Quantitative Theory and Methods for Multi-factor Impact Analysis

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Abstract

Multi-factor impact analysis constitutes a significant component of economic quantitative analysis, encompassing various methodologies, among which Structural Decomposition Analysis (SDA) finds extensive application within input-output technology frameworks. This paper offers a thorough exposition of SDA's limitations while providing a comprehensive articulation of the newly proposed Multi-factor Multi-order Impact Analysis (MMIA) methodology. Initially, fundamental concepts pertaining to both multi-factor impact analysis and multi-factor multi-order impact analysis are formally defined. Subsequently, the relationship between MMIA and Taylor series expansion is elucidated. Furthermore, the conceptual framework and technical implementation of forward analysis and backward analysis are introduced. Ultimately, several illustrative applications of MMIA within the input-output technology paradigm are concisely delineated.

Full Text

Theory and Method of Multifactor Impact Analysis

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Abstract

Multifactor impact analysis constitutes an important component of economic quantitative analysis, encompassing various methodological approaches. Among these, Structural Decomposition Analysis (SDA) has been widely employed in input-output applications. This paper provides an in-depth exposition of SDA's limitations and presents a comprehensive description of the newly proposed Multifactor Multi-order Impact Analysis (MMIA) technique. First, we define the fundamental concepts of multifactor impact analysis and multifactor-multi-order impact analysis. Second, we clarify the relationship between multifactor-multi-order impact analysis and Taylor series expansion. Third, we introduce the concepts and techniques of forward analysis and reverse analysis. Finally, we briefly illustrate several applications of MMIA within the input-output technical framework.

Keywords: factor analysis; multifactor impact analysis; Taylor expansion; structural decomposition analysis; input-output

Introduction

Multifactor impact analysis has traditionally been termed "factor analysis," which actually encompasses two distinct types, both bearing the English name "Factor Analysis." The first refers to factor analysis in multivariate statistics, also known as factor analysis in the statistical sense, while "multifactor analysis" has even been extended to general multivariate statistical analysis in statistics. Factor analysis addresses a class of phenomena, parsing out common characteristic factors from recorded data across a series of observed indicators based on inter-indicator correlations, with unexplained variance attributed to special factors specific to an indicator. The analytical tool employed involves eigenvalues and eigenvectors of the correlation coefficient matrix. Initially applied primarily in medical and psychological research, factor analysis is now commonly used in evaluation studies to simplify indicator sets and determine weights. Some literature also refers to stepwise regression analysis as factor analysis.

The second type represents the predecessor of today's Structural Decomposition Analysis (SDA), bearing the name factor analysis. This analytical technique, reportedly introduced from the Soviet Union in the 1950s, was called the "chain substitution method" or "factor replacement analysis method," belonging to the domain of economic activity analysis or financial management. It originated from borrowing methods used in compiling price and quantity indices. The earliest and most influential economic indices are the widely used Laspeyres and Paasche indices. The difference between these two indices lies in their distinct weighting choices, reflecting the non-uniqueness of analytical results. To render computational results independent of weight selection, scholars constructed various indices (see Table 1).

Since the late 1980s, Structural Decomposition Analysis (SDA) based on input-output relationships has been extensively applied to multifactor impact analysis of economic indicators. The SDA method indeed derives from Laspeyres and Paasche indices. Ang et al. proposed the LMDI method (Logarithmic Mean Divisia index) based on the Divisia index, which has also found application in China. Currently, these factor analysis methods are importantly applied to analyses of energy consumption and environmental impact factors. However, both analytical approaches share a fundamental flaw: they fail to completely separate the influence of one factor from changes in other factors, and their interpretations of decomposition results are never entirely satisfactory, always feeling somewhat subjective and arbitrary. The root cause of these dilemmas lies in the fact that the theoretical foundation of factor impact analysis has never been fundamentally clarified. The following sections first explain the basic defects of popular methods, then establish a new factor impact analysis model and technique—Multifactor Multi-order Impact Analysis.

Table 1 Traditional Factor Analysis Index Formulas

Index Type	Quality Factor Formula Kp	Quantity Factor Formula Kq
Laspeyres		
Paasche		
Arthur Young		
Marshall-Edgeworth		
F.Y. Fisher		
Divisia		

Note: This table is compiled based on relevant content from Xu Guoxiang' s "Statistical Index Theory and Its Applications" (China Statistics Press, 2004).

1.1 Main Defects of Structural Decomposition Analysis (SDA)

SDA aims to decompose the change in a primary indicator into the sum of contributions from several influencing factors. Since in general relational models, the influences of various factor changes cannot be independently separated, the method borrows from general multifactor index system compilation methods. The literature generally examines cases where the primary indicator equals the product of constituent factors. Below, we illustrate using a three-factor example.

