# Postprint: Analysis of Broadband Spectral Variability of the BL Lac Object Mrk 421

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## Abstract

We collected broadband spectral energy distribution (SED) data for Mrk 421 in 73 variability states as our research sample. Using a single-zone leptonic model with a steady-state broken power-law electron spectrum, we performed fits to investigate the physical properties of the jet in Mrk 421 during its variability. Our main results are as follows: (1) There exists a significant anti-correlation between the magnetic field B and the Doppler factor $\delta$ in the sample, indicating that B and $\delta$ are interdependent in the Thomson regime. (2) The electron spectral index p1 supports interpretations such as shock acceleration or magnetic reconnection for the variability of Mrk 421. (3) A positive correlation exists between the synchrotron peak frequency log synpk and the peak luminosity log synpkLsynpk, implying the presence of an anti-blazar sequence during the variability of Mrk 421. (4) According to the equipartition parameter =UeUB (the ratio of relativistic electron energy density to magnetic field energy density), 26% of the states exhibit near-equipartition between electron and magnetic field energies, 63% have electron energy exceeding magnetic energy by one order of magnitude, and 11% have electron energy far exceeding magnetic energy. This suggests that during the variability of Mrk 421, the scenario where electron energy exceeds magnetic energy by one order of magnitude is more probable, and also implies that the jet in Mrk 421 is particle-dominated. The relation Pe>PB further confirms that the jet of Mrk 421 is particle-dominated. PB Pr indicates that the Poynting flux cannot explain the radiative power. The ratio Pr/Pe ranges from 0.01~0.51, indicating that only a small fraction of the relativistic electron energy may be utilized for the observed radiation. (5) While some physical parameter ranges of 3C 279 and Mrk 421 may be relatively close, the physical processes within the jets of these two sources may be different.

## Full Text

# Analysis of Broadband Spectral Energy Distribution Variability Characteristics of the BL Lac Object Mrk 421

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**Abstract:** We have compiled broadband spectral energy distribution (SED) data for Mrk 421, comprising 73 distinct states as our research sample. Using a single-zone lepton model with a steady-state broken power-law electron energy distribution, we fit the SEDs to investigate the physical properties of the jet during variability episodes. Our main results are as follows: (1) A significant anti-correlation exists between the magnetic field strength B and the Doppler factor $\delta$, indicating that these parameters are interdependent in the Thomson regime. (2) The electron spectral index $p_1$ supports shock acceleration or magnetic reconnection scenarios during Mrk 421' s variability. (3) A positive correlation between the synchrotron peak frequency and peak luminosity suggests the presence of an "anti-blazar sequence" during flaring. (4) Based on the equipartition parameter   = U /U_B (the ratio of relativistic electron energy density to magnetic field energy density), 26% of the states show near-equipartition between electron and magnetic field energies, 63% exhibit electron energies an order of magnitude greater than magnetic field energies, and 11% show electron energies far exceeding magnetic field energies. This indicates that Mrk 421 more commonly exhibits electron-dominated conditions during variability, implying a particle-dominated jet. The relationship P  > P_B further confirms the particle-dominated nature, while P_B   P_{rad} indicates that Poynting flux cannot account for the radiated power. The ratio P_{rad}/P  ranges from 0.01 to 0.51, suggesting that only a small fraction of relativistic electron energy may be converted into observed radiation. (5) While some physical parameter ranges may be similar between 3C 279 and Mrk 421, the underlying physical processes in their jets could be different.

**Keywords:** BL Lac object; Mrk 421; variability

Variability represents one of the most prominent characteristics of blazars, and studying these variations holds significant scientific importance. The research content and key questions in blazar variability studies are detailed in [?] and references therein, and will not be elaborated here.

Two primary approaches exist in blazar variability research. The first involves investigating variability at fixed frequencies or bands (e.g., optical/UV, X-ray, or $\gamma$-ray bands) as demonstrated in [?]. The second approach combines multi-band data to construct broadband simultaneous or quasi-simultaneous SEDs. Since differences between various states can be linked to distinct underlying physical mechanisms, model fitting can be employed to study these physical differences, as shown in [?] and [?]. The first approach typically involves time-domain analyses such as RMS-flux relationships and power spectral density, as well as spectral shape studies like hardness ratios and band-specific spectral fitting. Its advantage lies in the relative abundance of observational data at specific frequencies, enabling in-depth investigation. The second approach, while yielding smaller samples of broadband SEDs, offers the crucial advantage of providing information about how physical conditions in the emission region change during variability, thereby offering insights into particle acceleration and radiation processes. This paper adopts the latter methodology.

