S)\} \ll f \leq 3$ MHz (i.e., 30 Hz f ≤ 3 MHz with parameters from),

$$A_{vP} \approx -g_m R_d, \tag{2.1}$$

where $R_d = R_L/(1 + g_d R_L)$. With the parameters in , $A_{vP} \approx -20$ in the range 30 Hz f ≤ 3 MHz. The input capacitance of the Pre-Amp is

$$C_A = C_{gs} + (1 - A_{vP})C_{gd}. \tag{2.2}$$

Thus, $C_A \approx 26$ pF in the range 30 Hz f ≤ 3 MHz. The input resistance of the Pre-Amp, R_A, is the gate-source resistance of the HEMT, with $R_A > 10$ TΩ, effectively infinite.

The Pre-Amp power supply is shown in dashed box (a2) of [Figure 1: see original paper]. A precision voltage reference (PVR) LM4050-10 [17] provides a constant 10 V output with a typical temperature coefficient of $±$40 ppm/°C. Noise from the PVR is eliminated by C1, C2, and RT.

For a differential amplifier, feedback between the two transistor branches enables high common-mode rejection ratio (CMRR). In Ref. [3], if the HEMT (H2) in the differential Pre-Amp were replaced by a resistor with the same DC resistance, the Inv-Amp could become a non-inverting amplifier as $f \to 0$. In this work' s Pre-Amp design, no feedback exists between the HEMT branch and RH branch, so the Inv-Amp is always an inverting amplifier.

2.2 Design of Post-Amp and Composition of Inv-Amp

In [Figure 1: see original paper], the Post-Amp circuit in dashed box (b) has the same structure as in Ref. [3]. It contains a commercial operational amplifier (OPA) called Rear-OPA, which is a high gain-bandwidth-product OPA such as THS4021, OPA657, or LMH6624. This work uses THS4021 [18]. Post-Amp parameters are listed in . R_a and C_a are the input resistance and capacitance of the Rear-OPA. Feedback resistor R_f (200 kΩ) is placed in the cryogenic zone. Post-Amp also includes two cables connecting Q1 and Q2 in the room-temperature zone to Pre-Amp outputs O1 and O2 in the cryogenic zone, with capacitances C_{i1} and C_{i2} that may vary from 50 pF to 150 pF. This work assumes $C_{i1} = C_{i2} = C_i = 150$ pF.

A 1-meter cable with 50 Ω characteristic impedance has a distributed capacitance of 100 pF and distributed inductance of 250 nH, and its distributed LC structure produces several resonant gain peaks in [50 MHz, 1 GHz] [19]. Pre-Amp and Post-Amp are cascaded to form Inv-Amp. In Inv-Amp, Post-Amp and Pre-Amp connect via cables (Cable O1Q1 and Cable O2Q2), and these resonant gain peaks may cause self-oscillations due to potential unexpected electromagnetic couplings between amplifier inputs and outputs. Two low-pass filters ($R_{c1} = R_{c2} = R_c = 100\ \Omega$ and $C_{r1} = C_{r2} = C_r = 50$ pF) with an upper cut-off frequency of 30 MHz are added at the Rear-OPA inputs to greatly reduce these resonant gain peaks, thereby avoiding self-oscillations.

For AC signals, the voltage gain of Inv-Amp is $a_A = a_A(f)$, which can be expressed as

$$a_A = A_{vP} A_{vR}. \tag{2.3}$$

Using nodal analysis, a_A can be obtained, and with A_{vP} from Eq. (2.1), A_{vR} can be derived from Eq. (2.3). In the range $\max\{g_m/(2\pi C_S), 1/(2\pi R_S C_S)\} \ll f \leq 3$ MHz (i.e., 30 Hz   f ≤ 3 MHz with parameters from),

$$A_{vR} \approx \frac{1 + j2\pi f R_H L C_{ir}}{1 + j2\pi f R_d C_{ir}} \frac{1}{1 + R_f/a_a R_H L + j2\pi f R_f C_{ir}} \tag{2.4}$$

where $R_{HL} = R_H R_L/(R_H + R_L)$, $C_{ir} = C_i + C_r$, and a_a is the voltage gain of the Rear-OPA. In this work, $R_f \gg R_{HL}$ and $R_a \gg R_{HL}$. Additionally, a_a can be approximated as $a_a = a_{a0}/(1 + jf/f_b)$ in (0, 40 MHz] for THS4021.

Using the parameters in , Inv-Amp performance was simulated with TINA-TI [20]. [Figure 2: see original paper] shows the $|a_A(f)|_{dB}$ and $\angle(a_A(f))$ curves from TINA-TI simulation. Nodal analysis calculations using equations with all components from [21] are also shown in [Figure 2: see original paper]. The calculated $|a_A(f)|_{dB}$ and $\angle(a_A(f))$ curves are identical to the simulated ones. The curves were also calculated using Eqs. (2.1), (2.3), and (2.4), and these results are nearly identical to the simulated curves in [3 kHz, 10 MHz], verifying the correctness of Eqs. (2.1), (2.3), and (2.4).

2.3 Frequency Compensation of Feedback Loop

To increase CryoSTM-TIA bandwidth, frequency compensation must be employed for the high feedback resistor R_F with parasitic capacitance C_F [1, 22].

In [Figure 1: see original paper], the compensated feedback network appears in dashed box (c). Setting $C_c = kC_F$ where $k > 10^3$, and adjusting $R_k = R_F/k$, we achieve $R_k C_c = R_F C_F$. The TIA output voltage $\dot{V}_o$ generates current $\dot{I}_F$ flowing to TIA input G, so the feedback network impedance is

$$Z_F(f) = \frac{R_k + \frac{R_F}{1+j2\pi f R_k C_k}}{1 + j2\pi f R_k C_k}$$

where C_k is the parasitic capacitance of R_k [1, 3, 22]. $Z_F(f)$ can be considered the feedback network impedance. Reference [22] experimentally demonstrated bandwidth extension of the feedback network with very high feedback resistor R_F of 10 GΩ up to MHz frequencies.

In (0, 1 MHz], with parameters from , $|Z_F(f)| \approx R_F/|1 + j2\pi f R_k C_k| > R_F/1.008$ and $|Z_F(f)| \leq R_F$, so $Z_F(f)$ can be considered equal to R_F.

[Figure 3: see original paper] shows experimental frequency compensation results. At 4.2 K, R_F is 1.16 GΩ and C_F is estimated as 3.36 pF. With $R_k = 380$ kΩ and $C_c = 10$ nF, the black curve shows the $|Z_F/R_F|_{dB}$ bandwidth extended to 905 kHz, while the red curve shows the Z_F phase is $-31.5°$ at 905 kHz.

To verify this compensation method, the following experiment was performed. A 500 MΩ resistor ($±$25 ppm/°C) at room temperature was selected as R_F, which becomes 1.16 GΩ at 4.2 K. R_k is a 390 kΩ resistor in series with a 0–20 kΩ potentiometer, and C_c is a 10 nF COG ceramic capacitor. To facilitate compensation adjustment, a 3 pF capacitor is paralleled with R_F, making $C_F \approx$ 3.36 pF. [Figure 3: see original paper] shows the frequency compensation results, where $|Z_F| = R_F/2$ at 905 kHz and $\angle(Z_F) = -31.5°$ at 905 kHz.

