Modeling of Grid-Following/Grid-Forming Devices from a Dual-Axis Current-Source Synchronous Machine Perspective and Synchronous Stability Analysis of Their Interconnected Systems

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Abstract

With the development of large-scale renewable energy and power electronics technology, power systems have gradually evolved into an interconnected configuration comprising grid-forming and grid-following devices, where typical grid-forming devices mainly include virtual synchronous machines and synchronous generators, while typical grid-following devices include phase-locked loop (PLL) based converters. Synchronous stability constitutes the foundation for large-scale power grid operation; however, due to significant disparities in synchronization mechanisms between grid-forming and grid-following devices, the analysis of synchronous stability in interconnected systems proves extremely challenging. To this end, this paper physically constructs an equivalent structure capable of unifying grid-forming and grid-following devices by leveraging the concept of a two-axis synchronous machine, mathematically proposes a general modeling methodology for characterizing device synchronization dynamics, and establishes an interconnected system synchronous stability analysis model based on the two-axis current-source synchronous machine. Furthermore, it provides the mathematical definition of synchronous stability for interconnected systems and derives sufficient conditions for ensuring synchronous stability. Finally, simulations implemented in MATLAB/Simulink validate the rationality of the proposed model and analysis.

Full Text

Abstract

With the large-scale integration of new energy sources and power electronics, modern power systems have gradually evolved into an interconnected architecture comprising grid-forming and grid-following devices. Grid-forming equipment primarily includes virtual synchronous machines and synchronous generators, while grid-following equipment mainly consists of phase-locked loop (PLL) converters. Synchronous stability is fundamental to the operation of large-scale power grids. However, due to significant differences in synchronization mechanisms between grid-forming and grid-following devices, analyzing the synchronous stability of interconnected systems is extremely challenging. This paper leverages the physical concept of dual-excitation winding synchronous machines to construct a unified equivalent structure for grid-forming and grid-following devices. Mathematically, a universal modeling approach is proposed to describe the synchronization characteristics of equipment, and a synchronous stability analysis model for interconnected systems is established based on current sources. Furthermore, the mathematical definition of synchronization stability in interconnected systems and several sufficient conditions for system synchronization stability are presented. Finally, simulation examples built in MATLAB/Simulink verify the rationality of the proposed model.

Keywords: Interconnected system; Synchronous stability; Dual-axis synchronous machine; Current source

0 Introduction

The development of new energy power systems is a crucial initiative for achieving China' s "carbon peak and carbon neutrality" goals and holds significant importance for driving the nation' s energy transformation [?]. The interconnection of grid-forming and grid-following Voltage Source Converters (VSCs) represents a primary characteristic of new energy power systems (hereinafter referred to as interconnected systems). Currently, grid-forming devices that independently establish voltage and frequency mainly include synchronous generators and Virtual Synchronous Generators (VSGs) [?]. Grid-following devices achieve synchronization with the grid by detecting voltage phase through calculation or closed-loop control; this study focuses on the typical PLL-synchronized converter (hereinafter referred to as PLL-VSC).

As converter penetration increases, the stability patterns and characteristics of interconnected systems have evolved, raising questions about the adaptability of traditional analysis methods [?]. Among these, synchronous stability, as the foundation of AC grid operation, requires renewed understanding and expanded analysis and control theories to ensure stable operation of interconnected systems. Therefore, accurately comprehending the synchronous stability characteristics of interconnected systems has become an urgent problem.

Unlike the "physical synchronization" dominated by relative rotor motion in synchronous generators, the synchronization mechanisms of VSGs and PLL-VSCs are determined by control algorithms, manifesting as "control synchronization" [?]. VSGs emulate synchronous generator rotor swing equations and achieve synchronization through power control, exhibiting "power angle" stability similar to synchronous generators under normal operating conditions [?]. However, converter current limiting constraints may cause VSGs to switch to current source operation during large disturbances, making their dynamic behavior more complex [?] and hindering system-level understanding of VSG synchronous stability characteristics under current limiting constraints.

Furthermore, a unified understanding of PLL-VSC synchronous stability mechanisms has not yet been established. In single PLL-VSC grid-connected systems, scholars have classified PLL-dominated sub/super-synchronous oscillations and loss-of-synchronization issues as PLL-VSC synchronous stability problems [?, ?]. For PLL-dominated oscillations, the widely adopted method in academia involves establishing frequency-domain impedance models and applying Nyquist criterion analysis [?]. Regarding PLL loss-of-synchronization, some papers have proposed generalized swing equation modeling approaches [?] and explored the application of equal-area criterion and direct methods in single-machine transient synchronization stability analysis [?]. However, these studies still struggle to provide intuitive physical understanding of dynamic response behaviors under interactions between the PLL and other control loops. When studying multi-PLL-VSC grid-connected systems, the coupling of electromechanical-electromagnetic multi-timescale dynamics in synchronization makes system models high-order and extremely difficult to analyze, resulting in relatively few studies. Modal decoupling methods can reduce multi-converter systems to equivalent single machines [?], lowering model order to simplify analysis and facilitate quantitative assessment of synchronization stability margins, but they fail to reveal the synchronization stability mechanisms among multiple machines.

