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Abstract

To investigate the characteristics of bottomhole pressure variation in heavy oil thermal recovery from carbonate reservoirs, considering the effect of temperature on oil viscosity and formation stress sensitivity, a triple-continuum triple-permeability mathematical model was established. The mathematical model constitutes a system of nonlinear variable-coefficient partial differential equations, which is discretized using a fully implicit finite difference scheme, and the nonlinear difference equations are solved using the Newton-Raphson iterative method. The Jacobian matrix of the difference equation system is tridiagonal. Leveraging the properties of tridiagonal matrices for matrix inversion enhances the iteration speed. Based on the computational results, well test type curves were plotted and parameter sensitivity analysis was conducted. The research results indicate that: the pressure response is divided into 6 stages; the sensitivity of heavy oil viscosity to temperature variation differs across different reservoirs. Specifically, higher sensitivity of viscosity to temperature variation leads to greater pressure values in the reservoir, earlier appearance of each crossflow stage, and it shortens the duration of crossflow from vugs to fractures while diminishing its effect, resulting in premature decline of the pressure derivative.

Full Text

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Triple-Continuum Triple-Permeability Model for Heavy Oil Thermal Recovery and Pressure Transient Analysis

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Abstract: To investigate the characteristics of bottom-hole pressure variation during heavy oil thermal recovery in carbonate reservoirs, considering the influence of temperature on oil viscosity and the stress sensitivity of formations, a triple-continuum triple-permeability mathematical model was established. The mathematical model is a system of nonlinear variable-coefficient partial differential equations. The fully implicit finite-difference scheme was used for discretization, and the Newton–Raphson iterative method was used to solve the nonlinear difference equations. The Jacobi matrix of the difference-equation system is a tridiagonal matrix. By using the properties of the tridiagonal matrix to obtain the inverse matrix, the iteration speed was improved. Based on the calculation results, test-well type curves were plotted and parameter sensitivity analysis was carried out. The results show that the pressure response consists of six stages; in different reservoirs, the sensitivity of heavy-oil viscosity dependence on temperature variation is different. Specifically, the higher the sensitivity of viscosity to temperature variation, the larger the numerical value of pressure in the reservoir, the earlier the appearance of each interporosity flow stage, and the shorter the time during which the cavity conducts interporosity flow toward the fractures, thereby weakening its effect and causing the pressure derivative to decline earlier.

Keywords: triple continuum; carbonate rock; heavy oil; variable-coefficient equation system; test well

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Triple permeability model in triple media and pressure dynamic analysis of thermal recovery of heavy oil

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Abstract: To study the characteristics of the wellbore pressure changes of thermal recovery of heavy oil in carbonate reservoirs, considering the effect of temperature on viscosity of oil and stress sensitivity of reservoirs, the triple permeability mathematical model in triple media has been constructed. The mathematical model is of nonlinear partial differential equations with variable coefficients. The difference equations have been discretized by the fully implicit finite difference scheme. Newton-Raphson method has been used for solving nonlinear difference equations. The Jacobi matrices of the difference equations are tridiagonal matrix. Using the properties of tridiagonal matrix, the iteration speed is improved. According to the calculation results, the typical well test curves have been plotted. Sensitivity of parameters has been analyzed. There are eight stages of pressure response. The sensitivities of viscosity depend on temperature are different in different reservoirs. The higher the sensitivities of viscosity depend on temperature, the greater the values of pressure in reservoirs. When the cross flow stages are earlier, the time of the cross flow between the frac-

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tures and vugs is shortened and the strength is weakened, causing the earlier decrease in the pressure derivative.

Key words: triple media; carbonate; heavy oil; equation with variable coefficient; well testing

In general, carbonate reservoirs have well-developed fractures and vugs, and their seepage laws differ from those of oil and gas reservoirs containing only one type of porosity. Seepage flow in carbonate rocks is more complex than that in single-porosity reservoirs, because fluid exchange exists among the matrix system (hereinafter referred to as matrix), the fracture system (hereinafter referred to as fractures), and the vug system (hereinafter referred to as vugs). The fluid-flow behavior is therefore more complicated than that in single-porosity reservoirs. To realistically reflect the reservoir characteristics of carbonate oil and gas reservoirs, carbonate rocks must be simplified as multiple media for study. References [1–2] proposed dual-continuum models. Reference [3] laid the groundwork for the establishment of a triple-continuum model. Reference [4] proposed the concept of triple-porosity media. Reference [5] pointed out that the influence of vugs must be considered in triple-continuum curve fitting. References [6–9] studied triple-permeability models and considered the effects of formation stress sensitivity and quadratic pressure-gradient terms. References [10–11] established and solved mathematical models for multilayer triple-continuum reservoirs considering the starting pressure gradient. Reference [12] investigated a horizontal-well model for oil–water two-phase unsteady interporosity flow in triple-continuum formations. Reference [13] considered pressure sensitivity and formation-permeability anisotropy, and established and solved an inclined-well mathematical model. Reference [14] established and solved a model in which the wellbore is connected with large vugs, while considering coupling between flow and wave motion.

With the development of the petroleum industry, many carbonate heavy-oil reservoirs have been discovered, and thermal recovery of heavy oil has been widely applied. Reference [15] first studied a multiple-continuum model for thermal recovery of carbonate heavy oil, divided the reservoir into a steam swept zone and a cold-oil zone, and established and solved a composite reservoir horizontal-well well-test interpretation model. However, after steam injection stimulation, the influence of temperature on fluid viscosity varies continuously from the wellbore to the reservoir boundary, and the permeabilities of the matrix, fractures, and vugs must be considered.

