Theoretical Explanations of the Operational Momentum Effect and Its Developmental Predictive Factors

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Abstract

Understanding the formation and development of operational biases is crucial for exploring the intrinsic mechanisms of arithmetic operation systems, as early arithmetic abilities form the foundation for children to comprehend and perform complex mathematical operations. Operational momentum bias refers to a systematic tendency where individuals overestimate addition outcomes and underestimate subtraction outcomes when performing basic mathematical calculations, which has been primarily explained by three theoretical frameworks: the attentional shift hypothesis, heuristic explanation, and compression explanation. Given the relative stability of the operational momentum effect in adult populations alongside inconsistent evidence observed across children at different developmental stages, the maturation of mathematical abilities and spatial attention can be integrated with these theoretical perspectives to illuminate developmental trajectories of the operational momentum effect. Future research should incorporate multiple experimental paradigms to delineate the developmental trajectory of the operational momentum effect, examine the relationship between numerical representation systems and the operational momentum effect, investigate the stability of the operational momentum effect across different operational symbols, explore the combined influence of multiple factors on the operational momentum effect, and design targeted interventions for mathematical abilities to mitigate this operational bias.

Full Text

The Theoretical Accounts and Developmental Predictors of Operational Momentum Effect

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Abstract: Understanding the formation and development of operational biases is crucial for exploring the underlying mechanisms of arithmetic computation systems. Early arithmetic abilities form the foundation for children to comprehend and perform complex mathematical operations. The operational momentum bias refers to a systematic tendency to overestimate addition outcomes and underestimate subtraction outcomes during basic arithmetic operations. Three main theoretical accounts have been proposed to explain this phenomenon: the attentional shift account, the heuristic account, and the compression account. Given the relative stability of operational momentum effects in adult populations alongside inconsistent evidence across different developmental stages in children, improvements in mathematical ability and maturation of spatial attention may illuminate developmental trajectories of operational momentum effects when considered in conjunction with these theoretical frameworks. Future research should integrate multiple paradigms to reveal developmental trajectories of operational momentum effects, examine associations between numerical representation systems and operational momentum, investigate the stability of operational momentum across different operation symbols, explore how multiple factors jointly influence operational momentum, and design mathematical ability interventions to reduce this operational bias.

Keywords: operational momentum effect, attentional shift account, heuristic account, compression account, development

Although complex mathematical operations are not essential for daily life, basic arithmetic is an indispensable part of everyday activities. For instance, people frequently judge product value-for-money by comparing quantities and prices, or select checkout lines by estimating customer numbers and cart contents. This capacity for approximate arithmetic is not limited to adults; even infants can roughly represent quantities of object sets and perform basic operations like addition and subtraction (Li et al., 2015; Liang et al., 2021; Barth et al., 2006; Cantlon & Brannon, 2007; McCrink & Wynn, 2004, 2009). Consequently, approximate arithmetic is considered a fundamental mathematical skill (Barth et al., 2005; Feigenson et al., 2004; Gilmore et al., 2007; Spelke, 2017). Interestingly, when individuals perform basic addition and subtraction on two quantities, operation signs systematically bias estimation outcomes in the direction of the operation—overestimating addition results and underestimating subtraction results. This operational bias is termed the *operational momentum* effect (McCrink et al., 2007), a name borrowed from research on *representational momentum*, which describes how observers' memory for a moving object's final position is displaced forward along its trajectory (Freyd & Finke, 1984; Hubbard, 2014, 2015).

