Quantum abstract neural automata

Authors: Guangcheng Xi

Date: 2024-01-23T00:00:00+00:00

Abstract

In the 1990s, I proposed and constructed the theory of Abstract Neural Automata (ANA) 1. Aim of present manu

Full Text

Quantum Abstract Neural Automata

Guangcheng Xi
The State Key Laboratory of Management and Control for Complex Systems
Institute of Automation, Chinese Academy of Sciences
Beijing, 100190, P.R. China

Abstract

In the 1990s, I proposed and constructed the theory of Abstract Neural Automata (ANA) [1]. The aim of the present manuscript is to explore the theory of ANA on quantum states. Some results have been obtained.

Keywords: Gibbs measure, Quantum abstract neural automata, Existence, von Neumann algebra, Isomorphic, Consciousness.

1. Introduction

In the 1990s, I proposed and constructed the theory of Abstract Neural Automata (ANA) [1]. The work accomplished includes: establishing abstract neural automata on N-dimensional lattices, Euclidean spaces, and compact Riemannian manifolds respectively; achieving concept generation, concept connection transfer, and thinking formation in abstract neural automata; investigating the origin of thought; demonstrating a positive correlation between brain intelligence and the volume of related brain regions; establishing the ergodicity of the genetic evolution of human brain intelligence; proving that abstract neural automata have a unique solution if and only if they go to infinity in both micro and macro directions (consciousness is expected to unify micro and macro); showing the equivalent proposition that abstract neural automata are Markovian; demonstrating that the limiting probability distribution of abstract neural automata is Gibbsian; establishing that abstract neural automata are evolutionary; proving that assuming the existence of information on the network of stochastic automata, abstract neural automata are forever-moving machines in the sense of evolution; and establishing the representation theory of knowledge by abstract neural automata.

The aim of the present manuscript is to explore the theory of ANA on quantum states (QANA). The precondition for writing this article is to ensure that quantum mechanics plays a role in brain function [2][3][4]. In particular, consciousness depends on quantum states of electrons within hydrophobic pockets in a class of brain proteins.

Proof of existence of QANA on the von Neumann algebra and on the C-*algebra* $\mathcal{A}$ *has been given in the present article. The following propositions are mutually equivalent: (1) There exists QANA on* C-algebra $\mathcal{A}$; (2) [text incomplete in original].

The conditional state of quantum measurement is Gibbsian; the brain of the observer begins the process of experiencing consciousness in the process of measurement. In particular, it is quite natural from the point of view of physics and mathematics to see that consciousness is the collapse of quantum states in the human brain system at the microscopic level, and is Gibbsian.

2. Definitions and a Theorem About Abstract Neural Automata [1]

Definition 1. A stochastic automaton a is defined as a triplet with a function-pair where Q is the space of states; Y is the set of inputs; Z is the set of outputs; $F : Y \times Q \times Q \to [0,1]$, such that $\sum_{q' \in Q} F(y,q,q') = 1$; and $G : Q \times Z \to [0,1]$, such that $\sum_{z \in Z} G(q,z) = 1$.

In fact, the transition probability from state q to state q' when input is y is given by $F(y,q,q')$, and the probability for output to be z when state is q is given by $G(q,z)$.

Definition 2. A network S of stochastic automaton a is defined as a pair consisting of d-dimensional integer lattices (d-dimensional neuron set) T and a function-pair (R,C), where R is a set of all automata defined as $\{a_t : t \in T\}$; in fact, a_t is an automaton a which corresponds to $t \in T$; C is a connected transformation of some network, when neuron t gives an output z, $C(z)$ is an input which is received by other neurons.

Definition 3. Abstract Neural Automata (ANA) which are on the network $S(T,(R,C))$ is a set of random elements $\{\xi(t), t \in T\}$, where $(\Omega, \mathcal{B}(\Omega), P)$ and $(Q = X, \mathcal{B}(X), \mu)$ are two probability spaces.

Definition 4. To every finite subset $\{t_1, \dots, t_n\} \subset T$ there corresponds a probability distribution $\mu_{t_1,\dots,t_n}$ on Q^n such that $\mu_{t_1,\dots,t_n}(x_1, \dots, x_n) = P\{\xi(t_i) = x_i, i = 1, \dots, n\}$, and satisfies the consistency condition. We refer to the family of probability distributions $\{\mu_L\}$ as the probability distribution of ANA.

