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Cooperative Obstacle Avoidance Methods for Multiple AUVs in Unknown Environments: Postprint

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Abstract

To address the challenge of simultaneously achieving obstacle avoidance and formation maintenance in path planning for multi-AUV (autonomous underwater vehicle) systems in unknown environments, a collaborative obstacle avoidance method based on leader-follower and behavior is proposed. First, a local path planning method for AUVs is designed by constructing collision risk and deviation-from-target evaluation functions. On this basis, and in combination with formation control methods, different behaviors and behavior selection modes are designed for the leader and followers respectively. Semi-physical simulation results demonstrate that the algorithm enables multi-AUV systems to achieve collaborative obstacle avoidance in unknown environments, with formation deviation and formation recovery time superior to those of traditional multi-robot obstacle avoidance algorithms. Experimental results verify the feasibility and effectiveness of the algorithm.

Full Text

Preamble

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Research on Multi-AUV Cooperative Obstacle Avoidance Method in Unknown Environment

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Abstract: Addressing the challenge of balancing obstacle avoidance and formation maintenance during path planning for multi-AUV (autonomous underwater vehicle) systems in unknown environments, this paper proposes a multi-AUV cooperative obstacle avoidance method based on the leader-follower and behavior-based paradigms. First, we design a local path planning method for AUVs by constructing evaluation functions for collision risk and deviation from the target. Building upon this foundation and integrating formation control methods, we design distinct behaviors and behavior selection patterns for both the leader and followers. Semi-physical simulation experiments demonstrate that the algorithm enables multi-AUV systems to achieve cooperative obstacle avoidance in unknown environments, with superior performance in formation deviation and formation recovery time compared to traditional multi-robot obstacle avoidance algorithms. The experimental results verify the feasibility and effectiveness of the proposed algorithm.

Keywords: path planning; multi-AUV; cooperative obstacle avoidance; leader-follower; behavior

0 Introduction

Multi-robot systems offer greater flexibility, broader exploration range, and enhanced robustness compared to single-robot systems, enabling them to accomplish tasks that are difficult for individual robots. AUVs serve as crucial tools for ocean exploration and development. As research on marine environments deepens, multi-AUV systems are increasingly applied in target search and collaborative localization. Within multi-robot systems, formation control has been extensively studied as a critical problem. Current mainstream formation control methods include the leader-follower approach [?], behavior-based methods [?], artificial potential field methods [?], and virtual structure approaches [?].

During mission execution, multi-robot systems often lack prior environmental knowledge, causing obstacles to appear on pre-planned paths. Robots must plan paths in real-time to avoid obstacles while preventing collisions with other robots due to route changes. Additionally, inter-robot distances must not become excessive, as this may compromise communication and even prevent formation recovery after obstacle avoidance. Therefore, to ensure robot safety and successful subsequent task execution, robots must collaborate to satisfy both obstacle avoidance and formation requirements simultaneously. Each robot in the system faces multiple constraints from obstacle positions and formation geometry, making the multi-robot cooperative obstacle avoidance problem particularly challenging.

Numerous scholars have investigated multi-robot cooperative obstacle avoidance based on formation control methods. Reference [?] treats the formation as a rigid structure within a leader-follower framework, achieving obstacle environment formation control through path planning for the leader and formation turning. While this simplifies multi-robot obstacle avoidance to a single-robot problem, it increases the complexity of the leader' s path planning algorithm and prevents the formation from passing through narrow regions due to strict formation maintenance. Reference [?] employs a behavior-based formation control method, assigning each robot behaviors such as moving toward the goal, obstacle avoidance, wall-following, collision avoidance, and formation keeping. Experiments demonstrate successful navigation through obstacle fields, but the weighted combination of behaviors makes formation maintenance difficult. Reference [?] utilizes artificial potential field methods for real-time multi-AUV obstacle avoidance, but with suboptimal formation performance. Furthermore, some researchers combine different formation control methods—such as integrating artificial potential fields with consensus algorithms [?, ?, ?], leader-follower with artificial potential fields [?, ?], and leader-follower with behavior-based methods [?, ?]—to compensate for the limitations of individual approaches and improve multi-robot obstacle avoidance effectiveness.