Although extensive research has been conducted on synchronous stability analysis for single-type devices, studies on interconnected scenarios remain in their infancy due to differences in synchronization mechanisms between grid-forming and grid-following devices. Current research can only investigate how factors such as grid-following renewable energy penetration and connection points affect synchronous generator angle stability in specific scenarios [?]. Specifically, at the device level, the diversity of converter control types and the complexity of cascaded control loops have prevented existing research from clarifying the relationships between different converter types or revealing the physical roles of multiple control loops in converter synchronization. At the system level, limited by the deep coupling between converter electromagnetic dynamics and the network, as well as differences in synchronization mechanisms between grid-forming and grid-following devices, synchronous stability analysis of interconnected systems is extremely difficult, with most existing research limited to synchronous generator angle stability.

Therefore, it is necessary to explore the commonalities in synchronization mechanisms among synchronous generators, VSGs, and PLL-VSCs and establish reasonable models as a foundation for interconnected system analysis. To address these problems, this paper' s main contributions are:

1. By leveraging the concept of "dual-axis synchronous machines," a unified equivalent structure for synchronous generators and converters is constructed. Both dynamics consist of control coordinate system phase dynamics and d-axis/q-axis dynamics, where synchronization dynamics can be understood as the swing of the control coordinate system phase, and other control loop dynamics are understood as circuit dynamic processes in the dq axes.

2. Based on requirements for model robustness and convenient stability analysis, a dual-axis current source synchronous machine model is established for both synchronous generators and converters, with their dynamic characteristic differences parameterized, providing a theoretical foundation for understanding interconnected system dynamics.

3. The mathematical definition of synchronization stability for interconnected systems is provided, simplified models for the network and interconnected system are established, and sufficient conditions for system synchronization stability under linearized models are given, laying the groundwork for converter synchronization control design and system synchronization stability analysis.

Terminology used in this paper: - Common coordinate system: The xy coordinate system of an ideal grid with rotation speed ω_0 and x-axis phase θ_s - Control coordinate system: The reference coordinate system for device dq-axis control, with d-axis phase θ. The synchronous generator control coordinate system coincides with the rotor dq axes, while the converter control coordinate system is determined by synchronization control - Orientation phase: The phase difference between device output voltage $\dot{V}$ or output current $\dot{I}$ and the control coordinate system d-axis phase, denoted as θ_V or θ_I - Virtual power angle: The phase difference between device control coordinate system phase and grid reference voltage phase, denoted as δ - d-axis/q-axis dynamics: Dynamics of electrical vector (voltage/current) d-axis and q-axis components in the control coordinate system - Voltage source/current source modeling: Selection of voltage or current as output variables in device state-space modeling - Dual-axis synchronous machine: A synchronous machine with excitation windings on both rotor dq axes, enabling simultaneous regulation of internal voltage dq components through excitation control

The relationships among these phases are shown in [Figure 1: see original paper].

1.1 Introduction to Device Operating Principles

Synchronous generators establish internal voltage through electromagnetic induction to achieve electromechanical energy conversion, while converters control electric field energy transfer through switching device operation to achieve DC-AC conversion. Although their energy conversion mechanisms differ, they share similar structures, with the typical structure shown in [Figure 2: see original paper] serving as this paper' s research object.

For comparison with converters, synchronous generators consider excitation windings, stator, rotor, and excitation control while neglecting damping windings. The converter model considers modulation stages under average modeling, filter inductors, and control systems.

Synchronous Generator: A synchronous generator consists of a stator armature and rotor, with the rotor' s direct and quadrature axes defined as the generator' s dq axes. The rotor d-axis has a single-axis excitation winding that receives excitation voltage under excitation control, as shown in [Figure 2: see original paper]. The d-axis dynamics include excitation winding dynamics and excitation control dynamics; the q-axis has no dynamics; the control coordinate system phase is generated by rotor swing.

Converter: The converter' s dq control is also performed in its own control coordinate system, where PLL-VSC and VSG dq-axis dynamics correspond to power-current dual-loop control and voltage-current dual-loop control dynamics, respectively. The control coordinate system phase is determined by PLL dynamics and virtual synchronous control dynamics, as shown in [Figure 2: see original paper].

1.3 Problem Formulation

Due to significant differences in synchronization mechanisms between PLL-VSC and grid-forming devices, their modeling approaches are completely different. This results in diverse interface variables between devices and the network in interconnected system models, as shown in [Figure 3: see original paper], lacking clear physical meaning. Additionally, VSG may switch between voltage source and current source operation after triggering current limiting, even exhibiting complex nonlinear instability phenomena [**?**]. This switching introduces discontinuous and non-analytical functions into both network and device models in traditional modeling, causing severe dimensional disasters in large-scale system analysis and exacerbating the difficulty of interconnected system synchronous stability analysis. Consequently, it is difficult to propose transient control improvement methods for VSG from a system requirements perspective.

Although grid-following and grid-forming devices have these differences, their overall structures are similar: both synchronous generator and converter dynamics consist of control coordinate system dynamics, d-axis dynamics, and q-axis dynamics. Their operating principle is that dq-axis dynamics determine the amplitude and orientation phase of output voltage/current, while the control coordinate system phase and orientation phase together constitute the output voltage/current phase. If a unified model could be established based on this similarity to understand the relationships among PLL, virtual synchronous, and rotor motion synchronization mechanisms, unify device-network interface variables, and avoid frequent switching of VSG modeling perspectives, the complexity of interconnected system stability analysis would be greatly simplified. Therefore, it is necessary to deeply investigate the differences between grid-forming and grid-following devices and explore unified modeling methods.