As a significant finding in the operational bias literature, the operational momentum effect was first discovered by McCrink et al. (2007) in adult participants using non-symbolic visual set arithmetic. Participants viewed two successive dot arrays on screen, performed addition or subtraction, and judged whether a centrally presented comparison dot array matched the correct result. Researchers tested five comparison quantities: substantially smaller, slightly smaller, correct, slightly larger, and substantially larger than the true outcome. Results showed that during addition, participants were more likely to accept slightly larger arrays as correct, whereas during subtraction, they more readily accepted slightly smaller arrays. Recent research has examined this operational bias and its underlying theoretical explanations, revealing its relative stability across various contexts. Studies using both symbolic (Arabic digits) and non-symbolic (dot arrays) tasks have consistently found operational momentum effects (Haman & Lipowska, 2021; Katz & Knops, 2014; Knops, Viarouge, & Dehaene, 2009; McCrink et al., 2007; Pinhas & Fischer, 2008). Investigations of different operation types have further demonstrated that adults show similar biases in multiplication and division, overestimating multiplication outcomes and underestimating division outcomes in non-symbolic estimation tasks (Katz & Knops, 2014), suggesting stability across operation types and tasks. Moreover, research on temporal sequences has revealed that adults overestimate summed durations and underestimate subtracted durations, indicating that operational momentum extends beyond visuospatial processing to temporal information processing and that arithmetic computation may follow a common computational principle (Bonato et al., 2021).

While operational momentum effects show consistent empirical support in adult studies (Dunn et al., 2019; McCrink & Hubbard, 2017; McCrink et al., 2007), findings across different age groups in children remain inconsistent (Barth et al., 2006; Cassia et al., 2016, 2017; Haman & Lipowska, 2021; Knops et al., 2013; McCrink & Wynn, 2004, 2009; Pinheiro-Chagas et al., 2018). Currently, it remains unclear when operational momentum effects first emerge and whether their magnitude strengthens or weakens with age. This review synthesizes recent research on operational momentum effects, summarizes theoretical explanations for their formation, and integrates developmental evidence to discuss predictive factors during childhood, thereby providing theoretical and practical insights into the underlying mechanisms of operational momentum effects in children and their arithmetic processing.

2 Theoretical Accounts of Operational Momentum Effect Formation

The formation of operational momentum effects represents a central research theme in approximate arithmetic bias. Three primary theoretical accounts have been proposed: the attentional shift account, the heuristic account, and the compression account (McCrink et al., 2007). These accounts differ in their emphasis on spatial-numerical associations and the depth of numerical processing, yet they are not mutually exclusive—operational momentum effects may result from multiple co-occurring mechanisms.

2.1 Attentional Shift Account

The attentional shift account represents the most widely accepted theoretical mechanism, proposing that operational biases arise from spatial attention shifts along a mental number line. The mental number line refers to a psychological representation where numbers are organized spatially from left to right in ascending order, with smaller numbers on the left and larger numbers on the right (Dehaene, 1992; Dehaene et al., 1993). According to this account, when the attentional focus shifts too far along the mental number line in the direction of the operation, it lands to the right of the correct outcome for addition and to the left for subtraction, resulting in overestimation and underestimation biases, respectively (Knops, Viarouge, & Dehaene, 2009; Knops et al., 2013; McCrink et al., 2007).

The specific computational process involves mapping the first operand onto the mental number line, then shifting the attentional focus by a distance corresponding to the second operand to reach the result. During mental calculation, this shift produces forward displacement along the mental number line—toward larger numbers for addition and multiplication, and toward smaller numbers for subtraction and division—thereby generating operational momentum effects (Katz & Knops, 2014; McCrink et al., 2007). Research has documented clear left/right attentional shifts during numerical operations (Liu et al., 2017; Masson & Pesenti, 2016; Masson et al., 2018; Zhu et al., 2018; Zhu et al., 2019). When participants selected correct results from seven on-screen options, they chose larger numbers in addition and smaller numbers in subtraction, with their selections also influenced by spatial location—preferring upper-left options for subtraction and upper-right options for addition. This suggests that approximate arithmetic involves dynamic changes in mental spatial-numerical representations (Knops, Viarouge, & Dehaene, 2009). Additionally, neural activation during central presentation of addition problems aligns with that observed during rightward saccades, indicating that addition naturally draws attention to the right side of the mental number line (Knops et al., 2009).

