

A Fringe Search Algorithm for Radio Sources in Space VLBI (Postprint)

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Abstract

China's Chang'e-7 lunar exploration mission will establish the first lunar-Earth space Very Long Baseline Interferometry (VLBI) experimental system, comprising a 4.1-meter aperture parabolic antenna aboard a relay satellite and ground-based radio telescopes. Unlike ground-based VLBI telescopes, the predicted delay model for radio sources can sometimes deviate significantly from the actual delay due to relay satellite orbital perturbations and equipment delays, which prevents the VLBI correlator from operating normally and necessitates the use of radio source fringe search methods to determine the actual delay model. As the 4.1-meter space antenna has a relatively small aperture and weak signal reception capability, long integration times are required to improve the signal-to-noise ratio; however, linear delay models cannot guarantee delay accuracy over such extended durations. To address this issue, a radio source fringe search algorithm for space VLBI is proposed. The algorithm first establishes a two-dimensional search grid for delay and delay rate, constructing a linear delay model using the a priori delay and delay rate corresponding to each grid point. This linear delay model is then applied for segmented correlation processing of the raw data. Subsequently, a two-dimensional Fourier transform is utilized to search for residual delay and delay rate, identifying the grid point associated with the maximum correlation amplitude. Finally, a quadratic delay model is reconstructed using the a priori delay and delay rate of this grid point together with the residual delay and delay rate. The validity of this radio source fringe search algorithm has been verified through processing actual Earth-space baseline data from the RadioAstron space VLBI mission.

Full Text

Preamble

A Radio Source Fringe Search Algorithm for Space VLBI

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Abstract

China's Chang'e-7 lunar exploration mission will establish the first lunar-terrestrial Space Very Long Baseline Interferometry (VLBI) test system, comprising a 4.1-meter parabolic antenna on a relay satellite and ground-based radio telescopes. Unlike conventional ground-based VLBI systems, the predicted delay model for radio sources may deviate significantly from the actual delay due to relay satellite orbital perturbations and equipment delays, preventing the VLBI correlator from functioning properly. This problem can be resolved by determining the actual delay model through radio source fringe search methods. However, the 4.1-meter space antenna has a small aperture and weak signal reception capability. To improve the signal-to-noise ratio, long integration times are required, but linear delay models cannot guarantee delay precision over extended periods. To address this challenge, we propose a radio source fringe search algorithm for Space VLBI. The algorithm first establishes a two-dimensional search grid for delay and delay rate, using the a priori delay and delay rate at each grid point to construct a linear delay model. This model is then applied to correlate the raw data in segmented time intervals. Next, a two-dimensional Fourier transform searches for residual delay and delay rate to identify the grid point corresponding to the maximum correlation amplitude. Finally, the quadratic delay model is reconstructed using the a priori delay and delay rate from this grid point combined with the residual values. Validation using actual RadioAstron Space VLBI ground-space baseline data confirms the correctness of the proposed algorithm.

Keywords: Space VLBI; fringe search; delay model; correlation processing; radio source

Introduction

Very Long Baseline Interferometry (VLBI) is a high-resolution, high-precision radio interferometry technique [1]. It employs highly stable independent local oscillator systems and data acquisition devices controlled by atomic clocks. Multiple telescopes simultaneously receive signals from a target source, record the data, and transmit it to a remote processing center for analysis. In VLBI observations, the distance between two radio telescopes is called the baseline, which

determines the maximum resolution of the system [2]. If a space telescope and ground-based telescopes form a space-ground VLBI system, the baseline length can exceed Earth's diameter, enabling even higher resolution. China's fourth-phase lunar exploration mission will utilize the 4.1-meter parabolic antenna on the Chang'e-7 relay satellite to conduct lunar-terrestrial Space VLBI experiments. Unlike conventional ground-based VLBI [3], orbital perturbations of the relay satellite and equipment delays may cause significant differences between the predicted radio source delay model and the actual delay, preventing the correlator from operating normally. Radio source fringe search can determine the high-precision delay model required for actual correlation processing in space-ground baseline VLBI.