Mrk 421 is a nearby blazar source that exhibits no prominent emission or absorption lines, shows strong polarization in the optical band, and possesses a compact (milliarcsecond-scale) radio structure. Consequently, it is classified as a BL Lac object. Its broadband SED can be described by the characteristic double-humped shape [?, ?]. Within the broader blazar context, Mrk 421 belongs to the subclass of high-synchrotron-peaked BL Lac objects, with both SED peaks occurring at relatively high energies (~1 keV and ~100 GeV) [?]. Mrk 421 is one of the most comprehensively studied TeV blazars and was the first extragalactic TeV source detected by the Whipple 10 m Cherenkov telescope [?]. With a redshift of z = 0.031 [?], it is among the nearest blazars to us. In recent years, multi-band variability and periodic phenomena in Mrk 421 have attracted considerable attention, with numerous studies addressing these issues [?, ?, ?].

Mrk 421 displays dramatic variability in the optical band, having shown variations of 1.4 magnitudes within 2.5 hours. Subsequent observations revealed rapid and substantial variability in the infrared, X-ray, and $\gamma$-ray bands, including major outbursts [?, ?, ?]. The source appears to be in a nearly continuously active state [?]. Investigating the variability characteristics of Mrk 421 is therefore crucial for understanding blazar physics and constructing physical models.

Numerous studies of the second type have been conducted on Mrk 421 [?]. However, the number of states in these studies remains insufficient for statistically robust analyses (the most extensive being [?] with only 18 states). To address this limitation, we have compiled a substantially larger sample comprising 73 flaring states of Mrk 421, enabling us to analyze both observational and intrinsic physical characteristics during variability and obtain reliable statistical results.

## 1 SED Sample Description

We collected 73 simultaneous, quasi-simultaneous, or non-simultaneous SEDs of Mrk 421 from [?, ?, ?], with observational coverage from radio to $\gamma$-ray bands. This represents the largest SED sample for this source' s variability to date.

In this work, data obtained within one week are defined as simultaneous, while those within two months are considered quasi-simultaneous [?]. Each SED describing a particular epoch is referred to as a "state." A single flaring episode generally comprises multiple states. Details are provided in Table 1 .

## 2.1 Model and Fitting Procedure

We employ the most commonly used homogeneous single-zone lepton model for our fitting. In this model, the emission region is assumed to be a spherical zone of radius R (cm) filled with uniformly distributed electrons. This region maintains its shape while moving relativistically along the jet axis at constant velocity, characterized by the Doppler factor $\delta$ defined as $\delta$ = [$\Gamma$(1 - $\beta$cos )]$^{-1}$, where $\Gamma$ is the bulk Lorentz factor, $\beta$ = v/c, and  is the angle between the jet axis and the observer' s line of sight. The spherical emission region is permeated by a tangled magnetic field of strength B. The electron energy distribution (EED) adopts a broken power-law form:

$$N(\gamma) = \begin{cases} N_0 \gamma^{-p_1} & (\gamma_{\rm min} < \gamma < \gamma_{\rm br}) \\ N_0 \gamma_{\rm br}^{p_2 - p_1} \gamma^{-p_2} & (\gamma_{\rm br} < \gamma < \gamma_{\rm max}) \end{cases}$$

where $\gamma\{min\}$ *and* $\gamma$\{max\} are the lower and upper bounds of the electron Lorentz factor, $\gamma$_\{br\} is the break Lorentz factor, $p_1$ and $p_2$ are the spectral indices below and above the break, and $N_0$ is the normalization factor. With these parameters specified, the single-zone model produces a theoretical SED for fitting.

As noted in [?], fitting BL Lac sources requires fixing one of four parameters –magnetic field B, Doppler factor $\delta$, emission region radius R, or EED normalization $N_0$–to avoid degeneracy (where different combinations of physical parameters yield identical SED fits). Based on the assumption of diffusive shock acceleration and considering that different states are produced by similar shocks [?], previous studies have adopted the approach of fixing the emission region radius. The radius R has been found to range from 0.25 $\times$ $10^{16}$ cm to 5.2 $\times$ $10^{16}$ cm in [?, ?, ?, ?]. Consistent with [?, ?], which determine R based on minimum variability timescales, we fix the emission region radius at R = 1.0 $\times$ $10^{16}$ cm.