Thus, the attentional shift account attributes operational momentum effects to biases in attentional shifts—specifically, the attentional focus moving too far along the mental number line in the operation direction, producing overestimation in addition and underestimation in subtraction.

Recent findings largely support this account. Jang and Cho (2022) found significant positive correlations between operational momentum effects and symbolic arithmetic ability, suggesting that stronger operational momentum may reflect more efficient application of arithmetic fundamentals and a more mature attentional system—consistent with the attentional shift account. Similarly, Pinheiro-Chagas et al. (2018) observed increasing operational momentum effects with age in 9- to 12-year-old children, attributing this to the Spatial-Numerical Association of Response Codes (SNARC) effect, which emphasizes how spatial positions influence numerical processing and relates to working memory and number polarity (Dai & Pan, 2021; Pan et al., 2019). However, while these findings link operational momentum to numerical processing and spatial attention, mapping numbers to space may involve complex processes (Dunn et al., 2019; Haman & Lipowska, 2021), making empirical validation of the underlying spatial attention mechanisms crucial. Furthermore, the attentional shift account allows flexible spatial mapping that can vary with mental number line orientation. For example, manipulating number line direction (left-to-right vs. right-to-left) reveals that operational momentum associates addition with the side containing larger numbers and subtraction with the side containing smaller numbers, rather than being fixed to "addition-right, subtraction-left" (Klein et al., 2014). This indicates that operational momentum effects relate to numbers' current (short-term) positions rather than habitual (long-term) locations (Pinhas et al., 2015).

2.2 Heuristic Account

The heuristic account emphasizes individuals' use of intuitive operation logic—specifically, that addition means "more" and subtraction means "less." Initially developed to explain operational momentum effects in infants, this account proposes that infants employ a simple heuristic: "if addition, accept larger results" and "if subtraction, accept smaller results," without involving spatial numerical representations (Dunn et al., 2019; McCrink & Wynn, 2009). McCrink and Wynn (2009) presented 9-month-old infants with videos showing object addition (second set joining first) or subtraction (second set leaving first), followed by correct, slightly smaller, or slightly larger outcomes. Infants looked longer at outcomes violating operational momentum—smaller results for addition and larger results for subtraction—demonstrating operational momentum effects in infancy.

Further research has examined how heuristics influence overestimation biases in addition. Charras et al. (2014) found that when operands were repeated (e.g., 24+24), addition results were underestimated, whereas non-repeated operands (e.g., 22+26) produced overestimation. Charras et al. (2012, 2014) attributed these differential biases to a heuristic balancing cognitive cost and approximate magnitude representation. Specifically, heuristics facilitate processing of identical stimuli; during addition, operands are mapped onto internal magnitude representations, triggering various errors. With repeated numbers, this mapping occurs only once, minimizing both error risk and encoding time, thereby producing underestimation (Charras et al., 2012; Charras et al., 2014; Gallistel & Gelman, 1992). Notably, this intuition-based logic related to processing costs only affects adults during approximate magnitude representation, as no differential biases emerge for repeated versus different operands when estimating relatively small numbers (Charras et al., 2014).

2.3 Compression Account

Based on behavioral and neuroscience research and Weber-Fechner' s law, the mental number line is logarithmically compressed, with representational distance between adjacent numbers increasing proportionally with magnitude (Dehaene & Changeux, 1993; Izard & Dehaene, 2008; Nieder & Miller, 2003; Piazza et al., 2010). The compression account suggests that operational momentum effects may result from necessary compression and decompression processes when performing arithmetic on this compressed number line. Specifically, operands are first represented as compressed logarithms on the mental number line. However, because this logarithmic conversion is not fully completed before computation—operands are not fully decompressed into linear metrics—individuals perform arithmetic on inadequately decompressed numbers. A small compression bias may persist, causing generated results to be substantially overestimated for addition and underestimated for subtraction, producing operational momentum effects. Conversely, precise decompression would yield arithmetically correct results.