2.4 Circuit Stability of the Proposed CryoSTM-TIA

In [Figure 1: see original paper], the signal source circuit appears in dashed box (d) with parameters shown in . The differential resistance of the tip-sample tunnel junction (TJ) is R_J, limited to no less than $10^{-3}R_F$ in this design. The TJ capacitance C_J is in parallel with R_J and estimated as several fF [3]. In this work, $C = C_A + C_I + C_J$. Since C_J is at least two orders of magnitude smaller than $C_A + C_I$, it can be neglected, giving $C \approx C_A + C_I$. The DC bias & modulated signal voltage source, denoted BMS, provides DC bias V_i and AC sinusoidal modulated signal voltage $\dot{V}_i$ for the CryoSTM-TIA. In simulations, C_I is always taken as 0.5 pF [16]. The TIA connects to the signal source circuit to form the CryoSTM-TIA, whose performance can be simulated with TINA-TI using parameters from .

The loop gain T_L of the proposed CryoSTM-TIA is [23]

$$T_L(f) = -a_A(f)\beta(f) = -a_A(f)/[1/\beta(f)],$$

where $\beta(f)$ is the feedback factor with reciprocal

$$1/\beta(f) \approx 1 + Z_F[1/R_J + 1/R_A + j2\pi f(C_A + C_I)]. \tag{2.5}$$

From Eq. (2.5), $|1/\beta(f)|$ and $\angle(1/\beta(f))$ can be calculated. Figure 4: see original paper shows calculated results for $|1/\beta(f)|_{dB}$ and $\angle(1/\beta(f))$ with $R_J = +\infty$, while Figure 4: see original paper shows results with $R_J = 1$ MΩ. Both figures show $|T_L|_{dB} = |a_A|_{dB} - |1/\beta(f)|_{dB} \leq -10$ dB for $f \geq 465$ kHz and $\angle(T_L) = \angle(-a_A) - \angle(1/\beta) = \angle(a_A) - 180^\circ - \angle(1/\beta) > -130^\circ$ for $f \leq 465$ kHz. Therefore, the CryoSTM-TIA is stable with gain margin exceeding 10 dB and phase margin exceeding 50°.

2.5 Voltage Gain and Transimpedance Gain of the Proposed CryoSTM-TIA

With the compensated feedback network from Section 2.3, Z_F can be considered equal to R_F in (0, 1 MHz]. Accounting for TJ capacitance C_J, the TJ impedance is $Z_J = R_J/(1 + j2\pi f R_J C_J)$, and $C \approx C_A + C_I$. When AC input voltage $\dot{V}_i$ is applied by BMS, the CryoSTM-TIA output voltage is $\dot{V}_o$, and the voltage gain is $A_v = \dot{V}_o/\dot{V}_i$. In (0, 1 MHz], nodal analysis gives

$$A_v \approx -\frac{1}{1 - j2\pi f R_F (C_A + C_I)}. \tag{2.6}$$

Setting $\dot{V}_i = 0$ and applying a sinusoidal current source $\dot{I}_i$ in parallel with the TJ, the CryoSTM-TIA output voltage $\dot{V}_o$ is generated. The transimpedance gain is $A_i = \dot{V}_o/\dot{I}_i$. In (0, 1 MHz],

$$A_i \approx -\frac{R_F}{1 - j2\pi f R_F (C_A + C_I)}. \tag{2.7}$$

Disconnecting the TIA from the signal source circuit and applying a sinusoidal current source $\dot{I}_{iT}$ to the TIA input, the output voltage $\dot{V}_{oT}$ is generated. The TIA transimpedance gain is $A_{iT} = \dot{V}_{oT}/\dot{I}_{iT}$. In (0, 1 MHz],

$$A_{iT} \approx -\frac{R_F}{1 - j2\pi f R_F C_A}. \tag{2.8}$$

Considering $R_A \gg R_F$ and $|a_A| \gg 1$, A_{iT} simplifies to Eq. (2.8). [Figure 5: see original paper] shows $|A_{iT}/R_F|_{dB}$ and $\angle(A_{iT}/R_F)$ calculated by Eq. (2.8) with parameters from , which agrees with TINA-TI simulation results in [10 mHz, 1 MHz], verifying Eq. (2.8). The upper cut-off frequency of $A_{iT}(f)/R_F$, its -3 dB frequency f_{hT}, is about 201 kHz. Comparing Eq. (2.7) with Eq. (2.8), since $C_A + C_I + C_J \approx C_A + C_I \approx C_A$ and $R_J \geq 10^{-3} R_F$, the CryoSTM-TIA upper cut-off frequency is approximately equal to f_{hT}.

2.6 Transient Response of the Proposed CryoSTM-TIA

Simulation results for the transient response of the proposed CryoSTM-TIA are shown in Supplemental file 3 [24]. For the CryoSTM-TIA, the transient response time t_r is defined as the time from applying an input step signal voltage until the output response stabilizes within a certain error. In simulations, a resistor with constant resistance R_0 replaces the tip-sample junction. For $R_0 \geq 30$ MΩ, $t_r < 5$ µs for 0.1% error. For 5 MΩ $\leq R_0 < 30$ MΩ, $t_r < 5$ µs for 1% error. For 1 MΩ $\leq R_0 < 5$ MΩ, $t_r < 5$ µs for 3% error.

3 Inherent Noise of the Proposed CryoSTM-TIA

For the proposed CryoSTM-TIA circuit shown in [Figure 1: see original paper], a differential equivalent circuit with all noise sources is used to calculate its equivalent input noise. Noise calculation details are provided in Supplemental file 4 [25].

3.1 Equivalent Input Voltage Noise and Equivalent Input Current Noise of Inv-Amp

The equivalent input noise voltage and current of the HEMT are denoted e_H and i_H, respectively. Resistors R_1, R_2, and R_f are in the 4.2 K cryogenic zone, and their noise above 1 kHz is thermal noise, which can be neglected [3]. Resistor R_h is also in the 4.2 K cryogenic zone, with thermal noise voltage e_{RH}. The thermal noise voltages of resistors R_{c1} and R_{c2} are e_1 and e_2. The equivalent input noise voltage and current of the Rear-OPA are e_a and i_a. These noise sources are independent. The equivalent input noise voltage and current of Inv-Amp are e_A and i_A, with e_A^2 as its equivalent input noise voltage PSD, i_A^2 as its equivalent input noise current PSD, and $e_A i_A^*$ and $i_A e_A^*$ as the cross-spectral densities. Using nodal analysis and the Wiener-Khinchin theorem while ignoring minor terms yields [25]:

$$e_A^2 = e_H^2 + \left(\frac{R_{HL}}{R_H}\right)^2 e_{RH}^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{3.1}$$

$$i_A^2 = i_H^2 + (2\pi f)^2 C_A^2 \left(\frac{R_{HL}}{R_H}\right)^2 e_{RH}^2 + (2\pi f)^2 \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{3.2}$$

$$e_A i_A^* = (i_A e_A^*)^* = -j2\pi f C_A \left(\frac{R_{HL}}{R_H}\right)^2 e_{RH}^2 - j2\pi f \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}. \tag{3.3}$$

With parameters from , for the Inv-Amp in this work: $e_A^2 = 0.14$ (nV)^2/Hz and $i_A^2 = 0.5$ (fA)^2/Hz at $f = 10$ kHz, and $e_A^2 = 0.09$ (nV)^2/Hz and $i_A^2 = 4.9$ (fA)^2/Hz at $f = 100$ kHz. For the Macro-OPA in Ref. [3]: $e_A^2 = 0.28$ (nV)^2/Hz and $i_A^2 = 20$ (fA)^2/Hz at $f = 100$ kHz.

In this work's Inv-Amp, resistor R_H replaces transistor H2 from the Macro-OPA in Ref. [3]. The noise generated by R_H is two orders of magnitude smaller than that generated by H2 in Ref. [3] (i.e., e_H^2). Therefore, the equivalent input noise of this work' s Inv-Amp is much lower than that of the Macro-OPA in Ref. [3].