Definition 5. The family of conditional probability distributions

$$\{\mu_L(x_1, \dots, x_n \mid \xi(t) = x(t), t \in T \setminus L)\}$$

is called a conditional distribution of ANA, where $L = \{t_1, \dots, t_m\} \subset T$.

Definition 6. When $L \subset T$, for arbitrary $x(t) \in Q$ for $t \in L$, for arbitrary function Φ, the conditional probability distribution

$$\mu_L(x_1, \dots, x_m \mid \xi(t) = x(t), t \in T \backslash L) = \frac{1}{Z(L)} \exp\{-\beta U(x_1, \dots, x_m \mid \xi(t) = x(t), t \in T \backslash L)\}$$

is called Gibbs distribution with a boundary condition, where constant $\beta > 0$, $0 < \beta < \infty$, is inversely proportional to the temperature of network S of stochastic automata; U is called potential.

Definition 7. ANA and its distribution are called Gibbs if its conditional distribution is given by equation (8).

Theorem 1. In the theory of ANA, suppose the collection of potential functions of various forms is a continuous potential function, and that Q is a compact metric space, then for given Φ, ANA exists on probability space $(Q^T, \mathcal{B}(Q^T), \mu)$.

3. Quantum Abstract Neural Automata (QANA)

To describe abstract neural automata at the quantum level (QANA), we associate with each neuron $a \in T$ a finite-dimensional Hilbert space $\mathcal{H}_a$. Hence, we have the Hilbert space $\mathcal{H}_\Lambda = \bigotimes_{a\in\Lambda} \mathcal{H}_a$ for $\Lambda \subset T$, and then the local C-*algebra* $\mathcal{A}_\Lambda = \bigotimes_{a\in\Lambda} \mathcal{L}(\mathcal{H}_a)$. *If* $\Lambda_1 \subset \Lambda_2$, *there exists an embedding* $j_{\Lambda_1,\Lambda_2} : \mathcal{A}_{\Lambda_1} \to \mathcal{A}_{\Lambda_2}$ *defined by* $j_{\Lambda_1,\Lambda_2}(A) = A \otimes I_{\Lambda_2\setminus\Lambda_1}$, *having the following property:* $j_{\Lambda_2,\Lambda_3} \circ j_{\Lambda_1,\Lambda_2} = j_{\Lambda_1,\Lambda_3}$. *Therefore, when* $\Lambda_1 \subset \Lambda_2$, *we have* $\mathcal{A}_{\Lambda_1} \subset \mathcal{A}_{\Lambda_2}$, *hence an increasing family. The set* $\bigcup_{\Lambda\in\mathcal{S}} \mathcal{A}_\Lambda$ *is a normed* -algebra; the completion of $\bigcup_{\Lambda\in\mathcal{S}} \mathcal{A}_\Lambda$ by means of this norm is known as the local C*-algebra of local observables.

Define $\mathcal{S} = \{\Lambda \subset T : |\Lambda| < \infty\}$, where $|\Lambda|$ is the cardinality of set Λ. Note that $\mathcal{S}$ is countable. On the family of sets $\mathcal{S}$, we have $\mathcal{A}_\Lambda \subset \mathcal{A}_{\Lambda'}$ when $\Lambda \subset \Lambda'$. Further, when $\Lambda_1 \cap \Lambda_2 = \emptyset$, there exists embedding $j_{\Lambda_1,\Lambda_1\cup\Lambda_2} : \mathcal{A}_{\Lambda_1} \to \mathcal{A}_{\Lambda_1\cup\Lambda_2}$ defined by $j_{\Lambda_1,\Lambda_1\cup\Lambda_2}(A) = A \otimes I_{\Lambda_2}$, having the following property: $[j_{\Lambda_1}(A_1), j_{\Lambda_2}(A_2)] = 0$ for all $A_1 \in \mathcal{A}_{\Lambda_1}, A_2 \in \mathcal{A}_{\Lambda_2}$. Therefore, when $\Lambda_1 \cap \Lambda_2 = \emptyset$, we have $\mathcal{A}_{\Lambda_1\cup\Lambda_2} = \mathcal{A}_{\Lambda_1} \otimes \mathcal{A}_{\Lambda_2}$.