However, recent developmental findings do not support the compression account. Four lines of evidence challenge this explanation. First, preschoolers show classic operational momentum effects in “$±1"*tasksbutnotin*"±5*or*6"*tasks.Accordingtocompressiontheory, compressi* or 6), producing larger operational biases, yet the opposite pattern emerged, suggesting other factors influence results (Haman & Lipowska, 2021). Similarly, operational momentum effects are larger for” 0” operations (e.g., 4+0) than for “1” operations (e.g., 3+1), which compression theory cannot explain (Pinhas & Fischer, 2008; Shaki et al., 2018). Second, intervention studies show that linear training on specific number lines improves precise mental number line representation but does not affect operational momentum effects (Kucian et al., 2011). Third, Knops et al. (2013) questioned the compression account, noting that if it were valid, children should show greater operational momentum in non-symbolic arithmetic because their numerical representations are more logarithmic (Berteletti et al., 2012; Siegler & Opfer, 2003). However, their study found reverse operational momentum in 6- to 7-year-olds, who overestimated subtraction more than addition. Finally, compression theory predicts that compression errors should decrease with age, reducing operational momentum effects, yet some studies find that operational momentum increases with age and mathematical ability (Jang & Cho, 2022; Pinheiro-Chagas et al., 2018). Thus, the compression account remains largely theoretical and requires further empirical validation.

2.4 Joint Action of Multiple Mechanisms

Accumulating evidence suggests that operational momentum effects cannot be explained by a single mechanism but rather result from multiple co-occurring theoretical processes (McCrink & Hubbard, 2017; Shaki et al., 2018). Some scholars propose that operational momentum effects may arise from the joint action of heuristic and attentional shift accounts, with the degree of heuristic influence related to the proportion of spatial attention allocated along the mental number line (Didino et al., 2019; McCrink & Hubbard, 2017). McCrink and Hubbard (2017) required participants to perform non-symbolic arithmetic while simultaneously processing unrelated spatial or non-spatial tasks to reduce attentional resources available for arithmetic. Results showed that reduced attention increased operational momentum effects, particularly for addition. This may occur because when fewer attentional resources are allocated to arithmetic, individuals rely more heavily on heuristics, and the pronounced increase in operational momentum for addition may reflect addition' s greater propensity to produce spatial attention shifts along the mental number line (McCrink & Hubbard, 2017).

Additionally, Shaki et al. (2018) proposed the Arithmetic Heuristics and Biases (AHAB) model, which posits that operational momentum effects involve at least three distinct and competing numerical cognitive mechanisms: the "more-or-less" heuristic, sign-space associations, and anchoring bias. The "more-or-less" heuristic reflects the general life experience that "addition produces larger results, subtraction produces smaller results" (McCrink & Wynn, 2009). Sign-space associations are based on the link between "addition-right space" and "subtraction-left space" (Hartmann et al., 2015; Pinhas & Fischer, 2008; Pinhas et al., 2014). Anchoring bias reflects the phenomenon that "if two operation outcomes are equal, the first operand in subtraction must be larger than in addition" (Shaki et al., 2018). This model not only addresses spatial influences on operational momentum and numerical representation but also suggests potential connections among these mechanisms (Fischer et al., 2018). For instance, reverse operational momentum effects in non-zero operations (when addends exceed zero) can be explained by anchoring bias exerting stronger influence than heuristics. However, in zero operations (when addends equal zero), anchoring bias affects addition and subtraction equally, yielding normal operational momentum effects (Shaki et al., 2018).

