# Research on the Nature and Capacity of Information

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## Abstract

[Objective] Analyze the definition and properties of information capacity based on the information sextuple model, propose formulas relating information to matter, energy, and time, and provide theoretical reference for information science, particularly for applications in quantum information technology.

[Methods] Based on the information sextuple model proposed by objective information theory, combined with quantum information theory, we studied information capacity effects under various combinatorial conditions through rigorous mathematical axiomatic methods, and derived the relationships between information capacity and mass, energy, and time.

[Results] We proved that defining information capacity using information entropy is merely a special case of the information capacity defined by objective information theory, estimated the information capacity that a single quantum carrier can bear, derived and presented formulas relating information to matter, energy, and time, and calculated and analyzed the information capacity that the universe could possibly carry to date.

[Limitations] The relevant theoretical methods need to be validated through empirical physics research and applications in complex information systems science.

[Conclusion] The connotation of information capacity in objective information theory can profoundly and accurately reveal the interrelationships among the three fundamental constituent elements of matter, energy, and information in the objective world, and possesses good universal significance and application prospects.

# Full Text

## Preamble

### An Investigation of the Essence and Capacity of Information

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**Abstract:** [Objective] This paper studies the relationship between information capacity and mass, energy, and time through analyses of the definition and properties of information capacity based on the information sextuple model, in order to provide a theoretical reference for research and applications of information science. [Methods] Based on four basic postulations regarding information, we propose the information sextuple model; through an axiomatic approach, we study the information capacity effect under a variety of combinatorial conditions; in accordance with quantum information theories, we derive the relationship between information and matter, energy, and time through mathematical axiomatic methods. [Results] We prove that information capacity based on information entropy is a special case of the definition of information capacity in Objective Information Theory. Furthermore, we estimate the information volume of a single quantum carrier, derive the relationship formula between information capacity and mass, energy, and time. Based on this, we estimate and comparatively analyze the information volume existing in the universe. [Limitations] These theoretical results are in need of validation with empirical physical studies and applications of complex information systems science. [Conclusions] The connotation of information capacity of Objective Information Theory can profoundly and accurately, to a certain extent, reveal the relationship among the three basic elements of the objective world—that is, matter, energy, and information—demonstrating good universal significance and application prospects.

**Keywords:** Information model; Information capacity; Information combination; Quantum information; Relationship formula of information, matter, energy, and time; Information volume in the universe

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## 1 Introduction

Despite humanity' s entry into the information age, understandings of information remain diverse and contested. Research into the essence and capacity of information can help clarify and unify these perspectives, thereby advancing the development of information science. Shannon's foundational work [2] established information entropy as the measure of information quantity, forming the basis of modern information theory. It assumes discrete events occur with known probabilities $p_1, p_2, \dots, p_n$, yielding the entropy formula $H(X) = -\sum p_i \log p_i$, which he emphasized serves as a core measure of information, choice, and uncertainty in information theory. To date, the vast majority of information theory research employs various types of information entropy as the fundamental formula for calculating information quantity, which has indeed played a crucial role in the development of information technology.

However, careful analysis reveals that information entropy as a definition of information measurement has significant limitations that make it ill-suited to the increasingly broad demands of information system construction and application. The main reasons are threefold: First, as indicated by its title "A Mathematical Theory of Communication," its conclusions primarily apply to typical communication scenarios. Modern information systems, however, involve far more complex processes beyond communication, including information acquisition, processing, storage, and application, whose patterns cannot be fully captured by communication theory alone. Second, it assumes the occurrence probabilities $p_1, p_2, \dots, p_n$ of discrete events are known, which represents only a rare special case even in practical communication applications. Third, using the information entropy formula to describe information quantity implicitly embodies the notion that "only change produces information." Yet the most common files transmitted, stored, and applied in today' s computers, internet, and databases —regardless of whether they change—constitute objective realities that information system developers must confront, and the information entropy formula is virtually powerless to address them. Therefore, information measurement based on information entropy is inadequate for the numerous problems modern information systems must face and solve, highlighting deeper issues arising from the lack of universal consensus on information in academia.

In response, reference [15] proposed the fundamental concepts of Objective Information Theory, establishing definitions and models of information, analyzing its basic properties, and defining nine categories of information measures. Reference [16] examined the applicability and rationality of Objective Information Theory using an air traffic control system as a case study. Reference [17] introduced the condition of recoverable information, modifying and expanding the definitions to eleven categories of measures including capacity, delay, breadth, granularity, type, duration, sampling rate, aggregation, pervasiveness, distortion, and mismatch. It further analyzed the potential efficacy of these eleven measures in information systems, constructed the dynamic configuration of information systems, and formed the basic framework of information system dynamics, providing comprehensive metric guidance and model support for information system architecture design and integration. This framework has been applied and validated in China' s Smart Court system-of-systems engineering [18].

This paper builds upon the fundamental concepts of Objective Information Theory, uses simple examples to illustrate the concrete connotation of information, proposes four basic postulates regarding the binary subjects of information, existence time, state expression, and enabling mapping. Based on these, it demonstrates that the information sextuple model constitutes necessary and sufficient conditions for defining information, proves that information capacity defined by Objective Information Theory can also express Shannon' s information entropy principle, and shows that defining information capacity using information entropy is merely a special case of the capacity definition in Objective Information Theory. According to the combinational and decompositional properties of information, it explains the capacity additivity of atomic information combinations, then based on the Margolus-Levitin theorem proves the information capacity that a single quantum carrier can bear within a certain time period. Using Einstein' s mass-energy conversion formula, it proves the relationship formula between information capacity and mass, energy, and time, and uses this to calculate and analyze the information capacity the universe may have borne to date. This demonstrates that the formula accurately reveals the interrelationships among information, matter, energy, and time, exhibits good universality, and thereby highlights the scientific significance of Objective Information Theory.

## 2 Information Model and Capacity Analysis

Although understandings of information vary widely, the various data, text, audio, video, and multimedia files collected, transmitted, processed, stored, and applied in today' s computers and internet represent the most typical and universally significant forms of information. Wiener [19] proposed that information is information, not matter nor energy, elevating information to the same level as matter and energy. Reference [15] argues that matter is fundamental existence, energy is the existence of motional capacity, and information is the objective reflection of things and their states of motion in both objective and subjective worlds, making matter, energy, and information the three fundamental elements constituting the objective world. Through simple example analysis, general patterns can be induced into concise postulates, and employing axiomatic methods to study information models, definitions, and capacity constitutes a reasonable and scientific research methodology.