Given that the inherent commonalities and differences between synchronous generators and converters are not yet clear, two problems need to be addressed to establish models suitable for synchronization stability analysis, which are answered in Sections 2 and 3, respectively.

Problem 1: Unified Understanding of Synchronization Principles. Synchronous generators achieve dynamic adjustment of control coordinate system phase through rotor motion, while converters achieve this through synchronization controls such as PLL/virtual synchronous control. How can we understand the similarities and differences between synchronous generator and converter synchronization mechanisms to explain their synchronization principles?

Problem 2: Unified Description Method for Synchronization Mechanisms. In synchronization stability analysis, synchronous generators are modeled as voltage sources, while converters have multiple modeling approaches: internal voltage modeling [?], output current modeling [?], etc. These modeling differences make interconnected system synchronization stability analysis extremely difficult. Does a reasonable modeling method exist that can describe synchronous generators and converters with unified mathematical equations, thereby facilitating analysis of interconnected system synchronization stability and guiding converter control design?

Drawing on traditional synchronous generator research and modeling approaches [?], this paper conducts preliminary exploration of interconnected system synchronization issues from the perspectives of equivalent physical meaning, detailed models, simplified practical models, and synchronization stability mechanism analysis under simplified models.

2 Equivalent Structure Based on the Dual-Axis Synchronous Machine Concept

This section constructs a unified equivalent structure for synchronous generators and converters based on the dual-axis synchronous machine concept: composed of a virtual "rotor" and dual-axis equivalent "windings" wound on the "rotor," as shown in Figure 4: see original paper. This type of dual-axis synchronous machine has been applied in Russian engineering practice [?]. Since excitation windings are wound on both rotor dq axes to simultaneously control the dq components of internal voltage, flexible control of internal voltage orientation phase can be achieved, demonstrating excellent performance in enhancing transient angle stability. In comparison, conventional synchronous machines have only single-axis excitation windings, whose internal voltage orientation phase is not control-determined but naturally aligned with the rotor d-axis, which can be considered a special case of dual-axis synchronous machines with q-axis excitation omitted.

2.1 Unified Understanding of "Rotor" Synchronization

As shown in Figure 4: see original paper, the control coordinate system dynamics of both synchronous generators and converters can be expressed as $\dot{\theta} = G_w(s)(M_{ref} - M)$. Here, M represents the device' s synchronization signal: for PLL-VSC, M is the voltage q-axis component; for VSG/synchronous generators, M is active power. $G_w(s)$ represents the dynamic process, expressed as either an inertial element or a PI element. Therefore, the converter' s control coordinate system can be understood as generated by "rotor" swing. The difference from synchronous generators is that the converter' s "rotor" driving quantity M can be freely selected rather than being active power. Under this equivalent virtual "rotor" structure, drawing on traditional synchronous generator synchronization principles, the synchronization processes of synchronous generators and converters can be uniformly described as: under the imbalance between the "rotor" driving quantity and its reference, the "rotor" continuously swings to adjust the control coordinate system phase until balance is achieved, realizing synchronization between the device control coordinate system and the grid (i.e., synchronization of the "virtual power angle" δ, as shown in [Figure 1: see original paper]).

2.2 Unified Understanding of dq-Axis Equivalent Circuits

Further comparing the dynamics of dual-axis synchronous machines and PLL-VSCs, as shown in Figure 4: see original paper (symbol meanings refer to Section 3.3), we find that PLL-VSC' s inner loop control and filter inductors, as well as dual-axis synchronous machine excitation windings and stator armature, all represent linear relationships between device voltage/current dq components, differing only in the form of control-determined dynamic equations. Notably, when dual-axis synchronous machine excitation control is modified to current control with flux and voltage feedforward, its dq-axis dynamics become similar to those of converters (refer to doubly-fed wind turbines operating in synchronous mode).

Note: Doubly-fed wind turbines are also typical dual-axis generator structures, differing in that their excitation voltage is AC. This paper does not discuss this in depth but will continue in future research.

Synchronous generators are typically understood as voltage sources with constant internal voltage amplitude (transient/subtransient voltage, hereinafter referred to as internal voltage), with rotor windings and stator armature equivalent to impedances in the dq-axis circuit, as shown in Figure 5: see original paper. Therefore, the stages representing converter voltage/current dq dynamics (PLL-VSC' s current inner loop and filter, VSG' s voltage-current dual loop and filter) can also be understood as "windings" and represented as dq-axis circuits, equivalent to either Figure 5: see original paper or Figure 5: see original paper.

The dynamic processes of synchronous generator and converter dq-axis circuits can be uniformly described as: output voltage or current quickly achieves stability in amplitude and orientation phase during circuit transients.

Note: The unified representation of synchronous generator and converter dq-axis dynamics aims to understand the operating principles of both devices connected to power systems from a circuit perspective. However, the energy injection methods differ: synchronous generators involve conversion from magnetic field energy to electric field energy, while converters directly inject energy from the DC side to the AC side through circuits. This difference in energy injection is one reason for their different dq-axis circuit dynamics. When considering synchronous generator damping windings, their mapping to circuits introduces more complex circuit dynamics, manifested as phase lag in equivalent reactance [**?**], without changing the essential nature of circuit dynamics.