In Eq. (3.2), $(1 + R_{HL}/R_d)^2 i_H^2$ is one order of magnitude smaller than $(2\pi f)^2(C_{gs}+C_{gd}+C_A R_{HL}/R_d)^2 e_H^2$. In Eq. (3.1), $(C_{gs}+C_{gd}+C_A R_{HL}/R_d)^2 i_H^2$ is two orders of magnitude smaller than $C_A^2 e_H^2$. Further ignoring minor terms in Eqs. (3.1), (3.2), and (3.3) yields:

$$e_A^2 \approx e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{3.4}$$

$$i_A^2 \approx i_H^2 + (2\pi f)^2 C_A^2 \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{3.5}$$

$$e_A i_A^* = (i_A e_A^*)^* \approx -j2\pi f C_A \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}. \tag{3.6}$$

3.2 Equivalent Input Voltage Noise and Equivalent Input Current Noise of the Proposed TIA

The equivalent input noise voltage PSD of the TIA is e_T^2, its equivalent input noise current PSD is i_T^2, its equivalent input noise voltage-current PSD is $e_T i_T^*$, and its equivalent input noise current-voltage PSD is $i_T e_T^*$ [25]:

$$e_T^2 = e_A^2 \approx e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{3.7}$$

$$i_T^2 = \frac{4k_B T}{R_F} + \frac{e_A^2}{R_F^2} + i_A^2 \approx \frac{4k_B T}{R_F} + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2 R_F^2} + i_H^2 + (2\pi f)^2 C_A^2 \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{3.8}$$

$$e_T i_T^* = (i_T e_T^*)^* = \frac{e_A^2}{R_F} + e_A i_A^* \approx \frac{e_H^2}{R_F} + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2 R_F} - j2\pi f C_A \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}. \tag{3.9}$$

3.3 Equivalent Input Current Noise of the Proposed CryoSTM-TIA

The equivalent input noise current PSD of the proposed CryoSTM-TIA is [25]:

$$i_{in}^2 = i_T^2 + \left[\frac{1}{R_J^2} + (2\pi f)^2 C_{IJ}^2\right] e_T^2 + \left(\frac{1}{R_J} + j2\pi f C_{IJ}\right) e_T i_T^* + \left(\frac{1}{R_J} - j2\pi f C_{IJ}\right) i_T e_T^*, \tag{3.10}$$

where $C_{IJ} = C_I + C_J$. Substituting Eqs. (3.7), (3.8), and (3.9) into Eq. (3.10) yields:

$$i_{in}^2 = i_H^2 + \frac{4k_BT}{R_F} + \left(\frac{1}{R_J} + \frac{1}{R_F}\right)^2 \left[e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}\right] + (2\pi f)^2 \left[C_A^2 + 2C_AC_{IJ}\right] \left[e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}\right]. \tag{3.11}$$

This can be simplified to:

$$i_{in}^2 = i_H^2 + \frac{4k_BT}{R_F} + \left(\frac{1}{R_J} + \frac{1}{R_F}\right)^2 e_A^2 + (2\pi f)^2 \left[C_A^2 + 2C_AC_{IJ}\right] e_A^2. \tag{3.12}$$

For the proposed CryoSTM-TIA, $R_F = 1$ GΩ, $C_A = 26$ pF, $C_I = 0.5$ pF, and $C_J = 10$ fF. R_F and the TJ are in the 4.2 K cryogenic zone. With $R_J = 1$ MΩ, i_{in}^2 and its four components are listed in , along with noise components from the CryoSTM-TIA in Ref. [3]. The CryoSTM-TIA proposed here has equivalent input noise current PSD of 0.62 $(\mathrm{fA})^2/\mathrm{Hz}$ at 10 kHz and 5.3 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz–significantly lower than Ref. [3].

For the CNRS-HEMT epi-wafer, Si doping in Al Ga_1 As creates two electronic states: a shallow, delocalized donor level associated with the normal substitutional site configuration (shallow donor state), and a more localized acceptor level called the DX^- center arising from lattice distortion. In Al Ga_1 As with $x > 0.22$, the DX^- bound state is more stable than the shallow donor state. For the CNRS-HEMT used here, $x = 0.37$. A large repulsion barrier E_{cap} exists for the shallow donor to DX^- transition, so DX^- centers freeze when $T < 120$ K [26, 27]. According to studies on CNRS-HEMT noise mechanisms [5], the "bias-cooling method" can control the density of frozen DX^- centers in the HEMT doping region. This unique method enables in-situ modification of the HEMT structure to reduce noise. By cooling the HEMT with positive gate-source voltage, more electrons freeze into DX^- centers. At low temperatures, the absolute gate-source voltage required to maintain the ideal operating point decreases, reducing gate leakage current and thus low-frequency noise from gate leakage [5]. This reduces the CryoSTM-TIA' s inherent noise. Moreover, if the HEMT used is not a CNRS-HEMT or its noise performance is inferior to , the "bias-cooling method" can compensate. This approach is particularly convenient for single-HEMT Pre-Amps.

4 Operating State Adjustment and DC Tunneling Current Measurements

For the CryoSTM-TIA in [Figure 1: see original paper] with parameters from , operating state adjustment proceeds as follows: (1) Disconnect Pre-Amp from

Post-Amp and ground the Pre-Amp input (HEMT gate G). Keep $R_p = 0$. Adjust R_{s1}, R_{h1}, and R_{t1} to set the HEMT at its ideal operating point ($V_{ds} = 100$ mV, $I_{ds} = 1$ mA) and equalize potentials at O1 and O2. That is, adjust R_{s1} and R_{h1} to achieve $R_H = R_S + V_{ds}/I_{ds} = R_S + 100\ \Omega$. (2) Cascade Pre-Amp and Post-Amp to form Inv-Amp, keeping input G grounded. Adjust R_p and R_{t1} to set Inv-Amp DC output voltage V_{om} to 0 while maintaining the HEMT at its ideal operating point. (3) Connect Inv-Amp output O and input G through the feedback network and disconnect input G from ground to form the TIA. Input G remains disconnected from the signal source circuit, so Inv-Amp input G and output O potentials stay at 0, since the HEMT input resistance R_A is effectively infinite. In the feedback network, $R_F + R_k \approx R_F = 1$ GΩ. (4) Connect the signal source circuit to the TIA to complete the CryoSTM-TIA.

As $f \to 0$, $a_A \to a_{A0}$, where a_{A0} is the Inv-Amp DC voltage gain. $|a_{A0}|_{dB} \approx (g_m R_f / g_d R_H)_{dB} \approx 94.5$ dB, consistent with the TINA-TI simulation result $|a_{A0}|_{dB} = 92$ dB shown in [Figure 2: see original paper]. When DC bias V_i is applied by BMS, the TJ DC resistance is R, the input G potential is V_G, and the CryoSTM-TIA output voltage is V_o. Clearly, $a_{A0}V_G = V_o$ and $(V_i - V_G)/R = (V_G - V_o)/R_F$. The DC bias on the TJ is $V = V_i - V_G$, and the DC tunneling current is $I = (V_i - V_G)/R$. An approximate current value is

$$I_s = -V_o/R_F. \tag{4.1}$$

The relative error is

$$E_r = |I_s - I|/|I| = 1/(1 - a_{A0}).$$

With $|a_{A0}|_{dB} \approx 92$ dB, $E_r < 30$ ppm, consistent with simulation results [28]. Since $a_{A0}V_G = V_o$, the DC bias on the TJ is

$$V = V_i - V_o/a_{A0}. \tag{4.2}$$

From Eq. (4.1), $V_o = -R_F I_s \approx -R_F I = -R_F V/R$. From Eq. (4.2), $V_i \approx V - R_F V/(Ra_{A0})$. Since the minimum R_J is not less than 1 MΩ, $R \geq 1$ MΩ, so $|R_F/(Ra_{A0})| \leq 1/40$. Thus, $V \approx V_i$. From measured V_o, we obtain $I \approx I_s$ and $V \approx V_i$. Therefore, scanning tunneling current spectra $I = I(V)$ ($V \in [V_L, V_H]$) can be obtained, where V_L and V_H are the measurement voltage limits.

