

## Bound States in the Continuum in Periodic Optical Systems

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### Abstract

Periodic optical systems, such as photonic crystals and optical metamaterials, can localize high-density electromagnetic field energy at the subwavelength scale and attain extremely small mode volumes, exhibiting tremendous application potential in the field of optical manipulation. In recent years, a strong light-matter interaction in periodic optical systems has been discovered, termed bound states in the continuum, which constitute a class of special electromagnetic eigenstates whose frequencies lie within the radiation continuum yet are completely localized, featuring numerous intriguing physical properties and rich application scenarios. This article provides a systematic review of the classification and theoretical framework of bound states in the continuum in periodic optical systems, and summarizes their fundamental physical properties and latest application developments. Bound states in the continuum in periodic optical systems are injecting new vitality into the development of integrated optics, information optics, biological optics, topological optics, and nonlinear optics.

### Full Text

## Bound States in the Continuum in Periodic Optical Systems

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### Abstract

Periodic optical systems, such as photonic crystals and optical metamaterials, can localize high-density electromagnetic field energy at subwavelength scales

and achieve extremely small mode volumes, offering tremendous potential for light manipulation applications. In recent years, a strong light-matter interaction has been discovered in periodic optical systems, known as bound states in the continuum (BIC). Optical BICs are special electromagnetic eigenstates whose frequencies lie within the radiation continuum yet remain completely localized, exhibiting fascinating physical properties and rich application scenarios. This paper systematically reviews the classification and theoretical framework of BICs in periodic optical systems, and summarizes their fundamental physical properties and latest application developments. BICs in periodic optical systems are injecting new impetus into the fields of integrated optics, information optics, bio-optics, topological optics, and nonlinear optics.

**Keywords:** Periodic optical systems; Bound states in the continuum; Theoretical system; Physical properties; Application development

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Classical mechanics suggests that electrons with sufficient kinetic energy will escape from their atomic systems, with this energy residing in the system's scattering continuum. However, some electrons possessing adequate energy remain bound within the system, never liberating themselves from the surrounding gravitational field. This seemingly paradoxical yet real phenomenon can be explained by quantum mechanics, as elucidated in 1929 by two physics pioneers, John von Neumann and Eugene Paul Wigner [1]. These electronic states are termed "bound states in the continuum" (commonly abbreviated as BIC). Similar phenomena based on this concept are quite universal, having been discovered in electromagnetic waves [2], water waves [3-5], acoustic waves [6-8], and elastic waves in solids [9,10]. Entering the 21st century, optical BIC phenomena have been studied across a wide range of dielectric systems including optical waveguides, optical fibers, photonic crystals, and optical metamaterials [11-32]. Conventional wisdom holds that electromagnetic eigenstates within the radiation continuum are radiative, and that completely trapped bound states should exist below the radiation continuum. BICs challenge this conventional understanding—they represent a class of electromagnetic eigenstates within the radiation continuum that exhibit no radiation whatsoever. Despite their frequencies and momenta potentially matching those of vacuum modes, they remain completely confined within the resonant system (see Section 2 for details). More intuitively, optical BICs are special electromagnetic resonance states within a resonant system that "should" radiate by intuition but actually do not—this description is not strictly accurate but facilitates an intuitive understanding of the profound concept of optical BICs.

Photonic crystals and optical metamaterials are typical periodic optical systems that interact with light at subwavelength scales, enabling localization of high-density electromagnetic energy within extremely small volumes. Optical metamaterials, in particular, demonstrate powerful light field manipulation capabilities, allowing customized control over light's frequency, phase, polarization, amplitude, orbital angular momentum, and spin angular momentum [33-45]. BICs

in periodic optical systems represent a strong light-matter interaction, though strictly speaking, an optical BIC is an ideal state because infinite periodic systems cannot exist. In practice, researchers primarily utilize its radiative leaky modes—quasi-BICs [46-53]. By adjusting system configuration, dimensions, periodic boundaries, or excitation wavevectors, BICs can be weakly coupled to the far field, allowing a small portion of energy to leak into the surrounding environment. This state is termed a quasi-BIC. The radiative loss in BIC-based systems can be precisely controlled by adjusting the interference among constituent resonant modes, thereby obtaining quasi-BICs with varying degrees of leakage [54-60]. While quasi-BIC modes are readily accessible, obtaining true BICs optically is extremely challenging because BIC modes have no coupling to free-space radiation and cannot be excited from the far field. This dilemma was only recently overcome when Joel K. W. Yang et al. proposed a technique and apparatus for precisely characterizing true optical BICs in nanoscale spaces. Their experiment combined cathodoluminescence and monochromatic electron energy loss spectroscopy in a scanning transmission electron microscope, using a nanoscale-focused electron beam to excite optical modes in the near field. Energy transferred during these radiative and non-radiative excitations was measured via monochromatic electron energy loss spectroscopy, while radiative losses in the far field were measured solely by cathodoluminescence. By comparing these spectra, true BIC and quasi-BIC optical modes could be distinguished [61]. It should be emphasized that true BICs may also be excited from the far field by introducing parity-time (PT) symmetric perturbations, which may harbor richer physical 内涵. This phenomenon was recently discovered in photonic crystal fibers [62], but has not been universally confirmed. Therefore, this paper only considers BICs that are conventionally non-excitabile from the far field.

Applications of BICs in periodic optical systems span integrated optics, information optics, bio-optics, topological optics, and nonlinear optics, primarily relying on two fundamental physical characteristics: tunable high quality (Q) factors and their special polarization distributions in momentum space [63-69], as detailed in Section 3 and illustrated in [Figure 1: see original paper]. High Q-factor and device miniaturization represent fundamental pursuits for quasi-BICs, enabling intense light-matter interactions within extremely small volumes to satisfy various applications. Recent advances indicate that the area of periodic systems supporting quasi-BICs can be as small as  $4\lambda^2$  (where  $\lambda$  is the operating wavelength) [70]. Leveraging the high Q-factor attribute, BICs can be applied to narrowband filtering [71], high-sensitivity sensing [72], molecular spectroscopic encoding imaging, image edge detection [73,74], nonlinear frequency doubling [75], microlasers [76], and micro-nano antennas [77]. Additionally, based on the special polarization distribution of BICs in momentum space, they can be used for vortex light generation in momentum space [78,79]. Furthermore, combining high Q-factor properties with momentum-space polarization characteristics may enable more intriguing applications such as narrowband asymmetric transmission [80]. These applications and principles will be elaborated in Section 4.

[Figure 1: see original paper] Fundamental Physical Properties and Applications of BICs in Periodic Optical Systems

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## 2.1 Concept and Classification of BICs

Consider a periodic slab dielectric in vacuum, as shown in [Figure 2: see original paper]a. The wavevector  $\mathbf{k}$  of radiative electromagnetic waves can be decomposed into perpendicular component  $k_{\perp}$  and parallel component  $k_{\parallel}$ . Geometrically, the frequency  $\omega = c|\mathbf{k}|$  of any radiative electromagnetic wave is always greater than  $c|k_{\parallel}|$ . Moreover, for any given parallel wavevector component  $k_{\parallel}$ , the perpendicular component  $k_{\perp}$  can take arbitrary values, making the frequencies of radiative electromagnetic waves continuous. In the frequency-wavevector ( $\omega$ - $\mathbf{k}$ ) spectrum, radiative electromagnetic wave frequencies occupy the region  $\omega > c|k_{\parallel}|$  (also called the light cone), i.e., the radiation continuum. Below the radiation continuum lies the region occupied by conventional bound states. Solutions of Maxwell's equations dictate that electromagnetic wave frequencies of bound states in dielectrics are discrete. Traditional viewpoints hold that all resonant modes within the radiation continuum are radiatively leaky, but BICs defy this principle. BICs are special resonant modes whose frequencies lie within the radiation continuum yet are completely bound without any energy leakage [81].