Fig. 1 Triple permeability flow scheme in triple-porosity media

Figure 1: Fig. 1 Triple permeability flow scheme in triple-porosity media

At present, a multiple-continuum well-test model that considers the above factors has not yet been proposed. This study considers the influence of temperature on fluid viscosity, as well as the effects of formation stress sensitivity and the quadratic gradient term. A triple-porosity triple-permeability model is established, solved numerically, and used to analyze the factors influencing bottomhole pressure.

1 Mathematical Model

Fractures and vugs are developed in carbonate formations, and fluids flow into the wellbore through the matrix, fractures, and vugs. Fluid exchange exists among the three fluid systems. During heavy-oil thermal recovery, steam injection creates a temperature difference between the near-wellbore zone and the far-field zone. Changes in temperature alter fluid flowability.

Assume that temperature variation affects only the viscosity of the heavy oil. Consider a single-layer circular reservoir producing at a constant rate from a well at the center. Heat conduction occurs in the reservoir due to the temperature difference between the inner and outer boundaries. Thermal convection caused by fluid flow is neglected. The porosity of each medium varies independently with pressure. The matrix, fractures, and vugs are saturated with fluid. The flow of the fluid in the three media all satisfies Darcy' s law. Before well opening, the pressure everywhere in the formation equals the original formation pressure p_0. Fractures and vugs are well developed, and the permeability of fractures is greater than that of vugs. Fluid in the matrix crossflows into fractures and vugs, and fluid in vugs crossflows into fractures; the crossflow is pseudo-steady. Based on these assumptions, this study considers pressure-sensitive effects and the quadratic gradient term. In the mathematical derivation and in the physical symbols appearing in the formulas, subscript 1 denotes fractures, subscript 2 denotes vugs, and subscript 3 denotes matrix. A schematic of the triple-continuum triple-permeability model is shown in Fig. 1.

图 1 三重介质三渗模型简图

Fig. 1 Triple permeability flow scheme in triple-porosity media

Assume that the temperature remains unchanged when steam is injected, and that the outer boundary considered is the reservoir temperature, which is treated as constant; the inner boundary is assumed to be the temperature of the injection well and is also constant. Let the wellbore radius be r_w, the reservoir radius be R, the temperature at the wellbore be T_w, and the temperature at the outer reservoir boundary be T_R. According to the steady-state heat-conduction equation, the temperature distribution in the reservoir satisfies the following boundary-value problem:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}T}{\mathrm{d}r}\right)=0, \\ r=r_w,\ T=T_w, \\ r=R,\ T=T_R. \end{cases} \qquad (1)$$

where r is radius, in m; T is temperature, in °C.

Solving Eq. (1) gives

$$T=T_w+\frac{T_R-T_w}{\ln(R/r_w)}\ln(r/r_w) \qquad (2)$$

Define the viscosity variation factor as

$$\beta=\frac{1}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}T} \qquad (3)$$

where μ is the fluid viscosity, in Pa • s; the unit of β is $^{\circ}\mathrm{C}^{-1}$. The viscosity-variation factor represents the sensitivity of fluid viscosity to changes in temperature.

From Eq. (3), one obtains

$$\mu=\mu_0\exp\left[\beta\left(T-T_R\right)\right] \qquad (4)$$

where μ_0 is the viscosity at the initial temperature, in Pa • s. The value of β is taken as $-0.031\,584\,5$ $^{\circ}\mathrm{C}^{-1}$.

Substituting Eq. (2) into Eq. (4) gives

$$\mu=\mu_0\exp\left[\beta\left(T_w-T_R+\frac{T_R-T_w}{\ln(R/r_w)}\ln(r/r_w)\right)\right] \qquad (5)$$

Considering the pressure-sensitivity effect, the permeability modulus is defined as [16]

$$\gamma_i=\frac{1}{K_i}\frac{\mathrm{d}K_i}{\mathrm{d}p_i},\quad i=1,2,3 \qquad (6)$$

where K_i $(i=1,2,3)$ is the permeability, in m^2. Assuming γ_i to be a constant, in Pa^{-1}, then

$$K_i = K_{i0} \exp\left[\gamma_i (p_i - p_0)\right], \quad i = 1, 2, 3 \tag{7}$$

where K_{i0} $(i = 1, 2, 3)$ is the initial permeability. p_i $(i = 1, 2, 3)$ denotes the fluid pressures in the three media.

The crossflow equations are

$$\begin{cases} q_{12} = \dfrac{\alpha_{12}\rho_0}{\mu}(p_2 - p_1) \\ q_{13} = \dfrac{\alpha_{13}\rho_0}{\mu}(p_3 - p_1) \\ q_{23} = \dfrac{\alpha_{23}\rho_0}{\mu}(p_3 - p_2) \end{cases} \tag{8}$$

where α_{12} is the crossflow coefficient between fractures and vugs with the dimension of unity; α_{13} is the crossflow coefficient between the matrix and fractures with the dimension of unity; α_{23} is the crossflow coefficient between vugs and the matrix with the dimension of unity; and q_{12}, q_{13}, and q_{23} are the corresponding crossflow rates, in $\mathrm{kg/(m^3 \cdot s)}$.