The drift of V_{om} can be considered as amplification of the Inv-Amp input offset voltage drift V_{OS}, where $V_{om} = a_{A0}V_{OS}$. For the TIA, the output drift voltage approximates V_{OS}. The estimated drift of $|V_{OS}|$ is less than 17 μV/°C [29]. The Pre-Amp power supply consumes less than 40 mW. A temperature control system based on TEC devices [30] can maintain temperature fluctuations of the power supply and Rear-OPA within 0.01 °C, ensuring V_{OS} fluctuations within 170 nV, i.e., TIA output fluctuations within 170 nV.

To verify the TIA design methods, we performed the work described in Ref. [31]. An N-Channel JFET (SST4393-T1) was selected to replace the CNRS-HEMT, and a TIA was designed and fabricated using the same circuit design method and topology. The electrical and noise performance of the JFET-based TIA at 77 K were measured [31], with experimental results basically consistent with calculations and simulations, verifying the design methods.

5 Applications for the Proposed CryoSTM-TIA in Spectra Measurements

For most applications, the CryoSTM-TIA transimpedance gain magnitude $|A_i(f)|$ must first be measured. $A_i(f)$ is given by Eq. (2.7). Since $|a_A(f)|_{dB} > 90$ dB in [1 kHz, 300 kHz] as shown in [Figure 2: see original paper], when $R_J \geq 10^{-3} R_F$, $|A_i(f)|$ in [1 kHz, 300 kHz] can be approximated as

$$|A_i(f)| = \frac{R_F}{|1 - j2\pi f R_F (C_A + C_I)/a_A(f)|}. \tag{5.1}$$

The measurement procedure is described in Ref. [3].

5.1 Measurements of Scanning Tunneling Differential Conductance Spectra by the Proposed CryoSTM-TIA

The TJ differential conductance $G_J = 1/R_J$ is a function of voltage V applied to the TJ. When the modulated signal voltage frequency f from BMS is low enough (e.g., $f < 1$ kHz), Eq. (2.6) gives $A_v \approx -R_J(V)/[R_J(V) + R_F]$. Therefore, $R_J(V) \approx [-1/A_v + 1/a_A(f)]R_F$. When measured $|A_v| \leq 1000$, $R_J(V) \approx R_F/|A_v|$ since $|a_A(f)|_{dB} \geq 85$ dB for $f \leq 300$ kHz as shown in [Figure 2: see original paper]. From measured $|A_v| = |\dot{V}_o|/|\dot{V}_i|$, differential conductance spectra $G_J(V) = 1/R_J(V)$ ($V \in [V_L, V_H]$) can be obtained.

Increasing the modulated signal frequency f can accelerate scanning tunneling differential conductance spectra measurements. In [1 kHz, 300 kHz], for $R_J \geq 1$ MΩ, Eq. (2.6) gives $|A_v(f)| \approx |Z_J(f)|/|R_F + Z_J(f)[1 - j2\pi f R_F(C_A + C_I)/a_A(f)]|$. Therefore, $1/|Z_J(f)| \approx |A_v(f)|/|A_i(f)|$, where $|A_i(f)|$ is given by Eq. (5.1). Both $|A_v(f)|$ and $|A_i(f)|$ can be measured. Since $1/|Z_J(f)| = \sqrt{1/R_J^2 + (2\pi f C_J)^2}$, selecting two different frequencies f_1 and f_2 in [1 kHz, 300 kHz] yields $|Z_J(f_1)|$ and $|Z_J(f_2)|$. R_J and C_J can then be solved from these measurements.

Because the CryoSTM-TIA inherent noise is very small, the modulated signal voltage amplitude $\dot{V}_i$ can be very small, significantly improving STS energy resolution.

5.2 Measurements of Scanning Tunneling Shot Noise Spectra by the Proposed CryoSTM-TIA

The measurement method for tunneling shot noise spectra using the proposed CryoSTM-TIA is essentially the same as described in Ref. [3] and is only briefly summarized here.

Before tip-sample approach in CryoSTM, R_J can be considered infinite and C_J zero. In this case, the CryoSTM-TIA output noise voltage PSD $S_{su}(f)$ can be measured, and the equivalent input noise current PSD $i_i^2(f)$ is

$$i_i^2(f) = S_{su}(f)/|A_i(f)|^2, \tag{5.2}$$

where $|A_i(f)|$ is obtained from Eq. (5.1).

To measure STSNS of a quantum system with the CryoSTM-TIA, the tip-sample distance is adjusted and interval $[V_L, V_H]$ is selected so that shot noise measurements at $V \in D_V = \{V|V_L \leq V \leq V_H, G_J(V) < 1\mu S\}$ are needed to study the quantum system' s physical properties [3]. In D_V, the output noise voltage PSD $S_{sum}(f, V)$ can be measured. The tunneling noise current PSD is $S_I(f, V)$. The CryoSTM-TIA equivalent input noise current PSD depends on R_J, making it a function of V, denoted $i_{in}^2(f, V)$. For $S_{sum}(f, V)$,

$$S_{sum}(f, V) = S_I(f, V) \cdot |A_i(f)|^2 + i_{in}^2(f, V) \cdot |A_i(f)|^2. \tag{5.3}$$

From Eqs. (5.2) and (5.3),

$$S_I(f, V) = \frac{S_{sum}(f, V) - S_{su}(f)}{|A_i(f)|^2} - \delta(f, V),$$

where $\delta(f, V) = i_{in}^2(f, V) - i_i^2(f)$. From Eq. (3.12),

$$i_i^2(f) = i_H^2 + \frac{4k_B T}{R_F} + (2\pi f)^2 \left[C_A^2 + 2C_A C_I\right] \left[e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}\right].$$

And $i_{in}^2(f, V)$ comprises the four terms shown in Eq. (3.12). According to , $|\delta(f, V)|$ is smaller than 0.3 $(\text{fA})^2/\text{Hz}$ in [10 kHz, 100 kHz) and 0.1 $(\text{fA})^2/\text{Hz}$ in [100 kHz, 200 kHz]. $\delta(f, V)$ can be neglected compared to $S_I(f, V)$ as long as the minimum $G_J(V)$ $(V \in D_V)$ is not too small. Therefore, $S_I(f, V)$ can be obtained as

$$S_I(f, V) \approx \frac{S_{sum}(f, V) - S_{su}(f)}{|A_i(f)|^2}$$

from measured $S_{sum}(f,V)$, $S_{su}(f)$, and $|A_i(f)|$. The STSNS $S_{Is}(I) = 2Fe|I|$ can then be extracted from $S_I(f,V)$, where $I = I(V)$ $(V \in D_V)$ is the DC tunneling current at bias V.

Compared to the CryoSTM-TIA in Ref. [3], the CryoSTM-TIA proposed here has the same transimpedance gain, bandwidth above 200 kHz, but much lower inherent noise. The inherent noise is only 5.3 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz with $\delta(f,V) <$ 0.1 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz, whereas Ref. [3] had 21 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz with $\delta(f,V) < 0.28$ $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz. Therefore, measurements of novel quantum states in various quantum systems are more accurate with this apparatus.

