An Enhanced Differential Evolution Based on Center Mutation for Environmental Economic Dispatch Postprint

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Full Text

An Enhanced Differential Evolution Based on Center Mutation for Environmental Economic Dispatch

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Abstract

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Keywords: Differential evolution; Multi-objective optimization; Center mutation; Environmental economic dispatch

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1 Introduction

Many real-world engineering problems can be formulated as multi-objective optimization problems. Since the first attempts to solve such problems using evolutionary algorithms, multi-objective evolutionary algorithms (MOEAs) have received extensive research attention and have been widely applied to numerous practical applications in recent years (Coello, 2006; Nam & Park, 2000). MOEAs benefit from the inherent ability of evolutionary algorithms to generate a set of solutions concurrently in a single run, thereby yielding multiple trade-off solutions for decision-makers.

Differential evolution (DE), proposed by Storn and Price in 1995 (1997), represents a new generation of evolutionary algorithms that has been successfully applied to a wide range of optimization problems. Known for its simplicity and powerful search capability, DE has proven effective for single-objective optimization. In recent years, researchers have extended DE to handle multi-objective optimization problems, leading to algorithms such as the Pareto differential evolution (PDE) algorithm (Abbass, 2002; Xue, Sanderson & Graves, 2003), Pareto-based multi-objective differential evolution (PMODE) (Madavan, 2002), differential evolution for multi-objective optimization (DEMO), and the adaptive differential evolution algorithm (ADEA) (Robic & Filipic, 2005; Qian & Li, 2008).

However, most DE-based multi-objective optimization algorithms suffer from premature convergence to varying degrees (Madavan, 2002; Robic & Filipic, 2005; Qian & Li, 2008). This paper presents a novel center mutation-based DEMO (CM-DEMO) for multi-objective optimization. The proposed algorithm features two key improvements: (1) a modified mutation operator that uses the center of all target vectors as the base vector and determines the difference vector direction based on the fitness values of three randomly selected vectors; and (2) an adaptive crossover probability mechanism that adjusts according to the distribution of fitness values in the population. To demonstrate its effectiveness, CM-DEMO is applied to an environmental economic dispatch problem involving four interconnected cascade hydroelectric plants and one thermal plant. Simulation results confirm the feasibility and effectiveness of the proposed method.

The remainder of this paper is organized as follows: Section 2 briefly reviews the principles of differential evolution that form the foundation of CM-DEMO. Section 3 details the proposed CM-DEMO algorithm. Section 4 presents a practical application study of CM-DEMO to an environmental economic dispatch problem. Finally, Section 5 concludes the paper.

2 Overview of Differential Evolution

The differential evolution algorithm is a population-based optimization method that employs three primary operators: crossover, mutation, and selection (Storn & Price, 1997). Several parameters must be appropriately tuned for effective performance. The algorithm maintains a population of NP n-dimensional real-valued parameter vectors in each generation g. According to Storn and Price, the standard DE strategy can be described as follows.

2.1 Mutation

For each target vector $\mathbf{x}_{i,g}$ in generation g, a mutant vector $\mathbf{v}_{i,g}$ is generated according to:

$$\mathbf{v}_{i,g} = \mathbf{x}_{r_1,g} + F \cdot (\mathbf{x}_{r_2,g} - \mathbf{x}_{r_3,g})$$

where integers r_1, r_2, and r_3 are randomly selected from the range [1, NP] and are mutually different from each other and from the target index i. The mutation parameter $F \in [0, 2]$ is a user-supplied real constant that controls the amplification of the differential vector.

2.2 Crossover

To increase the diversity of perturbed parameter vectors, crossover is introduced. The target vector $\mathbf{x}_{i,g}$ is combined with the mutant vector $\mathbf{v}_{i,g}$ to produce the trial vector $\mathbf{u}_{i,g}$ as follows:

$$u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if rand()} \leq CR \text{ or } j = j_{\text{rand}} \\ x_{j,i,g} & \text{otherwise} \end{cases}$$

where rand() generates a uniform random number in [0, 1], $CR \in [0, 1]$ is the crossover parameter, and j_{rand} is a randomly chosen index from $\{1, 2, \dots, n\}$ that ensures the trial vector receives at least one parameter from the mutant vector.