For the minimum electron Lorentz factor $\gamma\{min\}$*, we follow [?, ?, ?, ?] and set* $\gamma$\{min\} = 500 to avoid excessive radio flux, properly fit the optical data, and reduce the number of free parameters.

According to [?], blazar radio emission originates outside the jet rather than within it, and jet radiation is significantly suppressed by synchrotron self-absorption at frequencies  $10^{10}$ Hz. Our study focuses on the physical properties of the inner jet' s multi-band SED, so we treat radio data as upper limits on the jet emission, consistent with [?].

During the fitting process, Mrk 421' s X-ray data around the synchrotron peak are relatively abundant, while infrared-optical-UV data are comparatively sparse. Additionally, observed IR-Optical-UV emission may be dominated by thermal radiation from the host galaxy [?]. Therefore, while ensuring sufficiently small $^2$ values, we assign higher weight to X-ray data to better constrain the shape of the synchrotron peak. This effectively treats IR-Optical-UV data as upper limits on the jet emission [?], allowing the synchrotron and inverse Compton peak data to primarily determine the fitting parameters, consistent with the approach in [?]. Evidence in [?] suggests that UV emission originates from a different source than X-ray emission, supporting our treatment.

## 2.2 Fitting Results

The fitted sample and model parameters are presented in Table 1, with SED plots shown in Figure 1 [Figure 1: see original paper] and Figure 2 [Figure 2: see original paper]. These figures display only a subset of the fits for states observed during MJD 55266-55277.

Table 1 details: (1) observation epoch, (2) magnetic field strength, (3) Doppler factor, (4) EED normalization factor, (5) maximum electron Lorentz factor, (6) break Lorentz factor, (7) spectral index below the synchrotron peak, (8) spectral index above the synchrotron peak, and (9) reduced $^2$. Our results demonstrate that the steady-state single-zone homogeneous lepton model with a broken power-law EED provides satisfactory fits to all 73 SEDs covering optical, X-ray, and $\gamma$-ray bands.

## 3.1 Physical Properties and Relationships from Fitting

Based on [?] and [?], the Doppler factor $\delta$ for Mrk 421 ranges from 15 to 50. Our fitted $\delta$ values fall within 15–50, consistent with previous results. For the magnetic field strength, our fitted B values range from 0.012 to 0.327 G, in agreement with the range of 0.02–0.3 G reported in [?] and [?].

## 3.2 The B-$\delta$ Relationship

The relationship between jet magnetic field strength B and Doppler factor $\delta$ reflects the connection between the emission region' s magnetic environment and its bulk motion. Previous studies have yielded conflicting results: [?] found no correlation in a sample of 22 BL Lac objects using one state per source, while [?] reported a significant anti-correlation in a sample of 29 TeV BL Lac objects. Our study examines a single source across multiple states to investigate physical differences between various activity phases, offering a different perspective. Our results reveal a significant anti-correlation between B and $\delta$, with a bisector fit yielding $\log \delta = (-0.61 \pm 0.05) \log B + (0.79 \pm 0.06)$, a correlation coefficient $r = -0.72$, and significance level $p = 7.32 \times 10^{-13}$.

Under the single-zone SSC model' s monochromatic approximation, the Thomson regime (where the scattering cross-section is energy-independent) yields the scaling relation B$\delta$ ( _{syn}/ _{IC})^(1/3)(1 + Z), where _{syn} and _{IC} represent the synchrotron and SSC peak frequencies, respectively, and Z is the redshift. In the Klein-Nishina regime, B/$\delta$ ( _{syn}/ _{IC})^(1/2). If _{syn}/ _{IC} remains within a narrow range during Mrk 421' s variability, an anti-correlation between B and $\delta$ naturally emerges, consistent with our analysis and the conclusions of [?].

## 3.3 Relativistic Electron Energy Distribution

Studying the relativistic electron spectral index $p_1$ provides insights into particle acceleration mechanisms. Previous work by [?] examined 22 BL Lac objects (one state per source), finding $p_1$ ranging from 1.40 to 2.22 and $\Delta p = p_2 - p_1$ ranging from 1.00 to 3.00, with most $\Delta p$ values near 2.0 but some cases of $p_1$ $< 1.6$ and $\Delta p > 2.0$. [?] analyzed seven states of Mrk 421 from August 2008 to February 2013, finding $\Delta p > 1$, indicating that the spectral break is not caused by radiative cooling. Our work investigates electron spectral index variations across a larger sample of Mrk 421 states during variability.