For example, when investigating MBS existence in a magnetic flux vortex of iron-based superconductors using CryoSTM [4], the tunnel junction resistance must be large enough to rule out incoherent Andreev reflection under weak tunnel coupling conditions [32]. In Ref. [4], the tunneling current I is quite low (see Fig. 3(a) in Ref. [4] and Figs. S2(a) and (b) in its supplemental file). At tunnel junction bias $V = 0.3$ mV, I is only tens of pA, and the corresponding shot noise may be only a few $(\mathrm{fA})^2/\mathrm{Hz}$. For shot noise measurements in this system, the CryoSTM-TIA proposed here is clearly much more effective and accurate than that in Ref. [3].

6 Conclusion

This work presents a transimpedance amplifier (TIA) design for cryogenic scanning tunneling microscopy. The TIA integrated with the tip-sample component is called CryoSTM-TIA. The proposed CryoSTM-TIA has a transimpedance gain of 1 GΩ, bandwidth exceeding 200 kHz, and equivalent input noise current PSD of only 5.3 $(\mathrm{fA})^2/\mathrm{Hz}$ at 100 kHz.

In this CryoSTM-TIA, a single CNRS-HEMT Pre-Amp replaces the differential Pre-Amp with a pair of CNRS-HEMTs from Ref. [3]. This avoids the difficulty of matching identical HEMTs and, by eliminating one noisy HEMT, reduces the apparatus inherent noise to 1/4 of that in Ref. [3].

Furthermore, the inherent noise of the single HEMT can be in-situ reduced by the "bias-cooling method." With this apparatus, fast high-energy-resolution scanning tunneling spectroscopy measurements can be performed, and very low tunneling shot noise of quantum systems can be measured at the atomic scale. This apparatus can investigate novel physical properties of various quantum systems, such as detecting Majorana bound states in topological quantum systems.

Acknowledgment

This work is supported by the Open Research Fund Program of the State Key Laboratory of Low-Dimensional Quantum Physics (Grant No. KF202212). The author acknowledges stimulating discussions with Prof. Fang-Hao Liang of the School of Mathematics at Shandong University and Prof. Xing Liang of the School of Mathematics at the University of Science and Technology of China.

Declaration of Competing Interest

The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] C.J. Chen, *Introduction to Scanning Tunneling Microscopy*, Oxford Univ. Press, (1993).

[2] J.F. Ge, M. Ovadia, and J.E. Hoffman, Achieving low noise in scanning tunneling spectroscopy, *Rev. Sci. Instrum.* 90 (2019) 101401, https://doi.org/10.1063/1.5111989.

[3] Y.X. Liang, Low-noise large-bandwidth transimpedance amplifier for measuring scanning tunneling shot noise spectra in cryogenic STM and its applications, *Ultramicroscopy* 234 (2022) 113466, https://doi.org/10.1016/j.ultramic.2022.113466.

[4] D.F. Wang, L.Y. Kong, P. Fan, H. Chen, S.Y. Zhu, W.Y. Liu, L. Cao, Y.J. Sun, S.X. Du, J. Schneeloch, R.D. Zhong, G.D. Gu, L. Fu, H. Ding, and H.J. Gao, Evidence for Majorana bound states in an iron-based superconductor, *Science* 362 (2018) 333, https://doi.org/10.1126/science.aao1797.

[5] Y.X. Liang, Q. Dong, M.C. Cheng, U. Gennser, A. Cavanna, and Y. Jin, Insight into low frequency noise induced by gate leakage current in AlGaAs/GaAs high electron mobility transistors at 4.2 K, *Appl. Phys. Lett.* 99 (2011) 113505, https://doi.org/10.1063/1.3637054.

[6] Y. Jin, Q. Dong, A. Cavanna, U. Gennser, L. Couraud, and C. Ulysse, Ultra-low noise HEMTs for deep cryogenic low-frequency and high-impedance readout electronics, 12th IEEE International Conference on Solid-State and Integrated Circuit Technology (ICSICT) (2014).

[7] K.M. Bastiaans, D. Cho, T. Benschop, I. Battisti, Y. Huang, M.S. Golden, Q. Dong, Y. Jin, J. Zaanen, and M.P. Allan, Charge trapping and super-Poissonian noise centres in a cuprate superconductor, *Nat. Phys.* 14 (2018) 1183, https://doi.org/10.1038/s41567-018-0300-z.

[8] F. Massee, Y.K. Huang, M.S. Golden, and M. Aprili, Noisy defects in the high-Tc superconductor $Bi_2Sr_2CaCu_2O_8$, *Nat. Commun.* 10 (2019) 544, https://doi.org/10.1038/s41467-019-08518-1.

[9] C.J. Bolech and E. Demler, Observing Majorana bound states in p-wave superconductors using noise measurements in tunneling experiments, *Phys. Rev. Lett.* 98 (2007) 237002, https://doi.org/10.1103/PhysRevLett.98.237002.

[10] A. Golub and B. Horovitz, Shot noise in a Majorana fermion chain, *Phys. Rev. B* 83 (2011) 153415, https://doi.org/10.1103/PhysRevB.83.153415.

[11] H. Soller and A. Komnik, Charge transfer statistics of transport through Majorana bound states, *Physica E* 63 (2014) 99, https://doi.org/10.1016/j.physe.2014.05.020.

[12] C.W.J. Beenakker and D.O. Oriekhov, Shot noise distinguishes Majorana fermions from vortices injected in the edge mode of a chiral p-wave superconductor, *SciPost Phys.* 9 (2020) 080, https://doi.org/10.21468/SciPostPhys.9.5.080.

[13] K.M. Bastiaans, D. Cho, D. Chatzopoulos, M. Leeuwenhoek, C. Koks, and

M.P. Allan, Imaging doubled shot noise in a Josephson scanning tunneling microscope, *Phys. Rev. B* 100 (2019) 104506, https://doi.org/10.1103/PhysRevB.100.104506.
[14] K.M. Bastiaans, D. Chatzopoulos, J.F. Ge, D. Cho, W.O. Tromp, J.M. van Ruitenbeek, M.H. Fischer, P.J. de Visser, D.J. Thoen, E.F.C. Driessen, T.M. Klapwijk, and M.P. Allan, Direct evidence for Cooper pairing without a spectral gap in a disordered superconductor above Tc, *Science* 374 (2021) 608, https://doi.org/10.1126/science.abe3987.
[15] S. Cocklin and D.K. Morr, Scanning tunneling shot-noise spectroscopy in Kondo systems, *Phys. Rev. B* 100 (2019) 125146, https://doi.org/10.1103/PhysRevB.100.125146.
[16] Webpage of HF-STM-broadband cryogenic buffer amplifier by Stahl-electronics, https://www.stahl-electronics.com/products/amp/hfstm/. Its datasheet (pdf) shows the amplifier input capacitance, including HEMT input capacitance and tip-to-HEMT wire capacitance, is only 2.7–3.1 pF. The HEMT input capacitance C_A is at least 2.5 pF [6], so $C_I \leq 0.5$ pF is reasonable.
[17] Datasheet of LM4050-10, https://www.ti.com.cn/document-viewer/cn/lm4050-n/datasheet.
[18] Webpage of THS4021 OPA, https://www.ti.com/product/THS4021.
[19] Supplemental file 1.
[20] TINA-TI is a SPICE-based analog simulation program by Texas Instruments Inc., https://www.ti.com/tool/TINA-TI.
[21] Supplemental file 2.
[22] B. Michel, L. Novotny, and U. Dürig, *Ultramicroscopy* 42-44 (1992) 1647.
[23] S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*, McGraw-Hill Companies, Inc., (2002).
[24] Supplemental file 3.
[25] Supplemental file 4.
[26] D.J. Chadi and K.J. Chang, Energetics of DX-center formation in Al Ga$_1$ As alloys, *Phys. Rev. B* 39 (1989) 10063, https://doi.org/10.1103/PhysRevB.39.10063.
[27] W.F. Li, Y.X. Liang, and J.H. Wei, Bipolaron mechanism of DX center in Al Ga$_1$ As:Si, *Acta Phys. Sin.* 59 (2010) 8850, https://doi.org/10.7498/aps.59.8850.
[28] Supplemental file 5.
[29] Supplemental file 6.
[30] A suitable TEC device, https://datasheets.maximintegrated.com/en/ds/MAX1978-MAX1979.pdf.
[31] Y.X. Liang, Low-noise high-gain large-bandwidth transimpedance amplifier for STM at 77 K, submitted to *Ultramicroscopy*, https://chinaxiv.org/abs/202404.00274.
[32] L.Y. Kong and H. Ding, Emergent vortex Majorana zero mode in iron-based superconductors, *Acta Phys. Sin.* 69 (2020) 110301, https://doi.org/10.7498/aps.69.20200717.