[Figure 2: see original paper] The concept and classification of BICs. (a) Schematic diagram showing the positions of radiative resonance modes, BICs, and ordinary bound states in the band structure of periodic optical systems. Here, a symmetry-protected BIC is taken as an example, located at the high-symmetry point ( $\Gamma$  point) of wavevector space; (b) Diagram of classification, far-field decoupling approaches, and fundamentals of optical BICs in periodic systems.

Although BICs reside within the radiation continuum, they are completely decoupled from the far-field radiation continuum. There are two primary decoupling mechanisms, as shown in [Figure 2: see original paper]b. The first type of BIC arises from structural symmetry mismatch: microstructures with  $C_2$  symmetry can confine resonant modes with even-symmetric electromagnetic fields within the radiation continuum. These modes are completely orthogonal to odd-symmetric radiation modes, thereby decoupling from far-field radiation and forming BICs. These are called symmetry-protected BICs. Once the  $C_2$  symmetry of the structure is broken, the BIC couples with radiation modes of different symmetry, generating radiative leakage and forming quasi-BICs. The second type of BIC originates from interference cancellation: destructive interference between different electromagnetic modes within the structure, or between different waves of the same electromagnetic mode, indirectly achieves decoupling from far-field radiation. The former can be divided into Fabry-Pérot BICs and Friedrich-Wintgen BICs, while the latter generally corresponds to

single-resonance BICs. Adjusting structural parameters can control the degree of radiative cancellation; when radiation cannot be completely canceled, leaky quasi-BICs form. These BICs and quasi-BICs are achieved by tuning structural parameters, and the parameter conditions for BIC occurrence are difficult to predict accurately. Therefore, they are also called parameter-tuned BICs or accidental BICs.

Different theoretical explanations exist for different types of BICs, with mainstream approaches including symmetry theory, temporal coupled-mode theory, and coupled-wave theory—each suitable for explaining different BIC origins. Temporal coupled-mode theory is also appropriate for analyzing scattering problems of all BIC types. Additionally, electromagnetic multipole theory has emerged, which studies multipole radiation to identify symmetry-protected BICs and accidental BICs, but lacks further analytical capability for accidental BICs, such as distinguishing specific types. In summary, no single theory currently provides a unified description of all BIC origins; different BIC types require separate analysis.

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## 2.2 Symmetry-Protected BICs

When a physical structure possesses  $\pi$ -rotational symmetry (C2 symmetry), symmetry protection can completely prohibit coupling between certain bound modes of specific symmetry types and continuous modes of other symmetry types. As shown in [Figure 3: see original paper]a, the most common structure consists of circular holes etched in a square lattice, periodically arranged in two dimensions as a photonic crystal slab [81]. In two-dimensional periodic structures, electromagnetic modes can be represented by the wavevector parallel to the slab  $\mathbf{k}_{//} = (k, k)$ . Symmetry protection manifests as follows: when the structure is rotated by  $\pi$  about the z-axis (the normal direction of the photonic crystal slab), the parallel wavevector of certain electromagnetic modes remains invariant. To ensure this characteristic, the parallel wavevector of such modes typically resides at the  $\Gamma$  point in wavevector space, where these modes have no horizontal component, i.e.,  $\mathbf{k}_{//} = (0, 0)$ . At the  $\Gamma$  point, even-symmetric electromagnetic modes are completely decoupled from odd-symmetric modes. Due to the C2 symmetry of the structure, even-symmetric electromagnetic modes are fully confined within the structure, forming symmetry-protected BICs, while odd-symmetric modes radiate out of the structure. When the radiation frequency is below the diffraction limit (defined as  $\omega = 2\pi c/na$ , where  $c$  is the speed of light in vacuum,  $n$  is the refractive index of the ambient medium, and  $a$  is the periodic constant), radiation occurs only in the slab normal direction.

Away from the  $\Gamma$  point, at least one of the horizontal wavevector components  $k_x$  or  $k_y$  becomes non-zero, and  $\pi$ -rotation no longer ensures invariance of the horizontal wavevector. Without symmetry protection, BICs couple with radiation modes to form leaky quasi-BICs.

The core concept for obtaining quasi-BICs from symmetry-protected BICs involves breaking the C2 symmetry of the structure. This breaking can occur in wavevector space (as described above, where manipulating the horizontal wavevector away from the  $\Gamma$  point belongs to this category) or in real space. Real-space breaking of C2 symmetry to obtain quasi-BICs has been widely reported. As shown in [Figure 3: see original paper]b, common structures include: split rings or rectangular rings with varying opening sizes or positions [82,83], asymmetric rectangular bars of different lengths [84,85], a pair of elliptical bars inclined in a figure-eight configuration [86,87], asymmetric rectangular or circular blocks with added or removed portions [88-90], disks with off-center holes [91], and asymmetric crosses with displaced centers [92,93].

Perturbation theory is often employed to describe the radiation characteristics of symmetry-protected quasi-BICs. In this framework, any operation that breaks the C2 symmetry is treated as a perturbation to the original structure, leading to coupling between BICs and the radiation continuum. Quasi-BICs are regarded as eigen-resonant states resulting from the interaction between a closed resonant cavity and the radiation continuum. The radiative electric field can be expressed as  $\mathbf{E}(\mathbf{r}) = \mathbf{E}^{(0)}(\mathbf{r}) + \phi\mathbf{E}(\mathbf{r})$ , where  $\mathbf{E}^{(0)}(\mathbf{r})$  is the solution of Maxwell's equations with perfect magnetic boundary conditions (described as  $\mathbf{s} \times \nabla \times \mathbf{E}^{(0)}$ , where  $\mathbf{s}$  is the boundary normal vector), and  $\phi\mathbf{E}(\mathbf{r})$  describes the effect of structural perturbation [94].

An important physical quantity describing quasi-BIC resonance strength is the quality factor  $Q$ , defined as  $Q = \omega_0/2\gamma$ , where  $\omega_0$  is the resonant frequency and  $\gamma$  is the resonance damping rate. True BICs have no resonance damping, resulting in infinite  $Q$ . Since they cannot couple to far-field radiation, they also cannot be excited by far-field plane waves and cannot be observed spectroscopically. However, quasi-BICs result from interaction with far-field radiation and thus appear in spectra. Quasi-BICs exhibit resonance damping and show non-zero resonance linewidths in spectra. The quality factor of symmetry-protected quasi-BICs is closely related to the degree of C2 symmetry breaking. Yuri Kivshar's team derived the relationship  $Q = Q_0\alpha^{-2}$ , where  $Q_0$  is a structure-dependent constant and  $\alpha$  represents the asymmetry coefficient—a parameter measuring the degree of C2 symmetry breaking ([Figure 3: see original paper]c) [94]. Notably, the quality factor decreases with increasing asymmetry; when asymmetry becomes sufficiently large, the quasi-BIC resonance degenerates into an ordinary resonance.