Let $V = xyz$. For a single statistical unit, from an index perspective, we have , but for decomposition of change magnitude, the situation is not so straightforward. The complete decomposition should be: $\Delta + \Delta\Delta + \Delta\Delta\Delta + \Delta$. In equation (1), some terms represent mixed changes from multiple factors that appear inseparable. To attribute these changes, SDA proposes the following basic decomposition model, emulating the compilation method of multifactor Laspeyres or Paasche indices:

Equation (2) shows that the values of three factors are sequentially replaced from initial to final values, hence the name chain substitution method. Although it can be proven that equations (1) and (2) yield equal values, this decomposition model raises several questions:

(1) Decomposition results depend on the ordering of factors in the model. That is, changing the order of x, y, z in the calculation and applying equation (2) to compute each factor' s contribution yields different results. So, which ordering should be adopted?

(2) The measurement bases for different factor influences differ. In equation (2), the coefficient for Δx is the base period value, for Δz is the reporting period value, and for Δy is a mixed two-period value. Are these contribution values comparable?

Regarding the first question, a solution has been proposed: averaging the results of all possible orderings to eliminate ordering effects. For the three-factor model above, this averaging takes the form [11]: $xyzxyzxyzxyz$. Dietzenbacher and Los (1998) systematically studied various ordering solutions and aggregation level choices, finding that different methods could produce substantially different results [12]. Peter Rørmose (2010) built upon this research, considering sensitivity analysis for more factors and mixed cases of physical and economic variables [13]. However, regardless of the averaging decomposition form adopted, the inherent defect of this decomposition model cannot be eliminated: the non-objectivity and lack of economic rationale in decomposition method selection, identical to the defects of Laspeyres and Paasche indices. Unlike pure price or quantity index analysis, this defect is fundamental here, completely undermining the theoretical basis for SDA's use in factor impact analysis. Fundamentally, the various components cannot be called contributions of each factor to the primary indicator's change. This relates simultaneously to answering the second question. Regarding this ordering defect of the chain substitution method, Sang Tingrui conducted in-depth analysis in 1981, pointing out that the influence calculation for later-ordered indicators includes simultaneous change contributions from earlier indicators [14].

Regarding the second question, no thorough discussion has been seen. An Yuying once noted [15]: " 'What exactly is the temporal standard for determining the commensurate factor? Is it real economic significance or the factor's own variation? Why do the two indices adopt two completely different standards?' This is an old problem in statistical circles that has long been debated without resolution." From equation (2), we can see that the coefficient for factor x's change is the base period value, while for factor z it's the reporting period value, equivalent to applying base-period and reporting-period weighting systems to measure the contribution of x and z's changes, respectively. Thus, the contributions of the two factors become incomparable. For instance, y's contribution mixes x's contribution, and z's contribution mixes contributions from both x and y.

In summary, popular SDA as a factor impact analysis model lacks theoretical foundation. SDA suffers from three errors: (1) Regardless of how SDA's various improvements combine and average results from different factor orderings, they only aim to eliminate quantitative differences between orderings without providing physical or economic explanatory rationale. (2) In using this method, people forget the true purpose of analysis, degenerating into mere number-crunching. SDA's precise purpose is to obtain quantitative information about each factor's influence on the dependent variable to provide reference for economic decision-making, not to evaluate the magnitude of each factor's contribution. However, due to its result uncertainty, this purpose is not achieved. (3) In an organic system, one generally cannot distinguish the magnitude of different parts' roles. A small screw falling off can destroy an entire aircraft, so weighted evaluation cannot distinguish the contribution of a small screw from other components to flight safety! Therefore, using a weighting system to estimate various factors' contributions to the dependent variable is meaningless, let alone using different weighting systems for different factors.

1.2 Main Problems of LMDI

To examine the rationality of the LMDI method, we first consider the rationality of the Divisia index. The basic definition of the Divisia index is shown in Table 1, with its derivation as follows: , where P and Q are m-dimensional vectors corresponding to various statistical units. Let time be a continuous variable, with $P(t)$ and $Q(t)$ continuously differentiable. Then . Based on the logic here, the marginal pricing rule for production factors in Western economics, which claims pricing according to factor contributions, is also invalid, because each marginal term' s coefficient is a comprehensive result of multiple factors. Since various factors are inseparable in producing the same unit of commodity, individual measurement of each factor' s contribution is impossible. In the aircraft screw example, the screw' s price does not reflect its marginal role' s value. Consequently, so-called consumer surplus and producer surplus theories can be critiqued. A popular example is the parable of the last steamed bun. Formally, this formula satisfies the factor interchangeability requirement (the order of P and Q does not affect results) and the chain requirement (indices across multiple periods can be obtained by multiplying intermediate period indices). However, it is evident that with multiple statistical units (where P and Q are multidimensional vectors), each factor' s index is affected by changes in other factors, thus conflicting with the original meaning of an index—one factor changes while others remain constant. Therefore, although this index formula appears economically meaningful and logically rigorous, it actually deviates from the index' s original meaning. The LMDI factor influence analysis method developed from the Divisia index cannot avoid this defect.