Supplemental file 1: Voltage gain peaks for cables

Figure s1-1 The circuit scheme for measuring voltage gain peaks for cables.

A 1-meter cable with 50 Ω characteristic impedance has a distributed capacitance of 100 pF and distributed inductance of 250 nH, shown in Fig. s1-1. In

Fig. s1-1, $R = 1.5$ kΩ and the gain is V_o/V_{in}. Simulation results show its distributed LC structure produces several resonant gain peaks in [80 MHz, 1 GHz] as shown in Fig. s1-2. When the input signal V_{in} is the thermal noise of R (1.5 kΩ) at 300 K, we amplify V_o with Amplifier SA-220F5 (gain = 200, f_h = 200 MHz) and measure the cable gain peaks with an oscilloscope. Fig. s1-3 shows the measured gain peaks for the cable.

Figure s1-2 Simulation results of gain peaks for a 1-meter cable with distributed capacitance of 100 pF and distributed inductance of 250 nH.

Figure s1-3 The input signal V_{in} is the thermal noise of R (1.5 kΩ) at 300 K. Measured results of gain peaks for a 1-meter cable with distributed capacitance of 100 pF and distributed inductance of 250 nH.

Supplemental file 2: Voltage gain of Inv-Amp

Figure s2-1 Inv-Amp circuit with all components for Eqs. (s2.1–s2.8), with parameters from Table 1 in the article.

Using nodal analysis, Inv-Amp voltage gain $a_A(f)$ is calculated by the following equations. All components for Eqs. (s2.1–s2.9) are shown in Fig. s2-1, with parameters from Table 1 in the article.

[Equations s2.1–s2.9 would appear here with proper formatting]

Figure s2-2 Inv-Amp voltage gain $a_A(f)$. Solid curves show $|a_A(f)|_{dB}$, dashed curves show $\angle(a_A(f))$. Red curves are TINA-TI simulation results, green curves are nodal analysis calculations (Eqs. (s2.1–s2.9)), and black curves are calculations using Eqs. (2.1), (2.3), and (2.4) from the article.

Fig. s2-2 shows the Inv-Amp voltage gain magnitude $|a_A(f)|_{dB}$. The red solid curve is the TINA-TI simulation, and the green solid curve is the nodal analysis calculation. They are identical. The black solid curve is from Eqs. (2.1), (2.3), and (2.4) in the article. For $f \in$ [3 kHz, 10 MHz], all three curves are identical.

Fig. s2-2 also shows the Inv-Amp voltage gain phase $\angle(a_A(f))$. The red dashed curve is the TINA-TI simulation, and the green dashed curve is the nodal analysis calculation. They are identical. The black dashed curve is from Eqs. (2.1), (2.3), and (2.4) in the article. For $f \in$ [3 kHz, 10 MHz], all three curves are consistent.

Supplemental file 3: Transient response of the proposed CryoSTM-TIA

Figure s3-1 A resistor with constant resistance R_0 replaces the tip-sample junction for simulation. TINA-TI simulation results for CryoSTM-TIA transient response with different R_0 over a 10 µs interval. The dashed curve is the step input signal V_i/V_{inst} for different R_0. Solid curves are the output response

V_o/V_{ost}. Here, V_{ost} and V_{inst} are their values at 300 ms after applying the step input V_i, shown in Fig. s3-2. Transient response time $t_r < 5$ µs. (b) is a zoom-in of the output response V_o/V_{ost} from (a).

Figure s3-2 A resistor with constant resistance R_0 replaces the tip-sample junction for simulation. TINA-TI simulation results for CryoSTM-TIA transient response with different R_0 over a 300 ms interval. (a) Step input signal V_i/V_{inst}. (b) Output response V_o/V_{ost}. $V_o/V_{ost} = 1$ for $t \geq 100$ ms. (c) is a zoom-in of (b).

A resistor with constant resistance R_0 replaces the tip-sample junction for simulation. Fig. s3-1 shows TINA-TI simulation results for CryoSTM-TIA transient response with different R_0 over a 10 µs interval. The dashed curve is the step input signal V_i/V_{inst}, and solid curves are the output response V_o/V_{ost} for different R_0. Here, V_{ost} and V_{inst} are their values at 300 ms after applying step input V_i. Error is defined as $|V_o - V_{ost}|/|V_{ost}|$. For the CryoSTM-TIA, transient response time t_r is the time from applying the step input voltage until the output stabilizes within a certain error. As shown in Fig. s3-1 and Figs. s3-2(b)&(c), with $R_0 = 1$ GΩ and 30 MΩ, $t_r < 5$ µs for 0.1% error. With $R_0 = 5$ MΩ, $t_r < 5$ µs for 1% error. With $R_0 = 1$ MΩ, $t_r < 5$ µs for 2.5% error.

Supplemental file 4: Noise of the proposed CryoSTM-TIA

S4.1 Noises of STM-TIA

For Inv-Amp, the equivalent input noise voltage is e_A and equivalent input noise current is i_A, with harmonic components at frequency f denoted E_A and I_A. [Matrix equations] can be obtained by nodal analysis. By the Wiener-Khinchin theorem, [spectral densities] can be derived from [S4R1, S4R2]. The two matrix elements on the main diagonal are the equivalent input noise voltage PSD e_A^2 and current PSD i_A^2. The off-diagonal elements are the cross-spectral densities $e_A i_A^*$ and $i_A e_A^*$.

Inv-Amp connects to feedback resistor R_F to form TIA. The equivalent input noise voltage of TIA is e_T and its equivalent input noise current is i_T, with harmonic components E_T and I_T. The feedback resistance R_F is at temperature T, with noise voltage e_F and harmonic component E_F.

Figure s4-1 (a) TIA circuit with input short-circuit containing Inv-Amp equivalent input noise voltage e_A and current i_A, and output noise voltage e_{oTv}; (b) Noiseless TIA circuit with TIA equivalent input noise voltage e_T as input signal and output noise voltage e_{oTve}. Equivalence of these circuits means $e_{oTv} = e_{oTve}$.

For TIA, the circuit with all noise sources and input short-circuit is shown in Fig. s4-1(a), with output noise e_{oTv}. The noiseless circuit with TIA equivalent input noise voltage e_T as input is shown in Fig. s4-1(b), with output noise e_{oTve}.

To calculate TIA equivalent input noise voltage, equations are established based on circuit equivalence ($e_{oTv} = e_{oTve}$). By nodal analysis, $E_T = E_A$.

Figure s4-2 (a) TIA circuit with input open-circuit containing Inv-Amp equivalent input noise current i_A and output noise current e_{oTi}; (b) Noiseless TIA circuit with TIA equivalent input noise current i_T as input signal and output noise voltage e_{oTie}. Equivalence means $e_{oTi} = e_{oTie}$.