3 Developmental Predictors of Operational Momentum Effect

Early basic arithmetic abilities are crucial for children' s understanding of complex mathematical concepts and operations (Barth et al., 2005; Feigenson et al., 2004; Gilmore et al., 2007; Spelke, 2017) and serve as important predictors of future academic achievement (Duncan et al., 2007). Previous research indicates that infants and children can represent object quantities and perform basic addition and subtraction estimations using non-symbolic stimuli like dot arrays (Gilmore et al., 2007; McCrink & Wynn, 2004). Therefore, examining the developmental patterns and predictive factors of operational momentum effects in children is essential. While adult studies show consistent results, research on the formation and development of operational momentum effects in children remains limited, with inconsistent developmental patterns. The following sections review developmental studies of operational momentum effects to reveal their developmental trajectories and examine how various predictive factors influence their development.

3.1.1 Infancy

Operational momentum effects emerge during infancy (Cassia et al., 2016, 2017; McCrink & Wynn, 2009). Since young infants cannot perform true arithmetic, researchers have used magnitude ordering tasks to assess whether infants show prediction biases of overestimating "addition" (increasing sequences) and underestimating "subtraction" (decreasing sequences). Cassia et al. (2017) found that 4-month-old infants viewing increasing sequences expected more dots at a given position than actually presented, while those viewing decreasing sequences expected fewer dots—demonstrating prediction biases of overestimating next quantities in increasing sequences and underestimating them in decreasing sequences. Similar results were observed in 12-month-old infants (Cassia et al., 2016). Additionally, non-symbolic addition and subtraction tasks have revealed operational momentum effects in 9-month-olds, who overestimated addition outcomes and underestimated subtraction outcomes (McCrink & Wynn, 2009).

3.1.2 Childhood

Although operational momentum effects appear in infancy, findings from preschool to school-age children remain inconsistent. Specifically, operational momentum effects in 3- to 5-year-olds relate to mastery of counting principles (Haman & Lipowska, 2021), whereas 6- to 7-year-olds show reverse operational momentum effects (Knops et al., 2013), and 7- to 8-year-olds exhibit normal operational momentum effects (Jang & Cho, 2022). With increasing age, 9- to 12-year-olds show adult-like operational momentum effects (though 8-year-olds do not), with effect magnitude increasing with age (Pinheiro-Chagas et al., 2018). Researchers have explained this non-continuous developmental pattern from infancy to childhood by noting differences in attentional demands: infant studies rely on looking time measures requiring only passive viewing, whereas most child studies require active selection among multiple options and verbal or manual responses, consuming more cognitive resources and complicating developmental comparisons (Jang & Cho, 2022).

3.2 Developmental Predictors of Operational Momentum Effects in Children

The U-shaped developmental trajectory of operational momentum effects from preschool to school age may relate to multiple factors, including the acquisition of mathematical knowledge such as counting principles and operation understanding (Haman & Lipowska, 2021; Jang & Cho, 2022), as well as maturation of executive functions associated with brain development, where better spatial attention control may enable more accurate movement along the mental number line (Dunn et al., 2019).

3.2.1 Improvement in Mathematical Ability Mathematical ability refers to the capacity to acquire, process, and retain mathematical information or to learn and master new mathematical concepts and skills (Karsenty, 2020; Koshy et al., 2009; Vilkomir & O'Donoghue, 2009). Research indicates that operational momentum effects exist in infancy (Cassia et al., 2016, 2017), possibly reflecting innate mathematical intuition. As children acquire mathematical knowledge during the preschool years, their mathematical abilities improve, enabling them to apply counting principles and other mathematical concepts to estimate quantity changes. Children' s numerical concepts develop markedly during this period: from reciting numbers around age 2, to counting objects by age 3, to giving specified numbers of items and mastering other numerical concepts by ages 4-5 (Wynn, 1992). Individual differences exist in the development and formation of counting principles (Dowker, 2008), which may explain why operational momentum effect directionality differs between preschoolers and infants.