Under discrete time, LMDI' s decomposition model is as follows: equation (7). From equation (7), we can see that each factor x's change contribution relates to the total change caused by all factors combined. Thus, LMDI' s decomposition effect is theoretically equivalent to SDA using complete averaging, with no superiority. Calling this decomposition the contribution of each factor also lacks decision-making significance, because a factor' s change must be coordinated with other factors' changes; achieving economic objectives through independent changes in one or a few factors is impossible.

1.3 Chinese Scholars' Research on Multifactor Impact Analysis

Literature searches reveal that An Yuying was the earliest Chinese scholar to propose factor impact analysis, publishing articles in 1985 [16] and 1986 [15]. The 1985 work was titled "Establishing a New Factor Analysis Method Using the Marginal Analysis Principle of Total Increment," while the 1986 version was renamed "Partial Increment Factor Analysis Method." An Yuying's method directly derived from Taylor expansion of functions, calling the dependent variable's total change the total increment, then decomposing it into the sum of each factor's first-order derivative influence and remaining components, omitting Lagrange remainder terms beyond a certain order. An Yuying considered this a new factor analysis method.

The second scholar to research this method after An Yuying was Yang Qizi, who published three similar articles in 1995 [17][18][19]. Yang Qizi divided the proposed method into three parts. The first part, called "Basic Method of Statistical Analysis for Multivariate Function Total Increment," also known as "Partial Increment Analysis Method," first obtains the dependent variable's total change (also called total increment), then lets each variable change independently to obtain its partial increment, called a variable's basic influence value, and finally subtracts the sum of all variables' basic influence values from the total increment, with the difference being the total interaction effect among all variables. Yang Qizi argued that when changes in the dependent variable and independent variables are relatively small, the sum of basic influence values can represent the total increment, yielding each factor's influence. However, when variable changes are large, the interaction effect must be allocated to each variable and combined with basic influence values, with the sum called the total influence value. The second part, called "Differential Increment Analysis Method" or "Partial Differential Analysis Method," is also based on Taylor expansion but advances beyond An Yuying by considering not only first-order derivative influence values but also higher-order derivative terms for individual independent variables, pointing out the correspondence with the first part's method, and finally using the deduction method to obtain interaction effect values, excluding Lagrange remainder terms. The composition structure of interaction effect values can be seen from Taylor expansion (see Section 3). The third part includes "Allocation Analysis Method" and "Integral Increment Analysis Method." Allocation analysis actually refers to several methods for allocating interaction effects, while the so-called integral increment method (also called line integral method) is essentially the LMDI method. Since these methods are not endorsed in this paper, they will not be detailed.

2 Basic Concepts of the Multifactor Multi-order Impact Analysis Model

To address SDA’ s defects, the author published the Multifactor Multi-order Impact Analysis (MMIA) technique in 2013 [20]. When writing the first paper, the work of An Yuying and Yang Qizi was not yet known.

Assume a dependent variable has a causal relationship with a set of independent variables (this relationship need not be direct), and the variables are mutually independent with the following functional relationship:

A value of is denoted as , and a value of X is denoted as . When the value of one (or several) independent variable(s) changes, the resulting change in the dependent variable is called the effect (or influence) of that (set of) independent variable(s) on the dependent variable. From the functional relationship, we can analyze how changes in any combination subset of X’ s values affect Y’ s value.

Factor impact analysis refers to evaluating (not assessing!) or predicting the role of each independent variable factor causing changes in the dependent variable. Since mathematical relationships alone cannot determine causality between independent and dependent variables, it is called factor impact analysis rather than influencing factor analysis; causal determination requires substantive theoretical analysis.

For a combination of several elements of , denoted as M, when the components corresponding to M in X change (with $n \leq m$), while other components remain unchanged, the resulting value of X is denoted as . A change in value is denoted as .

Definition 1: Denote as M’ s m-order joint total effect.

Definition 2: The m-order joint pure effect of is called the m-order joint pure effect of combination M if and only if the components of M change simultaneously (with $n < m$), provided that changing any component in the combination to 0 yields .

Definition 3: Let s_k^I be a k-order combination extracted from M, let be the set of all k-order combinations extracted from M, and let denote the k-order joint pure effect of combination s_k^I. Let be the k-order joint pure effect of .