Given the complexity and diversity of obstacle environments and robot energy constraints, multi-robot systems are generally not treated as rigid structures. Under this premise, since each robot must simultaneously consider formation and obstacle constraints, the multi-robot system struggles to maintain its original formation and requires either local position adjustment for partial formation transformation or appropriate global formation transformation based on the obstacle environment. Current research on global formation transformation includes formation selection methods under obstacle constraints [?] and formation transformation strategies [?, ?]. Reference [?] introduces the concept of obstacle density, enabling robots to adjust positions locally based on perceived density. This approach yields adaptive formation transformation characteristics, but obstacle density proves difficult to compute in practice.

Based on the above analysis, this paper combines two formation control methods and proposes a multi-AUV cooperative obstacle avoidance method based on leader-follower and behavior-based approaches for unknown environments. The leader is responsible for formation selection and planning the overall path for the formation. Each AUV makes decisions through behavior selection, enabling the multi-AUV system to adaptively adjust its formation during obstacle avoidance and transform to appropriate formations as environmental conditions require. Simulation experiments demonstrate the algorithm's effectiveness across various underwater scenarios.

2.1 Leader-Follower Formation Control Method

The leader-follower approach is a common formation control method, primarily including $l-\psi$ control and $l-l$ control [?]. This paper adopts the $l-\psi$ control method, where followers must maintain specific distance and angle relationships with the leader. An AUV' s pose is represented as (x, y, ψ), where (x, y) are the coordinates in the fixed frame and ψ represents the heading angle.

As shown in Figure 2, with the leader's pose as (x_L, y_L, ψ_L), the follower's desired pose can be calculated based on leader information and formation parameters:

$$\begin{bmatrix} x_{ir}(t) \\ y_{ir}(t) \\ \psi_{ir}(t) \end{bmatrix} = \begin{bmatrix} x_L(t) + l_i \cos(\psi_L(t) + \varphi_i) \\ y_L(t) + l_i \sin(\psi_L(t) + \varphi_i) \\ \psi_L(t) \end{bmatrix}$$

[Figure 2: see original paper]

2.2 Multi-AUV Cooperative Obstacle Avoidance Control Architecture

As illustrated in Figure 3, this paper integrates behavior-based formation control with the leader-follower structure, assigning different behaviors to the leader and followers. The leader' s target is the final obstacle avoidance goal for the multi-AUV system, while followers' targets are virtual target points that satisfy the desired formation. At each time step, the leader selects behaviors and makes decisions based on environmental perception and the obstacle avoidance goal, planning the overall path for the formation while simultaneously performing formation selection and transmitting its current pose and formation parameters to followers. Followers calculate virtual target points in real-time and select behaviors based on their own sensory information.

[Figure 3: see original paper]

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1 AUV Local Path Planning Method Based on Forward-Looking Sonar

Robot path planning generally consists of environment modeling and path searching [?]. Current environment modeling methods primarily include grid-based, geometric representation, and topological representation approaches [?]. This paper employs geometric representation and, considering that polygons can approximate a broader range of contour shapes than circles, uses the minimum convex polygon of obstacle contours for environment modeling.

As shown in Figure 1, the AUV perceives its environment using a forward-looking sonar that returns sonar images containing information about the detected forward environment. The forward-looking sonar used in experiments has a horizontal opening angle of 130 degrees and an operating range of D_1. The sonar’ s detection region is evenly divided into N parts, with each part S_i $(i = 1, 2, ..., N)$ defined as:

$$S_i \in \left[\frac{13\pi}{18} - \frac{13\pi}{18N}(i-1), \frac{13\pi}{18} - \frac{13\pi}{18N}i\right)$$

The middle heading ψ_i of each divided detection region is designated as the representative heading for that region. An evaluation function is constructed to assess each ψ_i and determine the AUV’ s next heading.