2.3 Selection

During the selection operation, a competition is held between each target vector $\mathbf{x}_{i,g}$ and its corresponding trial vector $\mathbf{u}_{i,g}$. Based on their fitness values, the better individual is selected for the next generation. Assuming minimization of the fitness function, the selection operation can be expressed as:

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{u}_{i,g} & \text{if } f(\mathbf{u}_{i,g}) \leq f(\mathbf{x}_{i,g}) \\ \mathbf{x}_{i,g} & \text{otherwise} \end{cases}$$

3 An Enhanced Multi-objective Differential Evolution: CM-DEMO

This section analyzes the mutation and crossover operations in the standard algorithm and proposes the center mutation-based DEMO (CM-DEMO) algorithm. First, the CM-DEMO algorithm is described in detail, and then several widely used benchmark test problems are selected to evaluate its performance.

3.1 Center Mutation Operator

Equation (1) shows that the standard DE algorithm randomly selects two individuals to compute the difference vector, ignoring its directional information. While this provides some exploration capability, it slows down the convergence rate. To address this limitation, we propose a center mutation operator where each mutated individual is oriented around the center of the current population. The specific form of the mutation operator is described as follows:

$$\mathbf{v}_{i,g} = \mathbf{C}_g + F \cdot (\mathbf{x}_{o,g} - \mathbf{x}_{1,g}) + F \cdot (\mathbf{x}_{o,g} - \mathbf{x}_{2,g})$$

where $\mathbf{x}_{o,g}$ is the best individual among three randomly selected individuals, $\mathbf{x}_{1,g}$ and $\mathbf{x}_{2,g}$ are the other two random individuals, and $\mathbf{C}_g$ is the center of the population.

Equation (4) demonstrates that each mutant individual is calculated based on the fitness values of three randomly selected individuals and starts from the population center toward the best individual $\mathbf{x}_{o,g}$. The direction from $\mathbf{x}_{o,g}$ determines the direction of the difference vector, making the improved mutation operator retain a certain degree of randomness while incorporating guided directionality. Consequently, under the action of the center mutation operator, the new algorithm achieves faster convergence speed.

3.2 Adaptive Crossover Probability

In the standard DE algorithm, a fixed crossover probability CR is commonly used throughout the optimization process. However, for practical problems, this value should be adjusted appropriately based on the iteration progress and the fitness values of individuals to align the algorithm's behavior with the problem characteristics. Here, we present an adaptive crossover probability method (for minimization problems) as shown in Equation (5):

$$CR_i = \begin{cases} \max\left(\frac{F_i - F_{\min}}{F_{\max} - F_{\min}}, \frac{F_i - F_o}{F_i}\right) & \text{if } F_i > F_o \\ \min\left(\frac{F_i - F_{\min}}{F_{\max} - F_{\min}}, \frac{F_i - F_o}{F_i}\right) & \text{if } F_i \le F_o \end{cases}$$

where CR_i is the crossover probability for individual i, F_i and F_o are the fitness values of individuals $\mathbf{x}_i$ and $\mathbf{x}_o$, and $F_{\min}$ and $F_{\max}$ are the best and worst fitness values in the current generation.

Equation (5) shows that when $F_i > F_o$, the crossover probability should be increased to incorporate more mutant vector components in the new individual. The adjustment strategy compares the ratio of $(F_i - F_{\min})$ to $(F_{\max} - F_{\min})$ with the ratio of $(F_i - F_o)$ to F_i, selecting the larger value as the crossover probability. When $F_i \le F_o$, the crossover probability should be reduced to preserve more components of the original individual $\mathbf{x}_i$. In this case, the smaller of the two ratios is selected. This method effectively adjusts the composition of generated individuals based on fitness values during the iterative process, thereby improving the algorithm's search performance.

