# Atomic Time Scale Analysis Research Postprint

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## Abstract

Timing atomic clocks mainly include hydrogen masers and cesium atomic clocks. To further investigate the performance of time scale computation for different types of timing atomic clocks, this paper conducts research on all-hydrogen and hydrogen-cesium combined time scales. The article first classifies hydrogen masers according to Circular D published by the International Bureau of Weights and Measures (BIPM), then computes all-hydrogen time scales using theoretical methods for atomic time scales for each classification category, and presents analytical results. Subsequently, all-cesium time scales are calculated, and two different hydrogen-cesium combined clock group time scales are analyzed and investigated. The results demonstrate that time scales formed by hydrogen maser groups with smaller frequency drift exhibit smaller fluctuation ranges and superior stability compared to those formed by hydrogen maser groups with larger frequency drift. The stability of time scales formed by hydrogen-cesium combination outperforms that of all-cesium time scales, and time scales calculated from different hydrogen-cesium combined clock groups yield similar results.

## Full Text

## Analysis of Atomic Time Scale

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**Abstract**

Timekeeping atomic clocks primarily consist of hydrogen masers and cesium atomic clocks. To further investigate the time-scale performance characteristics of different types of timekeeping atomic clocks, this study examines all-hydrogen clock ensembles and hydrogen-cesium combined time scales. The paper first classifies hydrogen masers according to the d-bulletin published by the International Bureau of Weights and Measures (BIPM). Based on this classification, atomic time-scale theoretical methods are applied to compute all-hydrogen clock time scales, and the results are analyzed. Subsequently, all-cesium clock time scales are calculated, and two distinct hydrogen-cesium combined clock ensemble time scales are investigated. The results demonstrate that time scales generated from hydrogen maser ensembles with smaller frequency drift exhibit smaller fluctuation ranges and superior stability compared to those derived from ensembles with larger frequency drift. Furthermore, hydrogen-cesium combined time scales achieve better stability than all-cesium clock time scales, with different hydrogen-cesium combined clock ensembles producing similar results.

**Keywords:** hydrogen maser; cesium atomic clock; frequency drift; time scale; hydrogen-cesium combination

## 1. Introduction

Time is intimately connected to human society and exerts profound influence on science, technology, economy, communications, and metrology. In global navigation satellite systems, high-precision time represents a critical element due to the time-ranging principle. In basic research, precision time studies involve natural sciences such as astronomy and physics. In civilian applications, time synchronization and frequency calibration are widely employed in transportation and power systems. Consequently, research on high-precision timekeeping technology holds significant practical importance.

Atomic clocks constitute the essential foundation for timekeeping operations. They are primarily categorized into primary standard atomic clocks and timekeeping atomic clocks. The timekeeping atomic clocks at the National Time Service Center currently include hydrogen masers and cesium atomic clocks, which feature continuous signal output and diverse performance characteristics. Hydrogen masers exhibit excellent short-term stability, while cesium atomic clocks demonstrate superior long-term stability, making both indispensable components of timekeeping systems.

Coordinating atomic clock ensemble resources to generate a continuous, stable, reliable, and uniform time reference—known as an atomic time scale—reduces the uncertainty of individual atomic clocks and achieves higher frequency stability. The atomic time scale serves as a crucial reference for steering and controlling timekeeping systems and is vital for their continuous, reliable, and stable operation. This paper primarily investigates and analyzes the performance characteristics of time scales based on different types of timekeeping atomic clocks.

## 2. Atomic Time Scale Theory

Assuming $N$ atomic clocks participate in the atomic time scale calculation, the time scale $T(A)$ can be expressed as follows:

$$T(A) = \sum_{i=1}^{N} w_i \left[ h_i(t) + h_i'(t) \right]$$

where $h_i(t)$ represents the reading of clock $i$, $h_i'(t)$ denotes the phase correction term, and $w_i$ is the weight of clock $i$.

The phase correction term $h_i'(t)$ can be described using a quadratic model:

$$h_i'(t) = x_i(t_k) + B_i(t - t_k) + \frac{1}{2} C_i (t - t_k)^2$$

At time $t$, clock $i$ is characterized by three elements: phase, frequency offset, and frequency drift. This expression can also be written as:

$$\hat{h}_i'(t) = \hat{x}_i(t_k) + \hat{B}_i(t - t_k) + \frac{1}{2} \hat{C}_i (t - t_k)^2$$

where $\hat{x}_i(t_k)$ represents the phase estimate of clock $i$ relative to the reference frequency source at time $t_k$, $\hat{B}_i$ denotes the frequency offset estimate of clock $i$ relative to the reference frequency source during the interval $[t_k, t]$, and $\hat{C}_i$ represents the frequency drift estimate of clock $i$ relative to the reference frequency source during the same interval. The frequency and frequency drift estimates can be obtained through least-squares fitting methods.

The weights can be determined based on the absolute deviation between predicted and actual clock differences, expressed as:

$$w_i = \frac{1/\varepsilon_i}{\sum_{i=1}^{N} (1/\varepsilon_i)}$$

where $\varepsilon_i = |x_i(t) - \hat{x}_i(t)|$. This formulation, combined with the normalization constraint $\sum_{i=1}^{N} w_i = 1$, implements normalization control.