[Figure 1: see original paper]

When no obstacles exist along the heading toward the target, the AUV navigates directly toward it. When obstacles are present, the AUV must alter its heading to avoid collision. The closer the distance between the AUV and obstacles along a particular heading, the higher the collision risk. Therefore, the evaluation function comprises two components: the first measures deviation from the target heading, and the second measures collision risk with obstacles.

The deviation evaluation function is defined as:

$$\text{Goal}(\psi_i) = 180 - |\psi_{\text{goal}} - \psi_i|$$

where ψ_{goal} is the target heading. The distance evaluation function is:

$$\text{Dist}(\psi_i) = \begin{cases} \frac{D_s - D_i}{D_s}, & D_i < D_s \\ 0, & D_i \geq D_s \end{cases}$$

where D_i is the distance to obstacles along direction ψ_i and D_s is a set distance threshold.

Before computing the total evaluation function, both components are normalized:

$$\text{Goal}_{\text{norm}}(\psi_i) = \frac{\text{Goal}(\psi_i)}{\sum_{j=1}^{N} \text{Goal}(\psi_j)}$$

$$\text{Dist}_{\text{norm}}(\psi_i) = \frac{\text{Dist}(\psi_i)}{\sum_{j=1}^{N} \text{Dist}(\psi_j)}$$

The total evaluation function is:

$$\text{Evaluate}(\psi_i) = \alpha \cdot \text{Goal}_{\text{norm}}(\psi_i) + \beta \cdot \text{Dist}_{\text{norm}}(\psi_i)$$

2.3 Leader Behavior Settings and Formation Selection

Based on the proposed multi-AUV cooperative obstacle avoidance control architecture, this section details the leader' s behavior settings, behavior selection, and formation selection methods.

To ensure smooth AUV operation, the leader' s speed magnitude V_l is held constant during obstacle avoidance. Two behaviors are assigned to the leader: moving toward the goal and obstacle avoidance. Let the leader's decision velocity be V_L:

$$V_L = J \cdot \begin{bmatrix} V_{\text{move\_to\_goal}} \\ V_{\text{avoid\_obstacle}} \end{bmatrix}$$

where J is the behavior selection matrix with only one element equal to 1 at each time step and the rest 0. $V_{\text{move\_to\_goal}}$ and $V_{\text{avoid\_obstacle}}$ are the decision velocities for the respective behaviors.

1) Moving-to-Goal Behavior

The decision velocity for moving-to-goal behavior can be expressed as:

$$V_{\text{move\_to\_goal}} = \begin{bmatrix} u_g \\ \psi_g \end{bmatrix} = \begin{bmatrix} \rho \\ \arctan\left(\frac{y_g - y}{x_g - x}\right) \end{bmatrix}$$

where (x_g, y_g) is the target point coordinate, (x, y) is the current position coordinate, and ρ is the Euclidean distance between the current position and the target point.

2) Obstacle Avoidance Behavior

The leader's obstacle avoidance method follows the local path planning approach detailed in Section 1. To prevent collisions among followers during obstacle avoidance, the safety distance between the leader and obstacles is increased. Let the safety distance between followers and obstacles be $D_{\min}$, and the minimum allowed distance between followers be $d_{\min}$. Then set $D_{\min} = d_{\min} + 2d_{\min}$. The obstacle avoidance decision can be expressed as:

$$V_{\text{avoid\_obstacle}} = \begin{bmatrix} u_\psi \\ \psi \end{bmatrix}$$

where ψ is the decision heading from the local path planning method.

3) Behavior Selection

The leader' s behavior selection proceeds as follows:

a) Determine whether obstacles exist within the forward-looking sonar' s field of view. If none exist, execute the moving-to-goal behavior; otherwise, proceed to step b).

b) Calculate the distance D_s between the target heading and obstacles. If D_s is less than the set threshold, execute obstacle avoidance behavior; otherwise, execute moving-to-goal behavior.

4) Formation Selection

Two formations are configured: triangle (original formation) and inline formation. If the leader' s decision heading lies between the tangents from the current position to two obstacles, the leader determines to pass between them (Figure 4). In this case, calculate the minimum distance $d_{\min}$ between the two obstacles. If $d_{\min} \geq D_{\min}$, select the triangle formation; otherwise, select the inline formation.