3.3 Outline of CM-DEMO

Based on the above description of the improved algorithm, the outline of CM-DEMO is summarized as follows:

3.4 Performance Test

The benchmark test problems for evaluating our proposed method are selected based on significant past studies in multi-objective evolutionary algorithms. We chose four problems from the ZDT benchmark suite to test CM-DEMO (Zitzler & Thiele, 2000; Deb, Thiele, Laumanns & Zitzler, 2005).

For each test problem, the crossover probability CR was set to 0.5 and the scaling factor F was set to 0.5. To ensure fair comparisons, the population size NP was set to 100 and the algorithm was run for 250 generations.

Figures 1-4 show the Pareto fronts obtained by CM-DEMO compared with the true Pareto fronts for the four ZDT test problems. The results demonstrate that solutions obtained by CM-DEMO exhibit excellent convergence and wide distribution.

Table 2 presents the mean (in bold) and variance of the convergence and diversity metrics averaged over 10 runs (Deb & Jain, 2002; Van Veldhuizen, 1999). Results for other algorithms are taken from the literature (NSGA-II, SPEA2, PDEA, DEMO, ADEA, and DEMO/parent).

The convergence metric results (see Table 2) show that CM-DEMO achieves good convergence performance, performing better than PDEA, DEMO, and DEMO/parent, and significantly outperforming NSGA-II and SPEA2. On ZDT3, DEMO achieves comparable results to CM-DEMO, but on ZDT1 and ZDT6, CM-DEMO demonstrates superior performance. For the diversity metric Δ, Table 2 shows that CM-DEMO achieves much better results on all four test problems compared to the other referenced algorithms.

ZDT4 is a challenging optimization problem with numerous local Pareto fronts that can mislead the optimization algorithm. Table 2 reveals that NSGA-II, SPEA2, PDEA, and MODE all encounter difficulties in converging to the true Pareto front, while DEMO and CM-DEMO perform better than the other algorithms.

The Tamaki problem is a constrained test problem, and DTLZ1 is a high-dimensional problem ($M = 3$). Figures 5 and 6 show the Pareto fronts obtained by CM-DEMO for these problems.

[Figure 5: see original paper] [Figure 6: see original paper]

The results demonstrate that CM-DEMO handles constraints and high-dimensional problems effectively, accurately converging to the true Pareto fronts. After achieving satisfactory performance on benchmark problems, we now apply CM-DEMO to a practical environmental economic dispatch problem in power systems.

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4 Case Study

To validate the proposed methodology, a hydrothermal test system is used for case study (Naresh & Sharma, 1999). The system consists of a multi-chain cascade of four hydroelectric plants and three thermal units. The detailed system data are the same as those in the reference.

4.1 Hydrothermal Scheduling Problem

The environmental economic dispatch problem aims to simultaneously minimize thermal power plant operating costs and pollutant gas emissions while satisfying a series of equality and inequality constraints (Talaq, EI-Hawary & EI-Hawary, 1994; Yalcinoz & Köksoy, 2007; Abido, 2003).

4.1.1 Problem Objectives (1) Economic Objective

The generator cost curves are represented by quadratic functions of real power generation. The total fuel cost can be formulated as:

$$F_1 = \sum_{t=1}^{T} \sum_{i=1}^{N_s} \left[a_i + b_i P_{i,t}^s + c_i (P_{i,t}^s)^2 + |e_i \sin(f_i (P_{i,\min}^s - P_{i,t}^s))|\right]$$

where a_i, b_i, and c_i are the cost curve coefficients of the ith thermal unit, $P_{i,t}^s$ is the output power of the ith thermal unit at period t, $P_{i,\min}^s$ is the minimum output limit of the ith thermal plant, and e_i and f_i are the valve-point effect coefficients of the ith thermal plant.