Substituting the flow equations, crossflow equations, and state equations into the continuity equations (the detailed derivation is omitted), the partial differential equation system describing triple-continuum, triple-permeability flow with temperature effects is obtained as

$$\begin{cases} K_{10}\left[\dfrac{\partial^2 p_1}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial p_1}{\partial r} + \gamma_1\left(\dfrac{\partial p_1}{\partial r}\right)^2\right] + \alpha_{12}e^{\gamma_1(p_0-p_1)}(p_2 - p_1) + \alpha_{13}e^{\gamma_1(p_0-p_1)}(p_3 - p_1) \\ \quad = \mu_0\varphi_1 c_{t1} e^{\beta\left[T_w - T_R + \frac{T_R - T_w}{\ln(R/r_w)}\ln(r/r_w)\right] + \gamma_1(p_0-p_1)}\dfrac{\partial p_1}{\partial t} \\ K_{20}\left[\dfrac{\partial^2 p_2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial p_2}{\partial r} + \gamma_2\left(\dfrac{\partial p_2}{\partial r}\right)^2\right] + \alpha_{12}e^{\gamma_2(p_0-p_2)}(p_1 - p_2) + \alpha_{23}e^{\gamma_2(p_0-p_2)}(p_3 - p_2) \\ \quad = \mu_0\varphi_2 c_{t2} e^{\beta\left[T_w - T_R + \frac{T_R - T_w}{\ln(R/r_w)}\ln(r/r_w)\right] + \gamma_2(p_0-p_2)}\dfrac{\partial p_2}{\partial t} \\ K_{30}\left[\dfrac{\partial^2 p_3}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial p_3}{\partial r} + \gamma_3\left(\dfrac{\partial p_3}{\partial r}\right)^2\right] + \alpha_{13}e^{\gamma_3(p_0-p_3)}(p_1 - p_3) + \alpha_{23}e^{\gamma_3(p_0-p_3)}(p_2 - p_3) \\ \quad = \mu_0\varphi_3 c_{t3} e^{\beta\left[T_w - T_R + \frac{T_R - T_w}{\ln(R/r_w)}\ln(r/r_w)\right] + \gamma_3(p_0-p_3)}\dfrac{\partial p_3}{\partial t} \end{cases} \tag{9}$$

where φ_i $(i = 1, 2, 3)$ is the porosity; c_{ti} $(i = 1, 2, 3)$ is the total compressibility, in Pa^{-1}; and t is time, in s. This model comprehensively accounts for the permeability of the matrix, fractures, and vugs, and also considers the effects of formation stress sensitivity and the second-order gradient term. The influence of temperature on fluid viscosity is considered; this influence varies continuously from the wellbore to the outer boundary, and no reservoir partitioning is used.

The initial condition is

$$p_i(r,t)\big|_{t=0} = p_0 \, (r_w \le r \le R;\ i = 1,2,3) \qquad (10)$$

The outer boundary condition is a constant-pressure boundary condition, namely

$$p_i(r,t)\big|_{r=R} = p_0 \, (t > 0;\ i = 1,2,3) \qquad (11)$$

Considering wellbore storage and the skin factor, the inner boundary condition is

$$\frac{2\pi r_w h}{\mu}\left(K_1 e^{\gamma_1(p_1-p_0)}\frac{\partial p_1}{\partial r} + K_2 e^{\gamma_2(p_2-p_0)}\frac{\partial p_2}{\partial r} + K_3 e^{\gamma_3(p_3-p_0)}\frac{\partial p_3}{\partial r}\right)\bigg|_{r=r_w} - C\frac{\mathrm{d}p_w}{\mathrm{d}t} = q, \quad (t>0) \qquad (12)$$

$$p_w = p_1 - S\frac{q\mu}{2\pi K_1 h} \qquad (13)$$

$$p_w = p_2 - S\frac{q\mu}{2\pi K_2 h} \qquad (14)$$

$$p_w = p_3 - S\frac{q\mu}{2\pi K_3 h} \qquad (15)$$

where p_w is the bottomhole pressure, in Pa; h is the reservoir thickness, in m; q is the flow rate, in m^3/s; C is the wellbore storage coefficient, in $\mathrm{m}^4 \cdot \mathrm{s}^2/\mathrm{kg}$; and S is the skin factor.

Using the dimensionless variables defined in Table 1, Eq. (9) becomes

$$D_1\left[\frac{\partial^2 p_{\mathrm{D}1}}{\partial r_{\mathrm{D}}^2} + \frac{1}{r_{\mathrm{D}}}\frac{\partial p_{\mathrm{D}1}}{\partial r_{\mathrm{D}}} - \gamma_{\mathrm{D}1}\left(\frac{\partial p_{\mathrm{D}1}}{\partial r_{\mathrm{D}}}\right)^2\right] + \lambda_{12} e^{\gamma_{\mathrm{D}1} p_{\mathrm{D}1}}(p_{\mathrm{D}2} - p_{\mathrm{D}1}) + \lambda_{13} e^{\gamma_{\mathrm{D}1} p_{\mathrm{D}1}}(p_{\mathrm{D}3} - p_{\mathrm{D}1})$$

$$= \omega_1 e^{\beta_{\mathrm{D}}\left[\theta - \frac{\theta}{\ln(R/r_w)}\ln r_{\mathrm{D}}\right] + \gamma_{\mathrm{D}1} p_{\mathrm{D}1}}\frac{\partial p_{\mathrm{D}1}}{\partial t_{\mathrm{D}}},$$