Currently, during correlator processing for lunar exploration missions, fringe search algorithms are designed using the point-frequency characteristics of Differential One-way Ranging (DOR) signals from spacecraft [4]. However, radio source signals are white noise signals, making existing spacecraft signal search algorithms inapplicable. Existing radio source fringe search methods include the eFFT algorithm, CAF-W algorithm [5], and multi-grid fringe search algorithm [6]. The eFFT and CAF-W algorithms can calculate residual delay and delay rate but are primarily designed for delay model searches of strong sources. The multi-grid fringe search algorithm establishes hierarchical grids of varying sizes to search for target values across different ranges. This algorithm uses a linear delay model for correlation processing and searches for fringes in the delay-delay rate plane, making it suitable for scenarios with strong signals where fringes can be obtained through short integration times. Due to the weak signals received by small-aperture space antennas, obtaining interference fringes requires long integration times. Conventional linear delay models cannot guarantee correlator model delay precision over extended periods, necessitating the construction of a higher-precision quadratic delay model. Adding a quadratic delay term to the conventional multi-grid approach would expand the search space from two to three dimensions, increasing computational complexity by an order of magnitude.

This paper proposes a radio source fringe search algorithm for Space VLBI based on the multi-grid approach. First, a two-dimensional search grid for delay and delay rate is established. Using the a priori delay and delay rate information at each grid point, a linear delay model is constructed to process the raw data in segmented time intervals. Then, residual delay and delay rate are solved via two-dimensional Fourier transform and compensated to obtain the delay and delay rate for each time segment, calculating the correlation amplitude values for each grid point. Finally, the quadratic delay model is reconstructed at the grid point with maximum correlation amplitude. Application of this algorithm to actual RadioAstron Space VLBI observation data has achieved promising results.

1. Radio Source Fringe Search Algorithm

The current Chinese VLBI Network software correlator uses a fifth-order polynomial delay model for correlation processing. During fringe search, to reduce computational complexity, a linear delay model comprising constant and first-order terms is typically employed. However, as the time span increases, the error between the linear delay model and the fifth-order polynomial model also grows. Considering both computational load and model accuracy, we aim to construct a quadratic delay model comprising constant, first-order, and second-order terms.

1.1 Correlation Processing Algorithm

Assuming the target source RF signals received by two stations are $s_1(t)$ and $s_2(t)$, and the time delay difference for the same wavefront arriving at the two stations is τ , the baseband signal obtained from the RF signal is:

$$\begin{aligned} s_{b1}(t) &= s_1(t) \exp(-j2\pi f_{sky}t) \\ s_{b2}(t) &= s_2(t - \tau) \exp(-j2\pi f_{sky}(t - \tau)) \end{aligned}$$

where f_{sky} is the sky frequency.

The correlation processing uses Station 1 as a reference and applies a predicted model τ_c to compensate the delay for Station 2:

$$s_{b2c}(t) = s_2(t - \tau) \exp(-j2\pi f_{sky}(t - \tau + \tau_c))$$

After fringe rotation:

$$s'_{b2c}(t) = s_2(t - \tau) \exp(-j2\pi f_{sky}(t - \tau + \tau_c)) \exp(-j2\pi f_{sky}\tau_c)$$

Performing Fourier transforms on the signals from Station 1 and Station 2 yields:

$$\begin{aligned} S_1(f) &= \int s_{b1}(t) \exp(-j2\pi ft) dt \\ S_2(f) &= \int s'_{b2c}(t) \exp(-j2\pi ft) dt \end{aligned}$$

Let $P_1(f) = |S_1(f)|^2$ be the power spectrum of Station 1's signal after frequency conversion. The cross-correlation power spectrum between the two stations is:

$$P_{12}(f) = S_1^*(f)S_2(f) = |S_1(f)|^2 \exp(-j2\pi f(\tau - \tau_c))$$

From equation (7), when the residual delay $\tau - \tau_c$ approaches its minimum value, the cross-correlation power spectrum between the two stations reaches its maximum peak.

1.2 Residual Delay Search Algorithm

[Figure 1: see original paper]

According to the properties of two-dimensional Fourier transforms, after correlation processing, the two-dimensional Fourier transform of the cross-correlation spectrum yields a two-dimensional correlation function of residual delay and delay rate [7], as shown in Figure 1. The formula is:

$$\tilde{F}(\Delta\tau, \Delta\dot{\tau}) = \left| \int_{-B/2}^{B/2} \left\{ \int_{-T/2}^{T/2} P_{12}(f, t) \exp(-j2\pi f \Delta\dot{\tau} t) dt \right\} \exp(j2\pi f \Delta\tau) df \right|$$

where $\Delta\tau$ is the residual delay, $\Delta\dot{\tau}$ is the residual delay rate, B is the channel bandwidth, T is the data duration for one two-dimensional Fourier transform group, f is the sky frequency, $\tilde{F}(\Delta\tau, \Delta\dot{\tau})$ is the correlation amplitude value under different residual delays and delay rates, and $P_{12}(f, t)$ is the cross-correlation power spectrum after correlation processing.