[Figure 3: see original paper] Symmetry-protected BICs. (a) A circular-hole photonic crystal slab confines even-symmetric electromagnetic modes with frequencies below the diffraction limit while radiating odd-symmetric modes [81]; (b) Common structures supporting quasi-BICs due to C2 symmetry breaking; (c) The quality factor decreases with increasing structural asymmetry, following the relationship  $Q = Q_0\alpha^{-2}$  [94].

### 2.3 Resonance-Coupled BICs

Periodic geometries create photonic band structures, analogous to how periodic potentials in solids generate electronic band structures. Periodic slabs support guided resonances whose frequencies lie within the continuum of free-space radiation modes. These resonances typically have finite lifetimes because they can couple to free-space modes. Although these resonances are radiative, BICs can also form through destructive interference of far-field radiation resulting from coupling between different resonant modes. Resonance-coupled BICs in meta-surfaces can be divided into Fabry-Pérot BICs and Friedrich-Wintgen BICs.

By constructing high-reflectivity two-dimensional periodic structures such as photonic crystal slabs through guided resonance effects ([Figure 4: see original paper]a-b), and using two identical such structures as a pair of mirrors, BICs form when the round-trip phase shift of electromagnetic waves equals integer multiples of  $2\pi$ . This structure is equivalent to a Fabry-Pérot cavity formed between two resonant reflectors, where the round-trip phase condition creates standing waves that trap electromagnetic waves. These are therefore called Fabry-Pérot BICs [81,95]. Fine-tuning the distance between the two structures enables precise control from BIC to quasi-BIC. If the separation between the two two-dimensional periodic structures supporting Fabry-Pérot BICs is reduced to zero (effectively becoming a single periodic structure), radiative leakage from two resonances at the same location may also form BICs through destructive interference. These are called Friedrich-Wintgen BICs [96], as shown in [Figure 4: see original paper]c-f.

Temporal coupled-mode theory serves as an effective tool for explaining resonance-coupled BIC phenomena. In this theory, the key mathematical quantities determining Fabry-Pérot BIC frequencies and damping rates are expressed through a Hamiltonian matrix (detailed derivation in Section 2.4):

$$H = \begin{pmatrix} \omega - i\gamma & \kappa e^{i\phi} \\ \kappa e^{-i\phi} & \omega - i\gamma \end{pmatrix}, \quad (1)$$

where  $\kappa$  represents near-field coupling between two resonances of frequency  $\omega$ ,  $\gamma$  is the damping rate of a single resonance, and  $\phi$  is the propagation phase shift between the two resonators. The  $H$  matrix has two eigenvalues:

$$\omega_{\pm} = \omega \pm \kappa e^{\pm i\phi} - i\gamma, \quad (2)$$

When the round-trip phase shift equals integer multiples of  $2\pi$ ,  $\phi$  becomes integer multiples of  $\pi$ . One eigenvalue becomes  $\omega_{-} = \omega - i2\gamma$ , a complex number representing an eigenmode with frequency  $\omega$  and damping rate  $2\gamma$ . The other eigenvalue becomes a purely real number  $\omega_{+} = \omega$ , representing a BIC with zero damping ( $\gamma = 0$ ) and frequency  $\omega$ .

If the distance between the two resonators becomes zero (two periodic structures merging into one), Friedrich-Wintgen BICs may be supported. Here,  $\phi = 0$  in equation (1), and the two coupled resonances can no longer be guaranteed to have identical resonant frequencies and damping rates. The Hamiltonian matrix becomes:

$$H = \begin{pmatrix} \omega_1 - i\gamma_1 & -ik \\ ik & \omega_2 - i\gamma_2 \end{pmatrix}, \quad (3)$$

The frequency eigenvalues of the H matrix in equation (3) are:

$$\omega_{\pm} = \frac{(\omega_1 + \omega_2) - i(\gamma_1 + \gamma_2)}{2} \pm \frac{\sqrt{[(\omega_1 - \omega_2) - i(\gamma_1 - \gamma_2)]^2 + 4k^2}}{2}. \quad (4)$$

The condition for obtaining purely real eigenvalues is:

$$k = \pm \frac{\gamma_1 - \gamma_2}{\omega_1 - \omega_2} \cdot \frac{\gamma_1 + \gamma_2}{2}, \quad (5)$$

Substituting equation (5) into equation (4) reveals that  $\omega$  becomes purely real. At this point,  $\omega = (\omega_1 + \omega_2)/2 - (\gamma_1 + \gamma_2)(\omega_1 - \omega_2)/(\gamma_1 - \gamma_2)$ , yielding a BIC with zero damping ( $\gamma = 0$ ) and frequency  $\omega$ . Notably, when  $k = 0$  or  $\gamma_1 = \gamma_2$ , a BIC is obtained at  $\omega_1 = \omega_2$ . This condition was first derived by physicists Friedrich and Wintgen [97], hence the name Friedrich-Wintgen BIC. Physically, resonance-coupled BICs do not rely on structural symmetry. Their acquisition typically involves tuning a finite number of structural parameters; when continuously adjusted to a specific value, the structure supports BICs with infinite quality factors. On either side of this parameter value, the quality factor decreases, and BICs become quasi-BICs.

[Figure 4: see original paper] Resonance-coupled BICs. (a) Fabry-Pérot BIC supported by two identical highly reflective photonic crystal plates, and (b) its resonant mechanism: adjusting the distance  $d$  between the two plate resonators to form a standing wave [81]; (c) Schematic diagram of mode coupling in a single periodic structure device, (d) radiation spectrum of the coupling of two modes into a high-Q mode; (e) Frequency tuning spectrum: adjusting structural parameters causes the two eigenfrequencies to move and cross; near the crossover point, the resonance damping of one frequency disappears completely, forming a Friedrich-Wintgen BIC, (f) which appears as a smooth spectral line in the transmission spectrum [96].

## 2.4 Single-Resonance BICs

While destructive interference of multiple resonant modes can form resonance-coupled BICs, destructive interference of multiple waves within a single resonant mode can also form BICs. These are called single-resonance BICs, which are relatively rare but do exist. As shown in [Figure 5: see original paper]a-b, in 2013, Chia Wei Hsu et al. discovered a new type of BIC in a periodic slab with an array of square cylindrical holes [98]. This structure possesses time-reversal symmetry, C2 symmetry, and up-down mirror symmetry. In addition to obtaining a zero-leakage BIC at the  $\Gamma$  point on one band, another BIC with infinite resonance lifetime was obtained at a location away from the  $\Gamma$  point on the same band. During this process, the quality factor exhibited a trend of changing from extremely large to extremely small and back to extremely large. Since this BIC reappears away from the  $\Gamma$  point and is not constrained by structural symmetry compatibility, it does not belong to symmetry-protected BICs. Moreover, as it was discovered within a single TM resonant mode and does not rely on destructive interference between multiple resonant modes, it also does not belong to resonance-coupled BICs. This new type of BIC was subsequently defined as a single-resonance BIC.

An intuitive and simple theory can explain single-resonance BICs: the eigen-electromagnetic modes of periodic structures are Bloch resonant states, where wavefunctions can be expressed as Fourier series of plane waves. In the near-field region outside the structural slab, these waves exist as exponentially decaying evanescent waves, subsequently propagating as plane waves in the far-field region. The power of propagating waves is proportional to  $(|C_s|^2 + |C_p|^2)\cos\theta$ , where  $\theta$  denotes the propagation angle, and  $C_s$  and  $C_p$  represent the average amplitudes of the s and p components of the plane wave in the x-y plane. When the frequency is below the diffraction limit,  $C_s$  and  $C_p$  are zero-order Fourier amplitudes. Generally,  $C_s$  and  $C_p$  are complex numbers with more than two degrees of freedom, so adjusting only the two parameters  $k_x$  and  $k_y$  in wavevector space cannot achieve zero output propagating power. However, when the structure possesses time-reversal symmetry, C2 symmetry, and up-down mirror symmetry,  $C_s$  and  $C_p$  become purely real or purely imaginary, reducing the degrees of freedom of Fourier amplitudes to two. In this case, adjusting only  $k_x$  and  $k_y$  may make both  $C_s$  and  $C_p$  zero ([Figure 5: see original paper]c), thereby achieving zero output propagating power.