Recent research on 3- to 5-year-olds provides the first developmental evidence for this age group (Haman & Lipowska, 2021). The study measured operational momentum effects in both spatial and operational dimensions and analyzed their relationship with counting principle mastery. Results showed that counting principle mastery significantly affected operation-based operational momentum effects: only children who fully mastered counting principles showed classic operational momentum (overestimating addition and underestimating subtraction) in “$±$1” tasks. However, even children without full counting principle mastery showed directional biases linking quantity increases/decreases to rightward/leftward spatial directions. This indicates that spatial-directional operational momentum emerges earlier than operation-based operational momentum, with the latter being influenced by mathematical knowledge during the preschool period. Only children who grasped successor and predecessor concepts (understanding that adding or removing one object changes the total) showed classic operational momentum when estimating $±$1 operations. Additionally, classic operational momentum appears later in symbolic than non-symbolic tasks (Pinhas & Fischer, 2008), likely because symbolic tasks require understanding abstract symbols representing numerical meanings. On the other hand, research suggests that enhanced operational momentum effects relate to symbolic arithmetic ability, possibly reflecting stronger intuitive arithmetic capabilities and more robust arithmetic fundamentals (Jang & Cho, 2022).

3.2.2 Maturation of Spatial Attention Beyond mathematical ability improvement, another crucial predictor affecting operational momentum emergence and directionality is the maturation of spatial attention and its control in mapping onto the mental number line. Preschool and early elementary children tend to overestimate arithmetic outcomes when the second operand is relatively large (greater than 4), regardless of operation type (Haman & Lipowska, 2021; Knops et al., 2013). Interestingly, classic operational momentum effects re-emerge in 9- to 12-year-olds (Pinheiro-Chagas et al., 2018). This developmental change can be explained by the attentional shift account: maturation of visuospatial attention and attentional systems may be key factors influencing operational momentum effects across development. Knops et al. (2013) found that 6- to 7-year-olds with better spatial attention showed greater operational momentum effects in non-symbolic arithmetic, providing developmental support for the attentional shift account.

Furthermore, Dunn et al. (2019) provided supporting evidence by comparing preschoolers and adults. Participants viewed three dot arrays of identical quantities (ordered condition: e.g., 56, 56, 56) or three arrays doubling in value (unordered condition: e.g., 7, 14, 28) and predicted the next array. Adults showed greater overestimation of ascending sequences in unordered conditions compared to preschoolers, possibly reflecting maturation of spatial attention networks. The ability to shift spatial attention along the mental number line during arithmetic may change with age, with ages 9-12 representing a turning point when spatial attention networks become sufficiently mature to map onto the mental number line. Additionally, increasing mental length with age may influence children' s number line estimation representations (Cao et al., 2021). The study also found that preschoolers showed more pronounced spatial biases than operational biases, suggesting that operational momentum magnitude and directionality result from combined influences of visuospatial attention maturation and the degree of spatial attention shifting along the mental number line (Dunn et al., 2019).

4 Future Research Directions

Although increasing attention has been directed toward operational momentum effects in child development, theoretical explanations, predictive factors, and developmental trajectories remain under investigation, with several limitations requiring future enrichment, exploration, and improvement.

4.1 Integrating Research Tasks to Investigate Developmental Trajectories Across Age Groups

Despite operational momentum effects appearing in infancy and showing consistent empirical support in adults, findings across different developmental stages in childhood remain inconsistent (Zeng, 2020). These discrepancies may relate to task types (symbolic vs. non-symbolic stimuli), cognitive resource demands, and involvement of other mathematical knowledge. Moreover, no study has directly measured developmental trajectories of operational momentum effects. Consequently, although effects have been observed in infants and adults, underlying mechanisms may differ across developmental stages (Dunn et al., 2019). Task characteristics also influence operational momentum effects, possibly relating to familiarity and competence with different operation types—for instance, studies have found inconsistent operational momentum magnitudes for addition versus subtraction (Knops et al., 2009). Future research should employ consistent tasks across age groups to examine different operation types, enabling direct comparisons of operational biases across ages. Longitudinal studies with multiple time points or age groups are needed to comprehensively examine developmental trends. Additionally, adapting infant and child paradigms could enable investigation of operational momentum effects in younger children (ages 2-5), filling gaps in early childhood research.