$$
\begin{aligned}
&D_2\left[\frac{\partial^2 p_{\mathrm{D}2}}{\partial r_{\mathrm{D}}^2}+\frac{1}{r_{\mathrm{D}}}\frac{\partial p_{\mathrm{D}2}}{\partial r_{\mathrm{D}}}-\gamma_{\mathrm{D}2}\left(\frac{\partial p_{\mathrm{D}2}}{\partial r_{\mathrm{D}}}\right)^2\right]+\lambda_{12}e^{\gamma_{\mathrm{D}2}p_{\mathrm{D}2}}\left(p_{\mathrm{D}1}-p_{\mathrm{D}2}\right)+\lambda_{23}e^{\gamma_{\mathrm{D}2}p_{\mathrm{D}2}}\left(p_{\mathrm{D}3}-p_{\mathrm{D}2}\right)\\
&\quad=\omega_2 e^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}\ln r_{\mathrm{D}}\right]+\gamma_{\mathrm{D}2}p_{\mathrm{D}2}}\frac{\partial p_{\mathrm{D}2}}{\partial t_{\mathrm{D}}},\\
&D_3\left[\frac{\partial^2 p_{\mathrm{D}3}}{\partial r_{\mathrm{D}}^2}+\frac{1}{r_{\mathrm{D}}}\frac{\partial p_{\mathrm{D}3}}{\partial r_{\mathrm{D}}}-\gamma_{\mathrm{D}3}\left(\frac{\partial p_{\mathrm{D}3}}{\partial r_{\mathrm{D}}}\right)^2\right]+\lambda_{13}e^{\gamma_{\mathrm{D}3}p_{\mathrm{D}3}}\left(p_{\mathrm{D}1}-p_{\mathrm{D}3}\right)+\lambda_{23}e^{\gamma_{\mathrm{D}3}p_{\mathrm{D}3}}\left(p_{\mathrm{D}2}-p_{\mathrm{D}3}\right)\\
&\quad=\omega_3 e^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}\ln r_{\mathrm{D}}\right]+\gamma_{\mathrm{D}3}p_{\mathrm{D}3}}\frac{\partial p_{\mathrm{D}3}}{\partial t_{\mathrm{D}}}.
\end{aligned}
\tag{16}
$$

Equation (16) is a system of nonlinear variable-coefficient partial differential equations, which must be solved numerically together with the initial conditions and boundary conditions.

Table 1 Definitions of dimensionless variables

Tab. 1 Definitions of dimensionless variables

Variable name	Symbol ($i=1,2,3$)	Expression ($i=1,2,3$)
Pressure	$p_{\mathrm{D}i}$	$\frac{2\pi(k_{10}+k_{20}+k_{30})h}{\mu_0 q}(p_0-p_i)$
Bottomhole pressure	p_{wD}	$\frac{2\pi(k_{10}+k_{20}+k_{30})h}{\mu_0 q}(p_0-p_{\mathrm{w}})$
Time	t_{D}	$\frac{(k_{10}+k_{20}+k_{30})t}{\mu r_{\mathrm{w}}^2(\varphi_1 c_{t1}+\varphi_2 c_{t2}+\varphi_3 c_{t3})}$
Radial distance	r_{D}	$\frac{r}{r_{\mathrm{w}}}$
Permeability modulus [7]	$\gamma_{\mathrm{D}i}$	$\frac{\mu q\gamma_i}{2\pi h(k_{10}+k_{20}+k_{30})}$
Vug-fracture crossflow coefficient	λ_{12}	$\frac{\alpha_{12}r_{\mathrm{w}}^2}{k_{10}+k_{20}+k_{30}}$
Matrix-fracture crossflow coefficient	λ_{13}	$\frac{\alpha_{13}r_{\mathrm{w}}^2}{k_{10}+k_{20}+k_{30}}$
Vug-matrix crossflow coefficient	λ_{23}	$\frac{\alpha_{23}r_{\mathrm{w}}^2}{k_{10}+k_{20}+k_{30}}$
Storage ratio	ω_i	$\frac{\varphi_i c_{ti}}{\varphi_1 c_{t1}+\varphi_2 c_{t2}+\varphi_3 c_{t3}}$

Variable name	Symbol ($i=1,2,3$)	Expression ($i=1,2,3$)
Wellbore storage coefficient	C_{D}	$\dfrac{C}{2\pi h r_{\mathrm{w}}^{2}(\varphi_1 c_{t1}+\varphi_2 c_{t2}+\varphi_3 c_{t3})}$
Permeability ratio	D_i	$\dfrac{k_{i0}}{k_{10}+k_{20}+k_{30}}$
Viscosity variation factor	β_{D}	βT_{R}
Dimensionless temperature	θ	$\dfrac{T_{\mathrm{w}}-T_{\mathrm{R}}}{T_{\mathrm{R}}}$

The initial condition is

$$p_{\mathrm{D}i}(r_{\mathrm{D}},t_{\mathrm{D}})\big|_{t_{\mathrm{D}}=0}=0,\quad (1\le r_{\mathrm{D}}\le R/r_{\mathrm{w}};\ i=1,2,3) \qquad (17)$$

The outer boundary condition is

$$p_{\mathrm{D}i}(r_{\mathrm{D}},t_{\mathrm{D}})\big|_{r_{\mathrm{D}}=R/r_{\mathrm{w}}}=0,\quad (t_{\mathrm{D}}>0;\ i=1,2,3) \qquad (18)$$

The inner boundary condition is

$$C_{\mathrm{D}}\frac{\mathrm{d}p_{\mathrm{wD}}}{\mathrm{d}t_{\mathrm{D}}}-\left(D_1 e^{-\gamma_{\mathrm{D1}}p_{\mathrm{D1}}}\frac{\partial p_{\mathrm{D1}}}{\partial r_{\mathrm{D}}}+D_2 e^{-\gamma_{\mathrm{D2}}p_{\mathrm{D2}}}\frac{\partial p_{\mathrm{D2}}}{\partial r_{\mathrm{D}}}+D_3 e^{-\gamma_{\mathrm{D3}}p_{\mathrm{D3}}}\frac{\partial p_{\mathrm{D3}}}{\partial r_{\mathrm{D}}}\right)\Bigg|_{r_{\mathrm{D}}=1}=1,\quad (t_{\mathrm{D}}>0) \qquad (19)$$