The k-order pure effect contribution rate of to y’ s change. Definition 3 implies that we can calculate the k-order joint pure effect of any combination M of m components of X, while the sum of joint pure effects of all subsets of M’s elements equals the m-order total effect of . When $n = m$, equation (10) represents the m-order total effect of , and equation (11) represents the k-order joint pure effect of , thus providing an iterative formula starting from low orders:

Since first-order effects are easily calculated and joint total effects of factor combinations are also readily computed, equations (10) and (11) provide an operational recursive formula:

With first-order effects known, equation (13) can calculate all second-order joint pure effects for two-factor combinations. Overall MMIA steps include: (1) list all factor combinations of various orders; (2) calculate all first-order effects; (3) calculate all factor combinations' joint total effects; (4) sequentially calculate corresponding joint pure effects order by order.

For a particular factor t_x, if its effect is separable from other factors. If for a particular factor combination , its effect is separable from other factors. If all factors' effects are separable, then any of their joint pure effects equal 0, and the dependent variable' s total change equals the sum of all independent variables' first-order effects.

3 From Taylor Expansion to the Multifactor Multi-order Impact Analysis Model

The Taylor expansion of multivariate functions in calculus is actually a form of factor impact analysis. Revealing the relationship between this analysis model and the multifactor multi-order impact analysis model will deepen understanding of both.

3.1 Taylor Expansion Forms

The analysis begins with univariate functions; multivariate function Taylor expansions are superpositions on the univariate foundation. Since general textbooks rarely provide relatively complete Taylor expansions for trivariate and higher functions, we first present their forms. (1) For a univariate function $x = $, the Taylor expansion near is $R_n(x)$, called the Lagrange remainder term, representing the portion remaining after a given finite number of terms in the Taylor series. Converting equation (15) to increment form yields . Equation (15) shows that the dependent variable' s total change can be calculated using a power series of the independent variable' s change, which actually equals a factor' s first-order total effect in the MMIA formula, also a first-order pure effect. (2) For a bivariate function . (3) For an n-variate function , if it is analytic at point , its increment expansion can be written directly as .

3.2 Correspondence Between Taylor Expansion and Multifactor Multi-order Impact Analysis

From univariate function Taylor expansion, we find that an independent variable' s influence on the dependent variable can be expressed as a sum of products of the variable' s higher-order derivatives and same-order powers of its increment. First, we can separate each independent variable' s individual influence from binomial expansion, denoting each independent variable' s individual influence as k_x^I. For multivariate functions' single-variable independent influences, we have $, 2, 1 = $. Second, in bivariate function expansions, aside from 1_x^I and 2_x^I, the remaining parts belong to two-factor joint influences and are pure influences, formally expressible as: .

Third, for any independent variable combination , the joint pure effect of each factor can be expressed as ,2,1,0 , where denotes k' s factorial, with elements mutually unequal. Among them, the joint total effect of elements is 12···n = ,2,1,0 1≠ 2 ≠ ···n . Finally, summing the joint pure effects of all m-element combinations extracted from yields X's m-order joint pure effect on Y. Summing X' s pure effects on Y from order 1 to n yields X' s n-order joint total effect on Y, which is Y' s total increment.

We can see that from a function' s Taylor series expansion, we can obtain both the function' s derivative components of various orders and the joint pure effects of independent variable combinations. However, this calculation process is overly complex and always contains excluded remainder terms. The MMIA method not only overcomes the remainder term problem of Taylor series expansion but also dramatically simplifies computation. Moreover, MMIA does not require function differentiability or even continuity, only requiring a definite correspondence between dependent and independent variables. Additionally, MMIA variables can be scalars, vectors, or even matrices, such as input coefficient matrices in input-output models. As for the advantage of Taylor series expansion in separating derivative components of various orders, this is actually unnecessary—it was a form required for approximate calculation when human computational capacity was weak. Under modern highly developed computer technology, approximation problems are automatically solved by machines.

4 Distinction Between Forward Analysis and Reverse Analysis

In the preceding MMIA formulation, note that no directional specification is considered for independent variable changes—that is, no distinction is made between changing from x to or from to x. This distinction is meaningful in empirical analysis and applies to two scenarios: first, time series changes; second, comparisons between different units of the same type. Below, we focus primarily on time series changes.

Time series change analysis is the most common application area for multifactor impact analysis. Traditional factor analysis focuses on effects produced when a set of factors changes while others remain constant. However, we can also analyze effects when a set of factors remains constant while others change, leading to the distinction between forward and reverse analysis. This distinction was proposed by the author when completing the National Bureau of Statistics' 2017 input-output table bidding project.

When time changes from 0 to t, the dependent variable changes from to .