### 2.1 Simple Example of Information Concept

To facilitate understanding of the information concept and develop intuitive recognition of information models, we can analyze a penguin image file stored in a laptop computer. Figure 1 illustrates the active scene information of penguins stored in a laptop. The noumenon consists of the three penguins under the blue sky and white clouds in the scene, perhaps including the penguins' unknown subjective emotions. Assuming the image was captured at time $t_0$ with a shutter speed of 1/100 second, the noumenon' s state occurrence time is the interval $[t_0, t_0 + 0.01 \text{ seconds}]$. The noumenon' s state set is the charming postures of the penguins under the blue sky and white clouds during $[t_0, t_0 + 0.01 \text{ seconds}]$ as shown on the left side of Figure 1. The laptop computer on the right serves as the information carrier. Assuming the storage time runs from $t_1$ to $t_2$, any moment within the time interval $[t_1, t_2]$ can serve as the carrier' s reflection time, and the image format file stored in the laptop—a set of binary codes—constitutes the carrier' s reflection set.

This simple example reveals that the penguins' activity states and even subjective emotional information can be frozen and stored in the objective world through the laptop computer, with its content being the binary image format file that obeys objective scientific and technological laws. This represents the universal property of all information carried by modern information systems—existing independently of human subjective will—which is why we designate this epistemology of information as Objective Information Theory.

On the other hand, through appropriate processing of the image format file, the laptop can reproduce the image of three penguins under blue sky and white clouds on its display screen. Mathematically, this can be understood as performing an inverse mapping of the carrier' s reflection set to restore the noumenon' s state set. Thus, the most important property of information—recoverability—is concretely manifested. Assuming this image format file occupies 1M bits of storage space in the laptop, its information capacity is universally recognized as 1M bits, requiring neither probability estimation nor entropy formula calculation to determine its information capacity, and this capacity does not change because people are already familiar with the image.

People often regard only the binary image format file in the laptop as information, that is, treating only the carrier' s reflection set as information. Generally, this does not cause serious problems because in fact, people already implicitly possess knowledge of its noumenon when referring to the "information" in the carrier. However, in strict scientific terms, treating only the carrier' s reflection set as information severely limits our thorough investigation of information. In the above example, if we consider only the image format file in the laptop as an independent object, we cannot study critical issues such as authenticity, delay, and accuracy that are decisive for information. Therefore, only by incorporating the noumenon and carrier along with their existence times and states into a complete information model for comprehensive study can we fully reflect the various characteristics of information and provide adequate theoretical tools for solving many profound problems facing information science and technology development.

### 2.2 Information Model and Definition

Based on the fundamental understanding that information stands alongside matter and energy, and combined with the above example, to achieve the broadest possible consensus on information and establish a formal information model, we first propose four postulates regarding the information concept:

**Postulate 1 (Binary Subjects Postulate):** Information has two subjects—the noumenon and the carrier—where the noumenon possesses the essential connotation of information, and the carrier presents its objective form.

The important distinction between information and matter or energy is that matter can exist independently, energy can exist independently, but behind information's appearance always exists another "shadow."Hence, people often ask "Is this information true or false?" or "How accurate is this information?" —these questions essentially compare information' s "appearance" with its underlying "shadow." The fundamental significance of using information lies in its ability to restore or apply the truth behind its surface form. Here we particularly emphasize "objective form," requiring that information' s surface appearance should be independent of human subjective will, which demands that the carrier must be an entity in the objective world. In the example of Section 2.1, people appreciate the living states of the three penguin noumena precisely through the objective carrier of the laptop computer. This constitutes the basis for Postulate 1.

**Postulate 2 (Existence Time Postulate):** Both the connotation and form of information have their respective existence times.

Like matter and energy, information possesses temporality, manifested in that both the essential connotation of the information noumenon and the objective form of the carrier can emerge at certain times and disappear at others. Simultaneously, the existence times of noumenon and carrier have profound significance for the entire information. For instance, information delay depends on the relationship between the noumenon' s and carrier' s existence times. In the example of Section 2.1, for any moment $t$ within the time interval $[t_1, t_2]$, $t - (t_0 + 0.01)$ represents the delay of the image information in the laptop. Information delay has almost decisive impact in many information applications such as disaster forecasting, military intelligence, and document circulation. This constitutes the basis for Postulate 2.

**Postulate 3 (State Expression Postulate):** Both the information noumenon and carrier have their respective state expressions.

Another important distinction from matter and energy is that the essence of matter content remains matter, the essence of energy content remains energy, while the essence of information content is the state of things. Moreover, due to the binary nature of information subjects, neither the noumenon state nor the carrier state can be neglected. In the example of Section 2.1, the charming postures of penguins under blue sky and white clouds constitute the noumenon state, while the binary image file in the laptop constitutes the carrier state. Analyzing information authenticity and accuracy requires studying and comparing the relationship between noumenon state and carrier state. This constitutes the basis for Postulate 3.

**Postulate 4 (Enabling Mapping Postulate):** The information noumenon state can be enabling-mapped to the carrier state. The meaning of enabling mapping requires both establishing a surjective mapping from noumenon state to carrier state mathematically, and that the carrier state becomes objective reality because of the noumenon state.

If noumenon state and carrier state have no connection whatsoever, no information can be formed. Only when the noumenon state not only establishes a surjective mapping relationship with the carrier state mathematically but also forms a causal enabling relationship physically can it become information. In the example of Section 2.1, the penguin image information represents the mapping relationship established by information creators from the penguins' noumenon state to the laptop' s image file, and precisely because the penguins' noumenon state exists, the objective reality of the image file in the carrier emerges. This constitutes the basis for Postulate 4.

Postulates 1–4 employ axiomatic methods to induce the essential elements of information and their relationships, from which the information sextuple model can be derived.

**Lemma 1:** The necessary and sufficient condition for satisfying Postulates 1–4 is the existence of a non-empty information noumenon set $o$, noumenon occurrence time set $T_h$, noumenon state set $f(o, T_h)$, objective carrier set $c$, carrier reflection time set $T_m$, and carrier reflection set $g(c, T_m)$, such that $f(o, T_h)$ enabling-maps to $g(c, T_m)$. Denoting this mapping as $I$, we write $I = \langle o, T_h, f, c, T_m, g \rangle$, called the information sextuple model.

**Proof:** When Postulates 1–4 are satisfied, according to Postulate 1 there exist non-empty information noumenon set $o$ and objective carrier set $c$; according to Postulate 2 there exist non-empty noumenon occurrence time set $T_h$ and carrier reflection time set $T_m$; according to Postulate 3 there exists a noumenon state set depending on both $o$ and $T_h$, denoted as $f(o, T_h)$, and simultaneously a carrier state set depending on both $c$ and $T_m$, denoted as $g(c, T_m)$; according to Postulate 4, $f(o, T_h)$ enabling-maps to $g(c, T_m)$. When this mapping is denoted as $I$, we obtain the sextuple model $I = \langle o, T_h, f, c, T_m, g \rangle$. The necessity of the lemma is proved.