$$p_{\mathrm{wD}}=\left(p_{\mathrm{D1}}-r_{\mathrm{D}}Se^{-\gamma_{\mathrm{D1}}p_{\mathrm{D1}}}\frac{\partial p_{\mathrm{D1}}}{\partial r_{\mathrm{D}}}\right)\Bigg|_{r_{\mathrm{D}}=1} \qquad (20)$$

$$p_{\mathrm{wD}}=\left(p_{\mathrm{D2}}-r_{\mathrm{D}}Se^{-\gamma_{\mathrm{D2}}p_{\mathrm{D2}}}\frac{\partial p_{\mathrm{D2}}}{\partial r_{\mathrm{D}}}\right)\Bigg|_{r_{\mathrm{D}}=1} \qquad (21)$$

$$p_{\mathrm{wD}}=\left(p_{\mathrm{D3}}-r_{\mathrm{D}}Se^{-\gamma_{\mathrm{D3}}p_{\mathrm{D3}}}\frac{\partial p_{\mathrm{D3}}}{\partial r_{\mathrm{D}}}\right)\Bigg|_{r_{\mathrm{D}}=1} \qquad (22)$$

To eliminate the quadratic-gradient term, Pedrosa transformation [16] is applied to the system of equations and its initial and boundary conditions, giving

$$p_{\mathrm{D1}}(r_{\mathrm{D}},t_{\mathrm{D}})=-\frac{1}{\gamma_{\mathrm{D1}}}\ln\left[1-\gamma_{\mathrm{D1}}\eta(r_{\mathrm{D}},t_{\mathrm{D}})\right] \qquad (23)$$

$$p_{\mathrm{D2}}(r_{\mathrm{D}},t_{\mathrm{D}}) = -\frac{1}{\gamma_{\mathrm{D2}}}\ln\left[1-\gamma_{\mathrm{D2}}\xi(r_{\mathrm{D}},t_{\mathrm{D}})\right] \qquad (24)$$

$$p_{\mathrm{D3}}(r_{\mathrm{D}},t_{\mathrm{D}}) = -\frac{1}{\gamma_{\mathrm{D3}}}\ln\left[1-\gamma_{\mathrm{D3}}\zeta(r_{\mathrm{D}},t_{\mathrm{D}})\right] \qquad (25)$$

$$x = \ln r_{\mathrm{D}} \qquad (26)$$

where η, ξ, and ζ denote the transformed pressures.

After the Pedrosa transformation, the quadratic-gradient terms are eliminated, and the system of equations becomes

$$\begin{aligned}
\frac{D_1}{e^{2x}}\frac{\partial^2\eta}{\partial x^2} &= \omega_1(1-\gamma_{\mathrm{D1}}\eta)^{-1}e^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}x\right]}\frac{\partial\eta}{\partial t_{\mathrm{D}}} + \lambda_{12}\left[\frac{\ln(1-\gamma_{\mathrm{D2}}\xi)}{\gamma_{\mathrm{D2}}} - \frac{\ln(1-\gamma_{\mathrm{D1}}\eta)}{\gamma_{\mathrm{D1}}}\right] \\
&\quad + \lambda_{13}\left[\frac{\ln(1-\gamma_{\mathrm{D3}}\zeta)}{\gamma_{\mathrm{D3}}} - \frac{\ln(1-\gamma_{\mathrm{D1}}\eta)}{\gamma_{\mathrm{D1}}}\right], \\
\frac{D_2}{e^{2x}}\frac{\partial^2\xi}{\partial x^2} &= \omega_2(1-\gamma_{\mathrm{D2}}\xi)^{-1}e^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}x\right]}\frac{\partial\xi}{\partial t_{\mathrm{D}}} + \lambda_{12}\left[\frac{\ln(1-\gamma_{\mathrm{D1}}\eta)}{\gamma_{\mathrm{D1}}} - \frac{\ln(1-\gamma_{\mathrm{D2}}\xi)}{\gamma_{\mathrm{D2}}}\right] \\
&\quad + \lambda_{23}\left[\frac{\ln(1-\gamma_{\mathrm{D3}}\zeta)}{\gamma_{\mathrm{D3}}} - \frac{\ln(1-\gamma_{\mathrm{D2}}\xi)}{\gamma_{\mathrm{D2}}}\right], \\
\frac{D_3}{e^{2x}}\frac{\partial^2\zeta}{\partial x^2} &= \omega_3(1-\gamma_{\mathrm{D3}}\zeta)^{-1}e^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}x\right]}\frac{\partial\zeta}{\partial t_{\mathrm{D}}} + \lambda_{13}\left[\frac{\ln(1-\gamma_{\mathrm{D1}}\eta)}{\gamma_{\mathrm{D1}}} - \frac{\ln(1-\gamma_{\mathrm{D3}}\zeta)}{\gamma_{\mathrm{D3}}}\right] \\
&\quad + \lambda_{23}\left[\frac{\ln(1-\gamma_{\mathrm{D2}}\xi)}{\gamma_{\mathrm{D2}}} - \frac{\ln(1-\gamma_{\mathrm{D3}}\zeta)}{\gamma_{\mathrm{D3}}}\right].
\end{aligned} \qquad (27)$$

The initial conditions are

$$\eta\big|_{t_{\mathrm{D}}=0} = \xi\big|_{t_{\mathrm{D}}=0} = \zeta\big|_{t_{\mathrm{D}}=0} = 0 \qquad (28)$$

The outer boundary conditions are

$$\eta\big|_{x=\ln(R/r_{\mathrm{w}})} = \xi\big|_{x=\ln(R/r_{\mathrm{w}})} = \zeta\big|_{x=\ln(R/r_{\mathrm{w}})} = 0 \qquad (29)$$