For TIA, the circuit with all noise sources and input open-circuit is shown in Fig. s4-2(a), with output noise e_{oTi}. The noiseless circuit with TIA equivalent input noise current i_T as input is shown in Fig. s4-2(b), with output noise e_{oTie}. To calculate TIA equivalent input noise current, equations are established on $e_{oTi} = e_{oTie}$. By nodal analysis, $I_T = I_A + (E_A + E_F)/R_F$.

[Matrix equations] yield, by Wiener-Khinchin theorem, the two main diagonal elements as TIA equivalent input noise voltage PSD e_T^2 and current PSD i_T^2, and off-diagonal elements as cross-spectral densities $e_T i_T^*$ and $i_T e_T^*$.

The noise voltage PSD of R_F is $4k_BT/R_F$ [S4R1, S4R2]. TIA connects to the signal source circuit to form STM-TIA. The STM-TIA equivalent input noise current PSD can be obtained [S4R1-S4R3] by [matrix equation]. Here, $C_{IJ} = C_I + C_J$. Substituting equations yields the final noise expression.

S4.2 Noises of the proposed CryoSTM-TIA

S4.2.1 Equivalent input noise voltage and current of Inv-Amp

For Inv-Amp, HEMT equivalent input noise voltage and current are e_H and i_H, with harmonic components E_H and I_H. g_m is HEMT transconductance and g_d is channel conductance. Resistor R_H noise voltage is e_{RH} with harmonic component E_{RH}. Resistors R_1, R_2, and feedback resistance R_f have noise voltages e_{L1}, e_{L2}, e_f with harmonic components E_{L1}, E_{L2}, E_f. Resistors R_{c1} and R_{c2} have noise voltages e_1, e_2 with harmonic components E_1, E_2. Here $R_1 = R_2 = R_L$. All noise from R_H, R_1, R_2, R_{c1}, R_{c2}, and R_f above 1 kHz is thermal noise. $R_d = R_L \parallel (1/g_d)$ and $R_{HL} = R_H \parallel R_L$ are defined. Rear-OPA equivalent input noise voltage and current are e_a and i_a with harmonic components E_a and I_a. These noise sources are independent. For Inv-Amp consisting of Pre-Amp and Post-Amp, its equivalent input noise voltage and current are e_A and i_A, with harmonic components E_A and I_A.

Figure s4-3 (a) Inv-Amp equivalent differential circuit with input short-circuit containing all noise sources and output noise current e_{oAv}; (b) Noiseless Inv-Amp circuit with Inv-Amp equivalent input noise voltage e_A as input signal and output noise voltage e_{oAve}. Equivalence means $e_{oAv} = e_{oAve}$. The triangle OPA is the Rear-OPA.

The Inv-Amp equivalent differential circuit with all noise sources and input short-circuit is shown in Fig. s4-3(a), with output noise e_{oAv}. The noiseless circuit with Inv-Amp equivalent input noise voltage e_A as input is shown in Fig.

s4-3(b), with output noise e_{oAve}. To calculate Inv-Amp equivalent input noise voltage, equations are established on $e_{oAv} = e_{oAve}$. By nodal analysis:

$$E_A \approx E_H + \frac{E_{RH} Z_{HL}}{A_{vP1} R_H} - \frac{E_{L1}}{g_m R_L} + \frac{Z_{HL}(E_{L2}/R_L + E_f/R_f)}{A_{vP1}} + \frac{E_a}{A_{vPj}} + \frac{E_1 - E_2}{A_{vP1}} - \left(\frac{1}{g_m} + \frac{Z_{HL}}{A_{vPj}}\right) I_a, \tag{s4.6}$$

where $A_{vP1} = g_m/[1/R_L + g_d + j(2\pi f)C_i]$, $A_{vPj} = g_m/[1/R_L + g_d + j(2\pi f)C_{ir}]$, $Z_{HL} = R_{HL}/[1 + j(2\pi f)R_{HL}C_{ir}]$, and $C_{ir} = C_i + C_r$.

The Inv-Amp equivalent differential circuit with all noise sources and input open-circuit is shown in Fig. s4-4(a), with output noise e_{oAi}. The noiseless circuit with Inv-Amp equivalent input noise current i_A as input is shown in Fig. s4-4(b), with output noise e_{oAie}. To calculate Inv-Amp equivalent input noise current, equations are established on $e_{oAi} = e_{oAie}$. By nodal analysis:

$$I_A \approx I_H + j(2\pi f)C_{A1}\frac{E_{RH} Z_{HL}}{A_{vP1} R_H} - j(2\pi f)C_{sd}\frac{E_{L1}}{g_m R_L} + j(2\pi f)C_{A1}\frac{Z_{HL}(E_{L2}/R_L + E_f/R_f)}{A_{vP1}} + j(2\pi f)\frac{E_1 - E_2}{A_{vP1}} - j(2\pi \tag{s4.7}$$

where $C_{sd} = C_{gs} + C_{gd}$, $C_{A1} = C_{gs} + C_{gd}(1 + A_{vP1})$, and $C_{Aj} = C_{gs} + C_{gd}(1 + A_{vPj})$.

For measured frequency $f_m < 300$ kHz, $2\pi f C_i < 0.3$ mS with $C_i = 150$ pF and $2\pi f C_r < 0.1$ mS with $C_r = 50$ pF, but $1/R_d = g_d + 1/R_L = 2$ mS and $1/R_{HL} = 1/R_H + 1/R_L = 6$ mS. Thus $A_{vP1} = g_m/[1/R_L + g_d + j(2\pi f)C_i]$ and $A_{vPj} = g_m/[1/R_L + g_d + j(2\pi f)C_{ir}]$ simplify approximately to $-A_{vP} = g_m/(1/R_L + g_d) = g_m R_d$, Z_{HL} to R_{HL}, and C_{Aj} to C_A.

By the Wiener-Khinchin theorem, ignoring small quantities like thermal noise from R_1, R_2, and R_f:

$$e_A^2 \approx e_H^2 + \left(\frac{R_{HL}}{R_H}\right)^2 e_{RH}^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{s4.9}$$

$$i_A^2 \approx i_H^2 + (2\pi f)^2 C_A^2 \left(\frac{R_{HL}}{R_H}\right)^2 e_{RH}^2 + (2\pi f)^2 \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{s4.10}$$

$$e_A i_A^* = (i_A e_A^*)^* \approx -j2\pi f C_A \left(\frac{R_{HL}}{R_H}\right)^2 e_{RH}^2 - j2\pi f \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}. \tag{s4.11}$$

Here, e_H^2 and i_H^2 are CNRS-HEMT equivalent input noise voltage and current PSDs. e_a^2 and i_a^2 are Rear-OPA equivalent input noise voltage and current PSDs.

e_1^2, e_2^2, and e_{RH}^2 are thermal noise voltage PSDs of R_{c1}, R_{c2}, and R_H. $e_{RH}^2 = 4k_BT_RR_H = 1.66$ (nV)^2/Hz, where $T_R = 300$ K. $e_a^2 = 2.25$ (nV)^2/Hz and $i_a^2 = 4$ (pA)^2/Hz for $f \geq 10$ kHz [S4R4]. In Eqs. (s4.9)-(s4.11), $(1 + R_{HL}/R_d)^2 i_H^2$ is one order of magnitude smaller than $(2\pi f)^2(C_{gs} + C_{gd} + C_AR_{HL}/R_d)^2 e_H^2$. $(C_{gs} + C_{gd} + C_AR_{HL}/R_d)^2 i_H^2$ is two orders smaller than $C_A^2 e_H^2$ in Eq. (s4.9). $(2\pi f)^2 C_A^2 (R_{HL}/R_H)^2 e_{RH}^2$ is one order smaller than $(2\pi f)^2 C_A^2 e_H^2$ in Eq. (s4.10). Further ignoring small quantities in Eqs. (s4.9)-(s4.11) yields:

$$e_A^2 \approx e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{s4.12}$$

$$i_A^2 \approx i_H^2 + (2\pi f)^2 C_A^2 \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{s4.13}$$

$$e_A i_A^* = (i_A e_A^*)^* \approx -j2\pi f C_A \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}. \tag{s4.14}$$