Correspondingly, with the equivalence of "rotor" and "windings," PLL-VSC' s power/DC voltage control and VSG' s reactive power outer loop control can be equivalently understood as "excitation control" systems.

Through the above comparison, it can be found that converters and synchronous generators have completely identical equivalent structures, both describable by "dual-axis synchronous machines," composed of an equivalent "rotor" and dq-axis "windings," which dominate the device's synchronization process and circuit dynamic process, respectively.

3.1 Device Modeling Approach Based on Current Sources

Devices exhibit voltage source/current source characteristics in equivalent circuits, corresponding to voltage/current as output variables in state-space models. However, scientific justification is lacking for which modeling approach should be used for converters in synchronization stability analysis. This subsection proposes a unified current source modeling approach for synchronous generators and converters, considering both device model robustness and convenience of synchronization stability analysis.

(1) Device Model Robustness

A robust device model should not exhibit significant changes in stability analysis results when considering uncertainties such as modeling errors [**?**]. The robustness of linearized models is a necessary condition for the robustness of original nonlinear models, making linearized model robustness a fundamental requirement. For example, for frequency-domain models $y(s) = Z(s)u(s)$ or

$y(s) = Y(s)u(s)$ (when y is output voltage or current, $Z(s)$ or $Y(s)$ corresponds to impedance or admittance transfer function matrices, as shown in [Figure 5: see original paper]), model validity can be considered from both physical and mathematical perspectives:

- **Physical basis:** The smaller the internal impedance/admittance, the closer the device is to an ideal voltage/current source.
- **Mathematical basis:** The smaller the infinity norm (maximum singular value) of the impedance or admittance matrix, the higher the robustness of the model described by voltage source/current source.

Note: Larger singular values imply greater amplification of external errors by the device model, where errors from unmodeled parts may cause significant changes in analyzed stability margins [**?**]. From numerical simulation analysis perspective, truncation errors amplify within device models, causing error propagation [**?**]. Both cases will be further discussed in subsequent examples.

The robustness discussion of using current source or voltage source models for converters within the frequency range of synchronization stability analysis is provided in Appendix A. The main text only presents conclusions, as shown in .

(2) Convenience of Synchronization Stability Analysis and Control Design

Mixed voltage source/current source modeling in interconnected systems complicates network form and hinders understanding of system synchronization stability mechanisms. Therefore, unified modeling of converters and synchronous generators is necessary. As shown in , unified modeling as current sources ensures better robustness.

Additionally, extensive research on synchronization stability analysis of single PLL-VSC grid-connected systems has shown that current source modeling offers the advantage of transforming output synchronization into state synchronization compared to voltage source modeling, facilitating analytical methods [**?**, **?**]. Under Norton equivalence, the equivalent current phase of synchronous generators/VSGs remains a state variable, also convenient for analysis.

Modeling both converters and synchronous generators as current sources, the synchronous generator' s equivalent current is obtained through Norton equivalence of internal voltage [**?**], as shown in [Figure 6: see original paper]. When using sixth-order or fourth-order models, E_{dq} and X_{dq} are subtransient/transient parameters. VSG grid-side filter and transformer equivalent reactance can be regarded as Norton equivalent impedance, with magnitude similar to synchronous generator transient reactance, making the post-equivalence characteristics similar to synchronous generators.

More importantly, converters have current limiting constraints, which determine that they must operate in current source mode during short-term processes. VSG current limiting control causes frequent switching in dynamic models, which is unfavorable for synchronization stability analysis and poses challenges for VSG improved control [?]. To achieve grid-forming support functions while facilitating understanding of system characteristics before and after current limiting for improved converter transient synchronization control, understanding VSG synchronization from a current source perspective offers greater convenience. For example, under current source perspective modeling, VSG exhibits current source characteristics both before and after current saturation, with the only difference being whether a Norton equivalent ground inductance branch exists at the VSC grid connection node. Mature branch addition methods can conveniently handle network models. Subsequent research will delve into synchronization stability analysis of multiple VSGs under current limiting based on this foundation.

3.2 Mathematical Description Method for Device Synchronization Dynamics

This section uses PLL-VSC and synchronous generator comparison as an example, as shown in [Figure 7: see original paper]. VSG modeling follows the same principle and is not repeated here.

Unified Synchronization Signals: The synchronization signals for synchronous generators/VSGs and PLL-VSCs are active power and terminal voltage q-axis component, respectively. Some literature suggests that under unity power factor operation, PLL-VSC' s synchronization signal can be regarded as reactive power, with current amplitude as the synchronization loop gain coefficient [?]. However, when PLL-VSC operates at non-unity power factor, reactive power description becomes inaccurate. In this case, q-axis voltage can be expressed as $V_q = |\dot{V}|\sin(\theta_V - \theta) = |\dot{V}|\sin(\varphi_0 - \varphi)$, where $S = |\dot{V}||\dot{I}|$ is the port complex power magnitude, φ is the power factor angle, and φ_0 is the power factor angle reference value.

Synchronous generator/VSG synchronization signals can be expressed as $M = S_0 \cos(\varphi_0 - \varphi)$, where S_0 and φ_0 are the complex power magnitude and power factor angle at the steady-state equilibrium point.