The inner boundary conditions are

$$C_{\mathrm{D}}\frac{\mathrm{d}p_{\mathrm{wD}}}{\mathrm{d}t_{\mathrm{D}}} - \left(D_1\frac{\partial\eta}{\partial x} + D_2\frac{\partial\xi}{\partial x} + D_3\frac{\partial\zeta}{\partial x}\right)\bigg|_{x=0} = 1 \qquad (30)$$

$$p_{\mathrm{wD}} = \left[-\frac{\ln(1-\gamma_{\mathrm{D1}}\eta)}{\gamma_{\mathrm{D1}}} - S\frac{\partial \eta}{\partial x}\right]_{x=0} \qquad (31)$$

$$p_{\mathrm{wD}} = \left[-\frac{\ln(1-\gamma_{\mathrm{D2}}\xi)}{\gamma_{\mathrm{D2}}} - S\frac{\partial \xi}{\partial x}\right]_{x=0} \qquad (32)$$

$$p_{\mathrm{wD}} = \left[-\frac{\ln(1-\gamma_{\mathrm{D3}}\zeta)}{\gamma_{\mathrm{D3}}} - S\frac{\partial \zeta}{\partial x}\right]_{x=0} \qquad (33)$$

2 Numerical Solution

The first-order derivatives are discretized by a forward difference, and the second-order derivatives by a central difference. The variable-coefficient system of equations is discretized fully implicitly

17–18

, yielding

$$\frac{D_1}{\mathrm{e}^{2(j-1)\Delta x}}\frac{\eta_{j-1}^{n+1}-2\eta_j^{n+1}+\eta_{j+1}^{n+1}}{(\Delta x)^2} - \frac{\omega_1}{(1-\gamma_{\mathrm{D1}}\eta_j^{n+1})}\mathrm{e}^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}(j-1)\Delta x\right]}\frac{\eta_j^{n+1}-\eta_j^n}{\Delta t} + \lambda_{12}\left[\frac{\ln(1-\gamma_{\mathrm{D2}}\xi_j^n)}{\gamma_{\mathrm{D2}}} - \ln(1-\right.$$

$$\frac{D_2}{\mathrm{e}^{2(j-1)\Delta x}}\frac{\xi_{j-1}^{n+1}-2\xi_j^{n+1}+\xi_{j+1}^{n+1}}{(\Delta x)^2} - \frac{\omega_2}{(1-\gamma_{\mathrm{D2}}\xi_j^{n+1})}\mathrm{e}^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}(j-1)\Delta x\right]}\frac{\xi_j^{n+1}-\xi_j^n}{\Delta t} + \lambda_{12}\left[\frac{\ln(1-\gamma_{\mathrm{D1}}\eta_j^n)}{\gamma_{\mathrm{D1}}} - \ln(1-\right.$$

$$\begin{aligned}
&\frac{D_3}{\mathrm{e}^{2(j-1)\Delta x}}\frac{\zeta_{j-1}^{n+1}-2\zeta_j^{n+1}+\zeta_{j+1}^{n+1}}{(\Delta x)^2} \\
&= \frac{\omega_3}{(1-\gamma_{\mathrm{D3}}\zeta_j^{n+1})}\mathrm{e}^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}(j-1)\Delta x\right]}\frac{\zeta_j^{n+1}-\zeta_j^n}{\Delta t} + \lambda_{23}\left[\frac{\ln(1-\gamma_{\mathrm{D2}}\xi_j^n)}{\gamma_{\mathrm{D2}}} - \frac{\ln(1-\gamma_{\mathrm{D3}}\zeta_j^{n+1})}{\gamma_{\mathrm{D3}}}\right] \\
&\quad + \lambda_{13}\left[\frac{\ln(1-\gamma_{\mathrm{D1}}\eta_j^n)}{\gamma_{\mathrm{D1}}} - \frac{\ln(1-\gamma_{\mathrm{D3}}\zeta_j^{n+1})}{\gamma_{\mathrm{D3}}}\right]
\end{aligned} \qquad (34)$$

where n ranges from 0 to $T-1$; j ranges from 1 to $J-1$; and J denotes the maximum value of the step length.

The initial conditions are discretized as

$$\eta_j^0 = \xi_j^0 = \zeta_j^0 = 0 \qquad (35)$$

The outer boundary conditions are discretized as

$$\eta_J^n = \xi_J^n = \zeta_J^n = 0 \qquad (36)$$

After the inner boundary conditions are treated according to Ref.

7

, they are discretized as

$$\begin{aligned}&\frac{C_\mathrm{D}}{\Delta t}\left[-\frac{\ln(1-\gamma_{\mathrm{D}1}\eta_1^{n+1})}{\gamma_{\mathrm{D}1}} - S\frac{\eta_1^{n+1}-\eta_0^{n+1}}{\Delta x}\right] - D_1\frac{\eta_1^{n+1}-\eta_0^{n+1}}{\Delta x} - D_2\frac{\xi_1^n-\xi_0^n}{\Delta x} - D_3\frac{\zeta_1^n-\zeta_0^n}{\Delta x}\\ &\quad + \frac{C_\mathrm{D}}{\Delta t}\left[\frac{\ln(1-\gamma_{\mathrm{D}1}\eta_1^n)}{\gamma_{\mathrm{D}1}} + S\frac{\eta_1^n-\eta_0^n}{\Delta x}\right] - 1 = 0\end{aligned} \tag{37}$$