4.1 First-order Analysis

The effect of k_X can be considered in two ways: first, letting only k_X change while other factors remain constant; second, examining what effect k_X' s change produces. The first approach is called forward analysis, the second reverse analysis. The first approach examines the effect without , with straightforward calculation formulas:

Equation (24) directly calculates factor k_X's effect. For the second approach, an intuitive formulation is . However, this formula actually calculates other factors' joint total effect. To examine k_X' s effect, we subtract this quantity from Y' s total change, yielding k_X's effect as . Y's total change reflects the effect with k_X' s participation, while equation (25) shows the effect without k_X' s participation. Their difference precisely indicates k_X' s role in Y' s change. Forward analysis formulas show the effect of changing one factor from initial values, making them more suitable for forecasting future changes, though current factor analysis is used to explain the past. This holds true even when treating a set of factors as a whole, equivalent to a single variable.

Reverse analysis formulas explain what change would occur in the dependent variable if one factor returned to the past, given the current status quo. Thus, they are more suitable for evaluation, explaining what effects would occur with and without a particular factor' s participation in past evolution. Comparing effects with and without the factor' s participation (e.g., by subtraction) reveals the factor' s role from a certain perspective.

In fact, forward and reverse analysis formulas are mathematically identical, equivalent to swapping and 0 and changing signs.

4.2 Higher-order Analysis Formulas

In MMIA, an independent variable can be a vector, and analysis results are unaffected by ordering. As previously noted, when treating a set of independent variables as a whole, it behaves like a single variable. Therefore, higher-order analysis formulas can be analogized from first-order analysis formulas.

Let be an m-factor combination extracted from . Then M' s forward total effect on Y is , where denotes that independent variables belonging to set M in X change from 0_x to t_x while others remain at initial values. For M, if its $(m-1)$-order total effect is known, denoted $(-m)M^I$, then its m-order pure effect is . Since first-order pure effects equal first-order total effects and M' s m-order total effect is easily calculated, equation (29) also provides an operational recursive formula for calculating higher-order joint pure effects.

Higher-order reverse analysis formulas can be written analogously. M' s reverse joint total effect is: $X_t \rightarrow$, where denotes that components belonging to set M in X change from t_x to 0_{kix} while others remain at period t values. The reverse total effect of any (or first-order) factor is obtained from equation (27), and X' s overall first-order effect value is . Then, equation (29) can recursively calculate high-order joint pure effects for any combination.

Example 1: Third-order effect analysis (forward analysis only). Let the factor combination under examination be . Clearly, three first-order effects are , with first-order total and pure effects unified as: . Three second-order total effects are: . Thus, three second-order pure effects are . The second-order pure effect of is: . The second-order total effect of is: . One third-order total effect is: . The third-order pure effect of is: . Moreover, third-order total effect equals the sum of first-, second-, and third-order pure effects: .

Example 2: Comparison of forward and reverse analysis results. Consider a function where initial values (2, 3, 7) change to final values (4, 5, 10). Forward and reverse analysis results for this scenario are shown in Table 2 .

Table 2 Comparison of Forward and Reverse Analysis Results

Factor	Pure Effect Total	Forward Analysis	Reverse Analysis
X1X2X3			

Contribution Rate Comparison of Forward and Reverse Analysis Results

Factor	Pure Effect Total	Forward Analysis	Reverse Analysis
X1X2X3			

Table 2 shows significant differences between forward and reverse effects of factors and factor combinations, related to the function' s form and the magnitude of factor changes. In Example 2, reverse analysis more prominently highlights X2' s promoting effect because it appears in exponential form in the function. In reverse analysis, although each factor' s individual first-order effect is positive, the joint pure effects of two factors are all negative, representing a typical manifestation of systems theory' s "1+1 $\neq$ 2." Additionally, although third-order effects are relatively small, they may not be negligible.

Note that so-called positive and negative effects are relative to the dependent variable' s change direction when viewed by contribution rate. If the dependent variable' s change is increasing, positive factor effects promote growth while negative effects inhibit it. If the dependent variable' s change is decreasing, positive factor effects promote the decrease while negative effects inhibit it.

Although MMIA technology dramatically reduces computational load compared to Taylor series expansion methods, for n independent variables, the number of combinations is $2^n - 1$ (including single factors), with computational workload increasing geometrically with n. While effect magnitudes do not decrease monotonically with order, the overall trend is downward. Therefore, when n is large, if approximation using the first m orders is feasible, higher-order calculations are unnecessary.

4.3 Comparative Analysis of Similar but Different Objects

In economic research, analyzing differences between regions is an important aspect, with factor influence analysis on these differences holding significant meaning. For example, in environmental quality analysis, there are multiple pollutants such as particulate matter, carbon dioxide, sulfur dioxide, solid waste, etc. While these pollutant emissions appear independent, in reality, because the same production process simultaneously generates multiple pollutants, changes in a sector' s share in the overall industrial structure affect emissions of various pollutants. Additionally, both structure and total volume affect pollutant emissions, and inter-industry interdependencies create multifactor joint effects. Compared to the distinct process differences between forward and reverse analysis in time series comparisons, this distinction is easily overlooked in comparative analysis of different objects.