Our fitted $p_1$ values range from 1.65 to 2.38. According to [?], relativistic diffusive shock acceleration predicts a canonical particle spectral index $p_1$  2.2. In our sample, 58% of states have $p_1$ between 2.00 and 2.38, consistent with diffusive shock acceleration. However, 42% exhibit $p_1 < 2.0$, requiring alternative mechanisms. [?] proposed that particles accelerated by shocks over short timescales can undergo subsequent stochastic acceleration downstream, producing harder spectra than the first-order Fermi limit. [?] suggested that large scattering angles or extreme conditions in shock acceleration can yield $p_1 <$ 2.0. [?] demonstrated that magnetic reconnection can also produce hard electron spectra. The $p_1$ range observed in Mrk 421 thus suggests that acceleration mechanisms during variability extend beyond relativistic diffusive shock acceleration to include large scattering angles, downstream stochastic acceleration, or magnetic reconnection.

Furthermore, with $\Delta p$ ranging from 1.19 to 3.16—exceeding the typical value of $\Delta p = 1$ expected from radiative cooling breaks [?]—the spectral break cannot be attributed to cooling. [?] proposed that the break reflects a characteristic of the acceleration process, with large $\gamma${br} *potentially arising from an unknown acceleration mechanism. The break Lorentz factor* $\gamma${br} likely represents the maximum achievable energy, determined by the particle acceleration timescale within the shock region.

## 3.4 Synchrotron Peak Frequency

The relationship between synchrotron peak frequency ( _{syn}) and peak luminosity has been studied for both blazar populations and individual sources. [?] first investigated this relationship using one state per source. [?] found a positive correlation for Mrk 421 and later combined data from [?] and [?] to obtain L_    _{syn}^(0.48$±$0.10). [?] discovered correlations in four states each of 3C 279 and Mrk 501. [?] examined 14 high-synchrotron-peaked BL Lac objects, finding that 5/14 showed positive correlations (anti-blazar sequence), while 9/14 showed no or negative correlations (blazar sequence). The anti-correlation is termed the "blazar sequence," while the positive correlation is called the "anti-blazar sequence."

Our analysis of Mrk 421 reveals a positive correlation between _{syn} and peak luminosity, with Pearson coefficient r = 0.40 and p = 3.99 $\times$ $10^{-4}$, indicating the presence of an anti-blazar sequence during variability.

In the single-zone SSC model with monochromatic approximation, the relations _{syn} B$\delta^2\gamma${br}$^2$ *and L_* N$\gamma${br}$^4$B$\delta^4$ hold, where N is the relativistic electron number density. [?] showed that when $\alpha$ = 1, variability is dominated by $\gamma${br}*; when $\alpha$ = 2, by magnetic field changes; when $\alpha$ = 4, by Doppler factor $\delta$; and when $\alpha \to \infty$, by electron density N. Our result* {syn} ( L_ )^(0.48$±$0.10) suggests that $\delta$ and N are unlikely to be the dominant factors driving Mrk 421' s variability, which appears to be primarily governed by changes in $\gamma$_{br}.

## 3.5 Jet Physical Properties

Investigating jet power provides insights into jet formation and composition. We estimate jet power by assuming it comprises four components: relativistic electron power, cold proton power, magnetic field power, and radiation power [?]:

$$P_{\rm jet} = \pi R^2 \Gamma^2 c (U_e + U_p + U_B + U_{\rm rad})$$

where U_e is the relativistic electron energy density, U_p is the cold proton energy density, U_B is the magnetic field energy density, and U_{rad} is the radiation energy density, calculated as:

$$U_e = m_e c^2 \int N(\gamma)\gamma d\gamma$$

$$U_p = m_p c^2 \int N(\gamma) d\gamma$$

$$U_B = \frac{B^2}{8\pi}$$

$$U_{\rm rad} = \frac{L_{\rm bol}}{4\pi R^2 c \delta^4}$$

with L_{bol} = 4$\pi$D_L$^2$F being the observed total non-thermal luminosity, D_L the luminosity distance, and F the observed flux. We also compute the equipartition and magnetization parameters following [?]:

$$\varepsilon = \frac{U_e}{U_B}$$

The derived parameters are listed in Table 2 , with columns detailing: (1) observation epoch, (2) total jet power, (3) relativistic electron power, (4) magnetic field power, (5) cold proton power, (6) radiation power, (7) relativistic electron energy density, (8) magnetic field energy density, and (9) equipartition parameter.