$$\begin{aligned}&\frac{C_\mathrm{D}}{\Delta t}\left[-\frac{\ln(1-\gamma_{\mathrm{D}2}\xi_1^{n+1})}{\gamma_{\mathrm{D}2}} - S\frac{\xi_1^{n+1}-\xi_0^{n+1}}{\Delta x}\right] - D_1\frac{\eta_1^n-\eta_0^n}{\Delta x} - D_2\frac{\xi_1^{n+1}-\xi_0^{n+1}}{\Delta x} - D_3\frac{\zeta_1^n-\zeta_0^n}{\Delta x}\\ &\quad + \frac{C_\mathrm{D}}{\Delta t}\left[\frac{\ln(1-\gamma_{\mathrm{D}2}\xi_1^n)}{\gamma_{\mathrm{D}2}} + S\frac{\xi_1^n-\xi_0^n}{\Delta x}\right] - 1 = 0\end{aligned} \tag{38}$$

$$\begin{aligned}&\frac{C_\mathrm{D}}{\Delta t}\left[-\frac{\ln(1-\gamma_{\mathrm{D}3}\zeta_1^{n+1})}{\gamma_{\mathrm{D}3}} - S\frac{\zeta_1^{n+1}-\zeta_0^{n+1}}{\Delta x}\right] - D_1\frac{\eta_1^n-\eta_0^n}{\Delta x} - D_2\frac{\xi_1^n-\xi_0^n}{\Delta x} - D_3\frac{\zeta_1^{n+1}-\zeta_0^{n+1}}{\Delta x}\\ &\quad + \frac{C_\mathrm{D}}{\Delta t}\left[\frac{\ln(1-\gamma_{\mathrm{D}3}\zeta_1^n)}{\gamma_{\mathrm{D}3}} + S\frac{\zeta_1^n-\zeta_0^n}{\Delta x}\right] - 1 = 0\end{aligned} \tag{39}$$

The three nonlinear variable-coefficient difference equations in Eq. (34), together with their corresponding boundary conditions, Eqs. (37), (38), and (39), form three systems of equations. They are solved using the Newton-Raphson iterative method.

Taking, for example, the system composed of the first equation in Eq. (34) and Eq. (37), suppose the equations have the following form:

$$\begin{cases} f_1(\eta_0^{n+1}, \eta_1^{n+1}, \cdots, \eta_{J-1}^{n+1}) = 0 \\ \cdots \\ f_J(\eta_0^{n+1}, \eta_1^{n+1}, \cdots, \eta_{J-1}^{n+1}) = 0 \end{cases} \qquad (40)$$

Here, f_1 is the left-hand side of Eq. (37), and f_i $(i = 2 \sim J)$ is obtained by moving terms in the first equation of Eq. (34). Using vector notation, let

$$\eta^{n+1} = (\eta_0^{n+1}, \eta_1^{n+1}, \cdots, \eta_{J-1}^{n+1})^{\mathrm{T}} \in R^J \qquad (41)$$

$$F = (f_1, \cdots, f_J)^{\mathrm{T}} \qquad (42)$$

Then the system of equations can be written as

$$F(\eta^{n+1}) = 0 \qquad (43)$$

The Jacobi matrix is

$$J(\eta^{n+1}) = \begin{bmatrix} J_{11} & \cdots & J_{1J} \\ \vdots & \ddots & \vdots \\ J_{J1} & \cdots & J_{JJ} \end{bmatrix} \qquad (44)$$

where

$$J_{11} = \frac{\partial f_1}{\partial \eta_0^{n+1}} = \frac{\left(\dfrac{C_D S}{\Delta t} + D_1\right)}{\Delta x},$$

$$J_{12} = \frac{\partial f_1}{\partial \eta_1^{n+1}} = -\frac{\left(\dfrac{C_D S}{\Delta t} + D_1\right)}{\Delta x} + \frac{C_D}{\Delta t} \cdot \frac{1}{1 - \gamma_{D1}\eta_1^{n+1}},$$

$$J_{i,i-1} = \frac{\partial f_i}{\partial \eta_{j-1}^{n+1}} = J_{i,i+1} = \frac{\partial f_i}{\partial \eta_{j+1}^{n+1}} = \frac{D_1}{\mathrm{e}^{2(j-1)\Delta x}(\Delta x)^2},$$

$$(i = 2, 3, \cdots, J-1;\ j = i-1),$$

$$J_{J,J-1} = \frac{\partial f_J}{\partial \eta_{J-2}^{n+1}} = \frac{D_1}{\mathrm{e}^{2(J-2)\Delta x}(\Delta x)^2},$$

$$J_{ii} = -2J_{i,i-1} - \frac{\omega_1}{\Delta t\left(1 - \gamma_{D1}\eta_j^{n+1}\right)} \cdot \mathrm{e}^{\beta_D\left[\theta - \frac{\theta(j-1)\Delta x}{\ln(R/r_w)}\right]} \cdot \left[\gamma_{D1}\left(\eta_j^{n+1} - \eta_j^n\right) + 1\right] - (\lambda_{12} + \lambda_{13}) \,/\, \left(1 - \gamma_{D1}\eta_j^{n+1}\right),$$

$$(i = 2, 3, \cdots, J;\ j = i-1)$$

The new value after iteration is

Fig. 2

Figure 2: Fig. 2

$$\left(\eta^{n+1}\right)^{(k+1)}=\left(\eta^{n+1}\right)^{(k)}-\left(J\left(\eta^{n+1}\right)^{-1}\right)^{(k)}\left(F\left(\eta^{n+1}\right)\right)^{(k)} \tag{45}$$

where k is the iteration number.

During the calculation, the inverse of the Jacobi matrix must be computed at each iteration. Since the Jacobi matrix is tridiagonal, the characteristics of a tridiagonal matrix can be used to make the inversion simpler. The algorithm for inverting a tridiagonal matrix in Ref. [19] is adopted to improve the computational speed.

3 Results Analysis

The finite-difference equations describing triple-continuum, triple-permeability flow under the influence of temperature are nonlinear equations with variable coefficients. After discretization, the number of computational iterations is greater than that of the equation system that does not consider temperature effects; this phenomenon becomes increasingly obvious as the absolute value of β_D increases.