Conversely, if there exist non-empty information noumenon set $o$, noumenon occurrence time set $T_h$, noumenon state set $f(o, T_h)$, objective carrier set $c$, carrier reflection time set $T_m$, and carrier reflection set $g(c, T_m)$, and $f(o, T_h)$ enabling-maps to $g(c, T_m)$, then the sextuple model $I = \langle o, T_h, f, c, T_m, g \rangle$ can satisfy Postulates 1–4. The sufficiency of the lemma is proved.

Based on Postulates 1–4 and Lemma 1, we can provide the general mathematical definition of information in Objective Information Theory:

**Definition 1 (Mathematical Definition of Information):** Let $O$ denote the objective world set, $S$ the subjective world set, and $T$ the time set. Elements in $O$, $S$, and $T$ can be appropriately partitioned according to the specific requirements of the domain. The noumenon $o \in 2^{O \cup S}$, occurrence time $T_h \in 2^T$, state set of $o$ on $T_h$ denoted as $f(o, T_h)$, carrier $c \in 2^O$, reflection time $T_m \in 2^T$, and reflection set of $c$ on $T_m$ denoted as $g(c, T_m)$ are all non-empty sets. Information $I$ is the enabling mapping from $f(o, T_h)$ to $g(c, T_m)$, i.e., $I : f(o, T_h) \to g(c, T_m)$, or $I(f(o, T_h)) = g(c, T_m)$. The set of all information $I$ is called the information space, denoted as $\mathfrak{T}$, which constitutes one of the three fundamental elements of the objective world.

Special attention must be paid to the fact that $I$ must strictly satisfy all mathematical properties of a surjective mapping from $f(o, T_h)$ to $g(c, T_m)$. However, only when $f(o, T_h)$ itself stimulates or through external forces generates the corresponding $g(c, T_m)$ in the objective world can $I$ be called information. This is the fundamental reason why Definition 1 introduces the concept of "enabling mapping." That is, even if a surjective mapping from $f(o, T_h)$ to $g(c, T_m)$ exists mathematically, if the emergence of $g(c, T_m)$ has no physical connection whatsoever with $f(o, T_h)$, such a mapping cannot constitute information. In short, surjective mapping is a necessary but not sufficient condition for information. Ultimately, information is both mathematical and physical [20], which is the essence of information.

**Definition 2 (Recoverable Information):** Let information $I = \langle o, T_h, f, c, T_m, g \rangle$ also be an injection from $f(o, T_h)$ to $g(c, T_m)$, meaning that for any $o_\lambda, o_\mu \in o$, $T_{h\lambda}, T_{h\mu} \in T_h$, $f_\lambda, f_\mu \in f$, if $f_\lambda(o_\lambda, T_{h\lambda}) \neq f_\mu(o_\mu, T_{h\mu})$, then necessarily $I(f_\lambda(o_\lambda, T_{h\lambda})) \neq I(f_\mu(o_\mu, T_{h\mu}))$. In this case $I$ is an invertible mapping, i.e., there exists an inverse mapping $I^{-1}$ such that for any set of $c_\lambda \in c$, $T_{m\lambda} \in T_m$, $g_\lambda \in g$, there exists a unique set of $o_\lambda \in o$, $T_{h\lambda} \in T_h$, $f_\lambda \in f$ satisfying $I^{-1}(g_\lambda(c_\lambda, T_{m\lambda})) = f_\lambda(o_\lambda, T_{h\lambda})$, from which we obtain $I^{-1}(g(c, T_m)) = f(o, T_h)$. We then say information $I$ is recoverable, and also call $f(o, T_h)$ the recovered state of information $I$.

Evidently, based on $g(c, T_m)$ and $I^{-1}$ we can recover the state $f(o, T_h)$ of $o$ on $T_h$, which is the recoverability of information. In the real world, people' s search for the recovered state through information represents the most important property and significance of information.

### 2.3 Definition and Corollaries of Information Capacity

Enriching and perfecting the information measurement system to support the analysis of information system efficacy is the original intention of Objective Information Theory. Information capacity is the most concerned measure among them.

**Definition 3:** Let $I = \langle o, T_h, f, c, T_m, g \rangle$ be recoverable information, $O$ the objective world set, $T$ the time set, $g(O \times T)$ the state set on the objective world and time domain containing $g(c, T_m)$, and $(g(O \times T), 2^{g(O \times T)}, \sigma)$ constitute a measure space. Then the capacity of information $I$ with respect to measure $\sigma$, denoted $\text{volume}_\sigma(I)$, is the measure $\sigma(g(c, T_m))$ of $g(c, T_m)$, i.e.,

$$\text{volume}_\sigma(I) = \sigma(g(c, T_m)). \tag{1}$$

To maximize universality, information capacity is expressed using measure, encompassing both summation forms under finite-element conditions and integral forms under continuous integrable function conditions. Similarly, for the same object set, mathematics can define multiple different measures based on different concerns, such as Lebesgue measure, Borel measure, etc. [21]. Therefore, the information capacity defined here is not unique but can be defined differently according to various application contexts. In information systems, information capacity is typically measured in bits, the most comprehensible information measure for people.

**Corollary 1 (Minimum Recoverable Capacity of Random Event Information):** Let event $X$ take values from the set $\{x_i\}$ $(i = 1, \dots, n)$ randomly, with probability $p_i$ $(i = 1, \dots, n)$ for value $x_i$, where different values are independent and $\sum_{i=1}^{n} p_i = 1$. Let information $I = \langle o, T_h, f, c, T_m, g\rangle$ represent the encoding for transmitting the value of $X$ via a channel, in which case we call $I$ random event information. Here, the noumenon $o$ is the random event $X$, the occurrence time $T_h$ is the time when event $X$ occurs, the state set $f(o, T_h)$ is the values $x_i$ $(i = 1, \dots, n)$ of event $X$, the carrier $c$ is the channel transmitting $X$' s values, the reflection time $T_m$ is the time when channel $c$ transmits $X$' s values, and the reflection set $g(c, T_m)$ is the specific encoding by which channel $c$ transmits $X$' s values. If measure $\sigma$ represents the number of bits of $g(c, T_m)$, then the minimum capacity of $I$ as recoverable information is

$$\text{volume}_\sigma(I) = -\sum p_i \log_2 p_i.$$

**Proof:** Reference [2] establishes that communication semantics are irrelevant to engineering problems. Therefore, to minimize required channel bandwidth, the communication process transmitting $X$' s values does not need to directly transmit specific values $x_i$; it only needs to use different binary encodings to represent the event of $X$ taking value $x_i$ and transmit the corresponding encoding $(i = 1, \dots, n)$. That is, by selecting an appropriate encoding method for $g(c, T_m)$, the required channel bandwidth can be reduced.