Near the rated operating point, synchronous generator/VSG synchronization signals approximate $M \approx S_0 \cos\varphi_0 + S_0 \sin\varphi_0(\varphi - \varphi_0)$. It can be observed that both synchronous generator and converter synchronization signals are functions of power factor angle. Both converters and synchronous generators can be regarded as current phase tracking of the grid connection point voltage phase, ensuring the final phase difference is φ_0, i.e., the synchronization process can be understood as a power factor tracking process [?] (for synchronous generators, synchronization is angle tracking of its steady-state value, which after Norton equivalence manifests as power factor angle tracking).

Unified Synchronization Equations: Synchronous generator and VSG synchronization dynamics are second-order inertial elements $\frac{2H}{s} + D$, while PLL-

VSC synchronization dynamics are first-order inertial elements $\frac{k_{p,pll}}{s}$ or PI control $\frac{k_{p,pll}s+k_{i,pll}}{s}$. In control theory, both can be understood as feedback controllers. When designing controls using lead-lag transfer functions $\frac{Ks+a}{s+b}$, the unified synchronization mechanism' s time-domain equations are shown in [Figure 7: see original paper] c(4). First-order inertia and PI control can be unified, leading to the unified synchronization mechanism' s time-domain equations.

3.3 Mathematical Description Method for Device dq-Axis Equivalent Circuit Dynamics

Both converter and synchronous generator current dynamics can be written as $I_{dq} = G(s)I_{dq}^{ref} - Y(s)V_{dq}$, with corresponding equivalent circuit diagrams shown in Figure 5: see original paper. Specific expressions for $G(s)$ and $Y(s)$ are given in [Figure 7: see original paper] equations (a1)(b1). PLL-VSC current reference values are given by outer loop control (this section uses PQ outer loop as an example; other outer loops follow the same principle and are not elaborated here). Synchronous generator current reference values are given by excitation control, as shown in [Figure 7: see original paper] equations (a2)(b2), with detailed derivations in Appendix A. Here, $PI_O(s)$ and $PI_I(s)$ represent outer and inner loop PI control equations, T_v is the inner loop feedforward filter time constant, L_f is the filter inductor; T'_{dq} and T''_{dq} are dq-axis transient/subtransient time constants, X'_{dq} and X''_{dq} are dq-axis transient/subtransient reactances, and $G_f(s)$ represents excitation control equations. VSG can also be written in the form of equation (3), with the difference being that VSG' s $G(s)$ and $Y(s)$ represent voltage-current dual-loop control dynamics, which are not elaborated here.

The established current equations consider full-order dynamics of PLL-VSC and synchronous generators. In traditional power system analysis, synchronous generator models can be reduced to simplified practical models based on analysis requirements. For PLL-VSC, when analyzing different problems, simplified practical models can also be used based on requirements: when analyzing PLL-dominated stability, singular perturbation theory indicates that inner loop and filter inductor dynamics are much faster than the PLL. When their dominant eigenvalues are in the left half-plane, feedforward and inner loop fast dynamics can be neglected (4th order), i.e., $G(s) = G_0$, $Y(s) = Y_0$. When outer loop dynamics are much slower than the PLL, further simplification is possible by assuming I_{dq}^{ref} and V_{dq}^{ref} are constant (2nd order). Therefore, according to synchronization stability analysis requirements, PLL-VSC models can also be reduced to simplified models, as shown in .

3.4 Discussion

Based on the dual-axis synchronous machine concept and current source modeling approach, grid-forming and grid-following devices are uniformly represented as dual-axis current source synchronous machine models, with their differences transformed into parameter and power factor variations. For example, equivalent "windings"are both represented as second-order admittance forms ([Figure 7: see original paper] equations (a1) and (b1)), but with different parameters. Additionally, as shown in equation (1), the difference in "rotor"driving quantities between PLL-VSC and grid-forming devices can be understood as different power factors. When PLL-VSC operates at zero power factor, $\varphi_0 = 0$, $M = S_0 \cos\varphi$, and PLL-VSC is equivalently synchronized by active power. When PLL-VSC operates at unity power factor, it is equivalently synchronized by reactive power. Therefore, taking PLL-VSC with zero power factor as an example, when its control parameters are designed to be the same as corresponding grid-forming device elements, it can also exhibit grid-forming characteristics, as discussed in literature [?].

4.1 Definition of Synchronization Stability

In physics, synchronization is defined as phase adjustment among oscillators due to mutual interaction, and synchronization stability refers to the ability of each oscillator to maintain synchronization [?]. This paper focuses on frequency synchronization (phase angle locking), with specific meaning as follows: In a complex system containing n oscillators, the phase of the ith oscillator is a function of time t, expressed as $\theta_i(t)$, and frequency is expressed as $\omega_i(t) = \dot{\theta}_i(t)$. The frequency of each oscillator in the system will converge to a constant common frequency value, while the phase differences between oscillators remain constant.

The network also establishes state-space equations, connecting with device models through input/output interfaces. The interconnected system can be represented as shown in [Figure 8: see original paper]. Therefore, oscillator synchronization in this interconnected system means synchronization of each device under network coupling. With the comparison between dual-axis current source synchronous machine models and traditional synchronous generator models, interconnected system synchronization stability can be described as "virtual power angle [?]" synchronization stability, i.e., phase locking among device control coordinate system phases. Its mathematical definition is:

$$\lim_{t\to\infty} \delta_{ij}(t) = \lim_{t\to\infty} (\theta_{si}(t) - \theta_{sj}(t)) = \delta_{ij}, \quad \forall i, j$$

where θ_{si} and θ_{sj} are the control coordinate system phases of the ith and jth devices, respectively.