4.2 Examining Associations Between Approximate Number System Maturity and Operational Momentum Effects

The Approximate Number System (ANS) enables individuals to estimate non-symbolic quantities greater than four without counting (Feigenson et al., 2004; Gallistel, 2011). ANS acuity during non-symbolic arithmetic may influence operational momentum effects through mathematical ability. Compared to adolescents and adults (Knops et al., 2014), the ANS may serve as a foundation for acquiring symbolic number meaning in pre- and early school-age children (Chu et al., 2016; van Marle et al., 2014). ANS acuity not only predicts mathematical ability (Cao et al., 2016; Niu et al., 2018; Au et al., 2018; Chen & Li, 2014; Elliott et al., 2019; He et al., 2016; Lindskog et al., 2021; Park et al., 2016; Szkudlarek & Brannon, 2018) but may also constitute a primary source of individual differences in mathematical ability during this period (Chu et al., 2016; Rittle-Johnson et al., 2017). Furthermore, symbolic and non-symbolic ANS estimation play distinct roles in mathematical skills for preschool and school-age children: non-symbolic estimation uniquely predicts early mathematical skills, numerical operations, mathematical problem-solving, and computational fluency in preschoolers, whereas symbolic estimation uniquely predicts mathematical problem-solving and numerical operations in school-age children (Cai et al., 2018). Therefore, when individuals estimate arithmetic with symbolic or non-symbolic stimuli, operational momentum effects may emerge through ANS influences on mathematical ability, with effects on operational momentum magnitude or direction potentially changing with age (Liang et al., 2021). Additionally, parental and child ANS acuity show positive correlations, suggesting possible intergenerational transmission of the ANS that manifests as individual differences from early development (Navarro et al., 2018; Odic & Starr, 2018). Future research should examine how ANS acuity and estimation precision affect operational momentum effects across development, including whether operational momentum emergence in preschoolers relates to ANS acuity and whether estimation precision using the ANS influences operational momentum magnitude in school-age children.

4.3 Investigating Stability of Operational Momentum Effects Across Different Operation Symbols

Current non-symbolic operational momentum research has primarily focused on addition and subtraction (Knops, Viarouge, & Dehaene, 2009; Knops et al., 2014; McCrink et al., 2007), with few studies examining more complex operations such as multiplication, division, logarithms, fractions, or elementary algebra (e.g., inverse operations: 2+X=8). However, research shows that preschool and early elementary children can double or halve both discrete (dot arrays) and continuous (line segments) non-symbolic stimuli, and can even quadruple or multiply by small numbers (Barth et al., 2009; Qu et al., 2021; Szkudlarek & Brannon, 2021), supporting the existence of multiplication and division estimation abilities beyond basic addition and subtraction. Existing research has examined operational biases in adults performing symbolic (digits) and non-symbolic (dot arrays) multiplication and division, finding that non-symbolic multiplication outcomes are overestimated while division outcomes are underestimated—demonstrating operational momentum effects in non-symbolic multiplication and division (Katz & Knops, 2014). However, other studies have found no overestimation in multiplication but overestimation in division, attributing this to heuristics (the intuition that "multiplication makes bigger, division makes smaller") or anchoring bias (where the first operand in division is on average larger than in multiplication for equivalent outcomes) (Shaki & Fischer, 2017). Moreover, inverse operation problems can be solved by children through the ANS before formal equation instruction, but only in non-symbolic contexts—symbolic written or spoken number operations cannot solve such problems (Kibbe & Feigenson, 2015). Additionally, multi-digit calculation requires basic mathematical knowledge and strategies such as carrying (Nuerk et al., 2011). For example, operational momentum effects appear in multi-digit no-carry problems (e.g., 24+53) and zero problems (e.g., 24+0), where addition estimates exceed subtraction estimates, but not in multi-digit carry problems, suggesting different mechanisms may operate in carry problems (Lindemann & Tira, 2015). Based on these findings, operational momentum effects likely exist in complex operations beyond addition and subtraction, and future research should investigate these effects across different operation types to further clarify their specific manifestations and stability.