Using proof by contradiction, suppose there exists some encoding method for $g(c, T_m)$ such that $H' = \sigma(g(c, T_m)) < -\sum p_i \log_2 p_i = H$ while still maintaining $I$ as recoverable information (meaning the channel can completely transmit the source information), where $H$ is the information entropy of event $X$. Assume channel $c$ has bandwidth $W$. Then we have $W/H' > W/H$, meaning channel $c$ can completely transmit source information at a rate greater than $W/H$. This contradicts the conclusion of Theorem 9 in [2] that the channel transmission rate $W/H$ cannot be exceeded.

Therefore, no encoding method can make $\sigma(g(c, T_m)) < -\sum p_i \log_2 p_i$ while keeping $I$ recoverable. Thus, $\text{volume}_\sigma(I) = -\sum p_i \log_2 p_i$ is the minimum recoverable information capacity. The corollary is proved.

Corollary 1 shows that the information capacity defined in (1) can also express Shannon's information entropy principle, and that defining information capacity using information entropy is merely a special case of (1). Since the information capacity defined in (1) only requires $I$ to be a bijective surjection from $f(o, T_h)$ to $g(c, T_m)$ and $g(c, T_m)$ to be a measurable set with respect to measure $\sigma$, it has more general mathematical constraints than information entropy and therefore must have more universal application scenarios.

Based on Definition 1, we can also mathematically define the other ten categories of information measures including delay, breadth, granularity, type, duration, sampling rate, aggregation, pervasiveness, distortion, and mismatch. Moreover, like Corollary 1, each category can derive important mathematical corollaries consistent with classical information principles or possessing practical significance (Table 1) [17].

**Table 1: Corollaries of the Information Measurement System**

| Classical and Common Principles | Corollaries of Objective Information Theory |
|---|---|
| Shannon Information Entropy | The minimum recoverable capacity of random event information is its information entropy. |
| Serial Information Transmission Delay | The overall delay of serial information transmission equals the sum of delays at each stage. |
| Rayleigh Criterion for Optical Imaging | The granularity of optical imaging information is proportional to light wavelength and inversely proportional to photosensitive element width. |
| Recoverable Information Type Invariance Principle | Recoverable information can maintain information type unchanged. |
| Average Duration of Continuous Monitoring Information | The average duration of continuous monitoring information equals the mean time between failures of information acquisition equipment. |
| Nyquist Sampling Theorem | The minimum recoverable sampling rate for periodic information equals the highest frequency of noumenon state. |
| Recoverable Information Aggregation Invariance Principle | Recoverable information can maintain information aggregation unchanged. |
| Metcalfe' s Law | The value of a network system equals the product of its maximum information breadth and maximum pervasiveness. |
| Kalman Filtering Principle | Kalman filtering is the minimum distortion estimation method for discrete linear random systems. |

| Classical and Common Principles | Corollaries of Objective Information Theory |
| --- | --- |
| Average Search Length of Search Algorithms | The average search length for information with minimum mismatch in a finite information set is the average search length of the algorithm. |

## 3 Capacity Effects of Information Combination and Decomposition

The sextuple model expression $I = \langle o, T_h, f, c, T_m, g\rangle$ does not emphasize the set properties of information. However, simple analysis reveals the set properties of information, namely $I = \bigcup\langle o_\lambda, T_{h\lambda}, f_\lambda, c_\lambda, T_{m\lambda}, g_\lambda\rangle$, where $o_\lambda \in o$, $T_{h\lambda} \in T_h$, $f_\lambda(o_\lambda, T_{h\lambda}) \in f(o, T_h)$, $c_\lambda \in c$, $T_{m\lambda} \in T_m$, $g_\lambda(c_\lambda, T_{m\lambda}) \in g(c, T_m)$, with $\lambda \in \Lambda$ as an index set. Thus, information $I$ is actually a set composed of a series of sextuple elements. Information possesses set properties. Multiple sets can be combined into larger sets through "union operations," and a set can be expressed as the union of several sets and decomposed into smaller sets. Set combination or decomposition determines that information has fundamental combinational and decompositional properties. Undoubtedly, information combination and decomposition directly affect relevant information measures including capacity. Therefore, studying the specific concepts and forms of information combination and decomposition is significant for more profoundly understanding and applying information capacity measures.

### 3.1 Mathematical Definitions of Information Combination and Decomposition

Information combination and decomposition originate from the basic concept of sub-information.

**Definition 4:** For information $I' = \langle o', T'_h, f', c', T'_m, g'\rangle$ and $I = \langle o, T_h, f, c, T_m, g\rangle$, if set $I' \subseteq I$ and $I'(f'(o', T'_h)) = I(f'(o', T'_h))$, then we call $I'$ sub-information of $I$, denoted $I' \subseteq I$, read as "$I'$ is contained in $I$." When $I'$ is a proper subset of $I$, we call $I'$ proper sub-information of $I$, denoted $I' \subset I$, read as "$I'$ is properly contained in $I$."

Figure 2 intuitively illustrates the relationship between information $I$ and its sub-information $I'$. All information measure definitions in [17] target recoverable information. Therefore, when studying the measure effects of information combination and decomposition, the condition of information recoverability is crucial.

**Corollary 2 (Recoverability of Sub-information):** Let information $I' =$

$\langle o', T_h', f', c', T_m', g' \rangle$ be sub-information of $I = \langle o, T_h, f, c, T_m, g \rangle$. If $I$ is recoverable information, then $I'$ is also recoverable information.

**Proof:** To prove $I' = \langle o', T_h', f', c', T_m', g' \rangle$ is recoverable, we need only prove $I'$ is a one-to-one mapping from $f'(o', T_h')$ to $g'(c', T_m')$. In fact, if $I'$ were not one-to-one, there would exist two different sets $o_1', o_2' \in o'$, $T_{h1}', T_{h2}' \in T_h'$, $f_1', f_2' \in f'$ and one set $c_1' \in c'$, $T_{m1}' \in T_m'$, $g_1' \in g'$ such that $I'(f_1'(o_1', T_{h1}')) = I'(f_2'(o_2', T_{h2}')) = g_1'$. However, by the definition of sub-information, at this time $I(f_1'(o_1', T_{h1}')) = I(f_2'(o_2', T_{h2}')) = g_1'$. This contradicts the recoverability of $I$. Therefore, $I'$ must be recoverable information. The corollary is proved.

Based on the sub-information concept, we can introduce the concepts of information combination and decomposition.

**Definition 5 (Information Combination and Decomposition):** For information $I = \langle o, T_h, f, c, T_m, g \rangle$ and its two proper sub-information sets $I' = \langle o', T_h', f', c', T_m', g' \rangle$ and $I'' = \langle o'', T_h'', f'', c'', T_m'', g'' \rangle$, if set $I = I' \cup I''$ and for any $o_\lambda \in o$, $T_{h\lambda} \in T_h$, we have $I(f(o_\lambda, T_{h\lambda})) = I'(f'(o_\lambda, T_{h\lambda}))$ or $I''(f''(o_\lambda, T_{h\lambda}))$, then we call $I$ the combination of $I'$ and $I''$, and also call $I'$ and $I''$ the decomposition of $I$, denoted $I = I' \cup I''$.