Based on coupled-wave theory [99,100], the physical mechanism of single-resonance BICs has been further explained. In coupled-wave theory, single-resonance BICs originate from the coupling of all waves in all channels (including in-plane closed channels and out-of-plane radiation channels), with all waves coupled to open channels undergoing destructive interference ([Figure 5: see original paper]d). At the  $\Gamma$  point, due to structural geometric symmetry, all parameters are symmetric, and the weights and phases of different in-plane waves coupled to open channels satisfy interference cancellation conditions, achieving zero radiation. Adjusting wavevector parameters away from the

$\Gamma$  point may create new symmetries, allowing the weights and phases of different in-plane waves coupled to open channels to again satisfy interference cancellation conditions, suppressing overall radiation in open channels to near zero. Of course, if certain symmetries of the structure are not guaranteed, the degrees of freedom of Fourier amplitudes may increase, requiring more parameters for adjustment. Moreover, adjusting wavevector parameters is not the only method for obtaining single-resonance BICs. For example, in a metasurface based on open dielectric rectangular blocks, single-resonance BICs have been discovered in certain TM modes by adjusting the opening position.

[Figure 5: see original paper] Single-resonance BICs. (a) Adjusting the incident wavevector can re-enter the BIC, with Q-factor variations shown in (b). When the structure possesses time-reversal symmetry, C2 symmetry, and up-down mirror symmetry, (c) adjusting the two parameters  $k_x$  and  $k_y$  can make both s and p component amplitudes ( $C_s, C_p$ ) zero, thereby obtaining zero output propagating power [98]; (d) Schematic diagram of coupled-wave theory explaining the origin of single-resonance BICs, considering it derived from destructive interference of all waves coupled into the open channel [100].

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## 2.5 Temporal Coupled-Mode Theory

Quasi-BICs result from weak coupling between BICs and far-field radiation. Temporal coupled-mode theory has been widely used to provide simple analytical descriptions for optical resonant objects weakly coupled to input and output ports [101-104]. This theory works well in the weak coupling regime; in practice, when  $Q > 30$ , the analytical results derived from this theory are nearly exact, which matches the quasi-BIC scenario. Therefore, temporal coupled-mode theory has become one of the fundamental mathematical theories for explaining BIC phenomena [105]. Its power manifests in several aspects: for instance, it successfully predicts the existence of resonance-coupled BICs through finite mathematical steps and even predicts the unique wavevector-space polarization characteristics of BICs. More importantly, this theory is a phenomenological theory for describing resonant phenomena that completely disregards the origin of resonances. Regardless of BIC type, radiation occurs with quasi-BIC formation, where a fundamental element is the coupling between resonators and ports. Consequently, this theory is universally applicable for describing scattering characteristics of arbitrary quasi-BICs.

The importance of temporal coupled-mode theory is self-evident. Next, we demonstrate how it predicts the existence of resonance-coupled BICs. A crucial parameter is the Hamiltonian matrix representing resonant frequencies and decay rates (damping rates). A resonator's resonant mode is a spatiotemporal dynamic field, mathematically separable into an  $e^{i(\omega t)}$  term representing temporal characteristics. Let the dynamic field amplitude be  $a$ , its magnitude determined by the energy stored in the mode. First, assume the system is a

closed system without ports and with energy conservation. Taking the time derivative yields the dynamic relationship of its resonance amplitude:

$$\frac{da}{dt} = -i\Omega a, \quad (6)$$

where  $\Omega$  is a frequency matrix. An important prerequisite for temporal coupled-mode theory to predict BIC existence is that the system contains more than two resonances, so  $\Omega$  must be at least a second-order matrix:

$$\Omega = \begin{pmatrix} \omega_{01} & k \\ k & \omega_{02} \end{pmatrix}, \quad (7)$$

where  $\omega_{01}$  and  $\omega_{02}$  represent the resonant frequencies of two modes, and  $k$  denotes the coupling rate between modes (near-field coupling). When two ports are introduced into such a resonant system, incident waves  $I$  from the ports couple to the resonator, and resonances inside the resonator decay to the ports with decay rate  $\Gamma$ . The system's dynamic relationship can then be expressed as:

$$\frac{da}{dt} = -i\Omega a + Da, \quad (8)$$

where  $D$  represents the coupling coefficient matrix of resonances coupling to ports, which is the transpose of the matrix for ports coupling to the resonator. Both  $\Gamma$  and  $\Omega$  are  $2 \times 2$  matrices:

$$\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}, \quad (9)$$

If system losses are not considered and the decay rate is completely determined by radiative loss, then  $\gamma_1$  and  $\gamma_2$  represent the radiative losses of the two modes.  $\gamma_{12}$  and  $\gamma_{21}$  are complex conjugates, indicating far-field radiative coupling between the two modes. The system's Hamiltonian matrix is then:

$$H = \Omega + i\Gamma = \begin{pmatrix} \omega_1 - i\gamma_{11} & k - i\gamma_{12} \\ k - i\gamma_{21} & \omega_2 - i\gamma_{22} \end{pmatrix}. \quad (10)$$

Solving the eigenvalues of the Hamiltonian matrix yields eigenfrequencies and decay rates. Notably, since the description of quasi-BIC frequencies and damping is a complex number  $\omega - i\gamma$  with real central frequency  $\omega$ , the frequency term in equation (10) is generally made real by taking the Hamiltonian as  $H = \Omega + i\Gamma$ . On the other hand, since the temporal characteristic of resonances can also be mathematically described as  $e^{\hat{(-i\omega t)}}$ , the system Hamiltonian matrix becomes  $H = \Omega - i\Gamma$ , which corresponds exactly to the form proposed in Section 2.2.

Following equations (1-5), the condition for forming BICs with zero decay rate can be derived, which will not be repeated here. Regardless of the Hamiltonian matrix form, the eigenfrequency and decay rate values remain unchanged.