For any $a \in T$, we let τ_a be the translation of T by a. The translations $\{\tau_a : a \in T\}$ form a partition of T. For any $\Lambda \subset T$, we let $N(\Lambda)$ be the number of elements in Λ. At this time, for all $a \in T$, a finite subset $\Lambda_n \subset T$ is defined to tend to T in the sense of Van Hove, if

$$\lim_{n\to\infty} \frac{N(\partial^a \Lambda_n)}{N(\Lambda_n)} = 0,$$

where $\partial^a \Lambda_n = \{x \in T : \tau_a(x) \in \Lambda_n \text{ and } x \notin \Lambda_n\} \cup \{x \in T : \tau_a(x) \notin \Lambda_n \text{ and } x \in \Lambda_n\}$.

For arbitrary $a \in T$, suppose unitary mapping $U_a : \mathscr{H}_a \to \mathscr{H}_{a+\xi}$ satisfies conditions (1) $U_a U_b = U_{a+b}$; and $U_a \mathscr{A}_\Lambda U_a^* = \mathscr{A}_{\Lambda+a}$. Using this unitary operator, let

$$\tau_a^\Phi(A) = U_a A U_a^*, \quad A \in \mathscr{A}_\Lambda.$$

Hence τ_a^Φ is an automorphism mapping from $\mathscr{A}_\Lambda$ to $\mathscr{A}_{\Lambda+a}$ (i.e., $\tau_a^\Phi \in \mathrm{Aut}(\mathscr{A})$).

Suppose φ is a state on $\mathscr{A}$. Its restriction φ_Λ is known as a local normal state if it satisfies the condition: there exists $\rho_\Lambda \in \mathscr{L}(\mathscr{H}_\Lambda)$ such that $\varphi_\Lambda(A) = \mathrm{tr}(\rho_\Lambda A)$ for all $A \in \mathscr{A}_\Lambda$. Moreover, if φ satisfies condition $\varphi \circ \tau_a^\Phi = \varphi$, then φ is known as invariant (i.e., τ_a^Φ-invariant). The entropy of φ_Λ equals $S(\varphi_\Lambda) = -\mathrm{tr}(\rho_\Lambda \log \rho_\Lambda)$.

Theorem 2. Let $\Lambda \subset T$, and suppose $|\Lambda|$ is the volume of Λ, then we have: $S(\varphi) = \lim_{\Lambda\uparrow T} \frac{S(\varphi_\Lambda)}{|\Lambda|}$ is an affine and upper semi-continuous function on $\mathscr{S}_I$.

A mapping $\Phi : \mathscr{S} \to \mathscr{A}$ satisfying condition $\Phi(\Lambda) \in \mathscr{A}_\Lambda$ is called an interaction potential (or simply a potential). Let $\|\Phi\| = \sum_{\Lambda\ni 0} \frac{\|\Phi(\Lambda)\|}{|\Lambda|}$, and let $\mathscr{B}$ be the set of all Φ with $\|\Phi\| < \infty$. Then $\mathscr{B}$ becomes a real Banach space for norm $\|\cdot\|$. Further, Φ is known as finite range if the number of finite sets Λ for which $\Phi(\Lambda) \neq 0$ is finite. Let $\mathscr{B}_0$ be the set of all interaction potentials of finite range; the completion of $\mathscr{B}_0$ for norm $\|\cdot\|$ equals $\mathscr{B}$, expressed as $\overline{\mathscr{B}_0} = \mathscr{B}$. When $\Phi(\Lambda + a) = \tau_a^\Phi(\Phi(\Lambda))$, Φ is known as τ_a^Φ-invariant or translationally invariant; there exists some number $c(\Phi)$ such that $\sum_{a\in\Lambda} \Phi(\Lambda) = c(\Phi) I$.

For thermodynamic functions, we have

$$P_\Lambda(\Phi) = \frac{1}{|\Lambda|} \log \mathrm{tr}(e^{-H_\Lambda(\Phi)}), \tag{15}$$

$$H_\Lambda(\Phi) = \sum_{\Lambda'\subset\Lambda} \Phi(\Lambda'). \tag{16}$$

Theorem 3. Suppose φ is τ_a^Φ-invariant, then (1) there exists $P(\Phi) = \lim_{\Lambda\uparrow T} P_\Lambda(\Phi)$; $P(\Phi)$ is a continuous convex function on $\mathscr{B}$; and (2) $S(\varphi) = \inf_{\Phi\in\mathscr{B}}\{P(\Phi) - \varphi(A_\Phi)\}$.