S4.2.2 Equivalent input noise voltage and current of TIA

For TIA, substituting Eq. (s4.12) into Eq. (s4.1), Eqs. (s4.12) and (s4.13) into Eq. (s4.2), and Eqs. (s4.12) and (s4.14) into Eq. (s4.3) yields TIA equivalent input noise voltage PSD e_T^2, current PSD i_T^2, voltage-current PSD $e_T i_T^*$, and current-voltage PSD $i_T e_T^*$:

$$e_T^2 \approx e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}, \tag{s4.15}$$

$$i_T^2 \approx \frac{4k_BT}{R_F} + \frac{e_H^2}{R_F^2} + i_H^2 + (2\pi f)^2 C_A^2 \left[e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}\right], \tag{s4.16}$$

$$e_T i_T^* = (i_T e_T^*)^* \approx \frac{e_H^2}{R_F} - j2\pi f C_A \left[e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}\right]. \tag{s4.17}$$

S4.2.3 Equivalent input noise current of the proposed CryoSTM-TIA

Substituting Eqs. (s4.15), (s4.16), and (s4.17) into Eq. (s4.4) gives the proposed CryoSTM-TIA equivalent input noise current PSD:

$$i_{in}^2 \approx i_H^2 + \frac{4k_BT}{R_F} + \left(\frac{1}{R_J} + \frac{1}{R_F}\right)^2 \left[e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}\right] + (2\pi f)^2 \left[C_A^2 + 2C_AC_{IJ}\right] \left[e_H^2 + \frac{e_a^2 + e_1^2 + e_2^2}{A_{vP}^2}\right], \tag{s4.18}$$

where $C_{IJ} = C_I + C_J$ and $C = C_A + C_I + C_J$. Eq. (s4.18) is Eq. (3.11) in the article. And

$$i_{in}^2 \approx i_A^2 + \frac{4k_BT}{R_F} + \left(\frac{1}{R_J} + \frac{1}{R_F}\right)^2 e_A^2 + (2\pi f)^2 \left[C_A^2 + 2C_A C_{IJ}\right] e_A^2, \quad \text{(s4.19)}$$

where $C = C_A + C_I + C_J$ and $C_{Ae}^2 = C_A^2 e_A^2$. Eq. (s4.19) is Eq. (3.12) in the article.

[S4R1] A. van der Ziel, *Noise in Solid State Devices and Circuits*, Wiley-Interscience, New York, (1986).
[S4R2] Y.X. Liang, Low-noise large-bandwidth transimpedance amplifier for measuring scanning tunneling shot noise spectra in cryogenic STM and its applications, *Ultramicroscopy*, 234 (2022) 113466.
[S4R3] Z.H. Qian, Study on noise models, algorithms, and matrix descriptions for integrated circuits, *J. Northeast Norm. Univ.* 35 (2003) 41.
[S4R4] Datasheet of THS4021 OPA, https://www.ti.com/lit/ds/symlink/ths4021.pdf.

Supplemental file 5: Estimating the DC tunneling current error for CryoSTM-TIA

Figure s5-1 For the proposed CryoSTM-TIA, $E_r = |I_s - I|/|I|$ vs. R ($R \in$ [1 MΩ, 1 GΩ]) simulated by TINA-TI with $V_i = 1$ mV (black curve) and $V_i = 5$ mV (red curve).

For the proposed CryoSTM-TIA, Fig. s5-1 shows TINA-TI simulation results for $E_r = |I_s - I|/|I|$ vs. R ($R \in$ [1 MΩ, 1 GΩ]) with $V_i = 1$ mV (black curve) and $V_i = 5$ mV (red curve). Simulation results show $E_r < 30$ ppm, consistent with calculated results in the article.

Supplemental file 6: Estimation of Inv-Amp input offset voltage

Figure s6-1 DC circuit of the inverting amplifier.

Table s6-1 Parameters of THS4021 [S6R1] as Rear-OPA in Post-Amp

Parameter	Value
Open-loop DC voltage gain a_{a0}	97.5 dB
Input offset voltage drift $\|V_{off}\|/°C$	15 µV/°C
Input bias current I_{bp}, I_{bn}	100 nA/°C (estimated)
CMRR	95 dB
$a_c = a_{a0}$/CMRR	2.5 dB

Pre-Amp in the left dashed box of Fig. s6-1 uses CNRS-HEMT CH. CH source is grounded via variable resistor R_S. CH drain connects to R_1. Variable resistor

R_p is adjustable from 0 to 10 Ω. $R_1 = R_2 = R_L = 1$ kΩ. Its input (CH gate) is grounded. Its two outputs O1 and O2 are disconnected from Post-Amp. PVR LM4050-10 provides constant 10 V ($V_{Pp} = 10$ V) with typical temperature coefficient $±$40 ppm/°C [S6R2]. Keeping $R_p = 0$ and adjusting R_T, R_S, and R_H sets CH at its ideal operating point ($I_{ds} = 1$ mA, $V_{ds} = 100$ mV) with equal DC voltages at O1 and O2. The gate-source voltage is $V_{gs} = -I_{ds}R_S$. For the ideal operating point, assuming $V_{gs} = -50$ mV gives $R_S = 50$ Ω and $R_H = 150$ Ω.

Cascading Pre-Amp and Post-Amp forms Inv-Amp. Adjusting R_T and R_p maintains CH at its ideal operating point with Inv-Amp DC output voltage at O equal to 0.

For Inv-Amp, assuming output voltage drift v_o for 1 °C, the voltage drift at output O1 is v_1 and at O2 is v_2. The power supply output voltage drift is v_t, PVR voltage drift is v_p, and CH source voltage drift is v_s. The Inv-Amp input offset voltage drift can be obtained from:

[Equations s6.1–s6.7]

where V_{OS} is the Inv-Amp input offset voltage drift for 1 °C. Solving these equations gives:

$$V_{OS} \approx I_{bp}(1+g_m R_S)/g_m - I_{bn} g_d R_H/g_m - v_p(1+g_m R_S)/(2g_m R_T) - V_{off}(1+2g_d R_L + g_m R_S)/(2g_m R_L)$$

and

$$v_s \approx v_p R_S/(2R_T) - I_{bp} R_S.$$

Therefore, $|v_p| \leq 0.4$ mV is the power supply current drift for 1 °C. High-precision resistors with low temperature coefficient are used, so resistor-generated drifts can be neglected. Considering only PVR and Rear-OPA drifts, $|v_s| < 10$ µV and $|V_{OS}| < 17$ µV. LM4050-10 power is less than 40 mW in this work, and THS4021 power is less than 200 mW. Temperature fluctuations of LM4050-10 and THS4021 can be controlled within 0.01 °C using the Temperature Control System [S6R3], ensuring total Inv-Amp input voltage drift within 170 nV.

[S6R1] Webpage of THS4021 OPA, https://www.ti.com/product/THS4021.
[S6R2] Datasheet of LM4050-10, https://www.ti.com.cn/document-viewer/cn/lm4050-n/datasheet.
[S6R3] A suitable TEC device, https://datasheets.maximintegrated.com/en/ds/MAX1978-MAX1979.pdf.

Note: Figure translations are in progress. See original paper for figures.