As mentioned, temporal coupled-mode theory is a phenomenological theory for describing resonant phenomena. Although it cannot explain the origins of BIC types other than resonance-coupled BICs, it is suitable for describing scattering characteristics of arbitrary quasi-BICs. Consider a quasi-BIC resonant mode of any type with complex frequency  $\omega_0 - i\gamma$ . The frequency term  $\Omega$  and decay rate term  $\Gamma$  in equations (6-8) become single values rather than matrices. Equation (8) is rewritten as:

$$\frac{da}{dt} = -i\omega_0 a - \gamma a + Da, \quad (11)$$

For scattered waves  $O$ , contributions arise from two aspects: first, background scattering directly generated by coupling between input wave  $I$  and output wave, represented by background scattering matrix  $C$ ; second, output waves from BIC resonance coupling to ports, which is the product of coupling coefficients and resonance amplitude, represented by  $Da$ . The complete scattered wave can be expressed as:

$$O = CI + Da. \quad (11)$$

Here,  $S$  represents the total scattering matrix considering both contributions:

$$S = C + D \frac{1}{i(\omega_0 - \omega) + \gamma} D^T. \quad (13)$$

For scattering systems satisfying time-reversal symmetry, reflection and transmission coefficients in the background scattering matrix have a  $\pi/2$  phase difference, with reflection coefficient  $r$  and transmission coefficient  $t$ . For lossless systems,  $|r|^2 + |t|^2 = 1$ . Furthermore, the resonator and ports can be merged into a new closed resonant system. Due to energy conservation, this new closed resonant system also has amplitude  $a$ , so equation (11) can be written as:

$$\frac{da}{dt} = -i\omega_0 a - \gamma a + DI. \quad (12)$$

Thus, we obtain the total scattering matrix:

$$S = C + D \frac{1}{i(\omega_0 - \omega) + \gamma} D^T. \quad (14)$$

Additionally, due to overall energy conservation,  $D$ ,  $C$ , and  $\Gamma$  are not independent. Their constraint relationship is:

$$D^\dagger D = 2\Gamma, \quad (15)$$

$$CD^* = -D, \quad (16)$$

where  $D^*$  and  $D^\dagger$  are the conjugate and Hermitian conjugate of  $D$ , respectively. Considering the system's physical characteristics and determining matrix  $D$  through the relationship in equations (15-16), substituting into equation (14) yields the analytical expression for the total scattering matrix  $S$ .

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## 2.6 Electromagnetic Multipole Theory

Theoretical understanding of BIC properties continues to develop, with electromagnetic multipole theory being representative [106]. In recent years, electromagnetic multipole theory has attracted significant attention in nanophotonics as a research tool for investigating fundamental resonances. Multipole decomposition, the concrete manifestation of this theory, represents arbitrary field distributions as superpositions of fields created by a set of multipoles—mathematically expanding arbitrary fields in series, with contributions from different multipoles reflected in the multipole expansion results. Multipole expansion has been widely used to determine the polarization and radiation patterns of scattering fields from single particles and their clusters (including plasmonic and dielectric systems), applicable to various fields such as polarization control, dielectric nanoantennas, and optical demultiplexing. Multipole decomposition methods have explained many new optical phenomena, including anapole effects, optomechanical effects, and Kerker effects [107-109].

V. Savinov et al. summarized the characteristics of different multipoles and their far-field radiation physical images [110], making it easier to understand BIC formation from an electromagnetic multipole perspective. Multipoles are mainly divided into three families: electric multipoles, magnetic multipoles, and toroidal multipoles. Each family includes dipoles, quadrupoles, and octupoles, intuitively represented in [Figure 6: see original paper]a. Electric dipoles and magnetic dipoles are well-known and were the first two fundamental multipoles proposed in electromagnetics. An electric dipole consists of a pair of positive and negative charges moving along the dipole direction. A magnetic dipole is an oscillating ring current with magnetic moment at the center and perpendicular to the current loop plane. The concept of toroidal dipoles was only established in 2010 [111], generated by oscillating angular currents flowing along the meridian of a cylindrical ring. Intuitively, it can be viewed as countless magnetic dipoles connected head-to-tail, with the toroidal dipole moment at the cylindrical ring center and perpendicular to the ring plane. Researchers have long considered toroidal dipoles as the third electromagnetic field source independent of electric and magnetic dipoles because approximate treatment during

multipole expansion yields this independent term. However, recent studies have found that toroidal dipoles are contained within the electric dipole expression and are actually higher-order terms of electric dipoles [112,113].

These three types of dipoles form the basis for the next-level members of each family. For example, quadrupoles are created by two pairs of anti-aligned dipoles, and octupoles are created by anti-aligned quadrupoles. All multipoles have no electromagnetic radiation field along their dipole directions, with fields radiating in other directions.

Clarifying the radiation images of different multipoles makes intuitive understanding of BIC formation easier. As shown in [Figure 6: see original paper]b, in subwavelength structure arrays, each multipole contributes to the far field, and there may exist a direction where no multipole contributes to the far field, or where non-zero contributions from different terms ultimately sum to zero. Specifically ([Figure 6: see original paper]c), symmetry-protected BICs at the  $\Gamma$  point have no far-field radiation along the z-axis direction, with each multipole's field in each unit cell having no radiation in the z direction, resulting in zero total radiation. Away from the  $\Gamma$  point, for specific directions  $\mathbf{k}$ , although each multipole may radiate, the sum of radiation from all multipoles may be zero, forming interference-canceling BICs (also called parameter-tuned or accidental BICs).

[Figure 6: see original paper] Electromagnetic multipole theory of BICs. (a) Electromagnetic multipole structure and its radiation image [110]; (b) Schematic diagram of multipole radiation in periodic structures, and (c) multipole interaction mechanism of symmetry-protected BICs and accidental BICs [106].

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### 3 Fundamental Characteristics Underlying BIC Applications

Complex polarization distributions in momentum space and high quality factors are the two fundamental physical characteristics of BICs, laying the foundation for a series of optical BIC applications.

#### 3.1 Radiation Polarization Characteristics in Momentum Space

Polarization is one of the most fundamental properties of electromagnetic waves. Polarization control is crucial in many fields including three-dimensional imaging, optical communications, and quantum optics. Traditional waveplates and polarizers are bulky and functionally limited as polarization modulation elements. In recent years, increasing attention has focused on using compact sub-wavelength periodic structural devices to modulate light polarization, which are more suitable for on-chip devices. Photonic crystal slabs have attracted interest for polarization modulation applications due to their simple fabrication, designable band structures, and complex polarization characteristics topologically connected to BICs in momentum space [63], which benefit polarization modulation.

For two-dimensional photonic crystal slabs, a series of Bloch resonant modes with different frequencies and wavevectors form photonic bands. The radiation polarization states of these modes at any frequency band can be projected onto the structural plane and mapped onto the Brillouin zone, defining the polarization field in momentum space. Most of these Bloch modes are radiative modes unless interference cancellation or symmetry mismatch makes them non-radiative—i.e., BICs. BICs appear as vortex singularities (V-points) in this field where polarization states cannot be defined, while polarization states around BICs in the Brillouin zone are linear and exhibit vortex variations ([Figure 7: see original paper]a). This polarization characteristic may generate new physics in topological photonics.

To characterize system polarization properties, Bloch mode polarization states can be mapped onto the Poincaré sphere, with coordinates specified by Stokes parameters  $S_0$ ,  $S_1$ ,  $S_2$ , and  $S_3$ . The coverage of Poincaré curves characterizes the system's polarization properties. BIC-based photonic crystal slabs support only resonant states with nearly linear polarization, and the corresponding polarization pattern projected from the Brillouin zone onto the normalized Poincaré sphere is a band near the equator plus a singularity. The momentum-space polarization states of BIC photonic crystal slabs cover only a small portion of the Poincaré sphere surface, with no coverage at the poles (circular polarization states), indicating limited capability in full-Stokes polarization modulation and restricting photonic crystal slab applications.

Professor Jian Zi's team at Fudan University eliminated vortex polarization singularities (V-points) on photonic bands by breaking the in-plane  $C_2$  symmetry of photonic crystal slabs ([Figure 7: see original paper]b) [114]. When singularities are destroyed, the winding of polarization state principal axes is preserved, leading to the generation of paired circular polarization states (C-points) near the  $\Gamma$  point. Utilizing linear polarization state lines (L-lines) surrounding the original BIC position, the generation of C-points can even achieve full coverage of the Poincaré sphere surface ([Figure 7: see original paper]c). This phenomenon provides a new degree of freedom for polarization modulation, with C-points finding applications in light-matter interactions.