Traditional power system angle stability is an equivalent description of equation (4) (control coordinate system phase coincides with internal voltage phase), corresponding to the interconnected system from a current source perspective: converter inner loop-dominated stability is electrical resonance stability [?]. When the inner loop is stable and sufficiently fast (i.e., the aforementioned negligible inner loop dynamics), the current phases of simplified synchronous generators and converters become algebraic equations of "virtual power angle." In this case, synchronization stability is described as phase locking of converter output current and synchronous generator/VSG Norton equivalent current, as shown in Figure 9: see original paper.

Notably, both virtual power angle and simplified model current phases are state variables, allowing the complex synchronization problems of interconnected systems to be understood by drawing on traditional power system state synchronization characteristics [?]. In steady state, virtual power angle coincides with voltage phase, and in engineering practice, whether the phase difference between device voltages exceeds boundaries can still be used as a practical synchronization stability criterion.

4.2 Simplified Network Model

In interconnected systems, the dynamics of synchronous generators and converters are coupled through the network. Power frequency phasor models that neglect network dynamics (represented by power flow equations) introduce significant errors [?]. To address this, network models are established in the devices' own control coordinate systems to simplify interconnected system complexity while maintaining accuracy.

Without loss of generality and for simplicity of expression, in the system shown in Figure 9: see original paper, the network only considers inductance while neglecting resistance and ground capacitance. After node elimination, L_{ij} represents the equivalent inductance value between node i and node j in per-unit under power frequency. The network dynamic equations can be written as:

$$\begin{cases} L_{ij}\frac{dI_{d,i}}{dt} = V_{d,i} - \sum_{j=1}^{n} L_{ij}\begin{bmatrix}\cos\delta_{ij} & -\sin\delta_{ij}\\ \sin\delta_{ij} & \cos\delta_{ij}\end{bmatrix}\begin{bmatrix}I_{d,j}\\ I_{q,j}\end{bmatrix}\omega_j \\ L_{ij}\frac{dI_{q,i}}{dt} = V_{q,i} - \sum_{j=1}^{n} L_{ij}\begin{bmatrix}-\sin\delta_{ij} & \cos\delta_{ij}\\ \cos\delta_{ij} & \sin\delta_{ij}\end{bmatrix}\begin{bmatrix}I_{d,j}\\ I_{q,j}\end{bmatrix}\omega_j \end{cases}$$

where ω_j is the rotation frequency of the jth device' s control coordinate system; $V_{dq,i}$ and $I_{dq,i}$ represent the dq-axis components of node i' s voltage and injected current in its own control coordinate system; T_{ji} is the transformation matrix from node j' s control coordinate system to node i' s control coordinate system.

Detailed derivation of equation (5) is provided in Appendix B. Compared with electromechanical transient models, this method represents device frequency variation and line coupling in algebraic terms T_{ji}, avoiding large errors caused by neglecting differential dynamics in the former. Specific accuracy comparison is shown in subsequent examples. Similarly, after neglecting network differential dynamics, RL loads can be eliminated as passive nodes (see equation (6)), and loads such as energy storage absorbing power can be regarded as grid-connected devices and modeled practically in their own coordinate systems.

4.3 Simplified Model of Interconnected Systems

Taking the system shown in Figure 9: see original paper as an example, a system with n PLL-VSCs and m synchronous generators connected through a purely inductive network. Combining the simplified 2nd-order device model from and the network simplified model from equation (6), a closed-loop second-order equation for the ith device' s current phase θ_i and the network can be formed based on [Figure 7: see original paper] equation (c4):

$$\begin{cases} \frac{d\theta_i}{dt} = \omega_i \\ J_i \frac{d\omega_i}{dt} = -D_i\omega_i - \sum_{j=1}^{n+m} K_{ij}\sin(\theta_i - \theta_j - \alpha_{ij}) \end{cases}$$

where current amplitude is constant, and other variable meanings are shown in [Figure 7: see original paper]. Further, equation (7)' s equilibrium point can be shifted to the origin (simplification process in Appendix B) to obtain:

$$\begin{cases} \frac{d\Delta\theta_i}{dt} = \Delta\omega_i \\ J_i \frac{d\Delta\omega_i}{dt} = -D_i\Delta\omega_i - \sum_{j=1}^{n+m} K_{ij}\cos(\theta_{ij0} - \alpha_{ij})\Delta\theta_{ij} \end{cases}$$

where $\Delta\theta_{ij} = \Delta\theta_i - \Delta\theta_j$, $\omega_0 = 100\pi$ rad/s is the power frequency, and $\Delta\omega_i$ is the relative angular frequency, which is 0 in steady state. L_{ij} represents the equivalent inductance between node i and node j at power frequency, and α_{ij} is the relative power frequency. Equation (8) constitutes the simplified model for interconnected system synchronization stability analysis, hereinafter referred to as the simplified model.

Further linearizing equation (8) yields the following state-space representation of the interconnected system linearized model:

$$\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_{Im}N & -K_{Re}N - H \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\omega \end{bmatrix}$$

where $K_{Im}N$ and $K_{Re}N$ are defined in the original text, $H = \text{diag}\{D_1/J_1, ..., D_{m+n}/J_{m+n}\}$, and $K_{Im}N$ and $K_{Re}N$ are defined as specified in the original equations.