Evidently, combination and decomposition are inverse relationships. Figure 3 intuitively illustrates the relationship between information $I$ and its two decomposed sub-information sets. When $I$ is recoverable information, according to Corollary 2, both $I'$ and $I''$ are recoverable information, making all measure definitions applicable to the three information sets $I$, $I'$, and $I''$. Therefore, we can study the capacity effects of information combination and decomposition without any obstacles.

**Definition 6 (Overlapping Information):** Let $I' = \langle o', T_h', f', c', T_m', g' \rangle$ and $I'' = \langle o'', T_h'', f'', c'', T_m'', g'' \rangle$ be two information sets. If set $I' \cap I''$ is nonempty and for any $o_\lambda \in o$, $T_{h\lambda} \in T_h$, we have $I'(f'(o_\lambda, T_{h\lambda})) = I''(f''(o_\lambda, T_{h\lambda}))$, i.e., there exist $c_\lambda \in c = c' \cap c''$, $T_{m\lambda} \in T_m = T_m' \cap T_m''$ such that $I'(f'(o_\lambda, T_{h\lambda})) = g'(c_\lambda, T_{m\lambda}) = g''(c_\lambda, T_{m\lambda}) = I''(f''(o_\lambda, T_{h\lambda}))$, define $g(c_\lambda, T_{m\lambda}) = I(f(o_\lambda, T_{h\lambda})) = I'(f'(o_\lambda, T_{h\lambda})) = I''(f''(o_\lambda, T_{h\lambda}))$, then $I = \langle o, T_h, f, c, T_m, g \rangle$ is also information and is sub-information of both $I'$ and $I''$. We call it the overlapping information of $I'$ and $I''$, denoted $I = I' \cap I''$. In particular, if set $I' \cap I'' = \emptyset$, we say information $I'$ and $I''$ have no overlapping information.

Figure 4 intuitively illustrates the concept of information $I$ being overlapping information of two information sets $I'$ and $I''$. Based on the sub-information definition, Corollary 2 yields the following corollary regarding the recoverability of overlapping information.

**Corollary 3 (Recoverability of Overlapping Information):** Let $I'$ and $I''$ both be recoverable information, and $I = I' \cap I''$ be overlapping information. Then $I$ is also recoverable.

### 3.2 Capacity Additivity of Information Combination

**Definition 7 (Non-overlapping Information Combination and Decomposition):** For information $I$, $I'$, and $I''$, if $I$ is the combination of $I'$ and $I''$ and $I' \cap I'' = \emptyset$, then we call $I$ the non-overlapping combination of $I'$ and $I''$, and also call $I'$ and $I''$ the non-overlapping decomposition of $I$. In this case, we also call $I'$ and $I''$ complementary information of $I$ with respect to each other, denoted $I' = I/I''$ and $I'' = I/I'$.

Figure 5 intuitively illustrates the concept of non-overlapping information combination and decomposition. Non-overlapping combination and decomposition are special cases of information combination and decomposition, but when studying capacity effects, focusing on them can greatly simplify problems while not affecting the discovery of relevant patterns for cases with overlapping sub-information.

**Corollary 4 (Capacity Additivity of Non-overlapping Combined Information):** For information $I = \langle o, T_h, f, c, T_m, g\rangle$, $I' = \langle o', T'_h, f', c', T'_m, g'\rangle$, and $I'' = \langle o'', T''_h, f'', c'', T''_m, g''\rangle$, if $I$ is the non-overlapping combination of $I'$ and $I''$, and $g'(c', T'_m)$ and $g''(c'', T''_m)$ are both measurable sets with respect to measure $\sigma$, then $g(c, T_m)$ is also measurable with respect to $\sigma$, and the capacity of information $I$ with respect to measure $\sigma$ equals the sum of the capacities of $I'$ and $I''$ with respect to $\sigma$, i.e.,

$$\text{volume}_\sigma(I) = \text{volume}_\sigma(I') + \text{volume}_\sigma(I''). \tag{2}$$

**Proof:** Since $I = I' \cup I''$, we have $g(c, T_m) = g'(c', T'_m) \cup g''(c'', T''_m)$. And from the measurability of $g'(c', T'_m)$ and $g''(c'', T''_m)$ with respect to $\sigma$, we can deduce that $g(c, T_m)$ is also measurable with respect to $\sigma$ [21].

We now prove $g' \cap g'' = \emptyset$ by contradiction. If not, there would exist $c_\lambda \in c' \cap c''$, $T_{m\lambda} \in T'_m \cap T''_m$ such that $g(c_\lambda, T_{m\lambda}) = g'(c_\lambda, T_{m\lambda}) = g''(c_\lambda, T_{m\lambda}) \in g' \cap g''$. Since $I$ is recoverable information, there must exist a unique set $o_\lambda \in o$, $T_{h\lambda} \in T_h$ such that $I(f(o_\lambda, T_{h\lambda})) = g(c_\lambda, T_{m\lambda})$. Similarly, since $I'$ and $I''$ are also recoverable information and $I$ is the combination of $I'$ and $I''$, we have $I(f(o_\lambda, T_{h\lambda})) = I'(f'(o_\lambda, T_{h\lambda})) = I''(f''(o_\lambda, T_{h\lambda}))$. Thus $o_\lambda \in o = o' \cap o''$, $T_{h\lambda} \in T_h = T'_h \cap T''_h$, and $f(o_\lambda, T_{h\lambda}) \in f = f' \cap f''$, making $o' \cap o''$, $T'_h \cap T''_h$, and $f' \cap f''$ all non-empty sets, which contradicts the assumption that $I'$ and $I''$ have no overlap. Therefore, by the additivity property of measure, $\text{volume}_\sigma(I) = \sigma(g(c, T_m)) = \sigma(g'(c', T'_m)) + \sigma(g''(c'', T''_m)) = \text{volume}_\sigma(I') + \text{volume}_\sigma(I'')$. The corollary is proved.

Equation (2) shows that information capacity, like set measure, possesses additivity. This both aligns with common understanding and provides an important foundation for deeply studying the capacity effects of information combination and decomposition. Using set and measure properties, we can obtain the following corollary regarding the capacity composition of overlapping combined information.