[Figure 7: see original paper] Radiation polarization characteristics of BICs and quasi-BICs in momentum space [114]. (a) Radiation polarization states in momentum space with symmetry-protected BIC as an example; (b) Schematic diagram of radiation polarization state changes in momentum space after breaking  $C_2$  symmetry; (c) Radiation polarization states in momentum space of quasi-BICs can cover the entire Poincaré sphere.

**3.2 High Quality Factor Characteristics** In experiments, the Q-factors of periodic structures supporting quasi-BICs easily reach magnitudes of  $10^2$ – $10^4$ , yet there remains room for improvement. Factors limiting quasi-BIC Q-factors include material absorption, finite sample size, and scattering losses caused by fabrication imperfections—common issues for many high-Q on-chip

resonators. In 2019, Professor Chao Peng's group at Peking University dramatically increased quasi-BIC Q-factors by integrating multiple BICs in momentum space into a single BIC [115]. They considered a photonic crystal slab with circular holes in a square lattice, finding nine BICs in momentum space within the transverse electric mode, each appearing as a topological defect (vortex) in the momentum-space polarization field with topological charge  $\pm 1$ . Among these nine vortices, one is fixed at the center of the Brillouin zone due to symmetry, while the positions of the other eight move toward the center as the hole size increases. When the hole size increases further, the eight off-center BICs move toward the center until all nine BICs are isolated. The topological configuration of BICs controls the Q-factor of an isolated BIC, which decays quadratically with momentum space  $Q \propto 1/k^2$ . However, during the merging of all nine BICs, radiative loss is suppressed, changing the scaling to  $Q \propto 1/k^6$ . The Q-factor of the merged BIC is always several orders of magnitude higher than that of isolated BICs in all directions of momentum space (Figure 8: see original paper [c]), with experimental Q-factors reaching  $10^5$ . In 2022, building upon BIC merging [116].

[Figure 8: see original paper] Construction of quasi-BICs with high Q-factors and dynamic BICs. (a) Photonic crystal slab supporting nine BICs; (b) Adjusting the period size gradually merges the nine BICs into one; (c) Q-factor variation diagram of quasi-BICs before and after merging, where the merged quasi-BIC's Q-factor always exceeds that of isolated quasi-BICs [115]; (d-e) Schematic diagrams of dynamic BIC structure and function based on photo-doped rectangular and cylindrical silicon [117,119]; (f) Image of a patterned graphene-metal metasurface device that realizes dynamic switching between BIC and quasi-BIC by tuning the graphene Fermi level [120].

BICs represent a strong light-matter interaction. Beyond statically increasing quasi-BIC Q-factors, researchers also desire dynamic control over this interaction strength. Song Han et al. designed an all-silicon dielectric metasurface supporting quasi-BICs, using short-pulse optical pumping with photon energy greater than silicon's bandgap to generate tunable free carriers in the material. The recombination dynamics of photo-generated carriers can be customized on ultrafast timescales, enabling ultrafast dynamic control of quasi-BIC quality factors ([Figure 8: see original paper]d) [117]. The fundamental mechanism of this technique is modifying the complex eigenfrequency through photo-doping or equivalent complex refractive index, increasing non-radiative loss rates and reducing Q [118]. Kebin Fan et al. also used this principle, based on an all-silicon suspended cylindrical periodic slab supporting high-Q quasi-BICs, to control Q-factors by modifying silicon carrier density through photo-doping, achieving dynamic Q-factor variation exceeding two orders of magnitude ([Figure 8: see original paper]e) [119]. Beyond photo-doping modulation, electrical doping modulation has also been proposed. By introducing patterned graphene into metal metasurfaces supporting BICs or quasi-BICs, with graphene located beneath the metal structure layer and silicon as the substrate, applying voltage between the silicon substrate and metal injects carriers from silicon into the graphene between the metal structure and substrate, enabling dynamic switching of graphene conductivity between semiconductor-like and metal-like phases ([Figure 8: see original paper]f) [120]. When the graphene Fermi level is low, the

graphene effect is negligible (equivalent to no graphene), but when the graphene Fermi level increases to a level comparable to metals, the patterned graphene becomes equivalent to a metal with the same pattern. Tuning the graphene Fermi level from low to high is equivalent to transforming the metal structure from defective to complete, enabling switching of the entire metasurface between BIC and quasi-BIC states.

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#### 4.1 Applications in Narrowband Filtering and High-Sensitivity Sensing

The intuitive characteristic of a sharp quasi-BIC resonance peak within a spectral range naturally suggests filter applications in spatial optical communications. In 2014, J. M. Foley et al. reported a narrowband transmission filter based on a suspended silicon grating fabricated on silicon-on-insulator, operating in the long-wave infrared spectrum [71]. The working principle involves coupling incident light into two grating modes with overlapping frequencies but different coupling strengths. The strongly coupled mode produces a broadband reflective resonance, providing low background transmittance, while the symmetry-protected quasi-BIC as a weakly coupled mode generates narrow transmission peaks within the background, thereby achieving narrowband transmission filtering ([Figure 9: see original paper]a). Leveraging the high Q-factor characteristic of quasi-BICs also enables high-sensitivity sensing with low detection limits [121-123]. Changes in ambient refractive index cause quasi-BIC resonance peak shifts. Various biological and chemical molecules, such as proteins, cells, and glycerol, can cause ambient refractive index changes. For example, integrating microfluidic channels enables real-time, rapid liquid refractive index detection [72], and detection of cells, proteins, and glucose concentrations in biochemical fields ([Figure 9: see original paper]b-c) [124,125].

[Figure 9: see original paper] Applications of BICs in narrowband filtering and sensing. (a) Photonic crystal grating supporting quasi-BIC, where quasi-BIC produces narrow transmission peaks in low background transmittance, realizing spatial narrowband filtering [71]; (b) High-sensitivity molecular sensor based on a pair of inclined elliptical bars supporting quasi-BIC [124]; (c) High-sensitivity refractive index sensor based on crescent shape supporting quasi-BIC [125].

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#### 4.2 Imaging-Based Applications: Spectral Encoding and Edge Detection

Designing BIC operating frequencies to match molecular vibrational frequencies of biomolecules and using special geometric designs to obtain all resonant frequencies covering a broad band enables a multi-pixel array. Each pixel contains multiple identical units and corresponds to a specific operating frequency.

Biomolecules coated on the metasurface experience dramatic enhancement in reflectance spectra at corresponding positions when their vibrational frequencies match the metasurface BIC operating frequency, thereby identifying different biological samples and marking them with specific spectral barcodes. This concept was proposed by Andreas Tittl et al. in 2018 [73], using all-dielectric figure-eight inclined elliptical bars supporting symmetry-protected quasi-BICs. Scaling the unit cells shifts quasi-BIC frequencies. A finite number of identical unit cells form one pixel, with multiple pixels integrated into the same plane. Different pixels have different unit cell scaling ratios, corresponding to different operating frequencies, ultimately forming a multi-pixel array with a discrete spectral range covering  $1350\text{ cm}^{-1}$  to  $1750\text{ cm}^{-1}$  ([Figure 10: see original paper]a-b). This spectral range includes characteristic molecular stretching and bending vibrations found in hydrocarbons and amino acids, suitable for detecting and distinguishing absorption features of biomolecules, environmental pollutants, and polymers, covering applications in biosensing and environmental monitoring. Due to compact spectral frequency variation, this device can read molecular absorption features at multiple spectral points and convert the resulting information into barcode-like spatial absorption maps for imaging ([Figure 10: see original paper]c). This technology can resolve material absorption fingerprints without requiring spectral analysis, frequency scanning, or moving mechanical components, showing great potential in sensitive and versatile miniature mid-infrared spectroscopic devices.