Lemma 1. Suppose that M is a commutative von Neumann algebra, then there exists a locally compact Hausdorff space X which satisfies the second axiom of countability, and a measure μ, such that $M \cong L^\infty(X, \mu)$.

Lemma 2. A C*-algebra $\mathscr{A}$ is a von Neumann algebra if and only if it has a dual $\mathscr{A}^*$ as a Banach space.

Denote the set of linear functionals $\mathcal{A}^*$. Since C-*algebra $\mathcal{A}$ is a bounded normed linear space, its dual space is a Banach space from C*-algebra $\mathcal{A}$ to $\mathbb{C}$ (complex field). When $\|\varphi\| = 1$, φ is called a state on $\mathcal{A}$. $\mathcal{A}^*$ is called the state space. From Lemma 2, C*-algebra $\mathcal{A}$ is a von Neumann algebra. Hence from Lemma 1 and Lemma 2, we have the following:

Theorem 4. Let $\mathcal{A}$ be a von Neumann algebra; hence there exists QANA on von Neumann algebra $\mathcal{A}$.

Proof. Let $\mathcal{A} = L^\infty(X, \mu)$, then $\mathcal{A} \cong C(X)$, from the nature of $L^\infty(X, \mu)$, we have $C(X) \cong \mathcal{A}$. Therefore, $C(X)$ and $\mathcal{A}$ are isomorphic. Hence there exists ANA on $C(X)$ (Theorem 1), and there exists QANA on $\mathcal{A}$. QANA $= \varphi \circ \pi$ (see subsection 4).

Suppose the set of all potentials which are τ_a^Φ-invariant and have interaction of finite body is $\mathcal{B}_1$; there exists $\varphi \in \mathcal{S}_I$ (i.e., KMS state).

Definition 8. Let σ_t^Φ be a strongly-continuous one-parameter group of automorphisms of $\mathcal{A}$; $\beta \in \mathbb{R}$ (real number). One says that a state ψ is a KMS$_\beta$ state if it is invariant under σ_t^Φ, i.e., $\psi \circ \sigma_t^\Phi = \psi$, and for all $A, B \in \mathcal{A}$ there exists a function $F_{A,B}$ bounded and continuous on the strip $\{z \in \mathbb{C} : 0 \le \mathrm{Im}(z) \le \beta\}$ and analytic on $\{z \in \mathbb{C} : 0 < \mathrm{Im}(z) < \beta\}$ such that $F_{A,B}(t) = \psi(A\sigma_t^\Phi(B))$ for all $t \in \mathbb{R}$, and $F_{A,B}(t + i\beta) = \psi(\sigma_t^\Phi(B)A)$ for all $t \in \mathbb{R}$.

Proposition. Let $\mathcal{K}(\mathcal{H})$ be the algebra of compact operators on a Hilbert space $\mathcal{H}$, $\beta \in \mathbb{R}$ (real number), and H a self-adjoint operator such that $e^{-\beta H}$ is trace class. Then the Gibbs state with density matrix $\rho = \frac{e^{-\beta H}}{\mathrm{tr}(e^{-\beta H})}$ is the unique KMS$_\beta$ state for the one-parameter group generated by H.

Theorem 5. For $\Phi \in \mathcal{B}$, the following propositions are mutually equivalent: (1) There exists QANA on C*-algebra $\mathcal{A}$; (2) There exists KMS$_\beta$ state.

4. Coexistence of N and Q

The content of this section is to explain the coexistence of QANA and ANA in the brain and their interdependent relationship by means of the method of text [6].

Suppose that the human neural system at the ion level (QANA) is denoted by Q, while the human neural system at the level of neural network (ANA) is denoted by N, that the category von Neumann algebra of N is $\mathcal{A}$, and that the category von Neumann algebra of Q is $\mathcal{B}$. Let' s do linear mapping $\varepsilon : \mathcal{B} \to \mathcal{A}$ for some finite set. Hence we have

$$\varepsilon(\rho) = \bigoplus_{a \in A} \rho_a, \tag{17}$$

where $\rho_a = \mathrm{tr}_{\mathcal{B}a}(\rho)$. In fact, $\varepsilon : \mathcal{A} \to \mathcal{B}^+$ is a positive operator-valued measure on $\mathcal{A}$, POVM, that takes values in $\mathcal{B}^+$. A conditional state on $\mathcal{B}$:

$$\rho_\varepsilon = \frac{\varepsilon(\rho)}{\operatorname{tr}(\varepsilon(\rho))}. \tag{18}$$