The wellbore-pressure and pressure-derivative curves are plotted in double-logarithmic coordinates, as shown in Fig. 2. They can be divided into six flow stages: Stage 1 is the wellbore-storage stage, in which the pressure and pressure-derivative curves are straight lines with unit slope; Stage 2 is the skin-factor influence stage, in which the pressure-derivative curve exhibits a hump; Stage 3 is the vug-to-fracture channeling stage, in which a depression appears in the pressure-derivative curve; Stage 4 is the matrix-to-fracture channeling stage, in which a depression appears in the pressure-derivative curve; Stage 5 is the matrix-to-vug channeling stage, in which a depression appears in the pressure-derivative curve; Stage 6 is the stable-flow stage, in which the pressure derivative drops sharply and the wellbore-pressure curve is a horizontal straight line.

The parameter sensitivity analysis of the calculation results is as follows.

1) Because the sensitivity of heavy oil in different storage media to temperature changes is different, the absolute value of β_D is also different. After steam is injected to modify the reservoir, the mobility of the fluid after steam injection is enhanced, the reservoir energy consumed during production at a fixed rate is low, and the bottomhole pressure rises; the earlier the occurrence of each channeling stage, the shorter the time during which the vugs channel into the fractures and the weaker the effect; the pressure derivative attenuates earlier. These effects become increasingly evident as the sensitivity of heavy oil to temperature increases, as shown in Fig. 3.

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

Figure 2 Wellbore-pressure and pressure-derivative curves

Fig. 2 The wellbore pressure and pressure-derivative curves

Figure 3 Effect of β_D on dimensionless wellbore pressure and pressure derivative

Fig. 3 The effect of β_D on dimensionless wellbore pressure and pressure-derivative

2) The larger the permeability modulus, the more sensitive the pressure is to permeability changes. The larger γ_{D1} is, the larger the wellbore pressure is, and the larger the numerical value of the pressure derivative is, as shown in Fig. 4.

Figure 4 Effect of γ_{D1} on dimensionless wellbore pressure and pressure derivative

Fig. 4 The effect of γ_{D1} on dimensionless wellbore pressure and pressure-derivative

3) The channeling coefficient λ_{13} affects the time at which each channeling stage occurs. The larger its value, the shorter the time during which the vug-to-fracture channeling stage proceeds, and the longer the time during which the matrix-to-fracture channeling stage proceeds; moreover, the effect is more obvious,

As shown in Fig. 5.

Figure 5 Effect of λ_{13} on dimensionless wellbore pressure and pressure derivative

Fig. 5 The effect of λ_{13} on dimensionless wellbore pressure
and pressure-derivative

4) The storage ratio ω_i affects the time at which each channeling-flow stage appears and its duration. Figure 6 gives the influence of ω_i on the curve shape. When ω_2 is unchanged, the smaller ω_1 and the larger ω_3, the longer the duration of the vug-to-fracture channeling-flow stage; the later the matrix-to-fracture channeling-flow stage appears, and the shorter its duration.

Fig. 5

Figure 5: Fig. 5

Fig. 6

Figure 6: Fig. 6

Fig. 7

Figure 7: Fig. 7

Figure 6 Effect of ω_i on dimensionless wellbore pressure and pressure derivative

Fig. 6 The effect of ω_i on dimensionless wellbore pressure
and pressure-derivative

5) The wellbore storage coefficient C_D has a relatively large influence on the shape of the early-time curve. The larger C_D is, the smaller the early wellbore pressure is, and the longer the wellbore-storage stage lasts; the later the vug-to-fracture channeling-flow stage appears, the shorter its duration and the weaker the channeling-flow effect; the later the matrix-to-fracture channeling-flow stage appears and the shorter its duration, as shown in Fig. 7.

Figure 7 Effect of C_D on dimensionless wellbore pressure and pressure derivative

Fig. 7 The effect of C_D on dimensionless wellbore pressure
and pressure-derivative

6) The larger the skin coefficient S is, the greater the wellbore pressure is; the longer the stage influenced by the skin factor lasts and the larger the value of the pressure derivative is; the later the vug-to-fracture channeling-flow stage appears, the shorter its duration and the weaker its effect, as shown in Fig. 8.

Figure 8 Effect of S on dimensionless wellbore pressure and pressure derivative

Fig. 8 The effect of S on dimensionless wellbore pressure
and pressure-derivative

4 Conclusions

1) Considering a carbonate heavy-oil reservoir as a triple-continuum, triple-medium system, a well-test model is established. The pressure response is divided into six stages: the wellbore-storage stage, the stage influenced by the skin factor, the vug-to-fracture channeling-flow stage, the matrix-to-fracture channeling-flow stage, the matrix-to-vug channeling-flow stage,

Fig. 8

Figure 8: Fig. 8

and the steady-flow stage.

2) Different parameters have different influences on the various stages of the pressure-response curve. After steam injection is used to reconstruct the reservoir, the mobility of the fluid after steam injection is enhanced, the formation energy consumed during constant-rate production is low, and the bottomhole pressure increases; the various channeling-flow stages appear earlier, and the time during which vug-to-fracture channeling flow proceeds is shortened and its effect is weakened; the pressure derivative declines earlier.

3) The sensitivity of heavy-oil viscosity in different storage continua to temperature differs. This sensitivity is characterized by β_D. The larger its absolute value is, the more sensitive the viscosity is to temperature variation. The influence of β_D on bottomhole pressure and the pressure derivative becomes more obvious, and the number of iterations required in the calculation after differential discretization also increases.

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Note: Figure translations are in progress. See original paper for figures.