## 3. Experimental Analysis

### 3.1 Analysis of All-Hydrogen Clock Ensemble Time Scale Based on Different Frequency Drift Conditions

Eight hydrogen masers from the National Time Service Center, Chinese Academy of Sciences, were selected for this study. Table 1 presents the frequency drift values for these eight hydrogen masers during the first six months of the experimental data period, as published by BIPM. According to Table 1, H326, H340, H339, and H341 exhibit relatively large frequency drift values, while H067, H296, H082, and H080 show relatively small frequency drift values.

To investigate time scales under different frequency drift conditions, the ensemble comprising the four hydrogen masers with smaller frequency drift values is designated as HE1, while the ensemble consisting of the four hydrogen masers with larger frequency drift values is designated as HE2. The experimental data sampling interval is 1 hour, with a period length of one month.

Figure 1 [Figure 1: see original paper] compares the time scales generated by hydrogen maser ensembles with different frequency drift characteristics. Figure 2 [Figure 2: see original paper] illustrates the stability comparison of these time scales, and Table 2 provides the Allan deviation values for both cases.

As shown in Figure 1, the time scale computed from the hydrogen maser ensemble HE1 with relatively small frequency drift exhibits fluctuations within the range of -1 ns to +2 ns, demonstrating relatively minor variations without abrupt changes. In contrast, the time scale derived from the hydrogen maser ensemble HE2 with relatively large frequency drift shows fluctuations within the range of -2 ns to +8 ns, displaying larger variations and a significant trend term. This phenomenon occurs because frequency drift constitutes the quadratic term in phase estimation. When frequency drift values are relatively large, inaccurate estimation combined with quadratic accumulation leads to a pronounced trend term in the results.

As demonstrated in Figure 2 and Table 2, the time scale generated from the hydrogen maser ensemble HE1 with relatively small frequency drift achieves superior short-term and long-term stability compared to that from ensemble HE2. The hourly stability exceeds 2\$$\times 10^{\{\text{-14}\}, andthedailystabilityexceeds 4\times 10}$\{-15\}\$.

### 3.2 Hydrogen-Cesium Combined Time Scale Analysis

Four cesium atomic clocks (Cs3436, Cs3089, Cs2976, and Cs2980) from the National Time Service Center were selected for investigation. The data period spans one month with a sampling interval of 1 hour. The all-cesium clock ensemble time scale computed using the atomic time scale method is presented in Figure 3 [Figure 3: see original paper].

As shown in Figure 3, the time scale based on the all-cesium clock ensemble fluctuates within the range of -40 ns to +30 ns. Compared with the all-hydrogen time scale discussed in Section 3.1, this range is substantially larger, with more pronounced short-term fluctuations. This behavior is determined by the inherent performance characteristics of cesium atomic clocks, which exhibit excellent long-term stability but generally inferior short-term stability compared to hydrogen masers.

To further investigate hydrogen-cesium combined time scales, two integration strategies were formulated. The first combines the four cesium atomic clocks with the four hydrogen masers having smaller frequency drift values (HE1 from Section 3.1), designated as HCs1. The second combines the same four cesium clocks with the hydrogen masers having larger frequency drift values (HE2), designated as HCs2. The hydrogen-cesium combined time scale based on the HCs1 ensemble is shown in Figure 4 [Figure 4: see original paper], while that based on HCs2 is presented in Figure 5 [Figure 5: see original paper]. Figure 6 [Figure 6: see original paper] compares the stability of the all-cesium clock time scale with the two hydrogen-cesium combined time scales. Table 3 provides statistical values for these time scales, including maximum, minimum, and peak-to-peak values. Table 4 compares the Allan deviation values of the two hydrogen-cesium combined time scales.

As illustrated in Figure 4, the time scale based on the HCs1 ensemble fluctuates within the range of -6 ns to +4 ns. Figure 5 shows that the time scale based on the HCs2 ensemble fluctuates within the range of -4 ns to +7 ns. Both combined time scales exhibit small fluctuation ranges with similar trends. According to Table 3, the peak-to-peak value is 8.9352 ns for the HCs1-based time scale and 10.4800 ns for the HCs2-based time scale, whereas the all-cesium clock time scale reaches 65.3459 ns. This indicates that hydrogen-cesium combination produces effective time scales with comparable performance.

As demonstrated in Figure 6, hydrogen-cesium combined time scales achieve superior stability compared to the time scale generated solely from the cesium clock ensemble. Table 4 further reveals that the stability performance of the two hydrogen-cesium combined clock ensembles is comparable, confirming that hydrogen-cesium combined time scales exhibit excellent stability.

## 4. Conclusion

This study investigated and analyzed the characteristics of all-hydrogen clock ensemble time scales under different frequency drift conditions and compared the performance of various hydrogen-cesium combined time scales. Experimental results indicate that time scales generated from hydrogen masers with smaller frequency drift exhibit smaller fluctuation ranges and reduced trend terms, whereas time scales computed from ensembles with larger frequency drift display significant trend terms. Furthermore, hydrogen-cesium combined time scales—whether combining cesium clocks with hydrogen masers having large or small frequency drift—yield similar results with small fluctuation ranges and comparable stability values that are superior to those of all-cesium clock time scales.

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