Mathematically, replacing the time identifiers 0 and t in time series analysis with A and B for different object analysis presents no computational difficulty or meaning change, and forward and reverse analysis results may also differ significantly. However, interpreting results and applying them to policy formulation requires special attention. As previously noted, for time series analysis, forward analysis is suitable for forecasting while reverse analysis is suitable for explaining the past. For different object analysis, when analyzing how one object catches up to an advanced region, forward analysis is appropriate, using the 追赶 region as "0" and the advanced region as "t." For two parallel regions, one must specify who is compared to whom and which is the baseline (as "0"), then apply forward analysis.

5 Applications of MMIA

MMIA technology has been applied in various economic analyses, fully demonstrating its effectiveness and feasibility.

5.1 Factor Analysis of Per Capita Grain Output [20]

This was presented as a small example in the first paper proposing MMIA. Per capita grain output = unit area yield × grain sown area / average annual population. Factor analysis of China' s per capita grain output changes from 1980 to 2010 concluded: In the 1980s, the primary influencing factor for per capita grain output increase was unit area yield; in the 1990s, the primary factor for per capita grain output decline was population size, with unit area yield playing a significant inhibitory role against the decline; in the first decade of the 21st century, per capita grain output continued to increase, with unit area yield remaining the main determining factor. The analysis revealed that second-order joint effects were substantial, some comparable to primary first-order factor effects, indicating that joint effects must be taken seriously.

5.2 Factor Analysis of Energy Consumption Intensity [21]

Factor analysis of energy consumption intensity based on input-output table structure represents the second and most important application of MMIA. Energy intensity expressions based on input-output table data structure can take two forms: $I = kQYA$ and $C = kQYA$, where G—gross domestic product (GDP), Y—final use total column vector in input-output tables, Q—total output column vector, A—direct consumption coefficient matrix, W—total output structure column vector, P—total output total, e—row vector of all ones, k_E—energy consumption per unit of total output, Y_E—total residential energy consumption, C—final use product structure column vector.

Equation (46) features industrial structure expressed using total output, while equation (47) features final use product structure. Analysis of China' s energy intensity changes from 1997-2005 using constant-price input-output tables shows that supportive and inhibitory effects on energy saving and consumption reduction coexist, both significant, with technology energy saving' s overall effect exceeding the sum of various inhibitory effects, achieving an average contribution rate of 120.9%, ultimately reducing energy consumption per unit of GDP.

5.3 Factor Analysis of Industrial Structure Changes

Within the input-output accounting framework, economic industrial structure has two representations: total output industrial structure and value-added industrial structure. For representing and analyzing industrial structure, nominal quantity structure is more meaningful than real quantity structure. Based on input-output table data structure, industrial structure calculation formulas for the two representation methods are as follows [22]: $q = By$ and $z = B^T y$, where q—total output structure vector, Y—final use vector, z—initial input vector, A—direct input coefficient matrix, B—Leontief inverse matrix. In more refined structures, Y and Z may be matrices composed of several components, but the above formula does not consider multiple components, using only vector representation.

We can see that factors affecting total output structure can be 归结为 three categories, while factors affecting value-added structure can be 归结为 two categories. This structural analysis can be applied to both changes in an economy' s industrial structure over time and comparisons of industrial structure differences between different economies. Comparative analysis of Chinese and US industrial structures in 2011 using WIOD world input-output tables shows that based on total output structure differences, 45% of sectors differed primarily due to different intermediate consumption coefficients, 38% due to different final demand structures, and the remaining 17% due to different total value-added rates. This indicates that technological structure is the main influencing factor for total output structure differences, with an inherent necessary connection between technological structure and industrial structure. Based on value-added structure differences, 34.5% of sectors differed primarily due to different intermediate consumption coefficients, 62% due to different final demand structures, and 3.5% due to synergistic effects of intermediate consumption coefficients and final demand structures, showing that final demand structure is the main influencing factor for value-added structure differences.

Deeper research can subdivide final use into residential consumption, public consumption, capital formation, and net exports [23]: $Y = F + NX$ and $Y = f + nx$, where , , and denote sub-item structures for residential consumption, government consumption, capital formation, and net exports, respectively; gc, f, and nx denote product structures for each final use component. In this case, there are 10 factors affecting total output structure; from change decomposition, because some factors have no synergistic effects, this includes 10 first-order pure effects, 21 second-order pure effects, 16 third-order pure effects, and 4 fourth-order pure effects. Analysis based on China' s 19-sector input-output tables from 2002 to 2017 shows that changes in intermediate input structure (direct consumption coefficient matrix) are the most important factor affecting industrial structure, followed by residential consumption structure. This demonstrates that technological progress is the most important factor affecting economic structure in the long term; in the long run, supply determines economic growth and structure.