**Corollary 5 (Capacity Composition of Overlapping Combined Information):** For information $I = \langle o, T_h, f, c, T_m, g\rangle$, $I' = \langle o', T_h', f', c', T_m', g'\rangle$, and $I'' = \langle o'', T_h'', f'', c'', T_m'', g''\rangle$, if $I$ is the combination of $I'$ and $I''$, and $g'(c', T_m')$ and $g''(c'', T_m'')$ are both measurable sets with respect to measure $\sigma$, then $g'(c', T_m') \cap g''(c'', T_m'')$ is also measurable with respect to $\sigma$, and

$$\text{volume}_\sigma(I) = \text{volume}_\sigma(I') + \text{volume}_\sigma(I'') - \text{volume}_\sigma(I' \cap I''). \quad (3)$$

**Corollary 6 (Capacity Additivity of Countable Non-overlapping Information Combination):** Let recoverable information $I = \langle o, T_h, f, c, T_m, g\rangle$ be the non-overlapping combination of a countable information set $\{I_i = \langle o_i, T_{hi}, f_i, c_i, T_{mi}, g_i\rangle \mid i = 1, 2, ...\}$. Then the capacity of information $I$ with respect to measure $\sigma$ equals the sum of the capacities of all $I_i$ with respect to $\sigma$, i.e.,

$$\text{volume}_\sigma(I) = \sum \text{volume}_\sigma(I_i). \quad (4)$$

**Proof:** Since recoverable information $I$ is the non-overlapping combination of the countable information set $\{I_i \mid i = 1, 2, ...\}$, i.e., $I = \bigcup_{i=1}^{\infty} I_i$, and for any $i \neq j$, $I_i \cap I_j = \emptyset$, from the proof of Corollary 4 we know that for any $i \neq j$, $g_i(c_i, T_{mi}) \cap g_j(c_j, T_{mj}) = \emptyset$. Moreover, since $g(c, T_m) = \bigcup_{i=1}^{\infty} g_i(c_i, T_{mi})$, by the countable additivity property of measure we have $\text{volume}_\sigma(I) = \sigma(g(c, T_m)) = \sum_{i=1}^{\infty} \sigma(g_i(c_i, T_{mi})) = \sum_{i=1}^{\infty} \text{volume}_\sigma(I_i)$. The corollary is proved.

Equation (4) also applies when the upper limit of $i$ is finite, which is important for practical applications because all information in information systems composed of networks and computers is actually finite and can be analyzed for capacity using (4).

### 3.3 Capacity Calculation of Atomic Information and Its Combinations

Like matter can be decomposed into indivisible elementary particles and energy can be decomposed into indivisible quanta, any information can also be decomposed to its most fundamental, indivisible level. This introduces the concept of atomic information.

**Definition 8 (Atomic Information):** For recoverable information $I$ and $I'$, if $I'$ is proper sub-information of $I$ and there exists no other proper sub-information $I''$ of $I$ such that $I'' \subset I'$, then we call $I'$ atomic information of $I$.

Figure 6 intuitively illustrates the concept of atomic information. Atomic information does not specifically refer to information with atoms as noumenon or carrier. Just as elementary particles are the most minute and fundamental components constituting the entire material world, atomic information is the most minute and fundamental component constituting the entire information space, thus playing a crucial role in studying information composition and properties. All information can be viewed as the combination of all its atomic information, and atomic information sets have no overlap. Therefore, from Corollary 6 we obtain:

**Corollary 7 (Capacity of Atomic Information and Its Combinations):** For recoverable information $I = \langle o, T_h, f, c, T_m, g\rangle$, if all atomic information $I_\lambda = \langle o_\lambda, T_{h\lambda}, f_\lambda, c_\lambda, T_{m\lambda}, g_\lambda\rangle$ have $g_\lambda(c_\lambda, T_{m\lambda})$ measurable with respect to measure $\sigma$ ($\lambda \in \Lambda$, where $\Lambda$ is an index set). Then when $\Lambda$ is a countable set,

$$\text{volume}_\sigma(I) = \sum \text{volume}_\sigma(I_\lambda). \tag{5}$$

When $\Lambda$ is a finite set, we need only change the upper limit of summation in (5) to the cardinality of $\Lambda$. However, if $\Lambda$ is an uncountable set, (5) cannot be simply changed to an integral formula because measure generally does not possess additivity for uncountable sets.

Therefore, to simplify problems, research on atomic information should avoid extending to uncountable continuous sets as much as possible. Specifically, the number of atomic information in $I$ depends on the composition and characteristics of reflection set $g(c, T_m)$, which in turn depends on the composition and characteristics of carrier $c$ and reflection time $T_m$. Thus, if we can simplify sets $c$, $T_m$, and $g$ to finite or countable sets as much as possible, capacity calculation for information $I$ can be greatly facilitated. Fortunately, despite the vastness of the universe, the number of its elementary particles is considered finite. Therefore, for any information $I$, we can consider carrier $c$ to be composed of a finite set of elementary particles. However, time is a continuous variable, so $T_m$ may be an uncountable continuous set. Therefore, appropriate discretization in research will provide sufficient convenience for calculating information capacity. The following study on the relationship between information capacity and matter and energy will prove that information carriers, limited by physical properties, can only bear different information on discrete time sets, thereby practically achieving the unification of mathematical requirements and physical rules.

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## 4 Relationship Between Information Capacity and Mass, Energy, and Time

Information capacity is determined by the measure of the state of its carrier in the objective world during the reflection time. Any carrier in the objective world exists in the form of matter or energy, so the states of matter and energy determine the state of the information reflection set. Changes in matter and energy states all require energy support [22], so information capacity must be closely related to its carrier' s state and the energy it contains or is subjected to, as well as to the carrier' s reflection time—issues that have attracted attention for decades.

### 4.1 Information Capacity of a Single Quantum Carrier

Quantum information theory represents an important frontier achievement in contemporary information science and technology. It is generally believed that no radiation exists with energy lower than a single photon [23]. Individual quanta such as quarks, leptons and other fermions, and gluons, W bosons, photons, gravitons and other bosons [24] are fundamental elements indivisible in energy, and thus are also the most efficient carriers for bearing atomic information.

**Corollary 8 (Information Capacity of a Single Quantum Carrier):** For recoverable information $I = \langle o, T_h, f, c, T_m, g\rangle$, let carrier $c$ be a single quantum, and call $I$ single quantum carrier information. Define measure $\sigma$ as the number of distinguishable states of $c$, and $t = \sup T_m - \inf T_m$ as the reflection duration of information $I$. Then the upper bound estimate of the capacity of information $I$ with respect to measure $\sigma$ is

$$\text{volume}_\sigma(I) = \begin{cases} 4\Delta Et/h, & \text{when } t \text{ is sufficiently large} \\ 1, & \text{when } t \text{ equals or approaches } 0 \end{cases} \tag{6}$$

where $\text{volume}_\sigma(I)$ is in quantum bits (qubits), $\Delta E$ is the average energy of quantum $c$, and $h \approx 6.6262 \times 10^{-34}$ J • s is Planck' s constant.