Furthermore, Aleksandrs Leitis et al. proposed in 2019 using the same figure-eight elliptical bars [65] to distinguish absorption spectra of various biomolecules through spectral angle multiplexing. Controlling spectral position via incident angle of mid-infrared light and using a single metasurface to provide resonances at different frequency points enables detection of mid-infrared absorption fingerprint features of different surface-adsorbed molecules by matching reflectance signals at each incident angle with molecular absorption intensities at corresponding resonant frequencies. This method can be implemented using a broadband light source and detector without a spectrometer, reducing research costs and opening new avenues for high-sensitivity, label-free biosensing.

Changing the incident wavevector causes symmetry-protected BICs to leak into quasi-BICs, with leakage rate varying monotonically with wavevector, providing an application basis for image edge enhancement. Image processing based on edge enhancement is particularly useful for data compression, object detection, microscopy, and general computer vision, with potential applications in biological imaging, 3D reconstruction, and autonomous vehicles. Edge enhancement uses spatial differentiation, which can be based on electronic or optical architectures. Optical analog computing can process information directly using optical signals, performing large-scale real-time data processing with minimal power consumption. Traditional optics uses Fourier methods based on lens and filter systems for analog image differentiation, employing multiple conventional lenses (such as in 4f Fourier filtering), resulting in large form factors incompatible with compact integrated systems. Using periodic optical structures such

as metasurfaces and photonic crystals for optical image processing can significantly reduce optical system size. To effectively identify high-order diffraction components of image edges, periodic structures must have gradient-enhanced responses to incident light within a small wavevector range around the vertical direction. This response is mathematically represented by the modulation transfer function. Symmetry-protected BICs have sharp responses to incident wavevectors. Based on this characteristic, You Zhou et al. demonstrated that a specific frequency can be found near the BIC frequency supported by metasurface devices, where the required modulation transfer function can be easily constructed. Although this function does not directly depend on the BIC frequency, it is indeed induced by BIC effects ([Figure 10: see original paper]d-f) [74].

[Figure 10: see original paper] Imaging-based applications of BICs. (a) Molecular spectral encoded pixel array based on inclined elliptical bars supporting quasi-BIC; (b) Spectral range covered by operating frequencies of all pixels; (c) Schematic results of different molecularly encoded images [73]; (d) Edge detection based on metasurface supporting quasi-BIC; (e) Modulation transfer function at a frequency near the quasi-BIC frequency; (f) Edge imaging results [74].

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### 4.3 Applications in Nonlinear Frequency Conversion, Microlasers, and Nanoantennas

Light's polarization of matter includes not only first-order linear effects but also higher-order nonlinear effects. Generally, light-matter interaction is weak, primarily manifesting as first-order linear polarization. However, BICs represent a strong light-matter interaction whose effects are prominent even for second- and third-order nonlinear polarization. In 2018, Luca Carletti et al. theoretically proposed a strategy to dramatically enhance microscale nonlinear responses using the BIC concept. By constructing AlGaAs nanoantennas supporting BICs, second-harmonic conversion efficiency can be improved by at least two orders of magnitude compared with conventional techniques, with smaller device volumes, establishing a new theoretical foundation for efficient optical frequency conversion [75]. In 2019, Professor Jin Liu's team at Sun Yat-sen University experimentally proposed an ultra-high-Q symmetry-protected quasi-BIC metasurface based on silicon thin films with T-shaped rectangular block unit cells ([Figure 11: see original paper]a-c) [126]. Using light at the quasi-BIC frequency as the pump source, they successfully excited the material's nonlinear effects, generating strong second- and third-harmonic light. Compared with conventional excitation of unstructured silicon thin films, exciting the quasi-BIC silicon metasurface produced harmonic intensities several orders of magnitude stronger.

Cavities play a fundamental role in wave phenomena from quantum mechanics

to electromagnetics, determining the spatiotemporal physics of lasers. Generally, cavities are constructed by closing all channels through which waves can escape. Ideal laser cavities reserve only a single light escape channel, which serves as the output channel when optical energy resonantly enhances to a certain threshold inside the cavity, producing high-quality-factor laser output. BICs supported by periodic optical structures are micro-resonant cavities that confine light modes within structures, thus possessing extremely high quality factors. When frequencies are below the diffraction limit, only a single zero-order radiation channel normal to the structure surface exists. These attributes indicate that BIC metasurfaces have potential for laser generation. In 2017, Ashok Kodigala et al. designed a two-dimensional periodic suspended slab supporting BICs based on InGaAsP quantum well material [76]. The finite-sized slab actually supported finite-Q quasi-BICs, with quasi-BIC frequencies within the InGaAsP material's photoluminescence frequency range. Pumping with a 1064 nm nanosecond laser, the device transitioned from spontaneous emission to high-Q stimulated emission as pump power increased, forming a 1.55  $\mu\text{m}$  laser. Larger designed Q-factors corresponded to smaller pump thresholds.

For two-dimensional micro-laser cavities with identical periodic dimensions in  $x$  and  $y$  directions, the leaky channels of supported quasi-BICs are restricted to the direction perpendicular to the structural surface. If one period is carefully adjusted to support a diffraction order at the resonant wavelength, the radiative leakage channel opens. This approach enables control over laser emission directionality for application in micro-emitting antenna fields. Son Tung Ha et al. reported in 2018 an optically fed emitting antenna consisting of a two-dimensional gallium arsenide nanopillar array supported by a quartz substrate [127]. The nanopillars support vertical and in-plane dipole resonances. In one direction, the lattice period is sub-diffractive for an operating wavelength within the gallium arsenide photoluminescence band, while in the other direction the period is diffractive for this wavelength. This design opens a radiative channel, transforming the BIC mode in the sub-diffractive case into a leaky resonance with finite Q-factor, presenting an intersection with  $\mathbf{k}$  in the emission plane ([Figure 11: see original paper]g-h). Beyond controlling emission direction, Rémi Colom et al. used dielectric nanodisk arrays to construct high-Q quasi-BICs, where quasi-BIC frequencies overlap with dipole antenna emission frequencies of the nanodisk structure, theoretically proving that quasi-BICs can greatly enhance antenna emission efficiency [77].

[Figure 11: see original paper] Applications of BICs in nonlinear fields. (a) T-shaped quasi-BIC device for generating frequency-doubled light; (b-c) Comparison of third-harmonic and second-harmonic intensities with those of conventional devices [126]; (d) Quasi-BIC metasurface for generating microlasers; (e) Effect of pump power on emission wavelength and power; (f) Relationship between pump power and laser output power [76]; (g) Sample diagram of optical feed antenna based on quasi-BIC and (h) schematic diagram of light emission direction [127].