[Figure 4: see original paper]

2.4 Follower Behavior Settings and Behavior Selection

According to the proposed multi-AUV cooperative obstacle avoidance control architecture, followers must calculate virtual target points based on pose information and formation parameters transmitted by the leader, then select behaviors based on their own sensory information. This section details follower behavior settings and selection.

Four distinct behaviors are assigned to followers: moving-to-goal, obstacle avoidance, keeping, and following. Unlike the leader, followers require formation and formation transformation capabilities, thus their speed magnitude changes during obstacle avoidance. Let the follower' s decision velocity be V_F:

$$V_F = J \cdot \begin{bmatrix} V_{\text{move\_to\_goal}} \\ V_{\text{avoid\_obstacle}} \\ V_{\text{keep}} \\ V_{\text{follow}} \end{bmatrix}$$

where J is the behavior selection matrix and $V_{\text{move\_to\_goal}}$, $V_{\text{avoid\_obstacle}}$, V_{keep}, V_{follow} are the decision velocities for the respective behaviors.

1) Moving-to-Goal Behavior

The moving-to-goal decision can be expressed as:

$$V_{\text{move\_to\_goal}} = \begin{bmatrix} u_\psi \\ \psi_{vg} \end{bmatrix} = \begin{bmatrix} \varepsilon \cdot \rho_{vg} \\ \arctan\left(\frac{y_{vg}-y}{x_{vg}-x}\right) \end{bmatrix}$$

where (x_{vg}, y_{vg}) is the virtual target point coordinate, (x, y) is the current position coordinate, ρ_{vg} is the Euclidean distance between the current position and the virtual target point, u is the leader' s speed magnitude, and ε is a speed gain factor.

2) Obstacle Avoidance Behavior

The follower' s obstacle avoidance method follows the approach detailed in Section 1. When executing obstacle avoidance, the follower cannot maintain formation as it deviates from the virtual target direction. Under these circumstances, maintaining constant distance while changing only the angle relative to the leader is ideal for two reasons: a) ensuring communication distance between leader and follower, and b) preventing collisions between follower and leader.

As shown in Figure 5, construct a circle centered at the leader with radius l_i. When the line along the decision heading intersects the circle, the intersection point closer to the follower becomes its current target. When no intersection exists, expand the circle radius until tangency is achieved, with the tangent point serving as the current target.

The obstacle avoidance decision can be expressed as:

$$V_{\text{avoid\_obstacle}} = \begin{bmatrix} u_\psi \\ \psi_c \end{bmatrix}$$

where ψ_c is the angle between the ray from the current position to the current target and the x-axis, ρ_c is the Euclidean distance between the current position and the current target, and ψ is the decision heading from the local path planning algorithm.

3) Keeping Behavior

Under the $l - \psi$ formation control structure, when the leader' s heading angle changes, the follower' s virtual target point position may change abruptly. For followers on the inside of the leader' s turn, the target point may appear behind them. In such cases, turning to follow a rear target is unreasonable. The keeping behavior maintains the current heading and decelerates to wait for the target point. The keeping decision can be expressed as:

$$V_{\text{keep}} = \begin{bmatrix} u_\psi \\ \psi_c \end{bmatrix} = \begin{bmatrix} \cos(\psi_{vg} - \psi) \cdot \rho_{vg} \\ \psi \end{bmatrix}$$

where ρ_{vg} is the Euclidean distance between the current position and the virtual target point, and ψ is the AUV' s current heading.

4) Following Behavior

When the leader commands an inline formation, indicating narrow passable width ahead, followers track the leader' s trajectory directly after formation establishment. That is, followers store the position information transmitted by the leader as their own target point sequence.