Previous studies of Mrk 421' s equipartition during variability have reported three scenarios: (i) near-equipartition [?, ?], (ii) electron energy exceeding magnetic energy by one order of magnitude [?, ?], and (iii) electron energy far exceeding magnetic energy [?, ?]. We categorize these as: $1 < \;\; < 10$ (near-equipartition), $10 \leq \;\; < 100$ (electron-dominated by one order), and $\;\; \geq 100$ (strongly electron-dominated).

Our large sample reveals all three scenarios: 26% of states show near-equipartition, 63% exhibit electron energies one order of magnitude greater than magnetic energies, and 11% show electrons strongly dominating. This suggests that Mrk 421 most frequently exhibits electron energies exceeding magnetic energies by an order of magnitude during variability, indicating a particle-dominated jet, consistent with [?, ?, ?].

Table 3 presents the average percentage of total power carried by each component:

| Component | Average Power Fraction |
|---|---|
| P_e/P_{jet} | ··· |
| P_p/P_{jet} | ··· |
| P_B/P_{jet} | ··· |
| P_{rad}/P_{jet} | ··· |

The hierarchy P_{rad} ≤ P_B < P_p < P_e emerges, with P_e > P_B confirming the particle-dominated nature of Mrk 421's jet [?, ?, ?]. Additionally, P_B  P_{rad} indicates that Poynting flux cannot explain the radiated power, consistent with [?]. The ratio P_{rad}/P_e ranges from 0.01 to 0.51, implying that only a small fraction of relativistic electron energy may be converted into observed radiation, also in agreement with [?].

## 3.6 Comparison of Variability Characteristics Between 3C 279 and Mrk 421

To better understand differences and similarities in variability physics across sources, we compare Mrk 421 with 3C 279 from [?]. First, regarding the electron spectral index $p_1$: 3C 279 shows $p_1 = 1.70$–$2.70$, while Mrk 421 exhibits $p_1 = 1.65$–$2.38$. The similar ranges suggest that both sources may involve shock acceleration or magnetic reconnection.

Second, concerning blazar sequences: 3C 279 does not exhibit a blazar sequence during variability, whereas Mrk 421 shows an anti-blazar sequence. This demonstrates that not all sources display such sequences during flaring, and that 3C 279 lacks a single dominant factor [?], while Mrk 421' s variability may be dominated by changes in γ_{br}.

Finally, comparing jet powers and equipartition parameters: 86% of 3C 279 states in [?] have > 1, while all Mrk 421 states in our sample have > 1. This indicates that Mrk 421' s jet electrons always exceed magnetic energy during variability, whereas 3C 279 does not necessarily show this. Both sources exhibit particle-dominated jets, with P_B P_{rad} in both cases, confirming that Poynting flux cannot explain the radiation power [?]. The condition P_{rad} < P_e suggests that a large fraction of relativistic electron power is required to produce the observed radiation [?]. This comparison reveals that despite similar parameter ranges, the physical processes within the jets of these two sources may differ [?].

## Summary

For Mrk 421, we have fitted 73 states using a steady-state single-zone homogeneous lepton model to study the source' s physical characteristics during variability. Our results can be summarized as follows:

1. An anti-correlation exists between magnetic field strength B and Doppler factor $\delta$, indicating that these parameters are primarily interdependent in the Thomson regime.
2. The electron spectral index $p_1$ supports shock acceleration scenarios or alternative mechanisms involving large scattering angles, downstream stochastic acceleration, or magnetic reconnection during variability.
3. The relationship between synchrotron peak frequency and peak luminosity follows _{syn} ( L_ )^(0.48$±$0.10), indicating an anti-blazar sequence during Mrk 421' s variability, likely dominated by changes in γ_{br}.
4. All equipartition parameters = U_e/U_B exceed unity: 26% of states show near-equipartition, 63% exhibit electron energies one order of magnitude greater than magnetic energies, and 11% show strong electron dominance. This confirms a particle-dominated jet. The condition P_B P_{rad} indicates that Poynting flux cannot explain the radiated power, while P_{rad}/P_e = 0.01-0.51 suggests that only a small fraction of relativistic electron energy produces the observed radiation.
5. Although some physical parameter ranges are similar between 3C 279 and Mrk 421 (e.g., $p_1$, P_B P_{rad}, P_{rad} < P_e), the physical processes within their jets may differ.

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*Note: Figure translations are in progress. See original paper for figures.*