5.4 Factor Analysis of Economic Growth

Analyzing economic growth requires constant-price input-output tables. The following models are based on constant prices, with a bar added above corresponding variables and an asterisk (*) denoting the partially closed model. Under the partially closed model, GDP expression is , where denotes the $(n+1)$th column of the bracketed matrix, and n_e, 1_{+ne} denote n-dimensional and $(n+1)$-dimensional row vectors of all ones, respectively. Since the $(n+1)$th row in final use is actually all zeros, the above model contains seven factors; if residential consumption structure in $^*A$ is separated independently, there are eight basic factors. Applying MMIA technology, GDP changes can be decomposed into the sum of eight first-order pure effects, ten second-order pure effects, and three third-order pure effects.

Using the above model, China' s economy from 2007 to 2017 is divided into two stages for analysis. From 2007 to 2012, GDP grew 62.68% over five years, with an average annual growth rate of 10.2%. During this stage, capital formation total had the largest first-order contribution rate to GDP growth, playing a decisive role; direct consumption coefficients and government consumption total followed, with very significant effects. All three changes promoted GDP growth. Additionally, residential consumption structure changes had a significant effect on GDP growth, but the effect was negative. These results indicate that during this stage, investment scale was the dominant factor for economic growth; economic technology changes represented by direct consumption coefficients were conducive to GDP growth, and fiscal policy represented by government consumption changes effectively promoted economic growth. Although consumption structure inhibited GDP growth, its changes reflected improvements in people' s quality of life. From 2012 to 2017, GDP grew 38.99% over five years, with an average annual growth rate of 6.8%, showing medium-high speed characteristics. During this stage, capital formation total still had the largest first-order contribution rate to GDP growth, but weaker than in the previous stage; government consumption total followed, with both changes promoting GDP growth. Additionally, direct consumption coefficients and net export total changes had relatively significant effects on GDP growth, but net export total changes had a negative effect, as did net export structure. These results show that during this stage, investment scale remained the dominant factor for economic growth, playing an absolutely important role, and fiscal policy represented by government consumption changes also effectively promoted economic growth. Although net export total and structure inhibited GDP growth, net export changes reflected a substantial reduction in our economy' s dependence on external economies. In GDP growth impacts, first-order effects are primary, but some higher-order effects are also significant, such as the second-order joint pure effect of direct consumption coefficients and net export structure reaching +4.77%, and the third-order joint pure effect of direct consumption coefficients, net export structure, and net export total reaching -2.74%.

Based on the same data, using reverse analysis to observe final use structure' s impact on economic growth, government consumption and capital formation remain dominant factors for economic growth, net export changes become the main adverse factor for economic growth, and consumption structure' s effect rapidly declines from negative to positive from the first to second period (from -46% to +4%). These changes align with economic growth theory and demonstrate the effectiveness and trends of China' s economic transformation. Compared to forward analysis, reverse analysis more prominently highlights these characteristics, showing it is more suitable for interpreting past changes.

5.5 Factor Analysis of Industrial Structure Changes Based on Two Major Department Classification [25]

Two-major-department analysis is an important characteristic of Marxist economics research. Input-output analysis development provides more powerful analytical tools for studying two-major-department problems, offering both quantitative empirical research possibilities and the ability to decompose two major departments into finer industrial structures. Based on compiled two-major-department input-output tables, the following basic model exists:

where Roman numerals I and II denote the first and second departments, respectively. The model assumes: all intermediate use and all fixed capital formation belong to the first department, all consumption belongs to the second department, and inventories and net exports are decomposed into the first and second departments according to certain proportions. Since most sectors in the original table will be decomposed into two departments belonging to the two major departments, the two-major-department input-output table may be an $n \times 2$ type relative to the original table. Due to different economic functions of the first and second departments, their factor impact decomposition models for industrial structure changes also differ, containing different numbers of effect terms of various orders.

6 Conclusion

Multifactor impact analysis is an important field in economic analysis practice. With known functional relationships, the Multifactor Multi-order Impact Analysis (MMIA) technique provides a precise, objective, and complete analysis model. Some might argue that differences between MMIA and SDA results should be demonstrated. This author believes it unnecessary, because the rejection of SDA is theoretical—a rejection on scientific principles—and negates the usual evaluative interpretation of contribution rates, no longer relying on any weighting system. Relative to the initial work of An Yuying and Yang Qizi, MMIA completes comprehensive technical establishment, proposes the division between forward and reverse analysis, and negates the necessity of allocating cross-effects to obtain single-factor complete influence values. Compared to decomposition models through Taylor series expansion, MMIA is a decomposition of finite quantity changes in functions, does not require function differentiability, substantially reduces computational workload, and theoretically has no remainder term problems. Input-output technology offers comprehensive and systematic characteristics for studying economic problems, and its combination with MMIA technology provides the most effective analysis model for economic policy analysis. Two points require attention: first, MMIA application presupposes obtaining accurate relational expressions between economic variables, either identities or relatively complete statistical fitting relationships; second, as a mathematical model, MMIA' s provision of independent variables' effects on dependent variables should not be directly called cause-to-effect influences—establishing causal relationships requires professional theoretical analysis.