4.4 Sufficient Conditions for Interconnected System Synchronization Stability

In the interconnected system shown in equation (8), an important scientific question is how the system maintains synchronization stability in the neighborhood of equilibrium points, i.e., the conditions for interconnected system synchronization stability. The following uses linearization methods to analyze system synchronization, i.e., analyzing the synchronization characteristics of model (9).

The system shown in equation (9) has one zero eigenvalue and $2(m + n - 1)$ conjugate complex eigenvalues. The zero eigenvalue is analogous to the inertia center of traditional synchronous generators, representing the interconnected system frequency "center,"while other conjugate complex roots represent relative swing modes among devices. When the real parts of conjugate complex roots are less than 0, the system is synchronization stable. Physically, this means: within the neighborhood of equilibrium points, after disturbances, each device' s phase will eventually form a stable flow manifold satisfying $\Delta\omega_i = 0$, with each device' s frequency equal to the "center frequency" corresponding to the zero eigenvalue.

Theorem 1 (Small-Disturbance Synchronization Stability Condition): When parameters in $K_{Re}N$ and H are positive and sufficiently large (much larger than elements in $K_{Im}N$), the real parts of eigenvalues in equation (9) are less than 0, and the interconnected system can maintain synchronization stability; otherwise, instability may occur.

Proof: See Appendix C.

Notably, the case where grid-forming device inertia coefficients are much larger than PLL-VSC PLL time constants is a special case of equation (9), where the above sufficient conditions still hold. In this scenario, the system synchronization process is: PLL-VSC subsystem synchronization dynamics form a fast manifold, achieving synchronization stability first; grid-forming device subsystem synchronization dynamics form a slow manifold, during whose internal synchronization process, PLL-VSC phases can be considered to ideally follow grid-forming device phases until the grid-forming device subsystem achieves synchronization.

From the above theorem, it can be inferred that $K_{Im}N$ elements characterize the coupling damping effect between device frequency variation and the network, manifesting as negative damping in PLL-VSC. If the network is modeled as a traditional power frequency phasor-based electromechanical model, it is equivalent to $K_{Im}N = 0$ in equation (9), i.e., neglecting the negative damping effect under network coupling. Additionally, D_i and K_{ij} in devices are damping coefficients, presenting positive damping when greater than 0. When damping coefficients can compensate for network negative damping, synchronization stability at equilibrium points can be guaranteed; if damping coefficients are insufficient to compensate for network negative damping, synchronization instability will occur.

The above analysis is conducted under linearized models. How to extend traditional transient angle stability analysis methods to interconnected system transient synchronization stability analysis using simplified model (8) is a topic for future work.

5 Case Studies

Simulation models were built in MATLAB/Simulink to verify the rationality of the above theories, with parameters from references [**?**, **?**].

(1) Verification of Device Model Robustness

Device voltage source modeling and current source modeling robustness were verified in a single PLL-VSC grid-connected system, with network models denoted as $Y_{g,VSC}(s)$ and $Z_{g,VSC}(s)$. To characterize uncertainties in the network, multiplicative perturbations of the same magnitude but arbitrary direction were applied to obtain the uncertain network model $Y^{\Delta}_{g,VSC}(s)$. Using perturbations of the same strength as an example, models with larger singular values are more sensitive to external errors, causing larger errors in PLL-dominant eigenvalues. Results are shown in [Figure 10: see original paper]. The figure shows that under the same perturbation strength, voltage source modeling causes larger changes in PLL-dominant eigenvalues than current source modeling, consistent with singular value analysis results, demonstrating that current source models have better robustness for synchronization stability analysis.

Further, numerical robustness in simulation was verified using the above models. A current disturbance was injected at the PLL-VSC port at 0.05s, with simulations conducted using step sizes of 10^{-4}s, 10^{-5}s, and 10^{-6}s. Results are shown in [Figure 11: see original paper]. Voltage source modeling requires smaller step sizes to achieve numerical stability compared to current source modeling, indicating that current source models have better numerical robustness.

(2) Verification of Interconnected System Simplified Model Effectiveness

A two-area four-machine model [?] was built to verify the effectiveness of the simplified model (each area connects one PLL-VSC and one synchronous generator). A three-phase ground short-circuit fault was applied at $t = 0.1$s and cleared at $t = 0.2$s. Time-domain response trajectories were compared among detailed electromagnetic model (Model 1), traditional electromechanical model (Model 2), and the proposed simplified model (Model 3). [Figure 12: see original paper] shows the response waveforms of the converter' s q-axis voltage component. The simplified model shows similar first-swing trajectories to the electromagnetic model after fault clearing, while the traditional electromechanical model shows large deviations, verifying the simplified model' s effectiveness and demonstrating its ability to accurately describe synchronous stability dynamic characteristics of interconnected systems.

(3) Verification of Synchronization Stability Analysis

In the two-area four-machine model, a phase disturbance was applied to the converter at $t = 0.5$s, with the PLL output phase suddenly increasing by 1 rad. The proportional term parameter $k_{p,pll}$ of PLL-VSC was varied, and system stability was observed using the current phase difference θ_{12} between converter and synchronous generator as an example, as shown in [Figure 13: see original paper]. When $k_{p,pll} = 4$, the system exhibits oscillatory instability; when $k_{p,pll}$ is greater than 4, oscillations gradually converge and the system eventually stabilizes. Using the condition that diagonal elements of $K_{Re}N$ are greater than 10 times the maximum element of $K_{Im}N$, calculation yields $k_{p,pll} > 15$.