The conditional state of the POVM applied to ρ with outcome a is

$$\rho_a = \frac{\varepsilon_a(\rho)}{\operatorname{tr}(\varepsilon_a(\rho))}, \tag{19}$$

where $\varepsilon_a : \mathcal{A} \to \mathcal{B}_a$ is a positive operator-valued measure. In fact, we can see the linear map ε represented as an orthogonal decomposition: $\varepsilon = \bigoplus_{a\in A} \varepsilon_a$ as a set-valued measurement which is $\varepsilon_a(\rho) = P_a \rho P_a$, where P_a is a projection operator. The probability of the outcome a is given by $p(a) = \operatorname{tr}(\varepsilon_a(\rho))$. And the conditional state of ρ given outcome a is $\rho_a = \frac{\varepsilon_a(\rho)}{p(a)}$.

Since $H(\Phi)$ is an invariant subspace of ε, and reducing ε, we have

$$\varepsilon(\rho) = \bigoplus_{a\in A} \rho_a. \tag{20}$$

The process of a measurement is a process of decoherence. A state in which decoherence has been achieved is called a trivial quantum state—equilibrium state—classical state. At this time, $\rho = \bigoplus_{a\in A} \rho_a$; and $H(\Phi) \subset \mathcal{H}_a$, hence we have

$$\rho_a = \frac{e^{-\beta H_a}}{\operatorname{tr}(e^{-\beta H_a})}. \tag{21}$$

Therefore, the trivial conditional state is Gibbsian; conditioning on a measurement is also called "state collapse" or "wave function collapse" ; the brain of the observer begins the process of experiencing consciousness in the process of measurement [7].

In particular, it is proved mathematically that the trivial conditional state of quantum measurement is Gibbsian, thus proving that the consciousness of the observer is Gibbsian, which is consistent with the discussion on consciousness in article [7].

References

[1] Xi Guangcheng, Abstract neural automata, Kybernetes: The International Journal of System and Cybernetics, Vol. 27, No. 1, 1998, pp. 81-86; Abstract neural automata -further considerations, Kybernetes: The International Journal of System and Cybernetics, Vol. 27, No. 2, 1998, pp. 176-181; Variability of structure of abstract neural automata and the ability of thought, Kybernetes: The International Journal of System and Cybernetics, Vol. 32, No. 9\10, 2003, pp. 1549-1554; Abstract neural automata: Ergodicity of evolution of geneticovariable structure, Kybernetes: The International Journal of System and Cybernetics, Vol. 28, No. 8, 1999, pp. 939-944; Abstract neural automata on compact

Riemannian manifold, Kybernetes: The International Journal of System and Cybernetics, Vol. 32, No. 9\10, 2003, pp. 1540-1548; Abstract neural automata (ANA): existence of "empty" and origin of thought, Kybernetes: The International Journal of System and Cybernetics, Vol. 32, No. 3, 2003, pp. 388-393; Gibbsian representation of knowledge in infinite dimensional random neural network (IDRNN), ChinaXiv:201803.01556v2.

[2] Herbert S. Green; Information Theory and Quantum Physics, p. 171, Springer, 1998.

[3] Friedrich Beck and John C. Eccles; Quantum aspects of brain activity and the role of consciousness, Proc. Natl. Acad. Sci. USA, Vol. 89, pp. 11357-11361, December 1992, Biophysics.

[4] Stuart Hameroff, Alex Nip, Mitchell Porter, Jack Tuszynski; Conduction pathways in microtubules, biological quantum computation, and consciousness, BioSystems 64 (2002) 149-168.

[5] 梅垣壽春，大矢雅则; 量子论的ントロピー, 昭和 58 年 8 月.

[6] Greg Kuperberg; A concise introduction to quantum probability, quantum mechanics, and quantum computation, on Internet.

[7] Guangcheng Xi; Entropy-partition of Complex Systems and Emergence of Human Brain' s Consciousness, ChinaXiv:201705.00829v3.

Note: Figure translations are in progress. See original paper for figures.