5) Behavior Selection

Follower behavior selection proceeds as follows:

a) If the current formation is inline and already established, execute the following behavior; otherwise, proceed to step b).

b) Determine whether the virtual target point lies ahead of or behind the current position. If ahead, proceed to step c); otherwise, proceed to step d).

c) Calculate the minimum distance D_s between the target heading and obstacles. If D_s is less than the set threshold, execute obstacle avoidance behavior; otherwise, execute moving-to-goal behavior.

d) Calculate the minimum distance D_s between the current heading and obstacles. If D_s is less than the set threshold, execute obstacle avoidance behavior (using the current heading as the target heading for the avoidance algorithm); otherwise, execute the keeping behavior.

It should be noted that after the leader issues a formation transformation command, followers' behavior selection differs to enable rapid formation establishment. If no obstacles exist between the current position and the target point, the moving-to-goal behavior is selected until the new formation is established, after which the above behavior selection method resumes.

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3 Simulation Experiments

To verify algorithm feasibility, this paper conducted simulation experiments using a visualization platform and real underwater robot control software, including single-AUV obstacle avoidance experiments and three-AUV cooperative obstacle avoidance experiments. As shown in Figures 6 and 7, the visualization platform constructs realistic 3D scenes based on actual multibeam bathymetric data, chart data, etc., allowing configuration of multiple AUVs and real-time display of AUV poses. The platform can also set the operating range of forward-looking sonar and simulate and return forward-looking sonar images in real-time based on distances to obstacles at different angles within the detection range, similar to real sonar imaging principles. The platform supports three obstacle types: irregular shapes, cuboids, and cylinders. The underwater robot control software can simulate inter-AUV communication at a certain frequency.

Each AUV in the experiments maintains a constant depth of 100m. The simulation framework for each AUV is shown in Figure 8: obstacles are configured on the visualization platform, which returns each AUV' s forward-looking sonar information; each AUV makes decisions according to the proposed algorithm based on sensory information and target points to obtain velocity commands V; speed magnitude and direction (heading) are passed to corresponding PID controllers to obtain control inputs; these inputs are fed into the AUV model to obtain updated poses; finally, poses are transmitted to the visualization platform for real-time update and display. This simulation approach closely approximates real AUV conditions in terms of environmental perception, motion control, and communication.

[Figure 6: see original paper] [Figure 7: see original paper] [Figure 8: see original paper]

3.1 Single-AUV Obstacle Avoidance Simulation Experiments

To verify the proposed AUV local path planning algorithm, single-AUV obstacle avoidance simulation experiments were conducted with constant AUV speed. Setting $u = 1.5$, experiments were performed in discrete obstacle scenarios, different-type obstacle scenarios, complex scenarios, and seamount scenarios using the proposed algorithm. Planning results are shown in Figure 9 and Table 1. Performance metrics include minimum distance to obstacles, runtime, path length, and path smoothness (measured by total heading change during obstacle avoidance). Experimental results show that the AUV successfully navigates obstacle fields in different scenarios while maintaining adequate safety distances.

[Figure 9: see original paper]

To further validate the algorithm' s effectiveness, artificial potential field method and RRT algorithm were applied in the discrete obstacle scenario for comparison with the proposed algorithm. Results are shown in Figure 10 and Table 2. The experiments demonstrate that, with comparable safety distances, the proposed algorithm produces smoother paths and shorter runtime than the artificial potential field method. The RRT algorithm, due to its significant randomness, generates more tortuous paths.

[Figure 10: see original paper]

3.2 Multi-AUV Cooperative Obstacle Avoidance Simulation Experiments

To verify the proposed multi-AUV cooperative obstacle avoidance algorithm, simulation experiments were conducted with three AUVs. Algorithm parameters are selected as shown in Table 3.

As shown in Figure 11, the desired formation for the three AUVs is triangular, with the transformation formation being inline. Formation parameters are selected as shown in Table 4.

[Figure 11: see original paper]

3.2.1 Algorithm Simulation Based on the above algorithm and formation parameters, simulation experiments were conducted in three scenarios: discrete obstacles, simple scenario, and complex scenario. Scenario configurations and experimental results are shown in Figures 12-14 and Table 5. Performance metrics include average formation deviation and formation recovery time. Formation deviation δ is calculated from the current formation and desired formation:

$$\delta = \frac{1}{n}\sum_{i=1}^{n}|d_i - l_i|$$

where d_i is the current inter-AUV distance and l_i is the desired inter-AUV distance. Formation recovery time is defined as the time from when the last AUV finishes obstacle avoidance until the formation deviation falls below 7.5.