That is, when $k_{p,pll} > 15$, the system is necessarily small-disturbance stable, verifying the correctness of Theorem 1' s conclusion.

6 Conclusions and Outlook

1. Both synchronous generators and converters can be uniformly understood as "dual-axis synchronous machines." Their dynamic processes consist of "rotor" synchronization dynamic processes and "winding" circuit dynamic processes, differing only in "winding"dynamics and "rotor"driving variables.

2. From the dual-axis current source synchronous machine perspective, the synchronization characteristics of synchronous generators and converters can be uniformly characterized as state synchronization, with their dynamic characteristic differences understood as variations in certain parameters, facilitating guidance for converter synchronization control design under current limiting constraints.

3. In interconnected systems, when PLL-VSC' s PLL proportional coefficient or synchronous generator damping coefficient is small, network negative damping effects may cause system instability. Increasing PLL proportional coefficients and synchronous generator damping coefficients helps enhance interconnected system synchronization stability.

This paper provides preliminary exploration of interconnected system synchronization stability using the dual-axis current source synchronous machine concept. Future work needs to further explore other stability problem mechanisms and analysis methods for interconnected systems.

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Appendix A

When synchronous generators are modeled with internal voltage as the output variable, their impedance model $Z_{SG}(s)$ derivation can be found in [?], with equations given as (A1). When synchronous generators use Norton equivalence, the admittance model $Y_{SG}(s)$ is given in the main text [Figure 7: see original paper] equation (a1), differing from the impedance model in that subtransient voltage is substituted through equivalent current (A2).

When converters only consider the current inner loop and are modeled with current as the output variable, their admittance model $Y_{VSC}(s)$ derivation is in literature [?]. When converters use internal voltage as the output variable, their impedance model $Z_{VSC}(s)$ is given in [?].

Maximum singular values from 0.1-1000 Hz are shown in [FIGURE:A1], characterizing model robustness [?]. In the low-frequency range dominated by synchronous generator rotor dynamics, internal voltage voltage source modeling ($Z_{SG}(s)$) has the smallest maximum singular value, followed by Norton equivalent current source modeling ($Y_{SG}(s)$). Direct current output modeling has poor robustness. In the frequency range dominated by PLL (5-100 Hz), current output modeling ($Y_{VSC}(s)$) has the best robustness, while the other two modeling approaches have relatively poor robustness.

Appendix B

Define matrix B as the admittance matrix under the system' s electromechanical model. By eliminating passive internal nodes, only device (synchronous generator and converter) nodes are retained. Define N as device nodes and M as passive nodes. After node elimination, $B_{red} = B_{NN} - B_{NM}B_{MM}^{-1}B_{MN}$, and $X_{red} = B_{red}^{-1}$ is obtained by matrix inversion, where each element X_{ij} represents the equivalent reactance between node i and node j.

When node j injects current and other nodes are open-circuited, the dynamic equation for node i voltage response is expressed as equation (B1), where $V_{dq,i|j}$ represents node i' s voltage dq components in the jth device' s control coordinate system. Transforming voltage $V_{dq,i|j}$ to device i' s dq coordinate system yields equation (B2), where T_{ij} is a diagonal matrix.

By performing linear transformation on the state-space equation and considering small voltage phase angle differences between connected devices, approximations lead to the simplified forms. When diagonal elements of $K_{Re}N$ are less than 0 and their absolute values are much larger than elements of $K_{Im}N$, the non-diagonal elements of $K_{Re}N$ can be neglected, and $K_{Re}N$ can be approximated as a diagonal matrix with all elements less than 0.

Considering the lower-left element of the transformed matrix is approximately 0, the system can be solved. Since the matrix is positive definite and other matrices are diagonal positive definite, the quadratic eigenvalues have real parts less than 0, ensuring system synchronization stability. Moreover, by continuity of eigenvalues with respect to matrix elements, as long as elements in $K_{Re}N$ and H are large enough to keep system eigenvalues sufficiently far from the right half-plane, the interconnected system remains synchronization stable even when considering device voltage phase angle differences, line resistances, and $K_{Im}N$ elements.

Similarly, by superimposing voltage responses from n injected currents converted to node i' s control coordinate system, main text equation (5) is obtained.

From main text equation (7), equation (B4) is derived. By further neglecting quadratic trigonometric terms, the multi-machine form can be written as equation (B5), yielding main text equation (8).

Appendix C

Define linear transformation matrix P for the system. The characteristic equation of the transformed state-space equation can be described as a quadratic eigenvalue problem. Further considering small voltage phase angle differences between connected devices, approximations lead to simplified matrix forms. When diagonal elements of $K_{Re}N$ are sufficiently negative and their absolute values are much larger than elements of $K_{Im}N$, the non-diagonal elements can be neglected. The lower-left block of the transformed matrix is approximately zero, allowing solution of the system.

Since the resulting matrix equation involves positive definite matrices, the quadratic eigenvalues have negative real parts, proving system synchronization stability. The continuity property of eigenvalues ensures that as long as damping parameters are sufficiently large to maintain adequate distance from the right half-plane, the system remains stable even when accounting for additional factors.

Note: Figure translations are in progress. See original paper for figures.

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