**Proof:** According to the definition of information capacity, $\text{volume}_\sigma(I)$ is the number of distinguishable states experienced by single quantum carrier $c$ during reflection time $T_m$. A quantum' s state can be composed of its two mutually orthogonal basis states $|0\rangle$ and $|1\rangle$ and their coherent superposition states $a|0\rangle + b|1\rangle$, where $a$ and $b$ are complex numbers satisfying $|a|^2 + |b|^2 = 1$ [25]. Distinguishable states of a quantum must be mutually orthogonal [22]. By the Margolus-Levitin theorem [26], the minimum time delay for transitioning from one state to another orthogonal state depends on its average energy $\Delta E$; the transition time cannot be shorter than $\Delta t = h/4\Delta E$. Since each distinguishable state of a single quantum exactly carries one qubit of information [27], for carrier $c$ with average energy $\Delta E$ and reflection duration $t$, if $t < \Delta t$, $c$ can only present one state distinguishable from others, and $\text{volume}_\sigma(I) = 1$ qubit, meaning $c$ carries only one qubit of information. This proves the lower part of the equation.

More generally, for longer reflection duration $t$, we have

$$\text{volume}_\sigma(I) = \lfloor t/\Delta t\rfloor + 1 = \lfloor 4\Delta Et/h\rfloor + 1.$$

Since $h \approx 6.6262 \times 10^{-34}$, in the above formula, as long as $t$ is sufficiently large –for example, when $\Delta Et \geq 10^{-30}$–the difference between $\lfloor 4\Delta Et/h\rfloor + 1$ and $4\Delta Et/h$ will be less than the $10^{-4}$ order of magnitude and can be neglected, giving $\text{volume}_\sigma(I) \approx 4\Delta Et/h$ qubits.

The corollary is proved.

Corollary 8 shows that quantum carriers, limited by the energy they contain or are subjected to, can only bear a finite number of different information sets during any time period. Moreover, because the number of quanta in the universe is always finite and quantum carrier information already constitutes the most fundamental component of information, even considering the infinite extension of time into the future, we can mathematically confirm that any information can at most be composed of a countable set of different atomic information, and the capacity additivity of information combinations can always be guaranteed under any circumstances.

A single quantum possesses indivisibility, so its state at any moment as a carrier is the reflection set of an atomic information. The most direct case is $I = \bigcup\{I_i = \langle c, T_{mi}, g, c, T_{mi}, g\rangle \mid i = 1, 2, \ldots, n\}$, where $n$ is the number of orthogonal states experienced by single quantum $c$ in the time sequence $T_{mi}$ $(i = 1, 2, \ldots, n)$, and $g(c, T_{mi})$ is the state of $c$ at moment $T_{mi}$. Each $I_i$ is atomic information and a self-mapping from $f(o, T_{hi})$ to $g(c, T_{mi}) = f(o, T_{hi})$. Figure 7 shows that information $I$ actually reflects the state transformation of single quantum $c$ itself in the objective world, where $|0\rangle$ and $|1\rangle$ are the two basis states of quantum $c$. According to (5), $\text{volume}_\sigma(I) = \sum \text{volume}_\sigma(I_i) = n$ qubits, with the upper bound of $n$ given by (6).

A more general case is $I = \bigcup\{I_i = \langle o, T_{hi}, f, c, T_{mi}, g\rangle \mid i = 1, 2, \ldots, n\}$, where $n$, $c$, $T_{mi}$, $g(c, T_{mi})$, and $i$ are defined as in the previous example, while $f(o, T_{hi})$ $(i = 1, 2, \ldots, n)$ is the state set experienced by noumenon $o$ in the subjective or objective world during the occurrence time sequence $T_{hi}$. Each $I_i$ is also atomic information, and their combined information $I$ maps the states of other noumena onto the states of single quantum $c$ (Figure 8), enabling people to efficiently process various things and phenomena in the subjective or objective world using quantum information technology. Here also $\text{volume}_\sigma(I) = \sum \text{volume}_\sigma(I_i) = n$ qubits, with its upper bound also given by (6).

### 4.2 Information Capacity of General Carriers

Reference [28] proposed that all energy in the universe exists in the forms of radiation and matter. Any matter and radiation in the objective world can become information carriers, and the information capacity they can bear depends on the carrier' s physical characteristics and technological level [29]. Currently, the most commonly used carrier with the largest information capacity in information systems is silicon chips. According to the most advanced manufacturing technology, a silicon chip with mass approximately 1.6 grams can achieve a storage capacity of $10^{12}$ bits, showing that 1 kilogram of silicon chips can bear an information capacity of $6.25 \times 10^{14}$ bits.

Reference [30], based on thermodynamic principles and the mass-energy conversion formula, calculated that the minimum mass of matter required to store one bit is $m_{\text{bit}} = k_b T \ln(2)/C^2$, where $k_b \approx 1.38 \times 10^{-23}$ J/K is Boltzmann's constant,

$T$ is the absolute temperature of the information carrier, and $C \approx 2.9979 \times 10^8$ m/s is the speed of light in vacuum. Thus, if the carrier of information $I$ is 1 kilogram of matter, its information capacity without energy dissipation is always $\text{volume}_\sigma(I) = 1/m_{\text{bit}} = C^2/(k_b T \ln(2))$ bits, where measure $\sigma$ is bits. At room temperature of about 300 Kelvin, $\text{volume}_\sigma(I)$ is on the order of $10^{38}$ bits. Since this formula derives from classical thermodynamic principles, it only applies to equilibrium states of classical digital memory and cannot apply to quantum carriers such as electrons and photons.

For information with quanta as carriers, (6) gives the information capacity that a single quantum carrier can bear within a specific time period. Since quantum carrier information sets do not overlap, the capacity additivity of atomic information combinations expressed in (5) can help us estimate the information capacity that quantum carriers can bear.

**Theorem 1 (Information Capacity of Quantum Carriers):** Let recoverable information $I = \langle o, T_h, f, c, T_m, g \rangle$ have carrier $c$ composed of $N$ quanta. Carrier $c$ exists partly in matter form and partly in radiation form, with mass $m$ and energy $E_r$ respectively, and total energy $E$. Let measure $\sigma$ be the number of states of $c$ during reflection time $T_m$, and $t$ the reflection duration of information $I$. Then when $t$ is sufficiently large,

$$\text{volume}_\sigma(I) = 4Et/h = 4(mC^2 + E_r)t/h \text{ (qubits)},$$

and when $t$ equals or approaches 0, $\text{volume}_\sigma(I) = N$. For simplified expression, letting $I$ directly represent its own information capacity yields the concise information capacity calculation formula

$$I = \begin{cases} 4Et/h = 4(mC^2 + E_r)t/h, & \text{when } t \text{ is sufficiently large} \\ N, & \text{when } t \text{ equals or approaches } 0 \end{cases} \tag{7}$$

where $I$ is in qubits, $C$ is the speed of light, and $h$ is Planck' s constant.