(2) Emission Objective

Since SO_2 and NO emissions are generally proportional to generator fuel consumption, the total emission function used in this paper to represent SO_2 and NO emissions takes the same form as the fuel cost function. The total emission of the hydrothermal system can be expressed as:

$$F_2 = \sum_{t=1}^{T} \sum_{i=1}^{N_s} \left[\alpha_i + \beta_i P_{i,t}^s + \gamma_i (P_{i,t}^s)^2\right]$$

where α_i, β_i, and γ_i are the emission curve coefficients of the ith thermal unit.

4.1.2 System Constraints The problem is subject to the following equality and inequality constraints:

(3) Power Balance Constraints

$$\sum_{i=1}^{N_s} P_{i,t}^s + \sum_{j=1}^{N_h} P_{j,t}^h = P_{D,t} + P_{L,t}$$

where $P_{j,t}^h$ is the power output of the jth hydro plant at period t, N_h is the number of hydroelectric plants, $P_{D,t}$ is the load demand at period t, and $P_{L,t}$ is the total transmission line loss at period t.

(4) Real Power Output Limits

$$P_{j,\min}^h \le P_{j,t}^h \le P_{j,\max}^h$$

where $P_{j,\min}^h$ and $P_{j,\max}^h$ are the lower and upper generation limits of the jth hydroelectric plant, respectively.

(5) Reservoir Storage Volume Limits

$$V_{j,\min} \le V_{j,t} \le V_{j,\max}$$

where $V_{j,\min}$ and $V_{j,\max}$ are the minimum and maximum storage volumes of the jth reservoir, respectively.

(6) Hydro Plant Power Limits

$$Q_{j,\min} \leq Q_{j,t} \leq Q_{j,\max}$$

where $Q_{j,\min}$ and $Q_{j,\max}$ are the minimum and maximum water discharge rates of the jth hydroelectric plant, respectively.

(7) Initial and Terminal Reservoir Storage Volumes

$$V_{j,0} = V_{j,\text{initial}}, \quad V_{j,T} = V_{j,\text{final}}$$

where $V_{j,\text{initial}}$ and $V_{j,\text{final}}$ are the initial and final storage volumes of reservoir j.

(8) Water Dynamic Balance Equation with Travel Time

$$V_{j,t+1} = V_{j,t} + I_{j,t} - Q_{j,t} - S_{j,t} + \sum_{k=1}^{R_j} (Q_{k,t-\tau_{kj}} + S_{k,t-\tau_{kj}})$$

where $I_{j,t}$ is the natural inflow rate of the jth reservoir at period t, R_j is the number of upstream units directly above the jth hydroelectric plant, $S_{j,t}$ is the spillage of the jth reservoir at time t, and τ_{kj} is the water transport delay from reservoir k to j.

(9) Hydroelectric Generation Relation

$$P_{j,t}^{h} = C_{1j}V_{j,t}^2 + C_{2j}Q_{j,t}^2 + C_{3j}V_{j,t}Q_{j,t} + C_{4j}V_{j,t} + C_{5j}Q_{j,t} + C_{6j}$$

where C_{1j} through C_{6j} are the power generation coefficients of the jth hydroelectric plant, $Q_{j,t}$ is the water discharge rate of the jth reservoir at period t, and $V_{j,t}$ is the storage volume of the jth reservoir at period t.

4.2 Solution Methodology Based on CM-DEMO

This section describes the application of the proposed CM-DEMO method to solve the generation scheduling problem for this hydrothermal system, with particular attention to constraint handling.