4.4 Examining Joint Effects of Multiple Factors on Operational Momentum Effects

Recent research suggests that operational momentum effects may result from joint influences of multiple variables, including mathematical ability, visuospatial ability, and attentional process maturity (Cheng & Mix, 2014; Gunderson et al., 2012; Kucian et al., 2013; Thompson et al., 2013). For example, studies examining operational momentum effects in children with developmental dyscalculia (who lack logarithmic number concept understanding and show deficits in visuospatial attention and attentional functions) found no operational momentum effects in this population, suggesting that operational momentum requires understanding of arithmetic principles and conscious shifting of spatial attention along the mental number line (Kucian et al., 2013). Therefore, from the perspectives of spatial attention and mathematical ability, the association between spatial and mathematical abilities may jointly influence operational momentum effects. Spatial ability refers to the capacity to generate, retrieve, maintain, and process symbolic and nonverbal information such as shapes and positions (Hegarty & Waller, 2005; Linn & Peterson, 1985; McGrew, 2009), encompassing dynamic spatial abilities (involving transformation or movement, such as mental rotation) and static spatial abilities (involving understanding abstract spatial relations) (Mix & Cheng, 2012; Uttal et al., 2013). Tam et al. (2019) examined relationships between spatial and mathematical abilities, finding positive correlations with mental number line representation fully mediating the relationship between spatial ability and both computational ability and word problem-solving ability, highlighting the mental number line' s crucial role. Numerous studies have also documented associations between mathematical and spatial abilities (Frick, 2019; Mix et al., 2016; Zhang et al., 2017). However, this relationship is not simply linear: logical reasoning shows stronger associations with spatial ability than numerical or computational abilities, and both spatial ability and visuospatial memory correlate with mathematical ability to varying degrees (Xie et al., 2020). Consequently, different mathematical abilities may influence operational momentum effects through pathways associated with spatial ability, ultimately affecting spatial representation on the mental number line. Future research should investigate joint effects of multiple factors to further explore underlying mechanisms of operational momentum effects.

4.5 Designing Mathematical Ability Intervention Measures

Research has demonstrated links between the ANS and mathematical ability, with approximate arithmetic training significantly improving symbolic arithmetic performance and ANS acuity significantly affecting later computational ability (Elliott et al., 2019; He et al., 2016; Park & Brannon, 2013; Szkudlarek & Brannon, 2017). Compared to non-symbolic number comparison tasks, visuospatial short-term memory tasks, or number ordering training, non-symbolic approximate arithmetic training more effectively improves symbolic arithmetic fluency, though some studies have failed to replicate these benefits (Park &

Brannon, 2014; Szkudlarek et al., 2021). Individual ANS precision shows dynamic properties, with children' s approximate number performance influenced by direct experience. When task difficulty progresses from simple to complex, ANS precision improves, promoting mathematical ability through temporary adjustments that affect subsequent approximate number task performance (Wang et al., 2016; Wang et al., 2018; Wang et al., 2021). This suggests that future interventions targeting the ANS through approximate arithmetic training, with gradually adjusted task difficulty, could improve children' s mathematical ability and thereby influence operational biases and momentum effects. Additionally, meta-analytic research on working memory training indicates that it may effectively improve number sense through training targeting storage systems or central executive functions, including the phonological loop, visuospatial sketchpad, shifting, updating, and inhibition components. Such training includes single-system approaches (e.g., categorization working memory span tasks, N-back tasks) and multi-system approaches (e.g., Cogmed Working Memory Training) (Guo et al., 2018), which may help reduce operational biases and momentum effects by improving number sense. Future research should continue examining the effectiveness of different intervention measures to improve operational biases and momentum effects.

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