**Proof:** For recoverable information $I = \langle o, T_h, f, c, T_m, g \rangle$, $E$ is the total energy of its carrier $c$. According to the principle of energy conservation [24], the total energy of $c$ at any moment during reflection time $T_m$ should be $E$. Moreover, carrier $c$ consists of $N$ quanta. If the average energy of a single quantum in $c$ is $\Delta E$, then the number of quanta contained in $c$ during reflection time $T_m$ is always $N = E/\Delta E$. Without loss of generality, we can set $c = \bigcup_{i=1}^{N} c_i$, where $c_i$ $(i = 1, \dots, N)$ are individual quanta that exist throughout reflection time $T_m$ and serve as information carriers. Let $I_i$ denote the sub-information of $I$ with carrier $c_i$. Then each $I_i$ is single quantum carrier information, and $t$ is both the reflection duration of information $I$ and the reflection duration of each sub-information $I_i$. According to (6), we have $\text{volume}_\sigma(I_i) = 4\Delta Et/h$ (when $t$ is sufficiently large) or $\text{volume}_\sigma(I_i) = 1$ (when $t$ equals or approaches 0), for $(i = 1, \dots, N)$. Since each $c_i$ is a different quantum, the sub-information

$I_i$ of information $I$ do not overlap, and each $I_i$ consists of $\lfloor 4\Delta Et/h \rfloor + 1$ atomic information. According to (4) and (5),

$$\text{volume}_\sigma(I) = \sum \text{volume}_\sigma(I_i) = \sum 1 = N \text{ (qubits) when } t \text{ equals or approaches 0,}$$

or

$$\text{volume}_\sigma(I) = \sum \text{volume}_\sigma(I_i) = \sum 4\Delta Et/h = 4Et/h \text{ (qubits) when } t \text{ is sufficiently large.}$$

Furthermore, the total energy $E$ of $c$ should be the sum of the energies of its matter and radiation forms. According to the mass-energy conversion formula [31, 32], the energy of the matter form is $mC^2$, where $C$ is the speed of light. Thus $E = mC^2 + E_r$, giving $\text{volume}_\sigma(I) = 4(mC^2 + E_r)t/h$ qubits when $t$ is sufficiently large.

The theorem is proved.

Since (7) concisely encompasses the basic measures of information, matter, energy, and time, we refer to it as the relationship formula of information, matter, energy, and time. Imagining that an information carrier can be decomposed into pure quanta listed in [24], when its contained energy corresponds to matter with mass 1 kilogram, (7) shows that its information capacity in 1 second is $4C^2/h \approx 5.3853 \times 10^{50}$ qubits. At a particular moment, its information capacity is simply the number of quanta it contains. Assuming this carrier consists entirely of electrons, since a single electron' s mass is about $9.1 \times 10^{-31}$ kg, meaning 1 kilogram of carrier contains about $10^{30}$ electrons, its information capacity at that moment is about $10^{30}$ qubits.

### 4.3 Information Capacity the Universe May Have Borne to Date

The vast universe is filled with various information and is also considered an enormous quantum computer. For decades, many have studied how much information the entire universe may have borne to date [33-37]. According to Theorem 1' s relationship formula of information, matter, energy, and time and the principle of energy conservation [24], this question can be easily answered.

Standard inflation theory predicts the spatial flatness of the universe. Einstein' s general relativity determines that such a universe has a total energy density equal to the critical density $\rho_c = 3H_0^2/8\pi G$ [38], where $H_0 \approx 2.1 \times 10^{-18}$/s is the current value of the Hubble parameter and $G \approx 6.7 \times 10^{-11}$ m$^3$kg$^{-1}$s$^{-2}$ is the gravitational constant, giving $\rho_c \approx 7.9 \times 10^{-27}$ kg/m$^3$. On the other hand, the radius of the observable universe is approximately $L = 4.56 \times 10^{10}$ light-years [39], from which we can estimate the universe' s total volume as $V \approx (4/3)\pi L^3 \approx 3.35 \times 10^{80}$ m$^3$, and estimate the universe' s total mass as

$m = \rho_c V \approx 2.6 \times 10^{54}$ kg. According to common understanding, the universe' s age is about 13.7 billion years [40], meaning the information reflection duration $t \approx 4.3 \times 10^{17}$ seconds. Using (7), we obtain the universe' s information capacity to date as

$$I = 4mC^{2t}/h \approx 4\times2.6\times10^{54}\times(3.0\times10^{8})^2\times4.3\times10^{17}/(6.6\times10^{-34}) \approx 6.1\times10^{122} \text{ qubits.}$$

This estimate is nearly consistent with [34]' s estimation of $10^{123}$ logical operations in the universe. From the perspective of Objective Information Theory, any logical operation in the universe must produce specific states, and these states must contain objectively existing information. Therefore, we can consider the number of logical operations of the universe at a specific time as the total number of its states. On the other hand, this estimate differs from [34]' s estimation of $10^{90}$ bits for the universe' s information capacity, because the latter defines information capacity using entropy [41], which differs greatly from Theorem 1' s definition of information capacity. By comparison, the authors believe that defining information capacity using "the measure of all states of the carrier during reflection time" is more universal. More importantly, Theorem 1 establishes a general relationship among the three fundamental elements of the objective world—matter, energy, and information—and time. For any information carrier, given mass, energy, and time, we can substitute them into (7) to obtain the upper bound of information capacity.

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## 5 Conclusion

This paper builds upon the fundamental concepts of Objective Information Theory, proposes four postulates regarding information, and proves that the information sextuple model satisfies the necessary and sufficient conditions for defining information. It demonstrates that defining information capacity using information entropy is merely a special case of the capacity definition in Objective Information Theory, proves the upper bound of information capacity that a single quantum carrier can bear during reflection time, and establishes the relationship formula of information, matter, energy, and time. This profoundly and accurately reveals the interrelationships among the constituent elements of the objective world, showing that the theoretical system of Objective Information Theory has good universal significance and application prospects.

Objective Information Theory not only defines the capacity measure of information but also analyzes five basic properties of information—objectivity, recoverability, combinability, transmissibility, and associativity—and constructs ten other measures including delay, breadth, and granularity. This paper only conducts preliminary research on the essence of information and its capacity characteristics. Future work can further investigate the characteristics of other basic properties and measures of information and their interrelationships, while examining compatibility with existing information science principles and technical tools, to promote the establishment of a universally applicable and complete information science theory and a comprehensive information system dynamics system, providing more powerful theoretical tools for the development and application of information technology.

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*Note: Figure translations are in progress. See original paper for figures.*