4.2.1 Initialization In the initialization procedure, the population is created by generating NP solutions randomly. For all solutions in the initial population, each decision variable is randomly generated within the feasible range according to the constraints. For this problem, the variables are the discharge rate of each hydro plant and the power generation of each thermal unit. The population is initialized as:

$$x_{i,j} = x_{j,\min} + \mathrm{rand}() \cdot (x_{j,\max} - x_{j,\min})$$

where each dimension j is randomly generated between $x_{j,\min}$ and $x_{j,\max}$. Generally, newly generated individuals may not satisfy all constraints and must be modified using the constraint handling method described next.

4.2.2 Mutation, Crossover and Selection New values for water discharge rates and power generation are generated through mutation and crossover operations according to equations (8) and (2), respectively. Selection is then performed by calculating the fitness values of the different individuals. Individuals in the current population are evaluated in the objective space and assigned a scalar fitness value. Based on these fitness values, individuals are selected to form the new population, with individuals having lower fitness values having higher selection probability. It is worth noting that the constraint-handling approach implemented in this study penalizes infeasible solutions by assigning them a very high fitness value.

4.2.3 Constraint Handling First, after population initialization and crossover implementation, newly generated solutions may violate equality constraints (1) and (5). The penalty method is currently the most popular constraint handling strategy for such equality constraints, using penalty functions to punish infeasible solutions during selection and ensure priority for feasible solutions. However, this strategy may significantly degrade algorithm efficiency as it requires multiple runs to tune the penalty factors.

Second, we focus on handling the output capacity limits (2) and water release limits (4) when applying the proposed CM-DEMO method to solve the environmental economic dispatch problem.

Despite their popularity, penalty functions have several drawbacks, the main one being the need for careful fine-tuning of penalty factors to accurately estimate the degree of penalization required for efficient convergence to the feasible region. To retain the advantages of the penalty function approach while overcoming the difficulty of selecting penalty factors, this paper applies an effective constraint handling method for DE that does not require setting any additional parameters beyond those in the original DE. To balance computational efficiency and constraint handling, the following selection strategy is adopted by the proposed CM-DEMO method to choose better solutions while considering constraint violations during the selection operation:

1. If solution P_1 is feasible and solution P_2 is infeasible, P_1 is favored.
2. If both P_1 and P_2 are feasible, Pareto-dominance based selection is implemented. The selection operation is modified as follows:
   - If the candidate dominates the parent, replace the parent with the candidate.
   - If the parent dominates the candidate, discard the candidate.
   - If the candidate and parent are non-dominated, create a new population of size between NP and $2NP$, then add the non-dominated candidates and parents to it.
3. If both P_1 and P_2 are infeasible, the solution with smaller constraint violation is favored.

4.3 Simulation Results

Using the parameter settings listed in Table 3, the proposed method was implemented in Microsoft Visual C++ 6.0 and executed on a Pentium-4 2.0GHz computer to solve the optimal generation scheduling problem for this hydrothermal system. The hourly power generation for each hydro plant is shown in Table 4.

The hourly reservoir release and storage trajectories are shown in Figures 7 and 8, respectively.

[Figure 7: see original paper] [Figure 8: see original paper]

To validate the results obtained with the proposed CM-DEMO method, the same problem was solved using DEMO, NSGA-II, and SPEA-II methods. The results are summarized in Table 5 for convenient comparison.

The results clearly show that the total fuel cost and total emission obtained by CM-DEMO are significantly lower than the corresponding values from other methods. Additionally, during 20 independent simulations, CM-DEMO demonstrated that total fuel cost and total emission vary within a small range across trial runs. The simulation results confirm that the solutions obtained by CM-DEMO are optimal and satisfy all constraints completely for the environmental economic dispatch problem.

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5 Conclusions

This paper has proposed a modified multi-objective differential evolution optimization algorithm based on center mutation mechanisms. The algorithm replaces the original mutation and crossover operations with improved forms: center mutation and adaptive crossover. Simulation results and comparative analysis confirm the effectiveness and superiority of the proposed approach over other techniques in terms of solution quality and precision, providing an effective method for solving environmental economic dispatch problems.

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Note: Figure translations are in progress. See original paper for figures.