Discretizing equation (8) gives:

$$\tilde{F}(n, m) = \left| \sum_{k=0}^{N-1} \left\{ \sum_{l=0}^{M-1} P_{12}(k, l) \exp(-j2\pi \frac{km}{M}) \right\} \exp(j2\pi \frac{kn}{N}) \right|$$

where $\tilde{F}(n, m)$ is the correlation amplitude value obtained after two-dimensional Fourier transform [8], n and m are the discretized residual delay and delay rate, N is the number of Fourier transform points in the frequency domain, and M is the number of Fourier transform points in the time domain.

1.3 Fringe Search and Quadratic Delay Model Reconstruction Process

Based on the above principles, the radio source fringe search and quadratic delay model reconstruction process for Space VLBI is as follows:

[Figure 2: see original paper]

According to the properties of two-dimensional Fourier transforms, the ranges of residual delay and delay rate after correlation and two-dimensional Fourier transformation are $[-1/(2\Delta f), 1/(2\Delta f)]$ and $[-1/(2T), 1/(2T)]$, respectively. Therefore, the search step sizes for delay and delay rate should be within these ranges. Based on experience with processing actual data, the delay step size is set to $\Delta\tau_{step} = 1/(2B)$, and the delay rate step size is set to $\Delta\dot{\tau}_{step} = 1/(2T)$.

The fringe search step size is closely related to delay model accuracy and computational load. If the search step is too large, the resulting model may be too inaccurate for subsequent data processing. If the step is too small, the algorithm's full-grid search strategy—calculating each grid point sequentially—will

severely impact overall computational efficiency. Note that when the step size is small, multiple grid points may satisfy the accuracy requirements. Future work will investigate a two-dimensional random walk-based grid search strategy that stops calculations once an appropriate delay model is found, thereby improving efficiency effectively.

After determining the step sizes, the grid points are divided according to the search range and a priori delay model. Each grid point corresponds to different delays and delay rates, representing different delay compensation approaches in data processing. As shown in Figure 3 [Figure 3: see original paper], the delay and delay rate from the a priori model serve as the center point, and each grid point's corresponding values are calculated with equal step sizes.

Since weak signals cannot guarantee delay precision or obtain interference fringes during long integration times using linear delay models, the raw data is divided into I shorter segments for processing. Each segment constitutes one two-dimensional Fourier transform group with duration T . At each grid point, a linear delay model is constructed to compensate the raw data before correlation processing, with an integration time of T/I per segment. The data size for one two-dimensional Fourier transform group is $N \times M$. After obtaining the cross-correlation power spectrum between the two stations, equation (9) is applied to perform two-dimensional Fourier transform on each group's correlation spectrum, yielding I sets of residual delay, delay rate, and correlation amplitude values at each grid point. The grid point corresponding to the optimal solution is determined using the average of the maximum correlation amplitudes, and delay and delay rate fitting is performed at this grid point to obtain the optimal quadratic delay coefficients.

During fitting, adding the residual delay $\Delta\tau_i$ and delay rate $\Delta\dot{\tau}_i$ from each group yields that group's delay τ_i and delay rate $\dot{\tau}_i$:

$$\begin{aligned}\tau_i &= \tau_{0i} + \Delta\tau_i \\ \dot{\tau}_i &= \dot{\tau}_{0i} + \Delta\dot{\tau}_i\end{aligned}$$

VLBI observations have temporal continuity, and the delay model should also be continuous. However, data segmentation introduces discontinuities in delay between groups. When fitting quadratic delay coefficients, continuous constraints must be applied: keeping the delay τ_1 of Group 1 unchanged while correcting the delays τ_i of other groups to equal the end delay of the previous group:

$$\tau_m(i) = \tau_i + \sum_{k=1}^{i-1} \Delta\tau_k, \quad 1 < i \leq I$$

where τ_m is the corrected delay and I is the number of two-dimensional Fourier transform groups.

After the above delay correction, quadratic delay coefficient fitting is performed. Let the unknown quadratic delay coefficients be $[a_0, a_1, a_2]^T$. The delay polynomial to be solved is:

$$\tau_m(i) = a_0 + a_1 t_i + a_2 t_i^2$$

where t_i is the start time of each two-dimensional Fourier transform group.