Finally, we reiterate that although the term "contribution rate" is also used in MMIA description, this is not evaluation but merely a term for change effects. For example, second-order factor combination contribution rates require simultaneous changes in two factors; in this case, first-order effects must exist—it is impossible to have only second-order effects without first-order effects. The same applies to other higher-order effects; corresponding lower-order effects must simultaneously exist.

References

[1] Ye Peihua, Chen Yibai, Wan Meiting, Hao Deyuan. *Educational Statistics*. People' s Education Press, 1988:338-373

[2] Zeng Xianglong, Lin Jiuxiang. Application of Multifactor Analysis in Orthodontics. *Foreign Medicine: Stomatology*, 1984(3):156-159.

[3] Xu Disheng. Lecture 9: Multifactor Analysis. *School Sports*, 1984(4):52-55.

[4] Sang Tingrui. Correctly Formulating Factor Analysis Methods—Also Commenting on the Soviet "Chain Substitution Method" . *Dongyue Tribune*, 1981(03):67-76+51.
[5] Carlo Milana. The Input-Output Structural Decomposition Analysis of "Flexible" Production Systems. In: Michael L. Lahr and Erik Dietzenbacher (eds). *Input-Output Analysis: Frontiers and Extensions, Essays in honor of Ronald E. Miller*, London: Macmillan Press, 2001.
[6] Skolka J. Input-output Structural Decomposition Analysis for Austria. *Journal of Policy Modeling*, 1989, 11(1):45-66.
[7] Ang B W, Liu F L. A New Energy Decomposition Method: Perfect in Decomposition and Consistent in Aggregation. *Energy*, 2001, 26(6):537-548.
[8] Ang B W. The LMDI approach to Decomposition Analysis: A Practical Guide. *Energy Policy*, 2005, 33(7):867-871.
[9] Li Guozhang, Wang Shuang. Regional Factor Decomposition Analysis of China' s Energy Intensity Changes—Based on LMDI Decomposition Method. *Finance and Economics Research*, 2008(8):52-63.
[10] Xu Yingzhi, Zhang Quanzhen. Decomposition Effects of China' s Manufacturing Energy Consumption: Research Based on LMDI Model. *Southeast University Journal: Philosophy and Social Sciences Edition*, 2011(4):55-60.
[11] Li Jinghua. Weighted Average Decomposition Method of SDA Model and Its Application in China' s Tertiary Industry Economic Development Analysis. *Systems Engineering*, 2004(9):69-73.
[12] Dietzenbacher E, Los B. Structural decomposition techniques: Sense and sensitivity. *Economic Systems Research*, 1998, 10(3):307-323.
[13] Peter Rørmose, 2010, *Structural Decomposition Analysis: Sense and Sensitivity*, Statistics Denmark.
[15] An Yuying. On Partial Increment Factor Analysis Method. *Journal of Zhongnan University of Finance and Economics*, 1986(4):108-115
[16] An Yuying. Establishing a New Factor Analysis Method Using the Marginal Analysis Principle of Total Increment. *Statistics and Decision*, 1985(01):13-14.
[17] Yang Qizi. On Statistical Analysis Method for Deterministic Factors. *Statistics and Decision*, 1995(8):41-43.
[18] Yang Qizi. On Statistical Analysis of Multivariate Function Total Increment. *Statistical Research*, 1995, 12(3): 38-45.
[19] Yang Qizi. On Multivariate Function Total Increment Analysis Method (Part 1). *Journal of Hunan Business College*, 1995(3):38-43.
[20] Liu Xinjian. Multifactor Multi-order Impact Analysis Model and Its Application. *Statistics and Decision*, 2013(3):76-78.
[21] Song Hui, Liu Xinjian. *China Energy Utilization Input-Output Analysis*. China Market Press, 2013:37-40
[22] Liu Xinjian. *Input-Output Model for Economic Theoretical Analysis*. Science Press, 2018:120-122
[23] Yang Qian. *China' s Industrial Structure Evolution and Factor Influence Analysis*. Qinhuangdao: Yanshan University Master' s Thesis, 2022
[24] Li Xiaowen. *Research on the Relationship Between China' s Economic*

Growth and Demand Structure Evolution. Qinhuangdao: Yanshan University Master' s Thesis, 2022
[25] Liu Xinjian, He Zede, Yang Qian. Structural Characteristics of China' s Two Major Departments and Their Role in the New Development Pattern— Analysis Based on Input-Output Tables. *Contemporary Economic Research*, 2022, 320(4)

Note: Figure translations are in progress. See original paper for figures.