[Figure 12: see original paper] [Figure 13: see original paper] [Figure 14: see original paper]

Narrow Waterway Scenario

To verify the proposed formation transformation and following behavior, a narrow waterway scenario was configured with five sections of widths 60m, 40m, 28m, 23m, and 28m. Multi-AUV cooperative obstacle avoidance results are shown in Figures 15 and 16.

The experiment reveals that navigating the narrow waterway involves several stages: a) In the 60m-wide section, the formation maintains its original shape and passes safely, with followers beginning obstacle avoidance when detecting obstacles ahead within the threshold distance; b) In the 40m-wide section, both followers need to avoid obstacles, compressing formation width. When the leader calculates that the passable width ahead is smaller than the formation' s minimum compressible width, it commands transformation to inline formation and increases its safety distance from obstacles to 10m; c) After inline formation is established, followers execute following behavior, using the leader' s trajectory points as their own target sequence for trajectory tracking; d) When no obstacles exist in the leader' s sonar view, it begins moving toward the obstacle avoidance goal. Considering follower safety, the leader issues formation recovery command after 40s, and followers transform back to triangular formation; e) Upon reaching the obstacle avoidance goal, the multi-AUV system continues mission execution in the original formation on the planned route.

[Figure 15: see original paper] [Figure 16: see original paper]

3.2.2 Comparison Experiments To further validate the proposed multi-AUV cooperative obstacle avoidance algorithm' s effectiveness, it was compared with the artificial potential field method commonly used for obstacle environment formation control, with results shown in Figure 17.

[Figure 17: see original paper]

Table 6 shows that during multi-AUV obstacle avoidance, the proposed algorithm achieves smaller average formation deviation and faster formation recovery after obstacle avoidance compared to the artificial potential field method, benefiting subsequent task execution.

Some researchers argue that when encountering obstacles, multi-robot systems should prioritize obstacle avoidance over formation, splitting formation during avoidance and reassembling afterward. Based on this, the proposed algorithm was compared with formation-split obstacle avoidance, with results shown in Figure 18 and Table 7. Since formation-split avoidance lacks a formation deviation concept, inter-AUV distances are used for comparison. In the experimental scenario, formation-split avoidance causes AUVs to navigate around obstacles independently, resulting in excessive distances and potential communication loss. The large separation requires significant time for formation recovery after obstacle avoidance. Compared to formation-split avoidance, the proposed algorithm maintains inter-AUV distances near formation values while ensuring safety, enabling rapid formation recovery.

[Figure 18: see original paper]

3.2.3 Algorithm Scalability Experiments The above experiments validated the algorithm for three-AUV systems in various scenarios. To verify scalability, the number of AUVs was increased to five for experiments in simple and complex scenarios. The formation and parameters for five AUVs are shown in Figure 19 and Table 8.

[Figure 19: see original paper]

Algorithm parameters remain the same as for the three-AUV system. Experimental results in Figures 20 and 21 demonstrate that the proposed algorithm achieves good performance in both scenarios with five-AUV systems.

[Figure 20: see original paper] [Figure 21: see original paper]

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4 Conclusion

This paper proposes an AUV local path planning method based on forward-looking sonar and, building upon this, a multi-AUV cooperative obstacle avoidance method for unknown environments that combines leader-follower and behavior-based approaches. Behavior design considers not only AUV-obstacle collisions but also inter-AUV collisions. A target point calculation method for obstacle avoidance behavior is designed to maintain follower-leader distances near formation values during avoidance, ensuring communication and safety. Simulation experiments in various underwater scenarios demonstrate good performance, proving that the proposed method enables multi-AUV cooperative obstacle avoidance in unknown environments, balances formation requirements during avoidance, achieves rapid formation recovery after avoidance, and exhibits scalability. Future work will focus on physical validation using laboratory AUVs.

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