Differentiating the right side of equation (15) yields the delay rate polynomial:

$$\dot{\tau}_i = a_1 + 2a_2 t_i$$

Equations (15) and (16) are combined to form an overdetermined system, which is solved using orthogonal-triangular decomposition [9].

2. Algorithm Validation

The RadioAstron project is a Space VLBI (SVLBI) plan proposed by the Soviet Union in the mid-1980s. In 2011, Russia launched the RadioAstron space radio telescope (Spektr-R, Ra) satellite with a 10-meter aperture. Ra and multiple large ground-based radio telescopes constitute a Space VLBI system. Its receiving bands are 0.3 GHz, 1.6 GHz, 5.0 GHz, and 22 GHz, with orbital altitudes of 600 km at perigee and 330,000 km at apogee, performing interferometric measurements jointly with ground telescopes.

To validate the Space VLBI radio source fringe search algorithm, we processed interferometric data from a joint observation of radio source 0823+033 by the space radio telescope Ra and the Arecibo Radio Telescope (Ar) in 2014. The observation started at 23:07:00 on April 11, 2014, and ended at 23:07:59 on the same day. The Ar station used 2-bit quantization with 4-bit fan-out, while the space station Ra used 1-bit quantization with 2-bit fan-out. The sky frequency was 4836 MHz, bandwidth 16 MHz, integration time 2^{24} s, and the correlated data specification was $2^{12} \times 2^{12}$. Data points of $2^{12} \times 2^{12}$ were used as one group for two-dimensional Fourier transform.

From equations (10) and (11), the search step sizes for delay and delay rate were determined as $\Delta\tau_{step} = 1/(2B)$ and $\Delta\dot{\tau}_{step} = 1/(2T)$. Using the a priori delay model as the search center, the grid size for delay and delay rate search was 21×21 , with delay search range $[-10\Delta\tau_{step}, 10\Delta\tau_{step}]$ and delay rate search range $[-10\Delta\dot{\tau}_{step}, 10\Delta\dot{\tau}_{step}]$.

During algorithm validation, we first applied the conventional multi-grid fringe search algorithm using only linear delay models for correlation processing at each grid point to obtain interference fringe patterns at peak points. Then, we used the proposed radio source fringe search algorithm to construct a quadratic delay model and compared the results.

Figure 4 Figure 4: see original paper shows the correlation amplitude map obtained by direct fringe search using the linear delay model, with the peak point at grid position (15, 158) and correlation amplitude value 2.4×10^4 . Figure 4(b) shows the fringe phase at the peak point. No clear interference fringes are visible in the figure.

Figure 5 [Figure 5: see original paper] shows the correlation amplitude values at different grid points obtained by the Space VLBI radio source fringe search algorithm. The peak point is at grid position (11, 101) with correlation amplitude value 1.2×10^5 . After fitting, the quadratic delay coefficients are $[1.2 \times 10^{-6}, 2.4 \times 10^{-8}/s, 4.2 \times 10^{-11}/s^2]^T$. Using this quadratic model for correlation processing yields the interference fringes shown in Figure 6 Figure 6: see original paper. Figure 6(b) shows the fringe results after phase unwrapping. The results demonstrate that the proposed algorithm outperforms the multi-grid fringe search algorithm when processing RadioAstron data.

[Figure 5: see original paper]

The algorithm prototype was implemented on a Linux cluster platform using a two-level parallel architecture of MPI between compute nodes and POSIX multi-threading within nodes. The compute nodes use Intel Xeon E5-2620 CPUs at 2 GHz with 12 CPUs per node. The runtime is shown in Table 1. Further optimization can improve the execution speed.

Conclusion

In future Phase-IV lunar exploration missions, the lunar-terrestrial Space VLBI test system will be affected by relay satellite orbital perturbations and equipment delays, potentially causing significant differences between predicted and actual radio source delays. Building upon conventional multi-grid fringe search, this paper proposes a radio source fringe search algorithm tailored for Space VLBI characteristics. The algorithm first establishes a two-dimensional search grid for delay and delay rate, uses linear delay correlation processing, then searches for residual delay and delay rate via two-dimensional Fourier transform, and finally reconstructs the quadratic delay model. Validation using RadioAstron data demonstrates that while conventional linear delay model searches fail to achieve ground-space baseline fringes, the new algorithm successfully obtains clear Space VLBI radio source interference fringes. With further testing and refinement, this algorithm can be applied to China's subsequent Space VLBI missions.

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