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#### 4.4 Vortex Beam Generation and Polarization Control in Momentum Space

As mentioned in Section 3.1,  $\Gamma$ -point BICs have completely non-radiative properties and are polarization vortex singularities in momentum space with non-zero polarization topological charge  $q$ . Around the  $\Gamma$  point, the radiation polarization states of Bloch resonant modes are linear and wind around the  $\Gamma$  point in a vortex pattern. This linear polarization vortex feature can be decomposed into a left-circularly polarized beam and a right-circularly polarized beam carrying opposite phase topological charges of magnitude  $|2q|$  in momentum space, mathematically explainable through temporal coupled-mode theory [79]. Therefore, BIC metasurfaces can perform wavefront vortex transformations on input circularly polarized waves in momentum space. In 2020, Professor Qinghai Song's team at Harbin Institute of Technology and Professor Jian Zi's team at Fudan University successively demonstrated this interesting functionality experimentally ([Figure 12: see original paper]a-b) [78,79]. Furthermore, since photonic crystals are simpler and more lightweight than traditional structures for generating vortex electromagnetic waves such as Q-plates and vortex plates, and given the shortcomings of conventional methods for generating vortex terahertz beams, our research group is currently utilizing bound states in the continuum in momentum space of two-dimensional photonic crystals, combined with terahertz circularly polarized antennas, to fabricate terahertz vortex antennas with adjustable vortex topological charge at 220 GHz, enabling terahertz wave transmission to carry richer physical information. A sample schematic is shown in [Figure 12: see original paper]c.

Vortex generation based on BICs overturns conventional understanding of vortex control. Traditionally, vortex light generation focuses on real space, using bulky vortex phase plates. In recent years, researchers have developed compact phase metasurface devices to control vortex light generation in real space, but such devices have visible vortex geometric arrangements. BIC metasurfaces, as periodic structures in real space without visible vortex geometric configurations, intuitively cannot realize vortex beam generation. However, they indeed possess the ability to generate vortex light in momentum space. This special capability may give BIC metasurfaces advantages over conventional phase metasurfaces in secure electromagnetic communications.

Although  $\Gamma$ -point BICs are polarization singularities in momentum space, once BICs leak into quasi-BICs, the singularities are destroyed and their radiation polarization states at the  $\Gamma$  point in momentum space become defined. Appropriately adjusting structural geometric parameters may result in quasi-BIC  $\Gamma$ -point radiation polarization with significant ellipticity, meaning left- and right-circular polarization states decouple and chiral characteristics emerge. Adam Overvig et al. constructed quasi-BICs with broken in-plane  $C_2$  symmetry based on spatially tightly stacked structures, theoretically proving that such quasi-BIC  $\Gamma$ -point ra-

diation polarization states can be arbitrarily elliptical and proposed the concept of chiral quasi-BICs [128]. Maxim V. Gorkunov et al. constructed chiral quasi-BICs based on paired elliptical bars with broken out-of-plane  $C_2$  symmetry and in-plane  $C_4$  symmetry, theoretically showing that scattered waves under normal incidence have perfect narrowband circular dichroism. They derived an analytical expression for circular dichroism based on temporal coupled-mode theory, proving maximum circular dichroism at quasi-BIC frequencies [80]. However, the structures used in these two works lack up-down mirror symmetry (mirror symmetry about the  $z$ -plane), making them difficult to fabricate in practice. In 2022, to enhance the feasibility of fabricating chiral quasi-BICs, Professor Xi-angping Li's group at Jinan University proposed flat chiral quasi-BICs designed in flat structures with up-down mirror symmetry about the  $z$ -plane. The unit cells were based on dielectric pillars with broken in-plane  $C_2$  symmetry, experimentally verifying the narrowband asymmetric transmission capability of chiral quasi-BICs ([Figure 13: see original paper]a-b) [129]. Our research group further proposed construction methods for chiral quasi-BICs in flat structures and their scattering matrix analytical expressions. By simultaneously breaking in-plane  $C_2$  symmetry and in-plane mirror symmetry of rectangular dielectric blocks, the  $\Gamma$  point of quasi-BICs obtains elliptical radiation polarization states ([Figure 13: see original paper]c-d). In such structures, besides chiral quasi-BICs related to in-plane  $C_2$  symmetry breaking, chiral quasi-BICs related to interference cancellation have also been discovered [130]. The ultra-narrowband asymmetric transmission capability exhibited by chiral quasi-BICs may find applications in chiral molecular sensing.

[Figure 12: see original paper] Applications of BICs in vortex beam generation. (a) Schematic diagram of vortex beam generation in momentum space based on photonic crystal plate supporting BIC; (b) Radiation polarization distribution in momentum space (left) and measured electric field phase and intensity distribution under circularly polarized incidence (right) [78,79]; (c) Our group is combining BIC photonic crystal slab with THz circularly polarized antenna to fabricate THz vortex antenna with adjustable vortex topological charge.

[Figure 13: see original paper] Applications of BICs in asymmetric transmission. (a) Schematic diagram of planar structure device supporting chiral quasi-BIC and (b) its circularly polarized transmission spectrum and circular dichroism spectrum [129]; (c) Schematic diagram of metasurface supporting chiral quasi-BIC with simultaneous breaking of in-plane  $C_2$  symmetry and in-plane mirror symmetry (left), its polarization distribution in momentum space (right), and (d) its circular dichroism spectrum containing multiple chiral quasi-BICs [130].

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## 5 Conclusion

This paper systematically summarizes three major types of BICs emerging in periodic optical systems: symmetry-protected BICs, resonance-coupled BICs,

and single-resonance BICs. It elaborates on their physical and mathematical explanations based on symmetry theory, temporal coupled-mode theory, coupled-wave theory, and electromagnetic multipole theory. Furthermore, it reviews the fascinating physical properties of optical BICs in periodic systems, such as high Q-factor characteristics and special polarization distribution properties in momentum space. Based on these characteristics, optical BICs have enabled rich applications including narrowband filtering, high-sensitivity sensing, molecular spectroscopic encoding imaging, image edge detection, nonlinear frequency doubling, microlasers, micro-antennas, vortex light generation in momentum space, and narrowband asymmetric transmission.

High-Q optical quasi-BICs also have some limitations. On one hand, they are extremely sensitive to structural dimensional parameters—slight dimensional changes can cause enormous frequency shifts, requiring extremely high fabrication precision. On the other hand, intrinsic material losses enhance resonance damping, causing Q-factor degradation. Therefore, quasi-BICs based on metallic materials generally cannot achieve Q-factors above two orders of magnitude. High-refractive-index, low-loss all-dielectric materials are ideal choices for fabricating high-Q quasi-BICs, achieving Q-factors of three to four orders of magnitude. However, maintaining ultra-high Q-factors above four orders of magnitude generally requires extremely thin substrates or even substrate-free designs using mesoporous slab or suspended structures. Above terahertz frequencies, substrate-free structures are thin, especially suspended structures with extremely fine lateral dimensions, making fabrication very difficult with low success rates (even extremely thin substrates are hard to fabricate). Additionally, ultra-high Q-factor spectroscopy requires high-precision testing platforms, increasing testing costs. While Q-factor testing above four orders of magnitude can be accomplished in the visible to near-infrared bands, high-resolution spectroscopy testing for such high Q-factors is almost impossible in the far-infrared to terahertz and even microwave bands due to limitations in light source and spectrometer technologies, restricting multi-band applications of high-Q quasi-BICs.

Nevertheless, although many BIC applications remain limited to laboratory demonstrations due to high fabrication and testing difficulties and costs, continuous research on optical BICs will bring them closer to industrial applications. Future work may reveal more exotic characteristics and new applications, providing innovative vitality for the development of integrated optics, information optics, bio-optics, topological optics, and nonlinear optics.

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